RAILROAD CONSTRUCTION

THEORY AND PRACTICE

A TEXT-BOOK FOR THE USE OF STUDENTS
IN COLLEGES AND TECHNICAL SCHOOLS,
AND
A HAND-BOOK FOR THE USE OF ENGINEERS
IN FIELD AND OFFICE,

BY

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PREFACE TO SIXTH EDITION.

THE revision of the fifth edition has been so extensive that it has almost amounted to a rewriting of the book. Comparatively few pages have been left without some revision.

The last few years have seen a greater advance in the science of railroad construction than any similar period in its previous history. This has been largely due to the combined work of the several Standing Committees of the American Railway Engineering Association. The writer has received special permission to quote from the Association's publications and has availed himself of the privilege, because he considers that the decisions of such an Association are, in general, the highest authority obtainable.

Considerable new matter has been added on the general subject of railroad surveys, and the handling of surveying parties. One feature of the additions has been the emergency medical and surgical treatment which the engineer-in-charge, as responsible head of the party, must sometimes supply when regular professional advice is absolutely unobtainable and the engineer must choose between seeing the victim die (or become permanently injured), or assuming the unwelcome responsibility of applying simple instructions plus common sense. It usually means choosing the lesser of two evils. The author wishes to acknowledge his indebtedness to his friends, Dr. G. Victor Janvier and Dr. Henry P. DeForest, for advice and the revision of these sections, which may thus be depended on to be technically correct.

Those familiar with the former editions of this work will note that the computations previously given for the unit values of saving one foot (or mile) of distance, one degree of curvature, or one foot of rise-and-fall, have now been omitted. This is due to the belief, as expressed by the Economics Committee of the

Am. Rwy. Eng. Assoc., that all previously published methods of making such calculations are unreliable since they ignore certain operating conditions peculiar to each road, and that the application of such unit figures may lead to unwarranted conclusions. It may be that a method will be sometime devised by which some simple and satisfactory form of unit value may be used. At present, the most practicable method yet proposed is to compute the costs of operating two suggested routes on the basis of an assumed amount and kind of traffic and compare the results.

WALTER LORING WEBB.

PHILADELPHIA, PA.,
Nov., 1916.

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CHAPTER I.

RAILROAD SURVEYS.

THE proper conduct of railroad surveys presupposes an adequate knowledge of almost the whole subject of railroad engineering, and particularly of some of the complicated questions of Railroad Economics, which are not generally studied except at the latter part of a course in railroad engineering, if at all. This chapter will therefore be chiefly devoted to methods of instrumental work, and the problem of choosing a general route will be considered only as it is influenced by the topography or by the application of those elementary principles of Railroad Economics which are self-evident or which may be accepted by the student until he has had an opportunity of studying those principles in detail

The student-engineer should be warned against the hasty and inadequate surveying which has resulted in so much misconstruction in this country. This kind of surveying was especially common forty or fifty years ago, and the methods have more or less continued. The demand for railroad facilities was then so urgent that lax methods were tolerated. A general route would be selected which, at first sight, seemed most obvious and it would be immediately staked out in a manner suitable to a location survey. After correcting some of the most glaring faults, the survey was considered complete and the road was constructed accordingly. The cost of such a survey is comparatively small, but it is almost inevitable that the line is not as good as could have been obtained with a greater amount of

examination and study. The cost of construction and the future cost of operating such a line is always unnecessarily high. The money wasted in construction, plus the capitalized value of the annual waste in future operating expenses, is frequently a hundred times the cost of the extra study and surveying which would have avoided these faults. This has been unquestionably proved by the innumerable cases of reconstruction of portions of old lines which could have been constructed originally on the lines as revised at even less cost. The engineer is not always responsible for ill-advised hasty work. An impatient Board of Directors often insists on commencing to "throw dirt" before a proper survey has been made. The engineer should make, if necessary, the most earnest representations and even strenuous demands, that he be given the requisite time, opportunity and money to conduct his survey in such a manner as to investigate thoroughly every possibility for improving the alinement.

A railroad survey ordinarily consists of three parts: (*a*) the reconnoissance; (*b*) the preliminary survey, and (*c*) the definite location. As explained later, circumstances may modify the relative importance of these divisions, but under ordinary circumstances all three are necessary.

RECONNOISSANCE SURVEYS.

1. Character of a reconnoissance survey. A reconnoissance survey is a very hasty examination of a belt of country to determine which of all possible or suggested routes is the most promising and best worthy of a more detailed survey. It is essentially very rough and rapid. It aims to discover those salient features which instantly stamp one route as distinctly superior to another and so narrow the choice to routes which are so nearly equal in value that a more detailed survey is necessary to decide between them.

A map should be prepared, at a scale not smaller than one mile to the inch, which should show all general routes which are conceivably possible. It is particularly important that the mere lack of data should not exclude consideration of some general route which might be superior to the one or more obvious routes which have already been picked out.

2. Selection of a general route. The general question of running a railroad between two towns is frequently a financial rather than an engineering question. Financial considerations usually determine that a road must pass through certain more or less important towns between its termini. It is also possible that there may be certain topographical features in any route between two determined towns on the line, such as a low saddle in crossing a ridge or a difficult crossing of a large river, which, with the towns, may be considered as **control points,** and the problem may be narrowed down to the determination of the best route between these consecutive control points. But care should be taken that control points are not too hastily considered as fixed and unalterable, especially if it results in very unfavorable grades and alinement between consecutive points.

The reconnoissance survey should include the determination of the location and relative elevations of all these control points. These data should be obtained with sufficient accuracy to compute the necessary ruling grade and the general character of the alinement, and the map as thus amplified should be studied by comparing the several possible routes and eliminating all those which are unquestionably less favorable than others.

The engineer should avoid, especially in a rough and wooded country, the influence that an existing highway, or even a path through the woods or of a clearing of the trees, may have in determining the choice of routes. Mere ease of travel, as long as it is not glaringly wrong, has caused many prepossessions in favor of a certain route, when a much better line could be obtained by plunging through the woods or over swampy or rocky ground. As a first trial in selecting the route, the bearing of a line joining two consecutive control points should be determined and then an effort should be made to find a general route which will have the least possible variation from that straight line, without sacrificing the limits of ruling grade, curvature and general type or cost of construction which may have been fixed for the road.

A difficult line between two control points should be studied by beginning at either end for two independent studies. The very obvious route, starting from A toward B, may lead into very difficult construction, which may be avoided by com-

mencing at B and finally reaching A on a route which, while practicable, would not be considered attractive when starting from A.

When a railroad runs through a thickly settled and very flat country, where, from a topographical standpoint, the road may be run by any desired route, the "right-of-way agent" sometimes has a greater influence in locating the road than the engineer. But such modifications of alinement, on account of business considerations, are foreign to the engineer's side of the subject, and it will be hereafter assumed that topography alone determines the location of the line. The consideration of those larger questions combining finance and engineering (such as passing by a town on account of the necessary introduction of heavy grades in order to reach it), will be considered in later chapters.

3. **Valley route.** This is perhaps the simplest problem. If two control points to be connected lie in the same valley, it is frequently only necessary to run a line which shall have a nearly uniform grade. The reconnoissance problem consists largely in determining the difference of elevation of the two termini of this division and the approximate horizontal distance so that the proper grade may be chosen. If there is a large river running through the valley, the road will probably remain on one side or the other throughout the whole distance, and both banks should be examined by the reconnoissance party to determine which is preferable. If the river may be easily bridged, both banks may be alternately used, especially when better alinement is thereby secured. A river valley has usually a steeper slope in the upper part than in the lower part. A uniform grade throughout the valley will therefore require that the road climbs up the side slopes in the lower part of the valley. In case the "ruling grade"* for the whole road is as great as or greater than the steepest natural valley slope, more freedom may be used in adopting that alinement which has the least cost—regardless of grade. The natural slope of large rivers is almost invariably so low that grade has no influence in determining the choice of location. When bridging is necessary, the river

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* The *ruling grade* may here be loosely defined as the maximum grade which is permissible. This definition is not strictly true, as may be seen later when studying Railroad Economics, but it may here serve the purpose.

banks should be examined for suitable locations for abutments and piers. If the soil is soft and treacherous, much difficulty may be experienced and the choice of route may be largely determined by the difficulty of bridging the river except at certain favorable places.

4. **Cross-country route.** A cross-country route always has one or more summits to be crossed. The problem becomes more complex on account of the greater number of possible solutions and the difficulty of properly weighing the advantages and disadvantages of each. The general aim should be to choose the lowest summits and the highest stream crossings, provided that by so doing the grades between these determining points shall be as low as possible and shall not be greater than the ruling grade of the road. Nearly all railroads combine cross-country and valley routes to some extent. Usually the steepest natural slopes are to be found on the cross-country routes, and also the greatest difficulty in securing a low through grade. An approximate determination of the ruling grade is usually made during the reconnoissance. If the ruling grade has been previously decided on by other considerations, the leading feature of the reconnoissance survey will be the determination of a general route along which it will be possible to survey a line whose maximum grade shall not exceed the ruling grade.

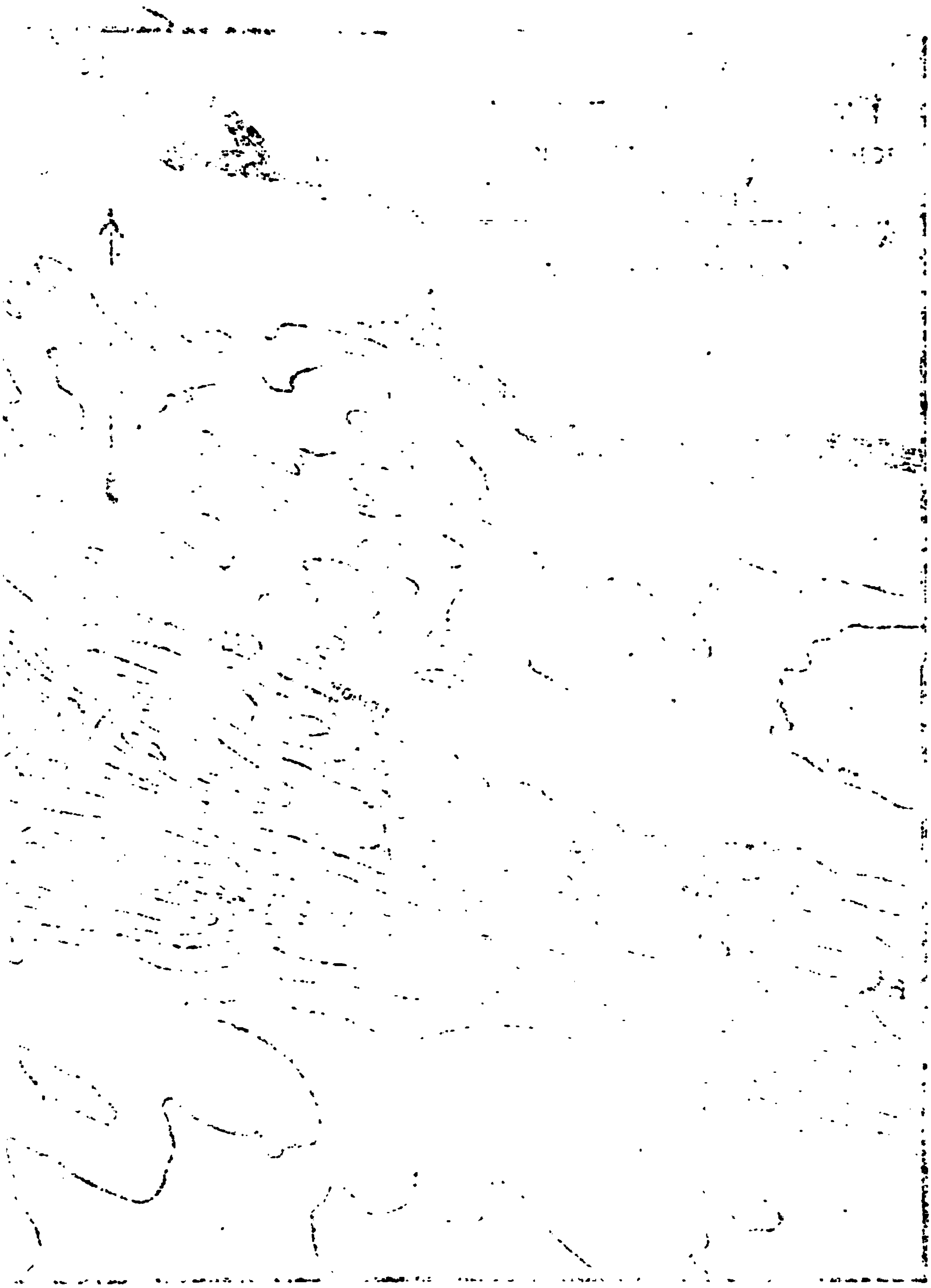
5. **Mountain route.** The streams of a mountainous region frequently have a slope exceeding the desired ruling grade. In such cases there is no possibility of securing the desired grade by following the streams. The penetration of such a region may only be accomplished by "development"—accompanied perhaps by tunneling. "Development" consists in deliberately increasing the length of the road between two extremes of elevation so that the rate of grade shall be as low as desired. The usual method of accomplishing this is to take advantage of some convenient formation of the ground to introduce some lateral deviation. The methods may be somewhat classified as follows:

(a) Running the line up a convenient lateral valley, turning a sharp curve and working back up the opposite slope. As shown in Fig. 1, the considerable rise between A and B was surmounted by starting off in a very different direction from the general direction of the road; then, when about one-half of the desired rise had been obtained, the line crossed the valley

and continued the climb along the opposite slope. (b) *Switchback.* On the steep side-hill *BCD* (Fig. 1) a very considerable gain in elevation was accomplished by the switchback *CD*. The gain in elevation from *B* to *D* is very great. On the other hand, the speed must always be slow; there are two complete stoppages of the train for each run; all trains must run backward from *C* to *D*. (c) *Bridge spiral.* When a valley is so narrow at some point that a bridge or viaduct of reasonable length can span the valley at a considerable elevation above the

Fig. 1.

bottom of the valley, a bridge spiral may be desirable. In Fig. 2 the line ascends the stream valley past *A*, crosses the stream at *B*, works back to the narrow place at *C*, and there crosses itself, having gained perhaps 100 feet in elevation. (d) *Tunnel spiral* (Fig. 3). This is the reverse of the previous plan. It implies a thin steep ridge, so thin at some place that a tunnel through it will not be excessively long. Switchbacks and spirals are sometimes necessary in mountainous countries, but they should not be considered as normal types of construction. A region must be very difficult if these devices cannot be avoided.



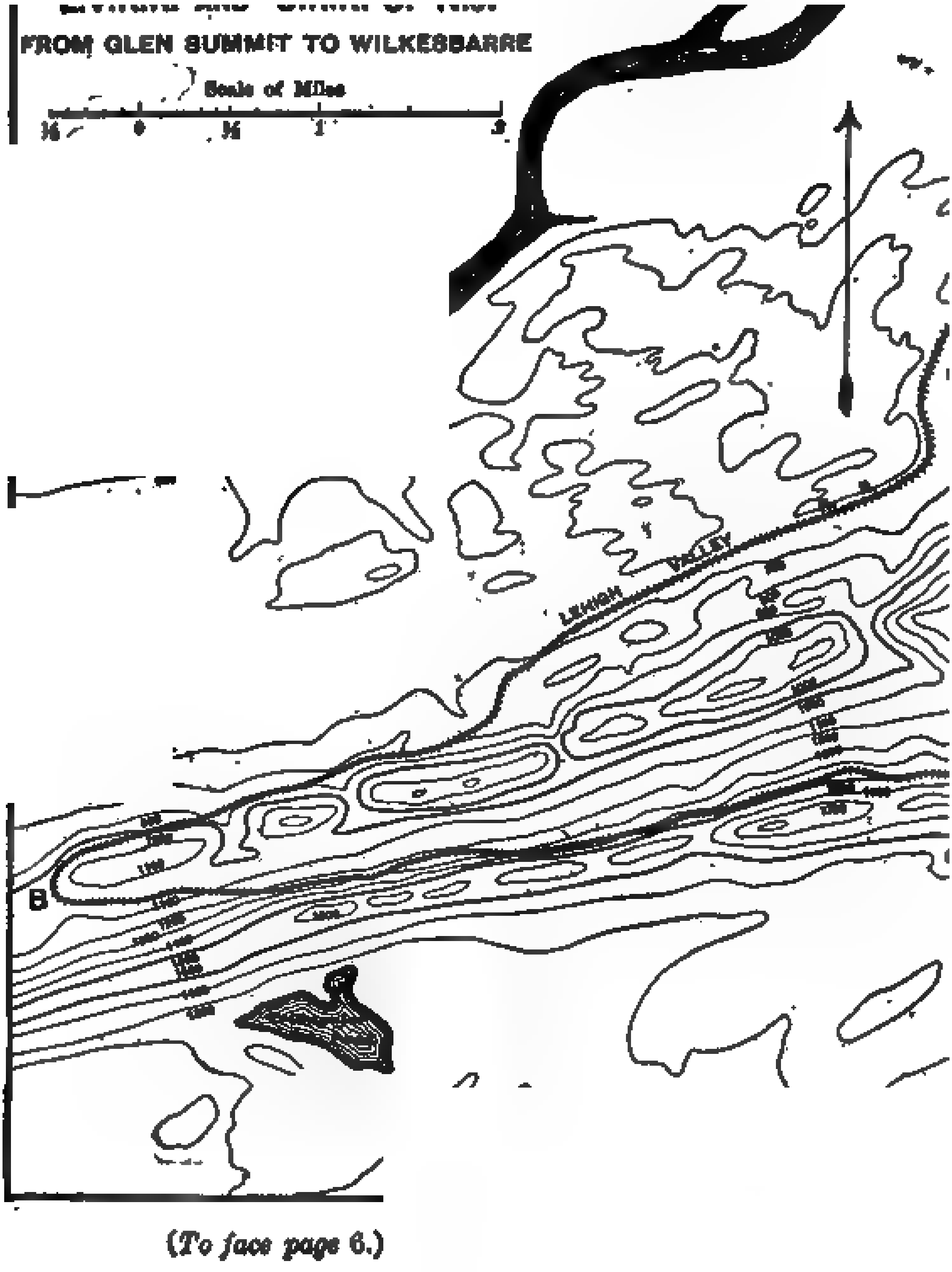


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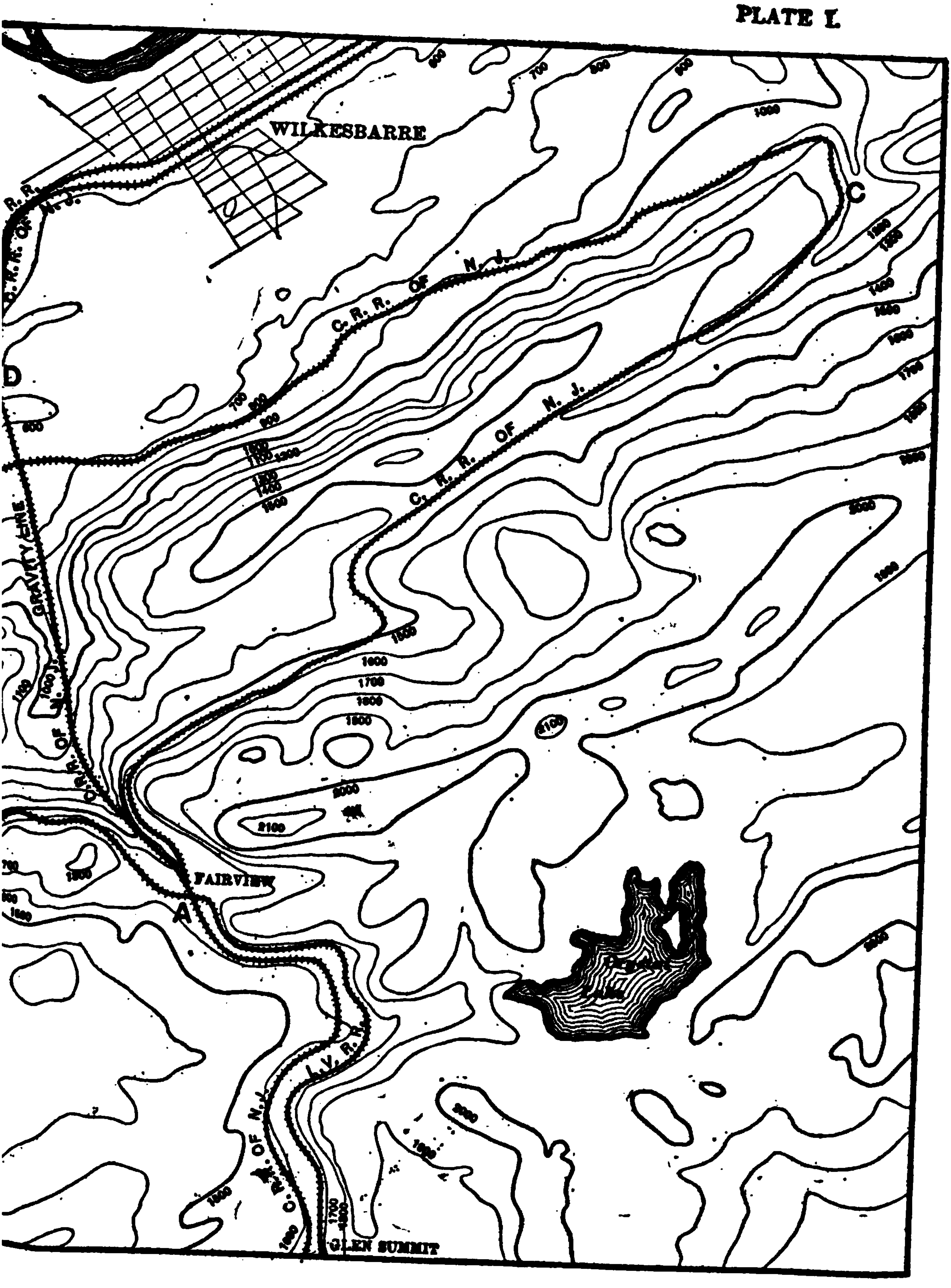
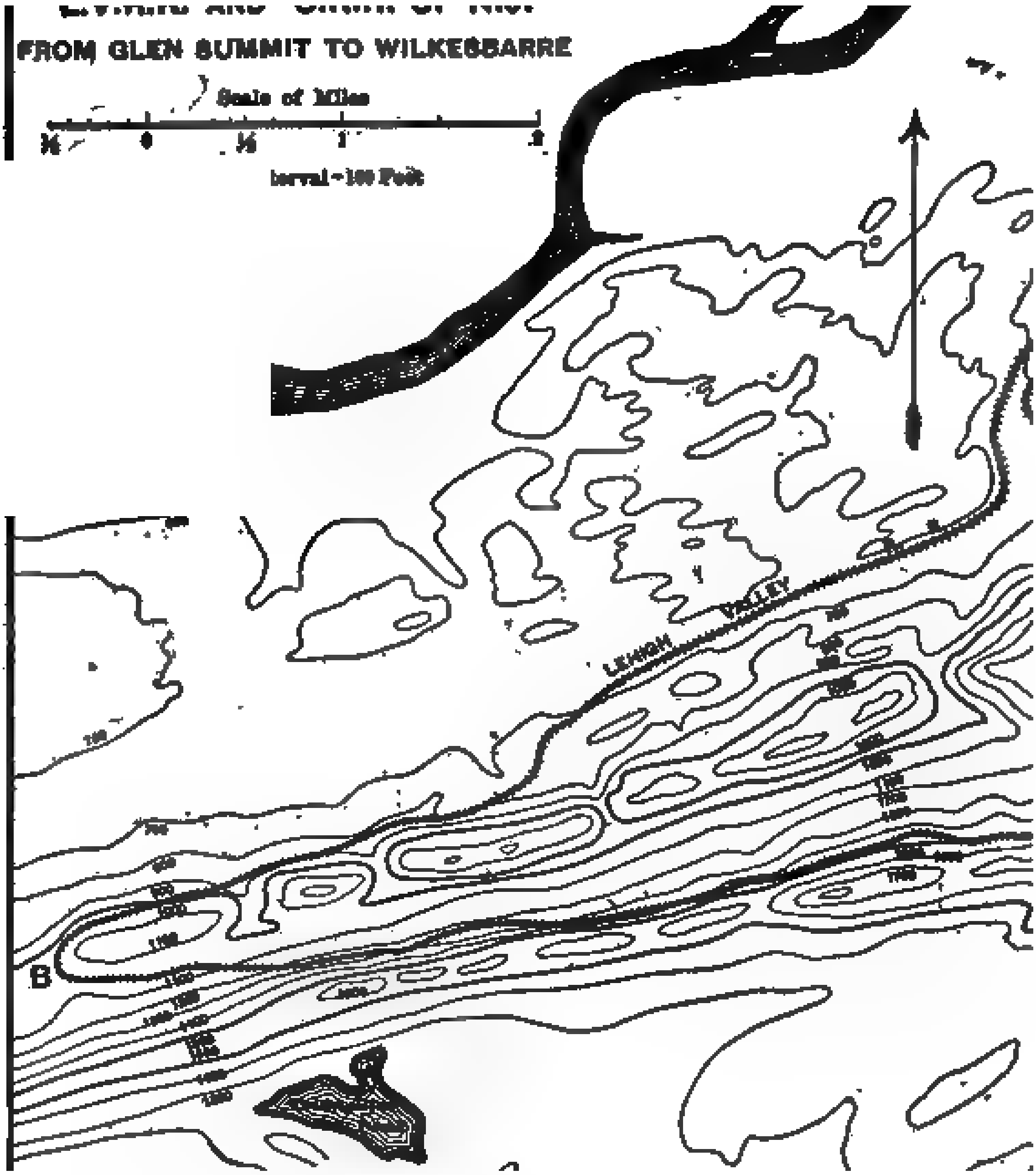
PLATE I.



WILKESBARRE
C. R. R. OF N. J.
C. R. R. OF N. J.
GRAVITY LINE
FAIRVIEW
GLEN SUMMIT
L. V. R. R.
A
C
D



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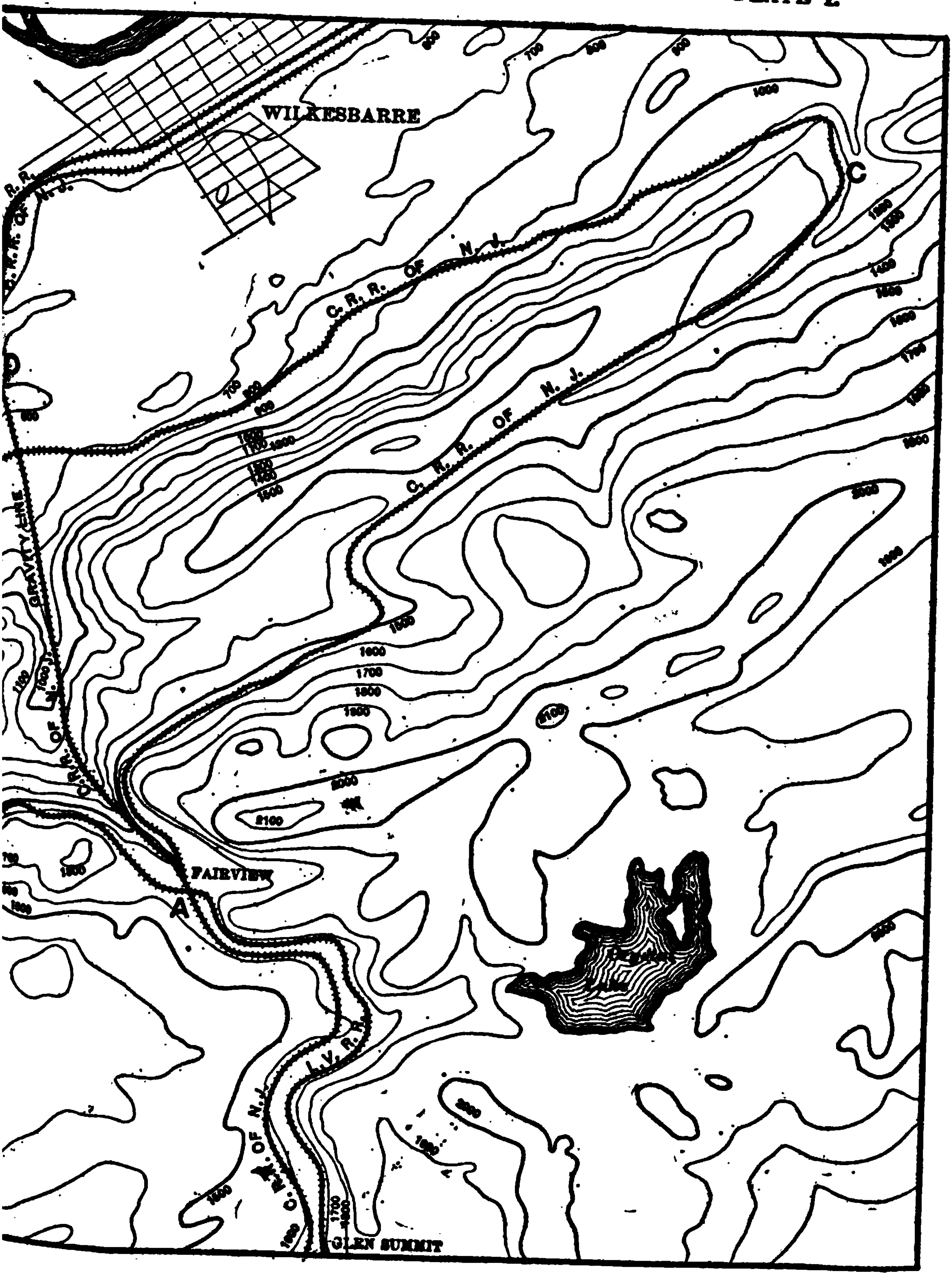
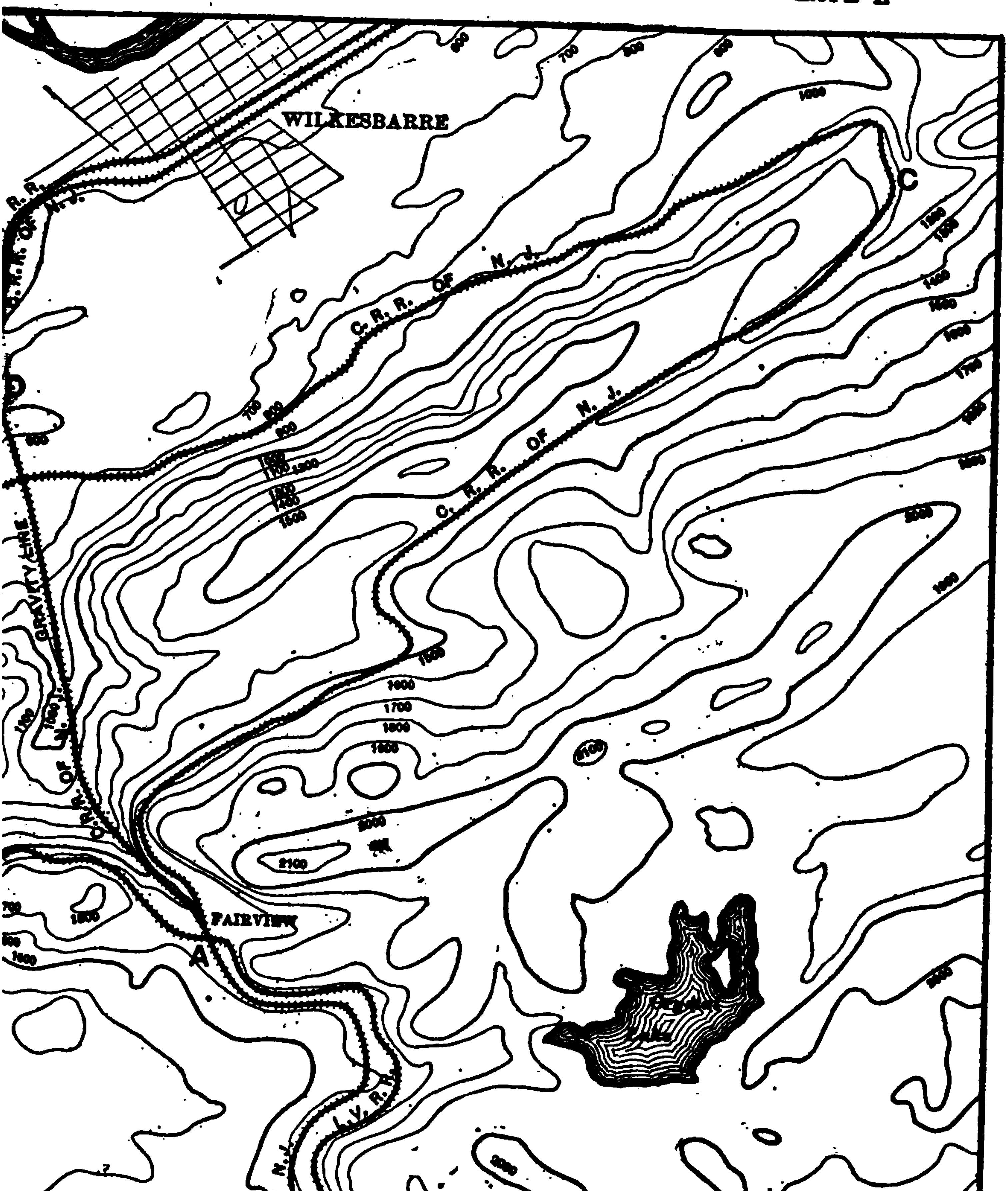

WILKESBARRE
C. R. R. OF N. J.
GRAVITY LINE
FAIRVIEW
GLEN SUMMIT
L. V. R. R.
A
C

(*To face page 6.*)


WILKESBARRE
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C. R. R. OF N. J.
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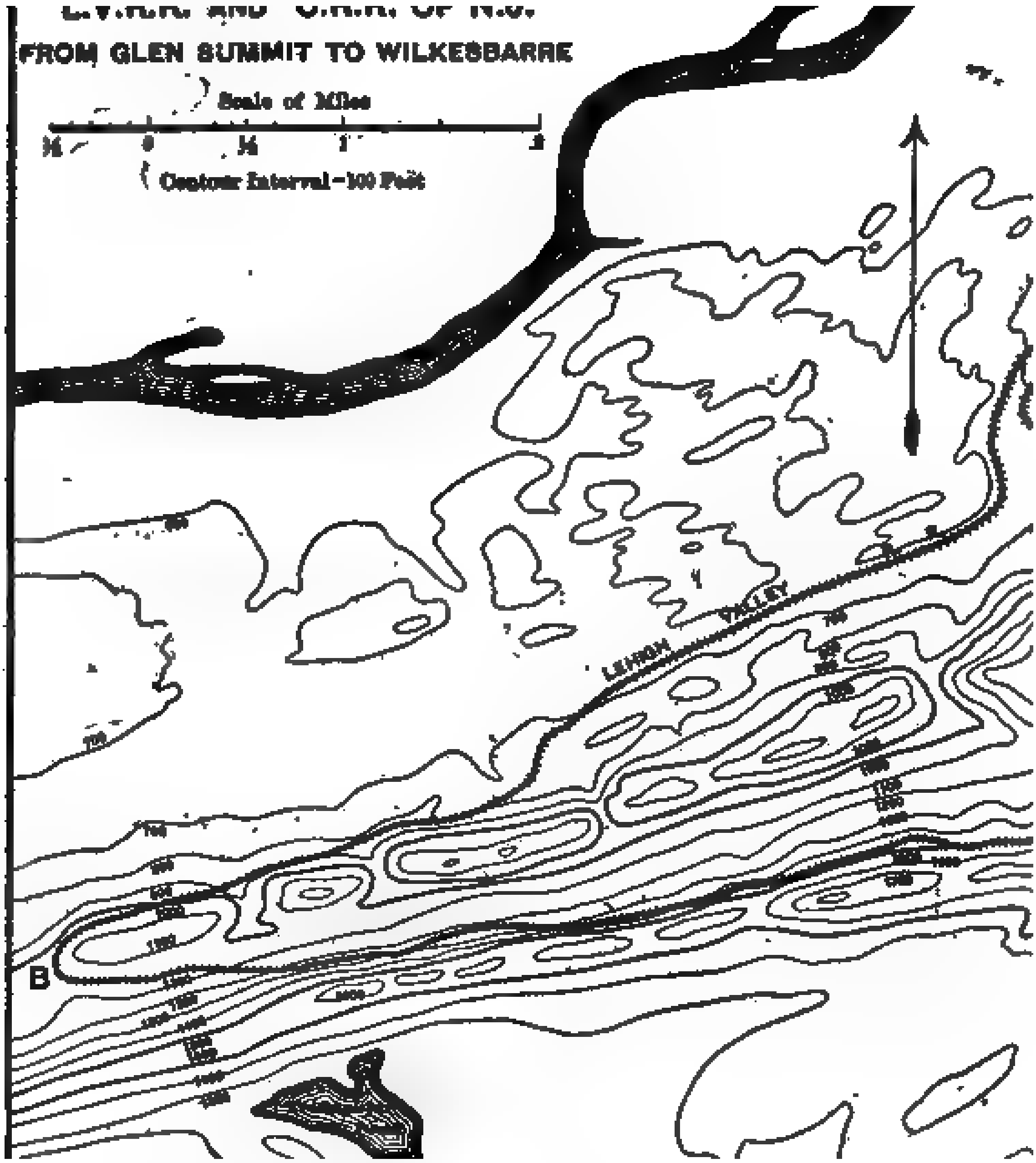

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PLATE I.

WILKESBARRE
C. R. R. OF N. J.
GRAVITY LINE
FAIRVIEW
GLEN SUMMIT
L. V. R. R.
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C
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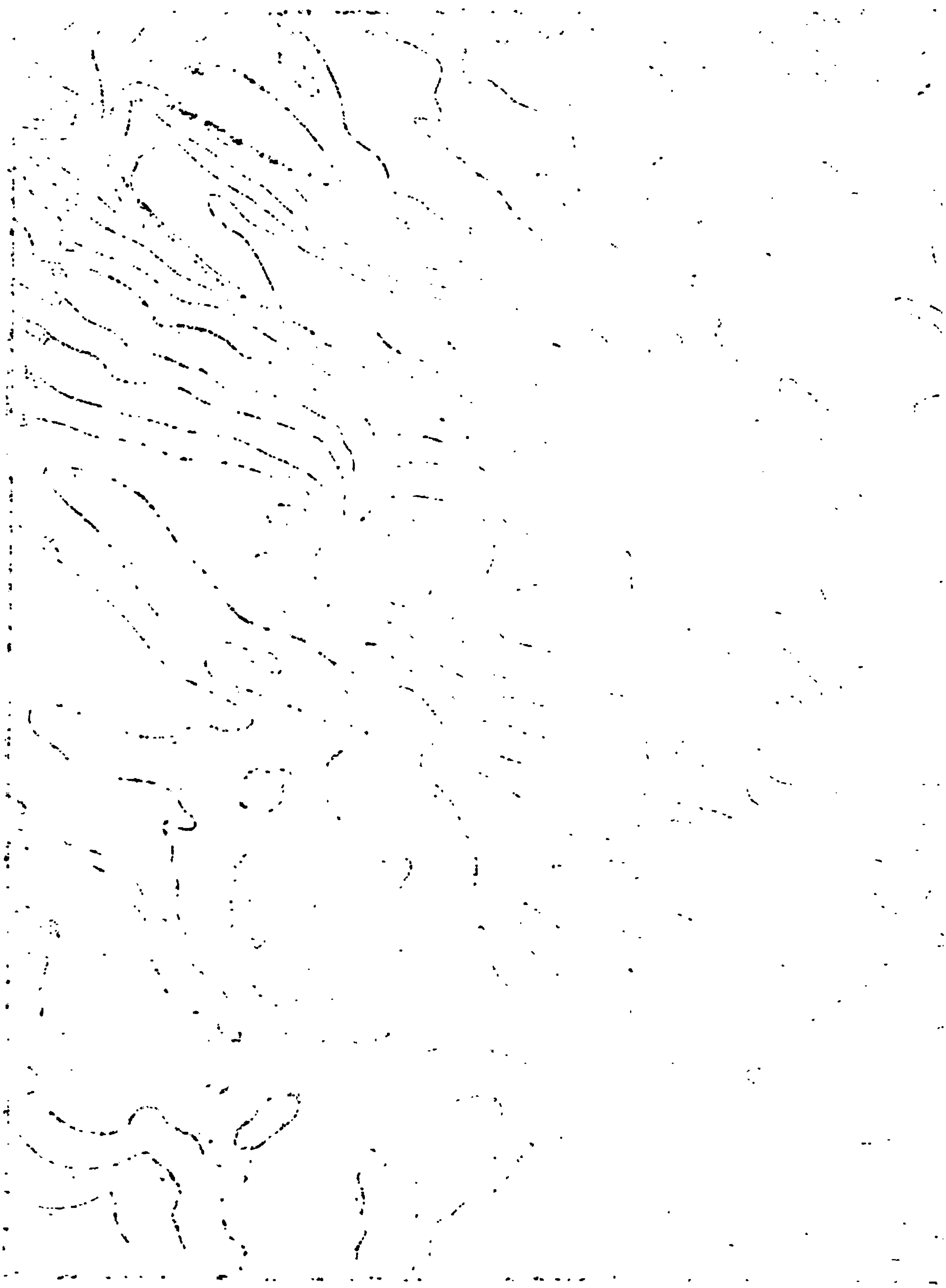

On Plate I are shown three separate ways (as actually constructed) of running a railroad between two points a little over three miles apart and having a difference of elevation of nearly 1100 feet. At A the Central R. R. of New Jersey runs *under* the Lehigh Valley R. R. and soon turns off to the northeast for about six miles, then doubles back, reaching D, a fall of about 1050 feet with a track distance of about 12.7 miles. The L. V. R. R. at A runs to the westward for six to seven miles,

FIG. 2.

FIG. 3.

then turns back until the roads are again close together at D. The track distance is about 14 miles and the drop a little greater, since at A the L V. R. R. crosses *over* the other, while at D they are at practically the same level. From B to C the distance is over eleven miles. From A directly down to D the C. R. R. of N. J. runs a "gravity" road, used exclusively for freight, on which cars alone are hauled by cable. The main-line routes are remarkable examples of sheer "development." Even as constructed the L. V. R. R. has a grade of about 95 feet per mile, and this grade has proved so excessive for freight work that the company has constructed a cut-off (not shown on the map) which leaves the main line at A, nearly parallels the

C. R. R. to C, and then running in a northeasterly direction again joins the main line beyond Wilkesbarre. The grade is thereby cut down to 65 feet per mile.

Rack railways and cable roads, although types of mountain railroad construction, will not be here considered.

6. Existing maps. The maps of the U. S. Geological Survey are exceedingly valuable as far as they have been completed. So far as topographical considerations are concerned, they almost dispense with the necessity for the reconnoissance and "first preliminary" surveys. Some of the State Survey maps will give practically the same information. County and township maps can often be used for considerable information as to the relative *horizontal* position of governing points, and even some approximate data regarding elevations may be obtained by a study of the streams. Of course such information will not dispense with surveys, but will assist in so planning them as to obtain the best information with the least work. When the relative horizontal positions of points are reliably indicated on a map, the reconnoissance may be reduced to the determination of the relative elevations of the governing points of the route.

7. Determination of relative elevations. A recent description of European methods includes spirit-leveling in the reconnoissance work. This may be due to the fact that, as indicated above, previous topographical surveys have rendered unnecessary the "exploratory" survey which is required in a new country, and that their reconnoissance really corresponds more nearly to our preliminary.

The perfection to which barometrical methods have been brought has rendered it possible to determine differences of elevation with sufficient accuracy for reconnoissance purposes by the combined use of a mercurial and an aneroid barometer. The mercurial barometer should be kept at "headquarters," and readings should be taken on it at such frequent intervals that any fluctuation is noted, and throughout the period that observations with the aneroid are taken in the field. At each observation there should also be recorded the time, the reading of the attached thermometer, and the temperature of the external air. For uniformity, the mercurial readings should then be "reduced to 32° F." The form of notes for the mercurial barometer readings should be as follows:

Time.	Merc. Barom.	Attached Therm.	Reduction to 32° F.	External Therm.	Corrected reading.
7:00 A. M.	29.872	72°	—.117	73°	29.755
:15	.866	73.5	.121	75	.745
:30	.858	75	.125	76	.733
:45	.850	76	.127	77	.723

The corrections in column 4 are derived from Table XI by interpolation.

Before starting out, a reading of the aneroid should be taken at headquarters coincident with a reading of the mercurial. The difference is one value of the correction to the aneroid. As soon as the aneroid is brought back another comparison of readings should be made. Even though there has been considerable rise or fall of pressure in the interval, the *difference* in readings (the correction) should be substantially the same provided the aneroid is a good instrument. If the difference of elevation is excessive (as when climbing a high mountain) even the best aneroid will "lag" and not recover its normal reading for several hours, but this does not apply to such differences of elevation as are met with in railroad work. The best aneroids read directly to $\frac{1}{100}$ of an inch of mercury and may be estimated to $\frac{1}{1000}$ of an inch—which corresponds to about 0.9 foot difference of elevation. In the field there should be read, at each point whose elevation is desired, the aneroid, the time, and the temperature. These readings, corrected by the mean value of the correction between the aneroid and the mercurial, should then be combined with the reading of the mercurial (interpolated if necessary) for the times of the aneroid observations and the difference of elevation obtained. The field notes for the aneroid should be taken as shown in the first four columns of the tabular form. The "corrected aneroid" readings of column 5 are found by correcting the readings of column 3 by the mean difference between the mercurial and aneroid when compared at morning and night. Column 6 is a copy of the "corrected readings" from the office notes, interpolated when necessary for the proper time. Column 7 is similarly obtained. Col. 8 is obtained from cols. 4 and 5, and col. 9 from cols. 6 and 7, with the aid of Table XII. The correction for temperature (col. 11), which is generally small unless the difference of elevation is large, is obtained with the

(Left-hand page of Notes.)

Time.	Place.	Aneroid.	Therm.	Corr. Aner.	Corr. Merc.
7:00	Office	29.628	73°	....	29.755
7:10	Δ0	29.662	72°	29.789	29.748
7:30	saddle-back	29.374	63°	29.501	29.733
7:50	river cross.	29.548	70°	29.675	29.720

aid of Table XIII. The elevations in Table XII are elevations above an assumed datum plane, where under the given atmospheric conditions the mercurial reading would be 30″. Of course the position of this assumed plane changes with varying atmospheric conditions and so the elevations are to be considered as *relative* and their difference taken. [See the author's "Problems in the Use and Adjustment of Engineering Instruments," Prob. **22**.] Important points should be observed more than once if possible. Such duplicate observations will be found to give surprisingly concordant results even when a general fluctuation of atmospheric pressure so modifies the tabulated readings that an agreement is not at first apparent. Variations of pressure produced by high winds, thunder-storms, etc., will generally vitiate possible accuracy by this method. By "headquarters" is meant any place whose elevation above any given datum is known and where the mercurial may be placed and observed while observations within a range of several miles are made with the aneroid. If necessary, the elevation of a new headquarters may be determined by the above method, but there should be if possible several independent observations whose accordance will give a fair idea of their accuracy.

The above method should be neither slighted nor used for more than it is worth. When properly used, the errors are compensating rather than cumulative. When used, for example, to determine that a pass *B* is 260 feet higher than a determined bridge crossing at *A* which is six miles distant, and that another pass *C* is 310 feet higher than *A* and is ten miles distant, the figures, even with all necessary allowances for inaccuracy, will give an engineer a good idea as to the choice of route especially as affected by ruling grade. There is no comparison between the time and labor involved in obtaining the above information by barometric and by spirit-leveling methods, and *for recon-*

(Right-hand page of Notes.)

Temp. at headqu.	Approx. field read.	Approx. headq. read.	Diff.	Corr. for temp.	Diff. elev.
—	—	—	—	—	—
75°	192	230	− 38	−(+ 2)	− 40
76	457	244	+213	+(+10)	+223
77	297	256	+ 41	+(+ 2)	+ 43

noissance purposes the added accuracy of the spirit-leveling method is hardly worth its cost.

8. Horizontal measurements, bearings, etc. When reliable maps are unobtainable, rapid exploratory surveys become essential. Since accuracy is sacrificed for rapidity in such surveys, more or less approximate methods are used. "An experienced saddle-horse, whose speeds at his various gaits have been learned accurately by previous timing," is quoted from Beahan * as one means of rapidly measuring distances. The percentage of probable error is evidently large. A **pedometer** (or pace-measurer) is probably more accurate, but its accuracy depends on a knowledge of the average length of the observer's pace. Due allowance must be made for the fact that the length of pace will vary very greatly depending on whether the surface is smooth and level, or is plowed ground, or marshy, or slippery, or consists of rough boulders covered with moss, or is a wilderness of brambles, fallen trees, bogs, etc. It will also depend on whether the observer is fatigued or is in fresh physical condition. Under such a variety of conditions the counting of steps for long distances is sometimes a farce. Even when the surface is fairly smooth and easy, precautions must be taken that paces are not counted during the pauses at important points while bearings are being taken and other data recorded. An **odometer** which records the revolutions of a wheel of known circumference is far more accurate. Such a machine has been made so that it may be trundled like a wheelbarrow and thus go through the woods and over ground that would be impassable to any horse-drawn vehicle. The attachment of an odometer to the wheels of a wagon is very tempting, since it permits the engineer to ride, but it is probably an unreliable method for the reason men-

* "The Field Practice of Railway Location," p. 34.

tioned in Art. 2—permitting the ease of travel over a road practicable for a horse and vehicle to deflect the engineer from his true course, which is perhaps over rough ground which is impassable for a vehicle.

When the country is quite open and clear of underbrush, very rapid work may be done by the **stadia method**, which is many times more accurate than any of the methods previously mentioned. Some of the accuracy possible with stadia may be sacrificed for extreme rapidity and sights may be made 1200 and even 2000 feet long. By taking very few, if any, "side-shots," the progress is very rapid and many miles per day may be covered, with the advantage that the three elements of distance, azimuth and relative elevation may be obtained with as great accuracy as is necessary for an exploratory survey. The method of using the stadia will be described later.

The **bearings** of the various lines forming the skeleton of the survey, and also the bearings of the courses of streams and of side lines from the stations on the skeleton line, may be taken most easily with a **prismatic compass**. This instrument has a circular card, or sometimes a metal ring, attached to the needle. The edge of the card is graduated into degrees and is usually numbered consecutively (instead of by quadrants), from 0° up to 360°. This is advantageous since the one number, without any qualifying letters, *NE* or *NW*, determines the quadrant definitely without danger of confusion or error. The observer sights through a narrow slit in the desired direction and, by means of the prismatic reflector, can read directly the number of degrees, measured *to the right*, and usually from the magnetic *South*. The makers of prismatic compasses do not always number the graduations in the same manner, and, therefore, the engineer, who is accustomed to one particular instrument, should carefully study the markings of any new instrument. In any case it should be remembered that the prism reflects the numbers on that side of the movable card or ring which is *toward the observer* rather than on the side toward the object sighted at. The prismatic compass has the special advantage that, like a sextant, it can be used when supported only by hand, while an ordinary sight compass of equal accuracy would require a tripod, or, at least, a Jacob's staff. The declination of the needle in that section of the country can be readily determined with sufficient

accuracy for the purposes of such a survey. Usually the declination may be ignored. Any errors due to local attraction are never cumulative, but apply only to the point where those individual observations are taken. The angle between two lines radiating from any station may be obtained by subtracting one bearing from the other.

Relative elevations may be obtained systematically, using a barometer, as already explained, but much filling in may be done with the use of a hand-level. Experience soon teaches an engineer that there are many optical illusions about the slopes of ground which have the practical effect of making the apparent slope different from the actual, and, in the case of low grade, may make an actual down grade appear as an up grade. For example, when looking along an actual but slight down grade, especially if there are no obstructions or natural objects which the eye can use as a comparative scale, the eye is apt to foreshorten the distance, which has the effect of lessening the apparent down grade and perhaps of making it appear as a slight up grade. The hand level will immediately detect such errors and its frequent use by a reconnoissance engineer will not only enable him to avoid many errors he might otherwise make, but will also be an effective means of training him to guard against such optical illusions. Such a simple and effective instrument should always be at hand and it should be tested with sufficient frequency to know that it is always as accurate as such an instrument can be. The bubble should be as sensitive as is practicable for an instrument which is held in the hand. A well-made hand level has a bubble of the right sensitiveness, but even a super-sensitive level may be utilized and still better work done by supporting it steadily on the top of a light wooden stick about five feet long.

9. Importance of a good reconnoissance. The foregoing instruments and methods should be considered only as aids in exercising an educated common sense, without which a proper location cannot be made. The reconnoissance survey should command the best talent and the greatest experience available. If the general route is properly chosen, a comparatively low order of engineering skill can fill in a location which will prove a paying railroad property; but if the general route is so chosen that the ruling grades are high and the business obtained is small and subject to excessive competition, no amount of perfection in

as in the previous method except that each course is limited to the practicable length of a stadia sight. The distance between stations should be checked by foresight and backsight—also the vertical angle. Azimuths should be checked by the needle. Considering the vital importance of leveling on a railroad survey it might be considered desirable to run a line of levels over the stadia stations in order that the leveling may be as precise as possible; but when it is considered that a preliminary survey is a somewhat hasty survey of a route that *may* be abandoned, and that the errors of leveling by the stadia method (which are compensating) may be so minimized that no proposed route would be abandoned on account of such small error, and that the effect of sueh an error may be usually neutralized by a slight change in the location, it may be seen that excessive care in the leveling of the preliminary survey is hardly justifiable.

A stadia party should include a locating engineer (or chief of party), and perhaps an assistant, a transitman, a recorder and four rodmen, beside axemen. The transitman should have nothing to do but attend to his instrument. After setting up the transit at an advanced station, a backsight should be taken to the previous station. If the vertical circle is full 360°, the telescope should be plunged and sighted on the backsight with vernier *A* reading the same as the foresight to the station occupied. If the vertical arc is semi-circular (or less), no vertical angle can be taken with the telescope plunged and, therefore, vernier A should read 180° more (or less) than the foresight. The lower plate should be very firmly clamped, and then, after loosening the upper plate, a reference sight and reading on some well-defined natural object should be taken. If there is any reason to suspect that the instrument has been disturbed while occupying that station, the reference point can be sighted at and the instrument can be re-alined, and re-leveled, if necessary, without sending a rodman back to the previous station. When taking a backsight the rod reading for distance should be taken first and immediately compared with the previously recorded foresight. Since the distance between stations will always be taken with especial care so as to avoid "blunders" of an even 10, 20 or perhaps 100 feet, the foresights and backsights should agree to within the proper limits of the stadia method. Similarly the vertical angle should agree with the previous reading, *but with opposite sign.* If especial care is

they will cause no vital error in the subsequent location survey. The transit method is essentially more accurate, but is liable to be more laborious and troublesome. If a large tree is encountered, either it must be cut down or a troublesome operation of offsetting must be used. If the compass is employed

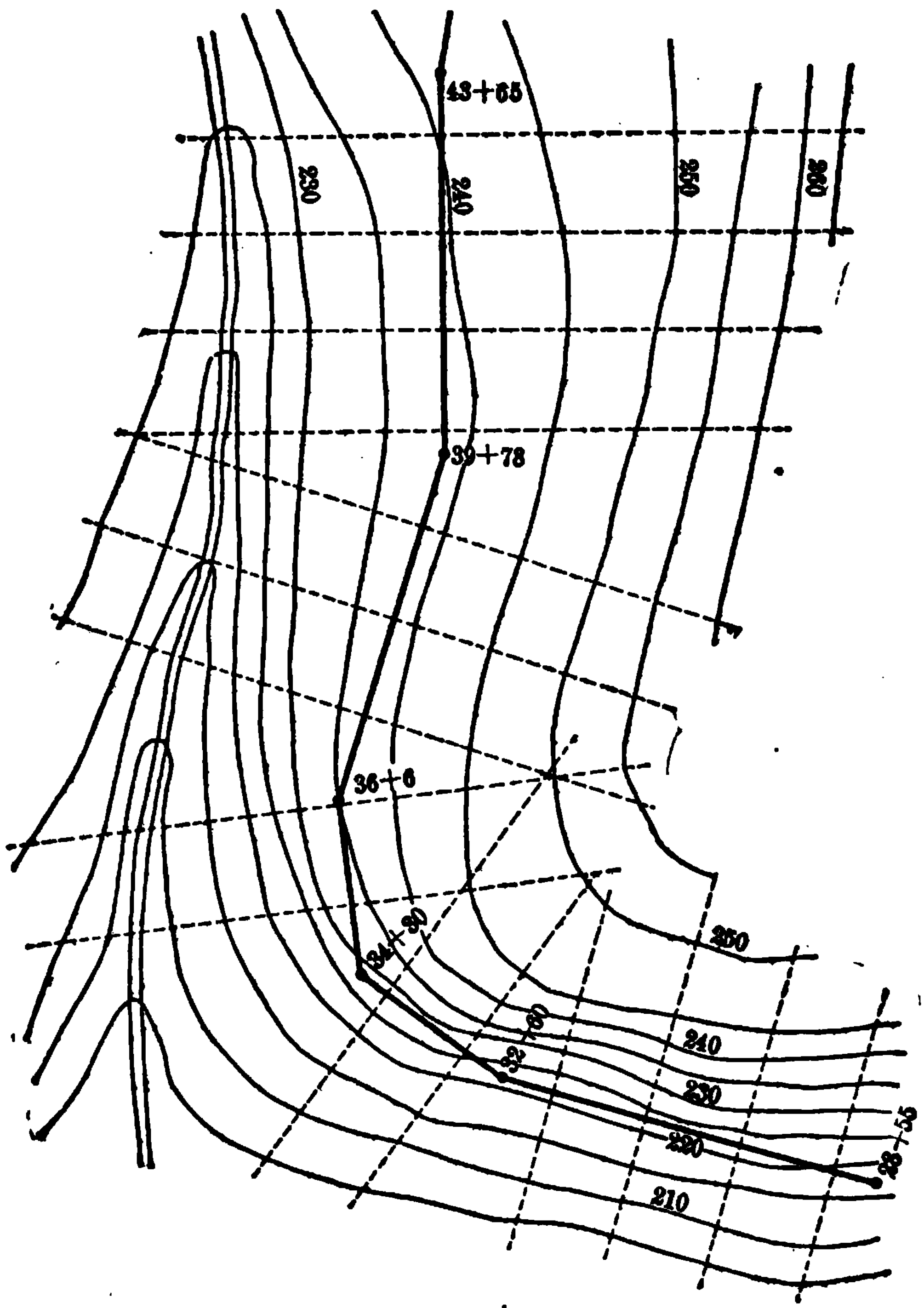

Fig. 4.

under these circumstances, it need only be set up on the far side of the tree and the former bearing produced. An error in reading a transit azimuth will be carried on throughout the survey. An error of only five minutes of arc will cause an offset of nearly eight feet in a mile. Large azimuth errors may, however, be avoided by immediately checking each new azimuth

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method is exceedingly rapid. Whatever error or inaccuracy occurs is confined in its effect to the one station where it occurs. The work being thus plotted in the field, unusually irregular topography may be plotted with greater certainty and no great error can occur without detection. It would even be possible by this method to detect a gross error that might have been made by the level party.

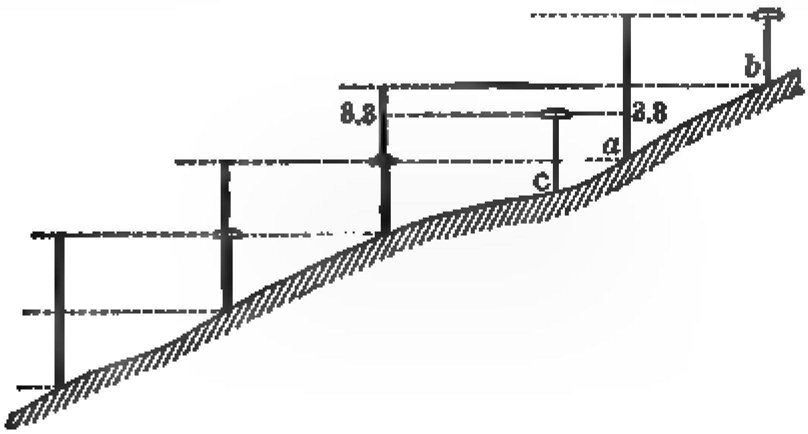

FIG. 5.

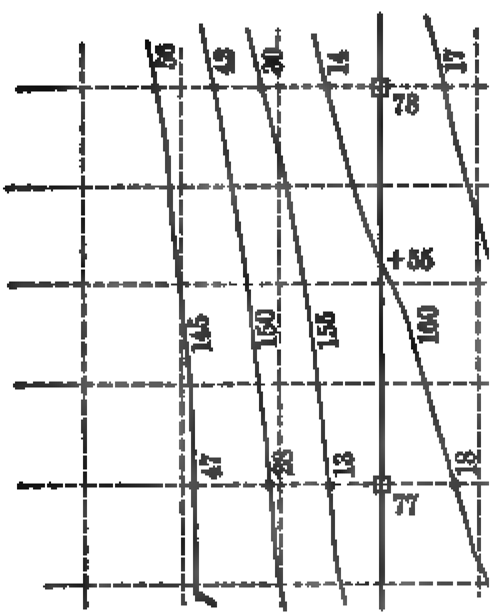

FIG. 6.

13. Stadia method. This method is best adapted to fairly open country where a "shot" to any desired point may be taken without clearing. The *backbone* survey line is the same

as in the previous method except that each course is limited to the practicable length of a stadia sight. The distance between stations should be checked by foresight and backsight—also the vertical angle. Azimuths should be checked by the needle. Considering the vital importance of leveling on a railroad survey it might be considered desirable to run a line of levels over the stadia stations in order that the leveling may be as precise as possible; but when it is considered that a preliminary survey is a somewhat hasty survey of a route that *may* be abandoned, and that the errors of leveling by the stadia method (which are compensating) may be so minimized that no proposed route would be abandoned on account of such small error, and that the effect of such an error may be usually neutralized by a slight change in the location, it may be seen that excessive care in the leveling of the preliminary survey is hardly justifiable.

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taken in leveling the instrument immediately before taking both foresights and backsights, these readings should agree to within one minute, or even 30 seconds, with a good transit. The height of the telescope above the ground at the new station must be measured, and the middle wire sighted at that reading on the rod (called the *H. I.*), when taking any vertical angle. Theoretically the rod reading for distance should be taken when the telescope is pointing at the proper vertical angle for that shot, but this will mean, in general, that both the upper and lower cross wires will read odd amounts and that an inconvenient subtraction must be made to get the difference, which is the "rod reading." But it may be demonstrated that no error of distance, amounting to the lowest practicable unit of measurement, can result if the telescope is raised or lowered just enough to set it on the nearest even foot mark. The routine of observing a shot is therefore as follows: (*a*) swing the instrument (the upper plate) horizontally until the telescope sights at the rod and clamp the horizontal motion—but very lightly and perhaps not at all; (*b*) raise or lower the telescope until the middle cross wire is sighting at the H. I., reading on the rod; a target on the rod may be set at the H. I. reading for each set-up and it will facilitate the work; (*c*) read the vertical angle and report it to the recorder, standing at hand; (*d*) raise or lower the telescope just enough so that the lower wire is on the nearest even foot mark and read (calling it out to the recorder) the number of even feet of interval from the lower to the upper wire and the odd amount at the top at the reading of the upper wire; (*e*) dismiss the rodman, who is then directed to another point by the chief of party; (*f*) read the azimuth on the horizontal plate. By that time another rodman has been located at a point where an observation is required, and the routine is repeated. The work of the transitman is thus very strenuous, without any recording work, and the progress of the party depends on him. He, therefore, should not be required to direct the party or even to record his notes, since every moment spent in that way delays the entire party by that amount. The recorder also has all that he can do to record the notes (with perhaps some sketches), as fast as the transitman calls them off. Usually four rodmen can be kept very busy, and they must be on the run between the successive points at which they hold their rods. One of the rodmen or one of the axemen, if axemen are employed, carries and

drives the stakes, which are only required at the instrument points. One or more axemen are generally useful in lopping off branches or cutting down saplings which interfere with desirable sights. The chief of party has plenty to do in directing the rodmén and axemen so that shots may be taken at points which will give the most significant information, and also in picking out the proper location for the advance station at some place from which a maximum of information may be observed with one set-up of the transit. A well-drilled organization and "team work" are necessary. The best work is done when every man is kept busy. Several hundred shots per day can be observed when it is considered advisable to obtain much detailed information and the average number of shots per set-up is large. On the other hand, when the stadia method is used for a rapid exploratory survey, only a few side shots (at some stations perhaps none at all) will be taken at each station. In such a case, the total number of shots taken during a day will be comparatively small, but the progress will be very rapid, and the salient features of several miles of a proposed route can be obtained in a day.

14. **Form for stadia notes.**

[Left-hand page.]

Inst. at	Azim.	Rod	Vert. angle	Diff. elev.	Elev.	Sighting at
Δ24.........	264° 27′	622	−0° 18′			Δ23
HI = 4.9....	83° 10′	528	+1° 16′			Δ25
El = 629.2...	184° 23′	264	−2° 18′			bend in creek
....	5° 47′	218(175)	+26° 20′			top of bluff

The usual six-column note-book can be utilized by ruling an extra line (shown dotted in the Form of Stadia Notes), in the fifth column, since the column is wide enough for both the "difference of elevation" and the "elevation." The "rod reading" (3d column) as recorded should include the $(f+c)$, which in almost all American transits equals 1.0 to 1.3 feet. Since the wire-interval ratio is almost invariably 1 : 100, the rod interval in hundredths of a foot is considered as the number of feet of distance, except that one even foot is added for the $(f+c)$. The sample figures given above are typical of all that needs to be taken in the field. The "difference of elevation" and the "elevation" are computed and entered later.

The "difference of elevation" may be mathematically computed from the formula

$$D = k\,r\,\tfrac{1}{2}\sin 2\alpha + (f+c)\sin\alpha,$$

in which D is the difference of elevation, k is a constant, usually 100, r is the rod intercept and α is the angle of elevation—or depression. The mathematical solution of such an equation for every shot that is taken (except the very few shots which are level) is very laborious and impracticable. But the work of reduction can be shortened by a justifiable approximation. By changing the factor of $(f+c)$ from $\sin\alpha$ to $\tfrac{1}{2}\sin 2\alpha$, the formula may be written

$$D' = [kr + (f+c)]\tfrac{1}{2}\sin 2\alpha.$$

The first term (that within the bracket) is the number recorded under "Rod" in the Form of Notes (622, 528, etc.). The second term ($\tfrac{1}{2}\sin 2\alpha$) may be taken from "Stadia Tables," of which many are published, although the tables usually give these numbers merely as the factors by which the distance is to be multiplied in order to obtain the "Difference of Elevation," and do not mention that the factor is really $\tfrac{1}{2}\sin 2\alpha$. The error of the approximation (when $(f+c) = 1$ foot) is less than 0.01 foot for a vertical angle of 15° and less than 0.1 foot for the unusual angle of 30°. Since 0.1 foot is the usual lowest unit of measurement for stadia elevations, probably 99% of all stadia work can use such an approximation without appreciable error. The special cases with high angles can be computed separately if it is considered necessary. The algebraic sign of the vertical angle should *always* be recorded, even if it is plus, or upward; the sign + is a positive statement that it *is* plus and that the sign was not forgotten. The difference of elevation likewise should always have a + or − sign. Adding the difference of elevation to the elevation of the station (or subtracting it), gives the elevation of each point.

Theoretically the true horizontal distance for all inclined sights is always less than the nominal distance, as given by the rod reading. The formula for true distance is

$$L = kr\cos^2\alpha + (f+c)\cos\alpha.$$

As before, we may use the approximation of combining the $(f+c)$ with the kr and say that

$$L'=[kr+(f+c)]\cos^2\alpha,$$

and that the *correction*, or the reduction from the nominal reading to the true distance, is

$$\text{Corr.}=[kr+(f+c)]\sin^2\alpha.$$

The error of this approximation is usually insignificant, as illustrated below. Since $\sin^2\alpha$ is very much less than $\cos^2\alpha$ for the usual small values of α, it is easier and more accurate to compute the smaller quantity and mentally subtract it from the nominal reading. When the vertical angle and the distance are both small, the horizontal correction is within the lowest unit of measurement (one foot), and should, therefore, be ignored. The engineer soon learns the approximate limits at which the combination of vertical angle and distance will make a correction necessary. In the above notes no correction is necessary except in the last case, the angle being 26° 20′. The exact mathematical computation is as follows, the rod interval being 2.17 and $(f+c)=1$,

$$L=217\cos^2 26^\circ 20'+1\cos 26^\circ 20'=175.20.$$

Using the approximate rule, the correction $=218\sin^2 26^\circ 20' =42.90$.

$$218-42.90=175.10.$$

The above calculations have been carried to hundredths of a foot for the sole purpose of illustrating that the discrepancy between the approximate and the theoretical value is only 0.10 foot, even for this unusually large angle, and considering that the rod interval is read only to the nearest 0.01 foot, which corresponds to one foot of distance, this discrepancy is utterly inappreciable.

15. **The reduction of stadia observations** is most easily accomplished by using a stadia slide rule, which has one logarithmic scale for distances and for the computed differences of elevation or corrections to distance, and also two other scales one of which gives values for $\frac{1}{2}\sin 2\alpha$, and the other gives values

for $\sin^2 \alpha$. Some scales give values of $\cos^2 \alpha$. To illustrate the difference, in the above case, it is evidently easier to read 43 (two significant figures) than to read 218, which has three figures. When the distance is over 1000 (four figures), the difficulty is even greater. The necessity for subtracting the correction is of no appreciable importance. In this case, the correction would be read from the slide rule as 43, and mentally subtracting 43 from 218, we write at once 175, which is recorded in parenthesis in the Rod column. The draftsman, when plotting the notes, uses this distance (175) instead of 218. Using a slide rule, two men can very quickly compute the differences of elevation for the entire day's work in a very short time. A very little practice will enable them to run down the list, picking out the observations, usually less than 10% of the total number, where the combination of distance and vertical angle is sufficiently great to make it necessary to compute a horizontal correction. The stadia slide rule is so small that it may readily be carried into the field and used there if desired, in which respect it has a great advantage over diagrams, which are sometimes used for the same purpose.

16. Stadia method vs. cross-section method. There is still a difference of opinion among engineers as to the choice of these two methods. When a large part of the route is thickly wooded, the cross-section method is preferable. In open country the stadia method is more rapid and more economical. Although it would be inadvisable to change from one method to the other every mile or so, a very considerable economy is possible by alternating the two methods according to the character of the country. The locating engineer can plan such change of method during his reconnoissance. The real efficiency of the stadia method is due to the fact that the preliminary survey should be considered as the topographical survey of an area or belt, and not the survey of a line, and that in open country the stadia method is the most efficient method of obtaining such topographical data. But the efficiency depends on the handling of the party. When a valley widens out with easy slopes and the possible area in which the location may lie is correspondingly widened, it is far easier and more accurate to widen the belt surveyed by stadia shots of 1000 feet if necessary.

17. "First" and "Second" preliminary surveys. Some engineers advocate two preliminary surveys. When this is done,

the first is a very rapid survey, made perhaps with a compass, and is only a better grade of reconnoissance. Its aim is to rapidly develop the facts which will decide for or against any proposed route, so that if a route is found to be unfavorable another more or less modified route may be adopted without having wasted considerable time in the survey of useless details. By this time the student should have grasped the fundamental idea that both the reconnoissance and preliminary surveys are not surveys of *lines* but of *areas*, that their aim is to survey only those topographical features which would have a determining influence on any railroad line which might be constructed through that particular territory, and that the value of a locating engineer is largely measured by his ability to recognize those determining influences with the least amount of work from his surveying corps. Frequently too little time is spent on the comparative study of preliminary lines. A line will be hastily decided on after very little study; it will then be surveyed with minute detail and estimates carefully worked up, and the claims of any other suggested route will then be handicapped, if not disregarded, owing to an unwillingness to discredit and throw away a large amount of detailed surveying. The cost of two or three extra preliminary surveys (*at critical sections* and not over the whole line) is utterly insignificant compared with the probable improvement in the "operating value" of a line located after such a comparative study of preliminary lines.

LOCATION SURVEYS.

18. **"Paper location."** When the preliminary survey has been plotted to a proper scale (usually 200 feet per inch), and the contours drawn in, a study may be made for the location survey. Disregarding for the present the effect on location of transition curves, the alinement may be said to consist of straight lines (or "tangents") and circular curves. The "paper location" therefore, consists in plotting on the preliminary map a succession of straight lines which are tangent to the circular curves connecting them. It may be assumed that the general route of the preliminary survey has been so well selected, as the result of the reconnoissance survey, that it is possible to construct a line without excessive earthwork between consecutive control points, and that the grades are within the ruling grade. If the preliminary

survey has been run by locating stations every 100 or 200 feet (see § 11 and Fig. 4), the profile of this line gives the first approximation toward the rate of grade, and from this may be determined whether one uniform grade between the control points is

FIG. 7. SINGLE GRADE BETWEEN CONTROL POINTS.

practicable, or whether two or more different grades must be used. If the stadia method was used, the profile of a line running through the station points will serve the same purpose. In Fig. 7 let *AMZ* represent, on a very small scale, the surface profile between two control points, *A* and *Z*, which are, perhaps,

FIG. 8. TWO GRADES BETWEEN CONTROL POINTS.

two miles apart. The upper dotted line shows the elevations of the highest points in the surveyed belt at each of the several stations, and the lower line the corresponding lowest points. If the straight line *AZ* does not go outside of these dotted lines, it indicates that the uniform grade *AZ* will have "supporting ground" for the entire distance and that such a grade is practicable and should be tentatively selected (or at least investi-

gated) for that stretch. If the straight line AZ passes outside the belt of the dotted lines, as in Fig. 8, it implies that there was some definite reason why no higher supporting ground could be found near M', or the preliminary survey, if properly made, would have covered that ground. It then becomes necessary to adopt two grades, such as AM' and $M'Z$. Three or more grades might prove necessary or desirable in some cases.

Having determined, at least tentatively and approximately, the rate of grade, set a pair of dividers at such a distance (to scale) that the distance times the rate of grade equals the contour interval. For example, with a contour interval of 5 feet and a 2% grade,

$$\text{distance} \times .02 = 5,$$

or

$$\text{distance} = 5 \div .02 = 250.$$

Then, with dividers set at 250 feet, put one leg where the line previously located crosses a contour and put the other leg where it reaches the contour next above—or below, if a down grade. Then step to the next contour and so on. If the desired starting point is not on a contour, the distance for the *first* step should be proportionately shortened. A strict application of this method would probably make a sidehill line run around short gullies where the curvature would need to be excessively sharp. To avoid such sharp curvature, these narrow gullies must be crossed by bridges, trestles or high embankments. To carry a grade across such a place, the length of step of the dividers should be doubled or trebled and the step should be to the second or third contour above or below. The line running through these successive points located on the contours will be practically a surface line which has nearly the desired grade. The cut and fill would be almost nothing—except "side-hill work," and the crossing of gullies. No accuracy need be expected on this preliminary trial since the distance is somewhat greater than the air-line distance AZ. It would, in general, be impossible to run a practicable combination of tangents and proper curves through these points, but such a line is very suggestive of a proper alinement which will fulfill the grade and curvature conditions and along which the cut and fill will be reasonably small.

If there are long stretches where, in each case, the line joining a group of consecutive points is nearly straight, the tangents will

predominate and should be located first and then connected by curves. If the line has numerous and long bends, it may be preferable to select the curves first and then connect them with tangents. For such work a series of curves, drawn to proper scale, varying by even degrees from 1° up to 15° or 20°, or whatever is the maximum allowable curvature, and drawn on any transparent material such as tracing cloth, celluloid or glass, is very useful, since different curves may be tried in turn until the curve which best fits the ground is discovered. The contours and other fixed features should have been inked in and then the trial lines and curves may be marked in lightly with a *soft* pencil, so that trial lines may be easily erased until a satisfactory line is obtained. The number of possible combinations is infinite, but certain conditions must be fulfilled which narrows the choice. (1) The connecting tangents must not be too short; 100, 200 and even 300 feet are used as limits. (2) The curvature must be within the adopted limit. If two consecutive curves, which are connected by a very short tangent, bend in the same direction, it is preferable that they should be combined into one simple curve, or into two branches of a compound curve, rather than to make a "broken-backed" curve. If they bend in opposite directions (making a reverse), even 300 feet is none too long for the transition curves which should be used, especially if the curves are sharp. Actual reverse curves (changing the direction of curvature without any separating tangent) should *never* be used, except on switch work and track where the speed is always slow. It would be far preferable to sharpen the curvature enough to introduce a tangent at least 100 feet long. The following considerations should be kept in mind.*

"(1) If the location could follow the grade line [or surface line] precisely, there would be no cuts or fills (practically speaking) on the center line.

"(2) Whenever the location lies on the $\left\{\begin{matrix}\text{down-hill}\\ \text{up-hill}\end{matrix}\right\}$ side of the grade contour [or surface line] there will be $\left\{\begin{matrix}\text{fill}\\ \text{cut.}\end{matrix}\right\}$

"(3) The further the location departs from the grade contour the greater will be the cut or fill, as the case may be."

* Course of Instruction on "Paper Location," by Prof. J. C. L. Fish, Stanford University.

After a location line has been selected which seems satisfactory from the standpoints of easy curvature, not too short tangents, a proper balance of cut and fill, and not too great cuts and fills, as will be approximately indicated by its distance from the surface line, the volume of earthwork may be estimated with sufficient accuracy for comparative purposes by drawing a profile of the surface location line and its roadbed line. Considering the ease with which such lines may be drawn on the preliminary map, it is frequently advisable, after making such a paper location, to begin all over, draw a new line over some specially difficult section and compare results. Profiles of such lines may be readily drawn by noting their intersection with each contour crossed. Drawing on each profile the required grade line will furnish an approximate idea of the *comparative* amount of earthwork required. A comparison of the areas of cut and fill on the profile will show the approximate balance in volume of cut and fill. If it is considered necessary to compute the volume with greater accuracy, it may be done by the use of Table XVII (see also § 126), applying the latter part of the table correctively to allow for side slope. After deciding on the paper location, the length of each tangent, the central angle (see § 51), and the radius of each curve should be measured as accurately as possible. Frequent tie lines and angles should be determined between the plotted location line and the preliminary line. When the preliminary line has been properly run, its "backbone" line will lie very near the location line and will probably cross it at frequent intervals, thus rendering it easy to obtain short and numerous tie lines.

19. Preparation of the notes. This and the actual transfer of the paper location to the ground is a problem in surveying which is so varied in its character that the ingenuity of the engineer is required to use the best method adapted to each particular case, but a few principles may profitably be kept in mind. (1) The scale of the paper location drawing is probably 200 feet per inch, unless the difficulties of the problem demand a larger scale for a particular stretch of the road, so that the paper location may be more accurate. Since a variation of 1/200 inch in the drawing means a variation of one foot on the ground, no close checking of the line on any tie-point need be expected. (2) Since a very small variation in alinement would, if persisted

in, throw the alinement very far from its desired location, it must be expected that there will be more or less adjustment of the paper location alinement (numerically) on nearly every tangent and curve. (3) The intersection of the preliminary line by a paper-location tangent (or the tangent produced) gives a possible tie-point. The position of this tie-point on the preliminary line must be scaled and the angle between the lines determined by measuring the chord of a long arc with its center at the point of intersection or by scaling the sine (or tangent) produced by a perpendicular from one line to the other from a point whose distance from the intersection is a convenient unit length. (4) When there is no intersection at some place where a tie is desired, a perpendicular offset from the preliminary line may be necessary. (5) When the paper location crosses the preliminary line at frequent intervals (say 500 to 1000 feet), it may be more simple to locate the tie-point intersections on the preliminary line and work from one to the other, taking up the inevitable inaccuracies by slight variations in the length of tangents or curves or by some one of the various methods detailed in § 63. When no practicable tie can be obtained for a considerable distance (say one-half mile), it may be desirable to determine the ordinates (latitudes and departures) of all the points on the preliminary and on the paper location between two consecutive intersections. In such a case the precision would depend entirely on the accuracy of scaling the positions of the two intersections and on the accuracy of the preliminary survey. While such a method requires considerable office computation, even that is cheaper than an extensive revision of a located line in the field. For a further development of this method, the student is referred to a course of instruction originally written by Prof. J. C. L. Fish, of Stanford University, and included in the sixth edition of "Surveying Instruments," by Webb & Fish, published by Wiley & Sons.

As previously stated, the above method has been developed as if the final located line were to be made up only of tangents and circular curves. But transition curves between the tangents and circular curves are essential for the easy operation of trains. Anticipating the more complete demonstration of the subject, § 41, *et seq.*, it may be stated that the effect of the transition curve, or "spiral," is to move the curve inward, or toward its center, or to move the tangent outward. The effect of this is

equivalent to offsetting the tangent outward, or offsetting the curve inward, and then connecting the tangent and circular curve by a transition curve which gradually crosses the offsetted distance. The amount of the offset varies with the degree of the central curve and the desired length of the transition curve, but it is seldom more than three or four feet, and is usually much less. No consideration need be given to these offsets when comparing several trial locations. It is only after the paper location has been settled and it is time to transfer this to the ground that it is necessary to compute these offsets and adjust the lines accordingly. Even then the offsets will seldom be so large that they would appreciably affect the paper location, but when the alinement is actually located on the ground, the proper offsets should be used and the alinement laid out as described in detail in § 80.

20. **Surveying methods.** A transit should be used for alinement, and only precise work is allowable. The transit stations should be centered with tacks and should be tied to witness-stakes, which should be located outside of the range of the earthwork, so that they will neither be dug up nor covered up. All original property lines lying within the limits of the right of way should be surveyed with reference to the location line, so that the right-of-way agent may have a proper basis for settlement. When the property lines do not extend far outside of the required right of way they are frequently surveyed completely.

The leveler usually reads the target to the nearest thousandth of a foot on turning-points and bench-marks, but reads to the nearest tenth of a foot for the elevation of the ground at stations. Considering that $\frac{1}{1000}$ of a foot has an angular value of about one second at a distance of 200 feet, and that one division of a level-bubble is usually about 30 seconds, it may be seen that it is a useless refinement to read to thousandths unless corresponding care is taken in the use of the level. The leveler should also locate his bench-marks outside of the range of earthwork. A knob of rock protruding from the ground affords an excellent mark. A large nail, driven in the roots of a tree, which is not to be disturbed, is also a good mark. These marks should be clearly described in the note-book. The leveler should obtain the elevation of the ground at all station-points; also at all sudden breaks in the profile line, determining also the distance of these breaks from the previous even station. This will in-

clude the position and elevation of all streams, and even dry gullies, which are crossed

Measurements should preferably be made with a steel tape, care being taken on steep ground to insure horizontal measurements. Stakes are set each 100 feet, and also at the beginning and end of all curves. Transit-points (sometimes called "plugs" or "hubs") should be driven flush with the ground, and a "witness-stake," having the "number" of the station, should be set three feet to the right. For example, the witness-stake might have on one side "137+69.92," and on the other side "P C 4° R," which would signify that the transit hub is 69.92 feet beyond station 137, or 13769.92 feet from the beginning of the line, and also that it is the "point of curve" of a "4° curve" which turns to the *right.*

Alinement. The alinement is evidently a part of the location survey, but, on account of the magnitude and importance of the subject, it will be treated in a separate chapter.

21. Form of Notes. Although the Form of Notes cannot be thoroughly understood until after curves are studied, it is here introduced as being the most convenient place. The right-hand page should have a sketch showing all roads, streams, and property lines crossed with the bearings of those lines. This should be drawn to a scale of 100 feet per inch—the quarter-inch squares which are usually ruled in note-books giving convenient 25-foot spaces. This sketch will always be more or less distorted on curves, since the center line is always shown as *straight* regardless of curves. The station points ("Sta." in first column, left-hand page) should be placed opposite to their sketched positions, which means that even stations will be recorded on every *fourth* line. This allows three intermediate lines for substations, which is ordinarily more than sufficient. The notes should read UP the page, so that the sketch will be properly oriented when looking ahead along the line The other columns on the left-hand page will be self-explanatory when the subject of curves is understood. If the "calculated bearings" are based on azimuthal observations, their agreement (or *constant* difference) with the needle readings will form a valuable check on the curve calculations and the instrumental work.

22. Number of men required in surveying parties. No fixed rules can be given. The general rule of economy and efficiency

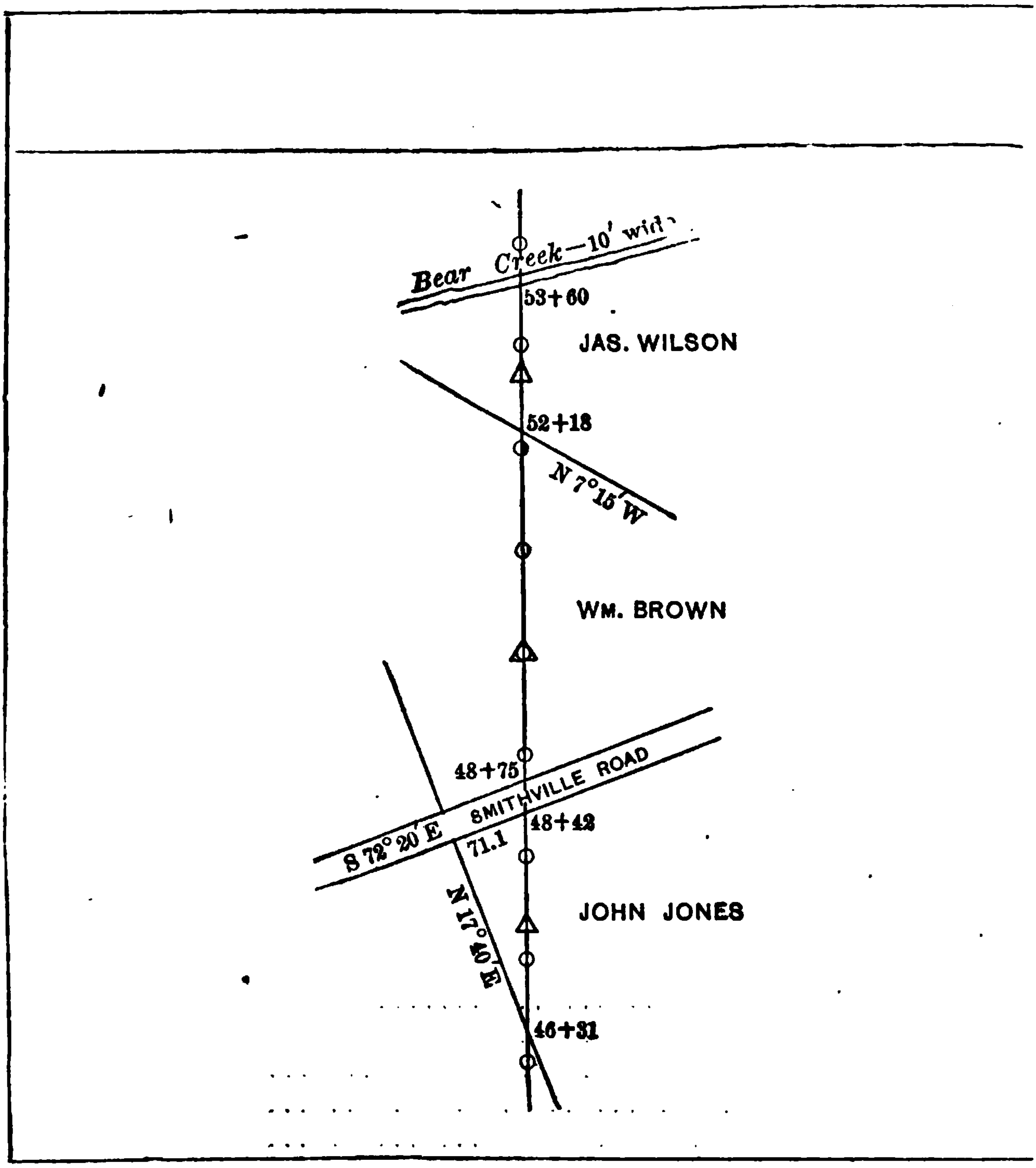
FORM OF NOTES.

[Left-hand page.]

Sta.	Aline-ment.	Vernier.	Tangential Deflection.	Calculated Bearing.	Needle.
54					
53 ⊙ + 72.2	P.T.	9° 11′	18° 22′	N 54° 48′ E	N 62° 15′ E
52		7 57			
51		6 15			
⊙ 50	[3° 24′ curve to right for 18° 22′; tang. dist., 272.5]	4 33			
49		2 51			
48		1 09			
⊙ + 32 47	P.C.	0°			
46				N 36° 26′ E	N 44° 0′ E

should govern, and that is, that the organization should be such that all desired data can be obtained at a minimum of cost. This general rule may be subject to the modification that the early completion of the survey is sometimes financially so important as to justify the maximum speed, almost regardless of expense. A common violation of the general rule of economy is the use of too few men, with the mistaken idea that it is economical. This requires the high-priced efficient men to waste their time on work which men at one-half (or even one-third) their salary could do sufficiently well, thus delaying the completion of the work or depreciating its quality by undue haste

[Right-hand page.]



or by neglect to obtain complete data. The work should be so organized that each man is constantly busy at the kind of work for which he is especially qualified, and that no men shall have to wait for others to complete their co-ordinate work. Even if 100% efficiency is unobtainable, it is very uneconomical to have nearly the whole party idle while one or two high-priced men do some work which must be done before the party can proceed but which could have been done by some extra lower-grade men without delaying the party. **Reconnoissance.** When the territory of the general route has been mapped by the U. S. Geol. Survey, there may be no need of instrumental work on the

reconnoissance, since the approximate ruling grades and general route may perhaps be determined directly from the map, and the purpose of the reconnoissance is the examination of physical features which would affect or modify the general route. In such a case the engineer does his technical work alone and only needs a guide and cook in case camping is necessary. When the reconnoissance partakes more of the nature of a hasty preliminary, distances, elevations and the necessary side topography being determined by rapid approximate methods, more men should be added, keeping in mind that the work should be so organized that each member of the party is kept busy at his own co-ordinate work, and that the chief engineer is not delayed in his own special work by spending his valuable time on a cheaper grade of work which an assistant could do sufficiently well. In other words, it is economical to add to the party an extra assistant whenever the work that he can do will so facilitate the work of the party as a whole that the value of the salaries and expenses saved will more than offset the assistant's salary and expenses. **Preliminary surveys.** No fixed list of members of a party is applicable to all conditions. The following list, with monthly salaries, is given by Mr. Fred Lavis* as having been used on each of five parties in surveying the Choctaw, Oklahoma & Gulf R. R. The list is very full but justifiably so.

Locating engineer	$150 to	$175
Assistant locating engineer	115	125
Transitman	90	100
Levelman	80	90
Draftsman	80	90
Topographers, two	80	90
Level rodman	50	
Head chainman	50	
Rear chainman	40	
Tapemen, two	30	
Back flagman	30	
Stake marker	30	
Axemen, three to five	25 to	30
Cook	50	
Cook's helper	20	
Double teams and driver, furnish their own feed, driver boarded in camp	65 to	90

* Methods of Railroad Location on the Choctaw, Oklahoma & Gulf R.R. Trans. Am. Soc. C. E., Vol. LIV, page 104.

Other organizations sometimes combine the first two positions on this list and possibly call him "chief of party." For the above work, the locating engineer was relieved altogether from the detailed direction of the party, which was handled by the assistant, and spent nearly all his time in studying the country so as to determine how the line should advance. In nearly all cases, such expense is justified, perhaps many times over, (1) by the saving of uselessly surveying an improper route, (2) by an improvement in the operating value of the route selected, or (3) by an improvement in route which makes a decrease in construction cost. Sometimes those controlling the financial side of the project insist that the chief of party shall also run the transit, as a measure of "economy." Such a policy cannot be too strongly condemned. The work of a transitman requires every instant of his time and every minute that he turns from his transit to direct the party or study the proper route is a minute delay for the entire party. It generally means also a deterioration in the quality of his work as a leader and as a transitman, in his effort to hastily do at one time work which requires the concentrated efforts of two men. In this survey (described by Mr. Lavis), the skeleton or backbone line was a broken line with angles every few hundred feet, and the topography was taken by right-angled offsets every hundred feet or oftener, substantially as described in § 11 and Fig. 4. These offsets were determined by a hand level and pacing by one of the two topographers. The other topographer, using a transit, with the other two tapemen "determined drainage areas, located property lines and section corners, got names of property owners, etc." When, as is usually the case, such essential work cannot be done by the main party without delaying their progress, there is a real economy in adding to the party these comparatively low-priced assistants. It may be noted that the above party includes two chainmen, back flagman and stake-marker, beside three to five axemen. The proper number of axemen manifestly depends on the amount of necessary cutting, but the chainmen or the stake-marker should not be depended on for such work. The steady march of the party should not be halted while a stake-marker or chainman stops his regular work to cut down a tree. One of the duties of the chief of party is to foresee the necessities of tree-cutting and clearing, so far in advance that, by the time the surveying members of the party have reached the spot, the

area is cleared. It is likewise false economy to dispense with the stake-marker and require the head chainman to do such work. A full corps of such men, properly drilled, can add 20 to 50% to the daily progress of the party and much more than save their cost.

MAINTENANCE OF SURVEY PARTIES.

23. Economy and efficiency. When considering the treatment and maintenance of surveying parties, it should be remembered that a false idea of economy is frequently responsible for making the parties too small, overworking the men, depriving them of physical comforts and even necessities, and that the result is a greater net cost and a great deterioration in the quality of the results. A party may cost $40 to $65 per day in salaries and expenses. Any policy which depreciates the net output of their work 20 to 50% (which is easily possible) in order to save a few dollars per day is manifestly poor policy. The men, especially those who must use their brains and who presumably have a finer nervous organism, have only a quite definite sum total of nervous energy. If a considerable part of that energy is spent in needlessly long tramps both morning and evening to and from work, or if that nervous energy is not maintained by plentiful and appetizing food and by sufficient and comfortable rest, there is a reduction in efficiency which is often far greater than any possible saving in expenses. This idea of developing the maximum efficiency of the party is the justification of the recommendations made below regarding outfit, equipment, and other details about managing a party.

24. Country hotels and farm houses. In settled sections of the country, country hotels and even farm houses are sometimes available where men can be provided with living facilities which are unobtainable in camp life and at less total expense. Such accommodations have the advantage that they obviate a considerable capital expenditure to purchase sufficient camp outfit. But if suitable accommodations are unobtainable over a considerable portion of the route and such accommodations as there are on the remaining distance are inconvenient and inadequate, it may be preferable to provide a camping outfit at once. Considering the fact that there is a real economy in making a survey with a large party and that such a party can

seldom if ever be accommodated in a single farmhouse, and that there is a lack of efficiency if the party is separated, the farmhouse plan is frequently impractical. But when villages are so located that there is always one within five miles of any point of the line, the house plan may be preferable, since the party may be taken to and from work in conveyances. The economy of employing conveyances may be judged by comparing the cost of the vehicles and the value of the time and energy saved. A five-mile tramp, carrying an instrument, following a full day's work surveying, will frequently incapacitate a man from doing effective work in the night-work which the higher grade men of the party must generally do. The day's work in the field must be begun later and ended earlier or else the time and strength spent in the morning and evening tramps are uneconomical drains on their total nervous energy.

25. **Camping Outfits: Tents.** The Choctaw, Oklahoma & Gulf R.R. survey, previously referred to, provided for each party one office tent, with fly, 14×16 feet, three tents, evidently without flies, 14×16 feet, and one cook tent 16×20 feet. The office tent had 5-foot walls; the others 4-foot. H. M. Wilson ("Topographical Surveying," p. 817) recommends 9×9 foot tents, with 4-foot walls. These are easier to erect but have only 36% of the floor area of the 14×16-foot tents and it would require 15 such tents to equal the floor area of the 5 tents described above. For a small party the smaller tents would be preferable. The canvas should be mildew-proof and free from sizing. A "sod-flap" about 8 inches wide, should be attached to the bottom of the wall. When this flap is weighted down with stones or heavy sticks the wind and weather is kept out. Dirt or sod should *not* be used for weights, since they rot the canvas. It pays to use tents which conform to the U. S. Army specifications. Some of the specifications as to material and workmanship are here quoted:

"*Materials.*—Body of tent to be made of Army standard $12\frac{4}{10}$ ounce cotton duck, $29\frac{1}{2}$ inches wide and the sod cloth of Army standard 8-ounce cotton duck, $28\frac{1}{2}$ inches wide.

"*Workmanship.*—To be made by machine in a workmanlike manner, all seams to be stitched with two rows of stitching, not less than six stitches to the inch, with three-cord twelve-thread Sea Island cotton, white.

"In making tents by hand, to have not less than two and one-

half stitches of equal length to the inch, made with a double thread of five-fold cotton twine, drab, well waxed.

"The seams should be not less than 1 inch in width, flat stitched, and no slack taken in them.

"*Grommet holes.*—Made with malleable iron rings, galvanized, to be worked with four-thread five-fold cotton twine, well waxed.

"*Sod cloth.*—To be 8 inches in width in the clear from the tabling, into which it is inserted 1 inch and extending from door seam to door seam around the tent.

"*Tabling.*—On foot of tent when finished to be 2½ inches in width." (Adopted July 14, 1911.)

A ditch should be dug outside the tent, at least on the up-hill side, if the ground is at all inclined. This will prevent rain-water from draining through the tent. Of course, the bottom of the ditch should have a uniform slope draining to an outfall amply clear of the tent.

26. Tent floors. Dry floors are almost essential to health. Sectional floors, about 3×9 feet per section, made by fastening boards to cross cleats, provide a perfectly dry floor and often repay their transportation. A mere layer of canvas, cut to proper shape and bound on the edges, is worth providing if the ground is dry when the tent is erected and can be kept from getting rainsoaked by proper outside drainage.

27. Tent stoves. For winter work, tents may be made quite comfortable with stoves. Oil stoves are convenient when the oil can be purchased without excessive cost for transportation. "Sibley" stoves, burning wood, are commonly used but they require smoke pipes which must pass through the canvas and this means that the holes must be properly protected with metal or asbestos. If a pipe elbow is provided, the pipe may be taken out through one end of the tent. This obviates a hole in the roof of the tent (and also the fly); it avoids a direct pour of rain on the fire or leakage into the tent around the pipe, and also the danger of sparks dropping on the canvas. A "Sibley" stove for mere heating is a sheet-iron frustum of a cone, about 3 feet high; diameter at bottom 18 to 30 inches; diameter at top 4½ to 6 inches, or so as to fit the stovepipe which is to be used. It has no bottom, or in other words, the bare earth forms the base. A door, large enough for the insertion of such fuel as it is designed to use, is placed in the side. Three or four lengths of pipe, one of which should have a damper, and an elbow,

should be provided. Draft at the bottom is obtained, and may be easily controlled, by packing earth around the base, leaving a small opening which may be easily enlarged or diminished to control the draft. **Cook stove.** A regular 6-hole cooking range, perhaps made of wrought-iron or sheet-steel, is essential to cook meals for twenty or more hearty men. Sporting outfitters supply all sizes of stoves, which must always be selected with due regard for the facilities for transportation. Oil stoves are commonly used. For still smaller parties, or when no cook stove can be permitted in the baggage, a primitive grid may be made from four sticks of *green* timber about 6 inches in diameter and 2 to 4 feet long. Notch two of them, each with a pair of notches about 10 inches apart. Place the other two sticks across the notches and they will steadily support a kettle or a frying pan. If the sticks are sufficiently green and the fuel quite dry the grid will last some time. A folding grid of iron bars may be obtained, which is but a small addition to the weight of the baggage. Another method is to suspend a kettle by a chain or long hook either from a tripod of sticks or from a horizontal stick lying in two forked sticks on each side of the fire.

28. Dining tables. These are justifiable for a large party when the baggage is necessarily great and camp wagons are a part of the equipment. Mr. Lavis, in the article previously referred to, describes a very good table from the standpoint of transportation. The table top consists of three loose planks $1\frac{5}{8}'' \times 12'' \times 18'\ 0''$. Two similar boards are used for seats. During transportation these boards are placed on the bottom of the wagon and, of course, project from the back where they form a support for stoves, etc., which can be roped on. These boards are supported on three trestles or horses, made as shown. For a much smaller party, a table may be improvised by utilizing two "mess-boxes," which carry the cooking utensils and tableware. These mess-boxes are about 20 inches wide and high and from 24 to 30 inches long. The covers are made to open 180° and may be fastened horizontally. An "inside cover," which can be utilized as a bread board, covers the entire inside area of the box. Two such boxes, set together and with the tops opened out, provide a fairly even surface four times the area of one box.

29. Cooking utensils, table-ware, tools, etc. The size of the party, the individual preferences of the person designing the

outfit and the facilities for transportation, vary such lists almost indefinitely. Agate ware has replaced china for plates and cups. Aluminum ware, although expensive, is preferable from a cooking standpoint and has the advantage of a very material reduction in weight. Out of the very great number of lists which have been published, the following list of articles is quoted as suggestive: Plates, cups, saucers, steel knives and forks, German-silver spoons, large and small, carving knives and forks, large cooking forks and spoons, pepper and salt boxes, tin pans about

FIG. 9.—CAMP DINING TABLE.

6 inches diameter by 1½ inches deep, utilized for serving soup, cereal, etc., pans and kettles of varying sizes which will "nest" and thus facilitate packing, tea kettle, coffee pot, frying pan, griddle, cake turner, pie plates,. dripping pan, chopping bowl and chopper, colander, flour sieve, coffee mill, broiler, corkscrew and can opener, rolling pin, folding table (similar to the drawing table described below), wash basins, kerosene oil can, alarm clock, spring balance. The last two articles are important. The cook is the first man up in the morning—usually before daylight—and may need the alarm clock. A single delay, of even ten minutes of such a party, would cost more than a very valuable clock. A spring balance is very essential to the proper

and economical use of provisions without waste. It pays to have a cook who is able to compute, weigh out and use an amount

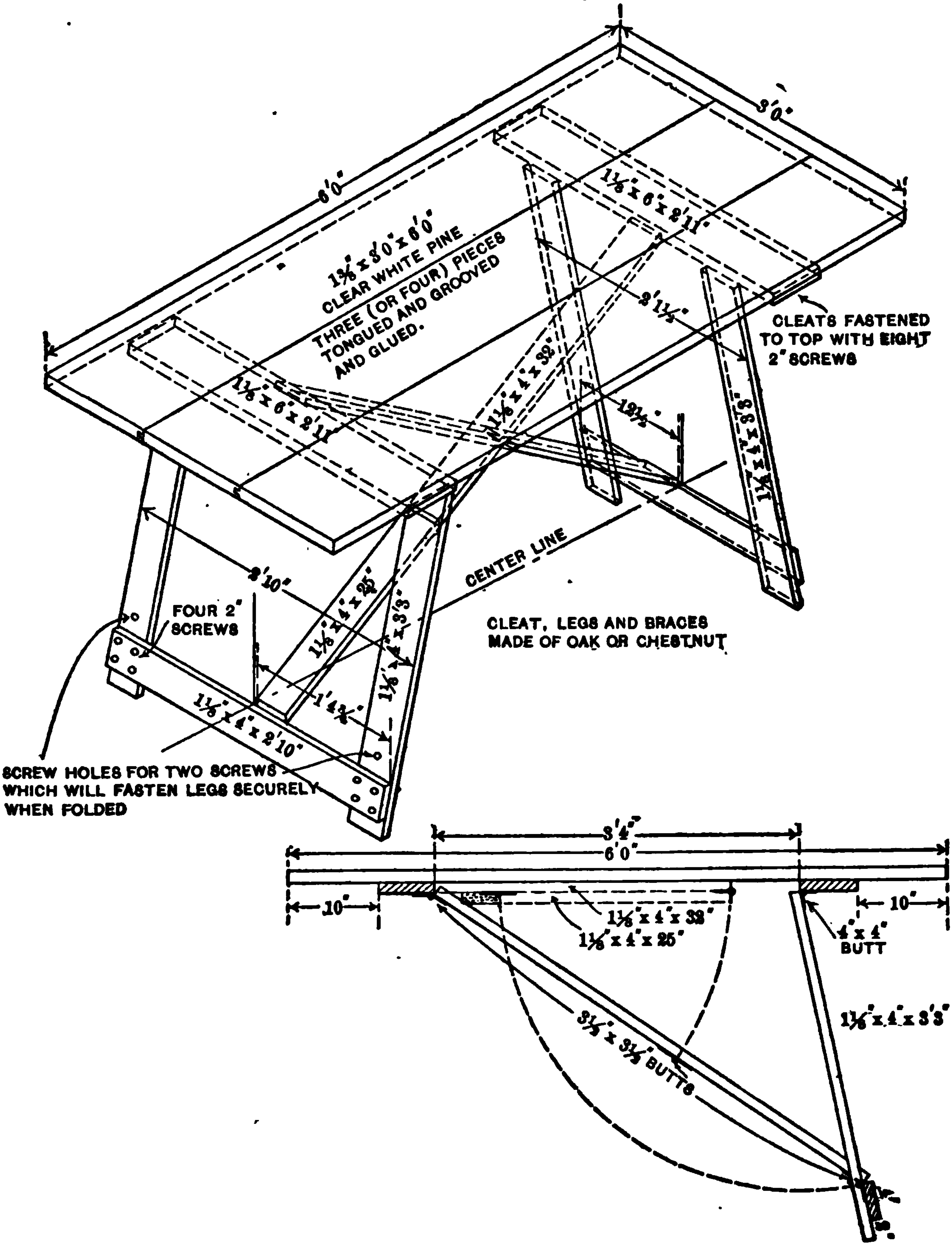


Fig. 10.—Folding Drafting Lable.

of each kind of provisions so that there will be sufficient, but no waste. Besides the above, dish towels are practically essen-

tial and tablecloths and napkins are easily carried. A table oilcloth may replace the ordinary tablecloth. Wash tubs and wash board facilitate the washing of table linen and also underwear, so essential to clean, healthy living. Illumination for night work must be provided. Reflecting lanterns will answer for all tents except the office tent, where good lamps, with cylindrical wick and center draft, or similar, should be provided. The farther the party travels from "civilization" the greater the necessity for providing for emergencies, breakages, etc. Axes are essential, apart from their use in the surveying work. Extra handles should be provided. A saw, brace and several sizes of bits, screw drivers, monkey wrench, files, pliers, hatchet, assorted screws and nails, pick, shovel, crowbar, whetstone, rope in various sizes, sailor's needles, palm and sewing twine, will all be useful and even invaluable in times of emergency. Canvas-covered canteens, for each member of the party, when passing through arid regions, may be essential.

30. **Drawing tables.** Complete topographic drawings, made in the field, are absolutely essential. Suitable drawing boards are, therefore, required. The design shown in Fig. 10 fulfills all the working requirements; it also is easily handled when packed up and is not readily broken. By packing them together in pairs, face to face, the surfaces are protected during transportation. The table consists essentially of a drawing board with stiffening cleats. The legs are hinged to the cleats, the braces for each pair of legs being of just such a length that when opened the legs stand at the desired angle. The braces are hinged and fold up, jackknife fashion, so that they nowhere project beyond the legs.

31. **Stationery and map chest.** Considering that the maps, drawings and notebooks may represent thousands of dollars, and that they are likely to be injured, if not irreparably ruined, by rain, when moving camp or during a cyclonic storm, a strong, water-tight chest, of ample capacity for all drawings and notebooks, should be provided. It should be required that *all* drawings and notebooks should be kept in the chest over night and at all other times, except such drawings and notebooks as are in actual use. The net inside length should be a little in excess of the longest roll or drawing, which is perhaps 36 inches. There should be a tray in the top with numerous compartments or boxes for the multitudinous small articles required by a drafts-

man. Handles should be provided for convenience and it should have a lock. A good "steamer" trunk of requisite size will answer the purpose, provided it is waterproof, and it would perhaps be cheaper than a chest of similar size, made to order.

32. **Provisions.** A "ration" is the estimated amount of food required per man per day. For men engaged in strenuous outdoor work, the food required is far more than that eaten ordinarily. Ration lists should average about 5 to 6 pounds of food per day per man. The amount that must be *transported* may be considerably less than this, in view of the fact that e.g., dried vegetables may be substituted for fresh vegetables in the ratio of 1 lb. of dried for 3 lbs. of fresh, the water used in cooking providing the other two pounds. For explorers, who carry their own provisions, and who must cut down every possible ounce of baggage, still further concentrations are possible.

Article	100 rations
Fresh meat, including fish and poultry, (*a*)	100 lbs.
Cured meat, canned meat, or cheese (*b*)	50 "
Lard	15 "
Flour, bread or crackers	80 "
Corn meal, cereals, macaroni, sago, or cornstarch	15 "
Baking powder or yeast cakes	5 "
Sugar	40 "
Molasses	1 gal.
Coffee	12 lbs.
Tea, chocolate or cocoa	2 "
Milk, condensed (*c*)	10 cans
Butter	10 lbs.
Dried fruits (*d*)	20 "
Rice or beans	20 "
Potatoes, or other fresh vegetables (*e*)	100 "
Canned vegetables or fruit	30 "
Spices	$\frac{1}{4}$ "
Flavoring extracts	$\frac{1}{4}$ "
Pepper or mustard	$\frac{1}{2}$ "
Salt	4 "
Pickles	3 qts.
Vinegar	1 "

"(*a*) Eggs may be substituted for fresh meat in the ratio of 8 eggs for 1 lb. of meat.

"(*b*) Fresh meat and cured meat may be interchanged on the basis of 5 lbs. of fresh for 2 lbs. of cured. [This ratio 5:2 is far higher than is usually allowed, 5:3 or even less is usually stated as the equivalent ratio.]

"(*c*) Fresh milk may be substituted for condensed milk in the ratio of 5 quarts of fresh for 1 can of condensed.

"(*d*) Fresh fruit may be substituted for dried fruit in the ratio of 5 lbs. of fresh for 1 of dried.

"(*e*) Dried vegetables may be substituted for fresh vegetables in the ratio of 3 lbs. of fresh for 1 lb. of dried."

The list at bottom of p. 43 is given by H. M. Wilson ("Topographic Surveying") as the ration list of the U. S. Geol. Survey. The quantities are those required to make up 100 rations, or the food for 5 men for 20 days, or for 100 men for one day. They are considered maximum. The sum total is about 525 lbs. or $5\frac{1}{4}$ lbs. per day per man.

Wilson states that the cost of the above list of rations should not average more than 45 to 55 cents per day for average conditions and with a maximum of 75 cents, but considering that this statement was written in 1900, some allowance may need to be made for higher prices since then.

The list given below represents the provisions actually supplied to a mining camp in British Columbia. The list has been reduced to the average quantity actually consumed per man per day. The food supply averaged nearly 6 lbs. per day per man.

Meat, etc.:		
Fresh beef	1.89	lbs.
Bacon	.076	"
Ham	.060	"
Codfish	.007	"
Canned salmon	.014	can
Breads, etc.:		
Pilot bread	.007	lb.
Flour	.894	"
Baking powder	.016	"
Corn meal	.037	"
Vegetables:		
Potatoes	1.421	lbs.
Turnips	.010	"
Carrots	.047	"
Beets	.016	"
Parsnips	.023	"
Rice	.043	"
Cabbage	.101	"
Dehydrated onions	.0014	"
" rhubarb	.0029	"
White beans	.0014	"
Bayo "	.027	"
Lima "	.013	"
Split peas	.006	"
Rowan "	.0014	"
Canned tomatoes	.016	can
" beans	.0043	"
" peas	.0014	"
Cereals:		
Pearl barley	.0004	lb.
Rolled oats	.117	"
Beverages:		
Tea	.021	lb.
Coffee	.036	"
Milk, condensed	.137	can

Fruit:		
Dried apples	.040	lb.
" pears	.033	"
" peaches	.029	"
" prunes	.020	"
" apricots	.007	"
" figs	.030	"
Dehydrated cranberries	.004	"
Currants	.021	"
Jam	.001	pint
Condiments, etc:		
Mustard	.001	lb.
Salt	.036	"
Pepper	.001	"
Vinegar, Klondyke	.0003	pint
Worcestershire sauce	.0043	"
Catsup	.0029	gal.
Miscellaneous:		
Sugar	.594	lb.
Lard	.030	"
Cheese	.016	"
Cornstarch	.007	"
Extract	.049	"
Curry powder	.0007	"
Cinnamon	.0009	"
Hops	.0001	"
Nutmeg	.0009	"
Ginger	.0014	"
Mapleine	.0011	oz.
Candied peel	.004	lb.
Butter	.014	"
Macaroni	.003	"
Sago	.011	"
Tapioca	.003	"
Baker's chocolate	.0014	"
Cocoanut	.0003	"
Pickles	.003	gal.

Supplies: candles, .03 lb.; gold dust, .003 lb.; soap, .024 bar.

The following list of provisions was bought to start a camp of 20 to 25 men on the Choctaw, Oklahoma & Gulf R. R. Survey. (F. Lavis, Trans. Am. Soc. C. E., Vol. LIV, p. 104.)

6 hams
6 pieces of bacon
50 lbs. fresh beef
1 case eggs
25 lbs. butter
25 " lard
100 " flour, hard wheat
100 " flour, soft wheat
100 " sugar
5 " baking powder
2 " tea
50 " coffee
50 " navy beans
25 " lima beans
12 " buckwheat flour
5 " macaroni
35 " cornmeal
1 cheese, about 15 lbs.
12 packages oatmeal
10 lbs. rice

100 cakes soap
1 gal. molasses
1 case condensed milk
1 doz. tomato catsup
¼ " Worcestershire sauce
1 gal. pickles
¼ doz. lemon extract
¼ " vanilla extract
1 box dried prunes
5 lbs. raisins
4 doz. assorted canned fruits
1 case tomatoes
1 bushel potatoes
1 kit salt mackerel
20 lbs. salt
¼ " mustard
1 " pepper
1 qt. vinegar
½ doz. yeast cakes

In addition to the above, there must be provided plenty of matches, kerosene oil and perhaps candles. As a matter of health conservation, and the prevention of piles, it is wise to provide toilet paper and to insist, if necessary, on its use. There is economy, when it is practicable, in making wholesale contracts for all provisions, rather than to buy haphazard from small local sources.

33. Beds. When baggage wagons accompany the party, as is virtually necessary to transport other essential equipment, it is desirable that they also transport army cots. These fold up so as to be easily transportable. It is a wise economy to obtain the regular army blankets, since they are what long experience has approved. Canvas covers should be provided for the bedding. This is essential to keep the bedding in even reasonably cleanly condition, especially while moving. The policy of requiring each member of the party to provide himself with cot, bedding and cover, and to care for them, is debatable. As a matter of business economy, the company should buy all cots and bedding wholesale. Requiring each one to purchase his own is virtually a reduction of salary, for, if a man leaves the party, he usually does not care to take his bedding with him, except in the hope of realizing something on it. But as all this is considered when accepting employment, the company virtually pays for the bedding by an increase of salary over what

they would have to pay if bedding were provided. There is the same reason for owning bedding as for owning dishes, etc. Sterilizing bedding by means of a formaldehyde candle, especially after a man has left the party, is a wise sanitary precaution and nullifies one of the strongest reasons for individual ownership.

34. **Transportation.** The route of travel of a mining engineer, a topographical engineer or an explorer, may be over country with every variety of surface and slope. But, since a practicable railroad route is necessarily on a low grade, except as it may pass over a ridge or mountain to be pierced by a tunnel, the question of grade does not ordinarily influence the method of transportation and wagons can ordinarily be used, provided the nature of the surface will permit. Strong and heavy wagons can usually pick their way between the camping places, even though long detours must be made to avoid swamps or other obstructions. The parties surveying the Choctaw, Oklahoma & Gulf R. R., previously referred to, used two teams regularly, one of which stayed with the topographical party. They used a third team for hauling supplies. Two teams of horses can help each other over a particularly bad place in the trail or in the case of accident. The wagons should have canvas tops, as a protection against rain, especially while moving. Transportation by dogs and sledges is only applicable under very limited and unusual conditions. It implies winter work, which is always uneconomical and inefficient compared with summer work, but in a very swampy country, where the transportation of any considerable amount of baggage is very difficult, and where it freezes during the winter to a comparatively smooth surface, such a method may be preferable in spite of short daylight hours and other disadvantages. "The Duluth, South Shore & Atlantic Railway employed toboggans during the construction of its road throughout the season of 1887." The description of this work, and much other useful information is given in a paper by Chas. H. Snow, Vol. XXIX, p. 164, in the Trans. Am. Inst. Mining Eng'rs. A reconnoissance through a comparatively unexplored country, made with the object of discovering a practicable low-grade route through a mountainous section, might require that all baggage shall be reduced to what may be handled in packs carried by horses, mules, Indian ponies or even by men. The question of the necessary method of transportation must always be studied before beginning a survey, since the entire question

of subsistence, and even many features of the method of work, must depend on what can be included in the baggage.

35. Clothing. While it may seem an unwarranted interference with personal liberty to control the clothing worn by members of the party, it becomes justifiable when the efficiency and progress of the party is impaired by bad health or disability, which is plainly due to neglect of proper precautions in the way of clothing or personal sanitation. Sore feet are responsible for a large part of the disablement of men. Washing the feet *every night*, especially when they have become wet, will often obviate blisters. Stockings should be heavy, made of "natural wool" and should fit tightly enough so that wrinkles will not form. Shoes should have heavy soles and should be made of such tough leather that they will not easily tear. Rubber boots should *not* be worn; they make the feet tender. Although a surveying trip is usually considered as the opportunity to use up discarded clothing, ordinary clothing is usually very unsuitable and quickly becomes unwearable. When camping conditions are rough and the work must last for several months, and possibly years, clothing made of specially suitable material is economical. The material should be tough, so that it will not easily be torn by brambles, etc. It should be waterproof so as to shed rain and yet should be porous. It should be so thoroughy shrunken that moisture cannot appreciably shrink it further. "Mackinaw" is a soft, rough cloth, all wool, thoroughly shrunken, light, warm and waterproof. It is especially suitable for cold weather. "Pontiac" is similar. "Khaki" is a twilled cotton and is especially suitable for warm weather clothing. "Jungle cloth" is somewhat similar, but is particularly noted for its toughness and durability.

Especial care should be taken in the choice of underclothing, so as to avoid sudden chills after becoming overheated. Woolen underclothing is almost essential. "Cholera bands," made of wool, should *always* be worn about the abdomen in tropical countries.

MEDICAL AND SURGICAL TREATMENT.

36. Responsibility of engineer-in-charge. Throughout any surveying trip, where camping is necessary, professional medical aid is usually unobtainable. There rests upon the engineer-in-

charge, as the head of the expedition, some measure of responsibility for the health and care of the party. When some member of the party is seriously injured by accident, bitten by a poisonous snake or insect, or stricken with a sudden and violent attack of disease, and competent medical assistance is absolutely unobtainable for several days or even weeks, the head of the party must choose between seeing the victim die or boldly performing some simple surgical operation or giving medical treatment which he would not dream of doing otherwise. It is the lesser of two evils and the engineer must not shirk his duty. Even though a doctor is *perhaps* obtainable after many days delay by despatching a messenger 50 miles for him, common sense first-aid work and the intelligent use of a few simple methods and remedies may save life or prevent or mitigate permanent disablement. The outfit should include a sufficient supply of the medicines and medical appliances which would most probably be required. All bottles should be carried in cases to prevent breakage and the corks or stoppers secured tightly. When practicable, the drugs should be in tablet form, rather than liquid, and a normal dose should be marked on each bottle or package. They should be doubly labeled and the labels varnished to prevent their coming off in a damp climate. All adhesive plasters, antiseptic gauze, and such appliances, should be kept carefully wrapped up and protected from air and moisture.

37. **Appliances.** The very simplest set of instruments should include a pair of good scissors, which can be made antiseptically clean by wiping off with alcohol; a knife with two razor-sharp blades; a probe; a small saw with detachable handle; a pair of mouse-tooth forceps; silk for ligatures, No. 2 catgut, needles and safety pins. There should be several rolls of sterilized gauze and "Z. O." adhesive plaster. A two-quart fountain syringe should be provided, also a hypodermic syringe and two needles. The engineer should thoroughly familiarize himself with the working and manner of use of this last instrument. Any engineer who is preparing to head an expedition into a region where medical attention is unobtainable should consider that he can very wisely spend time with some doctor friend in learning the elements of the use of all these appliances.

38. **Antiseptics.** The engineer should warn his men of the danger from the infection of even slight wounds and scratches, especially in hot climates. The best emergency treatment for

any scratch, nail gouge, or nail in the foot, is to apply pure tincture of iodine at the *base* of the wound by cotton on the end of a small stick or probe. A few of the many effective antiseptics are here mentioned: **Boric ointment**; one part of powdered boric acid added to nine parts of vaseline. **Carbolic ointment**; one part of carbolic acid to nineteen parts of vaseline. **Chinosol**; a $\frac{1}{1000}$ solution may be used for washing fresh wounds, burns, etc., or as a gargle for sore throat. **Iodoform** powder promotes rapid healing of sores and wounds; one part in eight parts of vaseline is a good healing ointment. **Permanganate of potash**; one grain gives a purple color to a gallon of water; if the water is impure, the purple color changes rapidly to brown and this is a rough test of organic impurity; the crystals are soluble in 20 parts of water; it is especially useful in the treatment of snake bites. In a snake-infested country, it is wise for each man to carry permanganate of potash crystals with him, for use in emergency. See "Snake bites."

39. **Drinking water.** Every chief of party should see to it that his party has a pure supply of drinking water and especially that this supply is not contaminated by excrement from the camp draining into it. If there is any doubt about the purity of the supply (especially if so indicated by the permanganate-of-potash test) it should be part of the duty of the camp cook to maintain a liberal supply of boiled and cooled water. A neglect of such a precaution might easily result in an epidemic of typhoid. In a region where all streams are contaminated, perhaps by decaying vegetation or other natural cause, it may be wise to provide canteens, which the cook should furnish each morning filled with sterilized water.

40. **Bleeding** from an artery or vein can sometimes be stopped by pressing the vessel with sufficient pressure to stop the flow and continuing the pressure until the blood coagulates. If the vein or artery is actually severed but is not too large, the bleeding may be stopped by the use of a pair of forceps; grasp and pinch the vessel and twist it around three or four times. In about ten minutes the forceps may be removed. If the vein or artery is larger, and especially when it is an artery, which may be recognized by spurts of bright red blood, it may be necessary to tie the vessel. This may be done with catgut ligature, which should previously be boiled to prevent infection. While preparing for this, bleeding should be stopped by temporary pres-

sure. This is most easily done when the bleeding vessel may be pressed against a bone. A tourniquet can be improvised for pressing a pad (or even a stone) against the vein or artery of a limb by using a stick and a piece of cloth, or, perhaps, a rope and a small block of wood. Fasten the cloth or rope into a loose loop around the limb and, running the stick through the loop; then twist it so that the pad is pressed down as desired. The rope can be so disposed as to press the block, which in turn presses the pad against the vein or artery.

41. Ailments and diseases; medicines. Colic or cramp. Essence of ginger, 5 to 20 drops, in a small amount of very hot water.

Diarrhœa. Remove the bowel irritant by a castor-oil purge; then, if diarrhœa continues, give 20 drops of chlorodyne and 10 drops of tincture of ginger, in two tablespoonsful of hot water, two or three times per day.

Purgatives. Epsom salts; dose, two teaspoonsful in a small glass of water. **Calomel**; dose, two to five grains; should be followed by **citrate of magnesia**. **Cascara sagrada**; dose, two to six grains. **Castor oil**; dose, one to three tablespoonsful, which may be made more palatable by mixing with an equal amount of glycerine, and then putting the mixture into a glass of lemonade. Any tendency to constipation, which leads to intestinal poisoning and appendicitis may be avoided by using a laxative, made as effective as necessary, about once a week.

Emetics. Common salt (two tablespoonsful), or **mustard** (one tablespoonful) or **Ipecacuanha** (30 grains) or **Zinc sulphate** (30 grains), dissolved in a glass of water. Tickling the throat with a feather may sometimes be effective. Strong "Ivory" soap suds is excellent.

Malaria. Five grains of quinine as a preventive; ten grains, three times a day, as an ordinary maximum dose. Larger doses are often given but it is dangerous unless under the care of a physician.

Cold-in-head. Rhinitis tablets, given as directed on bottle, are effective to break up an incipient cold. "**Dover's powder**"; dose, five to ten grains. Keep patient warm, with hot-water bottles and hot drinks.

42. Drowning; electric shock, asphyxiation. The trouble and the remedy is essentially the same in all three cases; respiration has been temporarily suspended and *must* be promptly restored

by artificial means. Loosen the patient's clothing, especially about the neck. In a drowning case, lay the patient on the ground, face down, straddle him and raise him at the hips so that the water in the air passages will drain out. Remove from the mouth any tobacco, false teeth or anything else that might obstruct breathing. Draw the tongue forward with forceps or a handkerchief. Then lay him face down, but with the face turned to one side so as to facilitate breathing, and with the arms extended forward. Then the operator, kneeling astride the patient, facing his head, and with the hands pressing on the lower ribs, *gradually* presses down so as to expel the air from the lungs. Then he suddenly removes the pressure by swinging back, and thus allows air to enter the lungs. Repeat the movements every four or five seconds, until natural breathing commences. Considering the fact that this method has successfully restored breathing after some *hours* of unsuccessful effort, and also that, in those cases, the patient would have died except for the persistency of the effort, the operator must not be discouraged because his efforts are not immediately successful. Promptness in beginning such treatment is so important that it is better to commence at once (even outdoors) rather than allow any material delay in order to get the patient to a house. The patient should be allowed plenty of air; crowding around him should be avoided. A blanket, extra clothing, hot bricks or stones, or hot-water bags, to restore heat to the body, will be of assistance, provided they do not interfere with the respiration operations. Do not attempt to make the patient swallow anything (e. g., a stimulant), until he is fully conscious; otherwise he will choke.

43. **Fractures.** Obtain medical aid if possible, but if this is unobtainable, except after a delay of many days or weeks, and it is uncertain even then, it may be preferable to take the chances of common-sense treatment, even if unskilled, rather than the certain permanent injury due to neglect of all treatment. Fractures are (a) **simple**, when the skin is not broken; (b) **compound**, when the skin is so broken that the fractured bone is more or less exposed to the air; and (c) **comminuted**, when there are two or more breaks of the same bone; a comminuted fracture may be simple or compound. Great care should be used in handling the patient immediately after the accident so that a simple fracture does not become compound. A broken limb should be

carefully straightened out and bound temporarily with the best improvised splints which are available until the patient can be removed to a bed. Even if amateur bone setting is decided to be advisable, setting should not be attempted if there is excessive swelling or tenderness. Apply ice or evaporating lotions to reduce any swelling. Splints should be made which are of proper length and are so rounded and padded with cloth that they cannot produce any concentrated pressure. Usually the dislocated bones are forced past each other, especially if the fracture is oblique rather than perpendicular, and it is always necessary to use considerable force, especially if it is a broken leg, to pull the bones back into position. The amateur must use his best common sense and knowledge of skeleton anatomy to restore the fragments to the same relative position they had previously, and then to secure them rigidly stiff with splints. Comparison with an unbroken arm or leg will be made even by a skilled surgeon, and such a comparison should be carefully studied by the amateur. While the binding should be as firm as it is safe to make it, it may be so tight as to produce swelling and even ulceration, and then the binding must be loosened. Compound fractures require the care of the flesh and skin wound in addition to the bone setting. The wound should be treated as described for wounds, but the splints and binding should be designed so that the wound can be properly dressed without loosening the splints. If the broken bone protrudes through the wound, it *must* be drawn back so that the wound can heal externally, even though the bone setting is beyond the skill of the amateur surgeon. Setting usually requires about six weeks, but, in the case of a limb, the joints above and below the break should be very carefully moved after about three weeks, so as to avoid stiff joints, special care being take that there is no strain on the healing bone.

44. Snake or insect bites. The majority of snake bites occur on the limbs. In such a case (1) **tie a cord or bandage** about the limb just above the wound as promptly as possible, so as to prevent the poisoned blood from getting into the system; (2) **cut into the wound** so as to induce free bleeding; (3) **suck the wound** to aid in drawing out the poisoned blood; there is little or no danger in this, provided the mouth is free from sores, and provided the mouth is immediately rinsed out, preferably with an antiseptic solution, such as a light purple solution of per-

manganate of potash; (4) **inject into the wound** a strong solution of **permanganate of potash,** which may be done hypodermically or, perhaps, even by rubbing into the wound crystals of the drug. When the case is very serious, on account of the known deadly character of the poison, and when no permanganate of potash is obtainable, heroic measures are sometimes necessary. Pure **carbolic acid,** or **caustic,** may be used, if available. Cauterizing the wound with white-hot iron, exploding a pinch of gunpowder over the wound, shooting away the infected part with a gun, or even summary amputation with a hatchet, may sometimes be considered the lesser of two evils. If the limb has been tightly tied, it will, of course, produce great pain, discoloration and swelling, which must not be continued too long. A second ligature should be tied a few inches above the first. When the limb becomes very swelled and painful, loosen the first ligature for about ten seconds and again tighten, and then loosen the second ligature for ten seconds and again tighten. After fifteen minutes, repeat the loosening and tightening. After about eight repetitions, the ligatures may be removed altogether. If the poison is partly sucked out, the remainder partly neutralized with chemicals, and does not get fully into the system for two hours, the danger is greatly diminished. Of course bites on the face or body cannot be tied up and can only be treated by sucking out the poison and by chemicals. Stimulation of the heart is usually essential, which may be done with one teaspoonful of **aromatic spirits of ammonia** in two tablespoonsful of water, or with alcoholic liquor, preferably whiskey. One 1-30th grain **strychnine** tablets, dissolved in two tablespoonsful of water, is also a stimulant. If a hypodermic is available, one tablet may be dissolved in thirty drops of sterile water and inserted in the back or arm, well under the skin.

45. **Wounds.** First, last and all the time, **prevent infection.** The marvelous success of modern surgery is due largely to antiseptic methods. Neglect of cleanliness almost inevitably induces blood poisoning. A perfectly clean cut, after being washed and sterilized with iodine, may be closed with adhesive plaster, taking stitches, if necessary, with sterilized catgut or silk or linen thread. The stitches may be removed in a week. But when the flesh is torn and, especially, when dirt and other matter, which is possibly poisonous or infectious, has been forced into the wound, there is great danger of blood poisoning, and

the wound must be cleansed. First, cover the wound itself with a pad which has been soaked in an antiseptic solution and then wash the skin (shaving off all hair), all around the wound, using first soap and then an antiseptic solution. Then cleanse out all foreign matter from the wound, using antiseptics and pack the wound with strip gauze, soaked in the antiseptic, so as to extend from the deepest part of it to the outside. This will drain the discharges. The dressing should be renewed every day until the wound shows a tendency to heal. A gaping torn wound should not be sewed up, except to bring the edges together temporarily.

CHAPTER II.

ALINEMENT

IN this chapter the alinement of the *center line* only of a pair of rails is considered. When a railroad is crossing a summit in the grade line, although the horizontal projection of the alinement may be straight, the vertical projection will consist of two sloping lines joined by a curve. When a curve is on a grade, the center line is really a spiral, a curve of double curvature, although its horizontal projection is a circle. The center line therefore consists of straight lines and curves of single and double curvature. The simplest method of treating them is to consider their horizontal and vertical projections separately. In treating simple, compound, and transition curves, only the horizontal projections of those curves will be considered.

SIMPLE CURVES.

46. Designation of curves. A curve may be designated either by its radius or by the angle subtended by a chord of unit length. Such an angle is known as the "degree of curve" and is indicated by D. Since the curves that are practically used have very long radii, it is generally impracticable to make any use of the actual center, and the curve is located without reference to it. If AB in Fig. 11 represents a unit chord (C) of a curve of radius R, then by the above definition the angle AOB equals D. Then

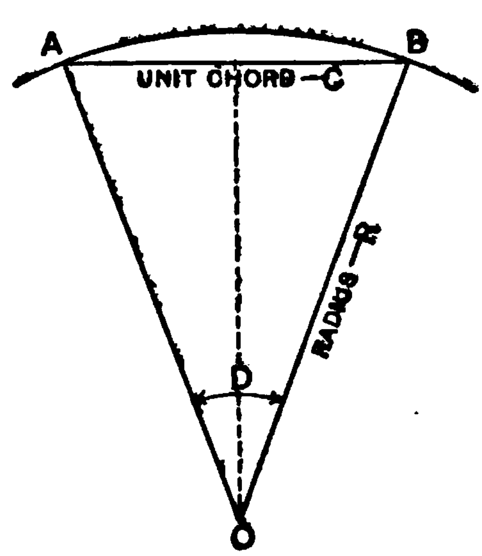


FIG. 11.

$$AO \sin \tfrac{1}{2}D = \tfrac{1}{2}AB = \tfrac{1}{2}C.$$

$$\therefore \quad R = \frac{\tfrac{1}{2}C}{\sin \tfrac{1}{2}D}, \qquad \qquad (1)$$

or, by inversion,

$$\sin \tfrac{1}{2}D = \frac{C}{2R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The unit chord is variously taken throughout the world as 100 feet, 66 feet, and 20 meters. In the United States 100 feet is invariably used as the unit chord length, and throughout this work it will be so considered. Table I has been computed on this basis. It gives the radius, with its logarithm, of all curves from a 0° 01′ curve up to a 10° curve, varying by single minutes. The sharper curves, which are seldom used, are given with larger intervals.

An approximate value of R may be readily found from the following simple rule, which should be memorized:

$$R = \frac{5730}{D}.$$

Although such values are not mathematically correct, since R does not strictly vary inversely as D, yet the resulting value is within a tenth of one per cent for all commonly used values of R, and is sufficiently close for many purposes, as will be shown later.

47. Metric Curves. The unit chord for railroad curves on the metric system is 20 meters. If a curve has a 100-foot chord and a central angle of 5°, the radius would, of course, be 1146.3 feet. Since 20 meters $=65.6174$ feet, a 20-meter chord between those same radial lines would subtend an arc with a radius of $.656174 \times 1146.3$ feet, or 752.16 feet. But this radius, measured in meters, would be $(.656174 \times 1146.3) \div 3.28087 = 229.26$ meters, which is $1146.3 \times .20$. In other words, the radius of any metric curve, measured in meters, is *numerically* one-fifth of the radius, measured in feet, of the same degree curve, but in actual length is a little less than two-thirds. This practically means that a 10° curve, metric, is actually very much sharper than a 10° curve, using foot-measure, or that the radius is about 66% as much. Therefore, in selecting curves for location, an engineer, who is accustomed to the foot-measure system, should remember that a 10° curve metric, for example, has approximately the same radius as a 15° curve, using foot-measure. While it is more convenient for an engineer, who is constantly using the metric system for curves, to have tables computed directly on

this basis, an engineer need not be dependent on such tables, since it is only necessary to divide the tabular quantities in the foot-table by 5 to obtain the corresponding quantities for the metric system. This applies not only to radii, but also to tangents, external distances and long chords for a 1° curve. A desired logarithm may be obtained by subtracting 0.6989700 from the foot-table logarithm.

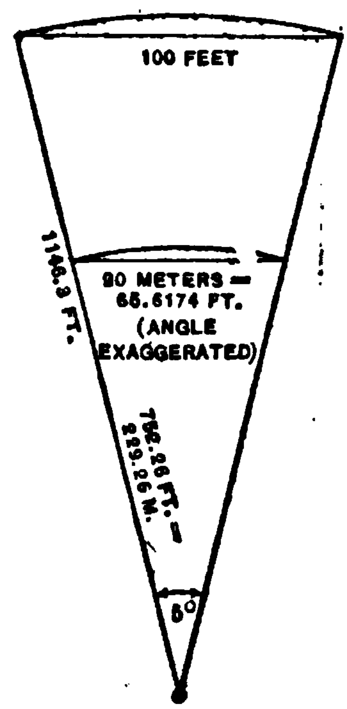


FIG. 12.

For example, anticipating the explanation in Art. 53, what is the tangent distance of a 6° metric curve, when the central angle is 32° 40′. From Table II, we find that by the foot-system the tangent distance for a 1° curve when the central angle is 32° 40′ is 1679.1 feet; then for a 6° curve it is $1679.1 \div 6 = 279.85$ feet; for a 6° metric curve it is $279.85 \div 5 = 55.97$ meters. The radius of the 6° metric curve $= 955.37 \div 5 = 191.074$ meters, which is in actual length about 66% of 955.37 feet.

As another illustration of the transformation from the foot-system to the metric system, or vice versa, the degree of a curve, by the foot system, may be multiplied by .66 and obtain approximately the degree of the equivalent curve by the metric system. For example, a 6° curve, foot system, has about the same actual radius as a $6 \times .66 = 3.96°$ metric curve, or about a 4° curve.

48. Length of a subchord. Since it is impracticable to measure along a curved arc, curves are always measured by laying off 100-foot chord lengths. This means that the actual arc is always a little longer than the chord. It also means that a *subchord* (a chord shorter than the unit length), will be a little longer than the ratio of the angles subtended would call for. The truth of this may be seen without calculation by noting that two equal subchords, each subtending the angle $\frac{1}{2}D$, will evidently be slightly longer than 50 feet each. If c be the length of a subchord subtending the angle d, then, as in Eq. 2,

$$\sin \tfrac{1}{2}d = \frac{c}{2R},$$

or, by inversion,
$$c = 2R \sin \tfrac{1}{2}d. \quad \ldots \ldots \ldots \quad (3)$$

The *nominal length* of a subchord $=100\frac{d}{D}$. For example, a *nominal* subchord of 40 feet will subtend an angle of $\frac{40}{100}$ of D^c; its *true length* will be slightly more than 40 feet, and may be computed by Eq. 3. The *difference* between the nominal and true lengths is maximum when the subchord is about 57 feet long, but with the low degrees of curvature ordinarily used the difference may be neglected. With a 10° curve and a nominal chord length of 60 feet, the true length is 60.049 feet. Very sharp curves should be laid off with 50-foot or even 25-foot chords (nominal length). In such cases especially the true lengths of these subchords should be computed and used instead of the nominal lengths.

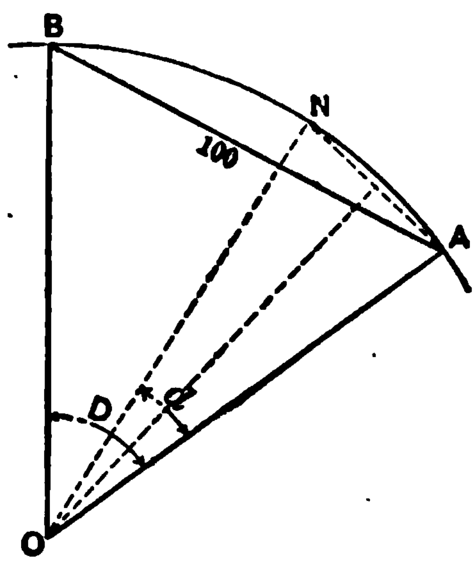


FIG. 13.

For example, assume that a 12° curve begins at Sta. 26+30. The first subchord will be nominally 70 feet and actually 70.066 feet. Assume that the central angle between the tangents is 39° 36′. Then the nominal length of curve is $39.6° \div 12° = 3.30$ stations. $3.30 - .70 = 2.60$, the nominal length of curve beyond the first station point on the curve. The final subchord is nominally 60 feet, but its actual length is 60.070 feet.

The values of these subchords for even degrees between 5° and 30°, and for nominal chord lengths of 10, 20, 30, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90 and 95 feet, are given in Table IIa. The excess values increase approximately as the square of the degree of curvature, but for intervals of 1° simple interpolation will be sufficiently accurate for intermediate values.

49. Length of a curve. The actual mean length of the two rails will be more than the nominal length of the curve, as defined above, and even more than the sum of the full 100-foot lengths and the true lengths of the subchord lengths at the ends. In the above numerical case the mean rail length is

$$39.6° \times \frac{\pi}{180°} \times R = 39.6° \times \frac{\pi}{180°} \times 478.34 = 330.604.$$

The sum of the two full-chord lengths and the two subchords is $70.066+200+60.070=330.136$. A large part of the excess $(330.604-330.136=.468)$ is the excess length (.183) of each arc of a 12° curve over the 100-foot chord. The remainder is the excess of the 70-foot and 60-foot arcs over the true chord lengths. But this excess length is of little practical importance. In the above case (a 12° curve) it adds about 0.2% to the length of rail that must be bought. The excess varies approximately as the square of the degree of curvature. The percentage of excess for the entire length of a road is utterly insignificant and is swallowed up by the 2% excess which is usually allowed for wastage in rail cutting.

50. **Curve notation.** The notation adopted by the Amer. Rwy. Eng. Assoc. indicates any point where there is a change of alinement by two letters, the first of which denotes the alinement on the side toward station zero and the second that away from station zero. Thus, the beginning of a curve, or the change from a tangent to a simple curve, is noted as *TC*; the other end of the curve, or the change from a simple curve to a tangent is noted as *CT*. But, since the use of two letters to indicate a point, or the use of four letters to indicate a line joining the two points, is cumbersome in the algebraic solutions and demonstrations which follow (demonstrations which the A. R. E. A. do not give), the author has decided to retain the old notation, rather than to try to conform to the A. R. E. A. notation. The A. R. E. A. system also indicates the central angle of a curve, or the angle between the two tangents, by *I*. In the first edition of this work, the author, following Searles, indicated the central angle by Δ. To make even this change, for the sake of conformity, would require a change in all the mathematical work and figures involving curves throughout the book. In Fig. 14 both notations are given, the A. R. E. A. notations being given in parentheses. Both notations are also shown in Fig. 36, which illustrates a transition curve or spiral. It should be noted that some of the notations coincide for some of the elements.

51. **Elements of a curve.** Considering the line as running from *A* toward *B*, the beginning of the curve, at *A*, is called the *point of curve* (*PC*). The other end of the *curve*, at *B*, is called the *point of tangency* (*PT*). The intersection of the tangents is called the *vertex* (*V*). The angle made by the

tangents at V, which equals the angle made by the radii to the extremities of the curve, is called the *central angle* (Δ). AV and BV, the two equal tangents from the vertex to the PC and PT, are called the *tangent distances* (T). The chord AB is called the *long chord* (LC). The intercept HG from the middle of the long chord to the middle of the arc is called the *middle ordinate* (M). That part of the secant GV from

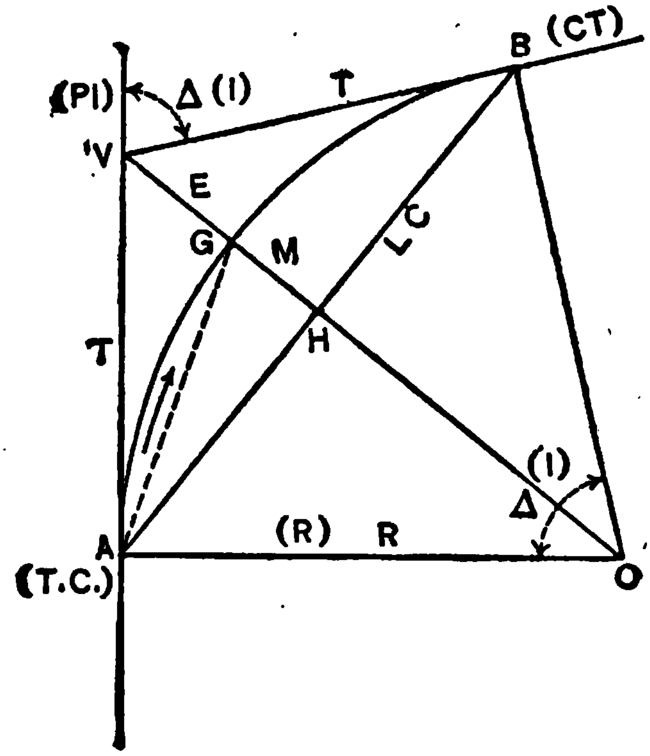


FIG. 14.

the middle of the arc to the vertex is called the *external distance* (E). From the figure it is very easy to derive the following frequently used relations:

$$T = R \tan \tfrac{1}{2}\Delta. \quad \dots \dots \dots \quad (4)$$

$$LC = 2R \sin \tfrac{1}{2}\Delta. \quad \dots \dots \dots \quad (5)$$

$$M = R \operatorname{vers} \tfrac{1}{2}\Delta. \quad \dots \dots \dots \quad (6)$$

$$E = R \operatorname{exsec} \tfrac{1}{2}\Delta. \quad \dots \dots \dots \quad (7)$$

52. Relation between T, E, and Δ. Join A and G in Fig. 14. The angle $VAG = \tfrac{1}{4}\Delta$, since it is measured by one half of the arc AG between the secant and tangent.

$$AGO = 90° - \tfrac{1}{4}\Delta.$$

$$AV : VG :: \sin AGV : \sin VAG;$$

$$\sin AGV = \sin AGO = \cos \tfrac{1}{2}\Delta;$$

$$T : E :: \cos \tfrac{1}{2}\Delta : \sin \tfrac{1}{2}\Delta;$$

$$T = E \cot \tfrac{1}{2}\Delta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The same relation may be obtained by dividing Eq. 4 by Eq. 7, since $\tan a \div \text{exsec } a = \cot \tfrac{1}{2}a$.

53. **Elements of a 1° curve.** From Eqs. 1 to 8 it is seen that the elements of a curve vary directly as R. It is also seen to be very nearly true that R varies inversely as D. If the elements of a 1° curve for various central angles are calculated and tabulated, the elements of a curve of $D°$ curvature may be approximately found by dividing by D the corresponding elements of a 1° curve having the same central angle. For small central angles and low degrees of curvature the errors involved by the approximation are insignificant, and even for larger angles the errors are so small that *for many purposes* they may be disregarded.

In Table II is given the value of the tangent distances, external distances, and long chords for a 1° curve for various central angles The student should familiarize himself with the degree of approximation involved by solving a large number of cases under various conditions by the exact and by the approximate methods, in order that he may know when the approximate method is sufficiently exact for the intended purpose. The approximate method also gives a ready check on the exact method.

A closer value may be obtained by using the "Corrective Table" found at the end of the main table. The correction is *always* additive and is usually very small and often even too insignificant for attention. A glance at the corrective table will show whether a correction need be made and an easily computed interpolation will show its amount. For example, what is the tangent distance for a 6° curve having a central angle of 42° 15′? Interpolating between 2209.0 and 2218.6, we have 2213.8 as the tangent distance for a 1° curve. Dividing by 6, we have 368.97 as the approximate tangent distance. Interpolating in the corrective table, we have .14 as the correction for a 5° curve and a

central angle of 42° 15′, and .28 as the correction for a 10° curve. Interpolating for 6° between these values of .14 and .28, we have .17, which added to 368.97 equals 369.14. The precise value, computed from Eq. 4, is 369.12. If the approximate value, even after correction, is not considered sufficiently accurate, Eq. 4 should be used. The student should appreciate that the discrepancy of even .02 in the above calculation is not due to any real error in the main table or the corrective table, but is due to the fact that the tangent distances are only computed to the nearest tenth of a foot for values over 1000 feet, and this will produce such discrepancies. The table should not be used where precise values are required.

54. Exercises. (*a*) What is the tangent distance of a 4° 20′ curve having a central angle of 18° 24′?

(*b*) Given a 3° 30′ curve and a central angle of 16° 20′, how far will the curve pass from the vertex? [Use Eq. 7.]

(*c*) An 18° curve is to be laid off using 25-foot (nominal) chord lengths. What is the true length of the subchords?

(*d*) Given two tangents making a central angle of 15° 24′. It is desired to connect these tangents by a curve which shall pass 16.2 feet from their intersection. How far down the tangent will the curve begin and what will be its radius? (Use Eq. 8 and then use Eq. 4 inverted.)

55. Curve location by deflections. The angle between a secant and a tangent (or between two secants intersecting on an arc) is measured by one half of the intercepted arc. Beginning at the *PC* (*A* in Fig. 15), if the first chord is to be a full chord we may deflect an angle *VAa* ($=\frac{1}{2}D$), and the point *a*, which is 100 feet from *A*, is a point on the curve. For the next station, *b*, deflect an *additional* angle *bAa* ($=\frac{1}{2}D$) and, with one end of the tape at *a*, swing the other end until the 100-foot point is on the line *Ab*. The point *b* is then on the curve. If the final chord *cB* is a subchord, its *additional deflection* ($\frac{1}{2}d$) is something less than $\frac{1}{2}D$. The last *deflection* (*BAV*) is

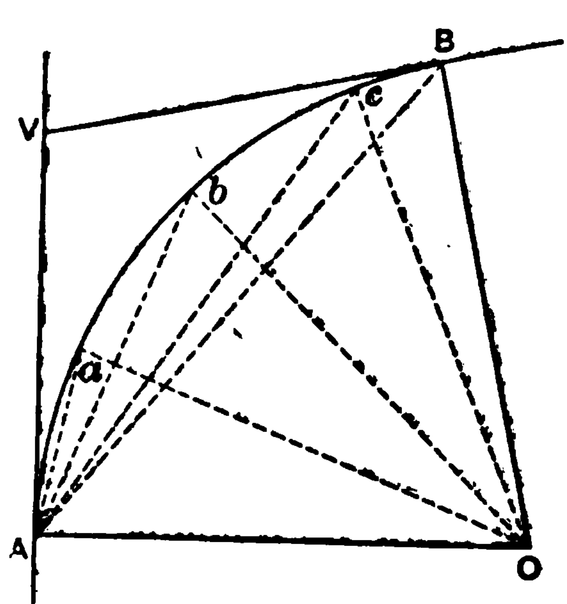


FIG. 15.

of course $\frac{1}{2}\Delta$. It is particularly important, when a curve begins or ends with a subchord and the deflections are odd quantities, that the last additional deflection should be carefully computed and added to the previous deflection, to check the mathematical work by the agreement of this last computed deflection with $\frac{1}{2}\Delta$.

Example. Given a 3° 24′ curve having a central angle of 18° 22′ and beginning at sta. 47+32, to compute the deflections. The nominal length of curve is 18° 22′ ÷ 3° 24′ = 18.367 ÷ 3.40 = 5.402 stations or 540.2 feet. The curve therefore ends at sta. 52+72.2. The deflection for sta. 48 is $\frac{68}{100} \times \frac{1}{2}(3° 24')$ $= 0.68 \times 1°.7 = 1°.156 = 1° 09'$ nearly. For each additional 100 feet it is 1° 42′ additional. The final additional deflection for the final subchord of 72.2 feet is

$$\frac{72.2}{100} \times \tfrac{1}{2}(3° 24') = 1°.2274 = 1° 14' \text{ nearly.}$$

The deflections are

P. C	Sta. 47+32		0°
	48	0° + 1° 09′	= 1° 09′
	49	1° 09′ + 1° 42′	= 2° 51′
	50	2° 51′ + 1° 42′	= 4° 33′
	51	4° 33′ + 1° 42′	= 6° 15′
	52	6° 15′ + 1° 42′	= 7° 57′
P. T.	52+72.2	7° 57′ + 1° 14′	= 9° 11′

As a check 9° 11′ = $\frac{1}{2}$(18° 22′) = $\frac{1}{2}\Delta$. (See the Form of Notes in § 21.)

56. Instrumental work. It is generally impracticable to locate more than 500 to 600 feet of a curve from one station. Obstructions will sometimes require that the transit be moved up every 200 or 300 feet. There are two methods of setting off the angles when the transit has been moved up from the *PC*.

(a) The transit may be sighted at the previous transit station with a reading on the plates equal to the deflection angle from that station to the station occupied, but with the angle set off on the *other* side of 0°, so that when the telescope is turned to 0° it will sight along the tangent at the station occupied. Plunging the telescope, the forward stations may be set off by deflecting the proper deflections from the tangent at the station occupied

This is a very common method and, when the degree of curvature is an even number of degrees and when the transit is only set at even stations, there is but little objection to it. But the degree of curvature is sometimes an odd quantity, and the exigencies of difficult location frequently require that substations be occupied as transit stations. Method (*a*) will then require the recalculation of all deflections for each new station occupied. The mathematical work is largely increased and the probability of error is very greatly increased and not so easily detected. Method (*b*) is just as simple as method (*a*) even for the most simple cases, and for the more difficult cases just referred to the superiority is very great.

(**b**) Calculate the deflection for each station and substation throughout the curve as though the whole curve were to be located from the *PC*. The computations may thus be completed and *checked* (as above) before beginning the instrumental work. If it unexpectedly becomes necessary to introduce a substation at any point, its deflection from the *PC* may be readily interpolated. The stations actually set from the *PC* are located as usual. RULE. **When the transit is set on any forward station, backsight to ANY previous station with the plates set at the deflection angle for the station sighted at. Plunge the telescope and sight at any forward station with the deflection angle originally computed for that station.** When the plates read the deflection angle for the station occupied, the telescope is sighting along the tangent at that station—which is the method of getting the forward tangent when occupying the *PT*. Even though the station occupied is an unexpected substation, when the instrument is properly oriented at that station, the angle reading for *any* station, forward or back, is that originally computed for it from the *PC*. In difficult work, where there are obstructions, a valuable check on the accuracy may be found by sighting backward at *any* visible station and noting whether

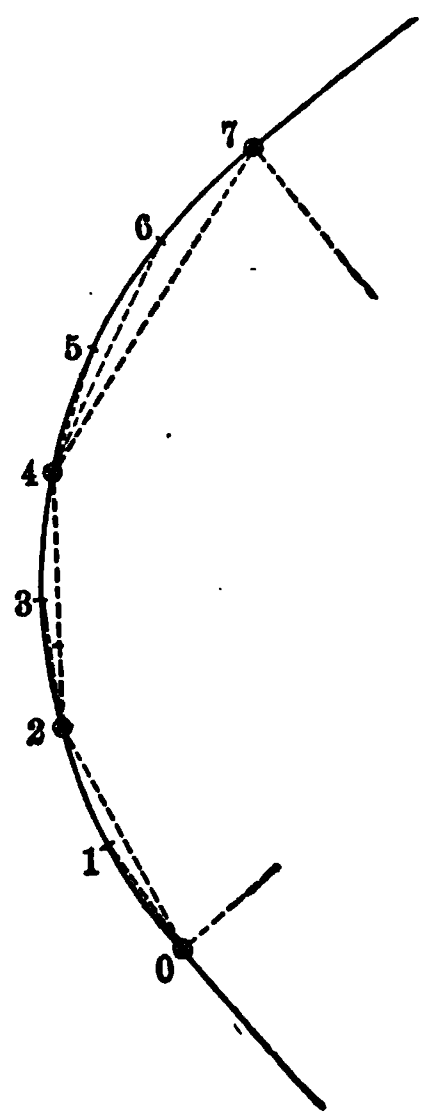


FIG. 16.

its deflection agrees with that originally computed. As a numerical illustration, assume a 4° curve, with 28° curvature, with stations 0, 2, 4, and 7 occupied. After setting stations 1 and 2, set up the transit at sta. 2 and backsight to sta. 0 with the deflection for sta. 0, which is 0°. The reading on sta. 1 is 2°; when the reading is 4° the telescope is tangent to the curve, and when sighting at 3 and 4 the deflections will be 6° and 8°. Occupy 4; sight to 2 with a reading of 4°. When the reading is 8° the telescope is tangent to the curve and, by plunging the telescope, 5, 6, and 7 may be located with the originally computed deflections of 10°, 12°, and 14°. When occupying 7 a backsight may be taken to any visible station with the plates reading the deflection for that station; then when

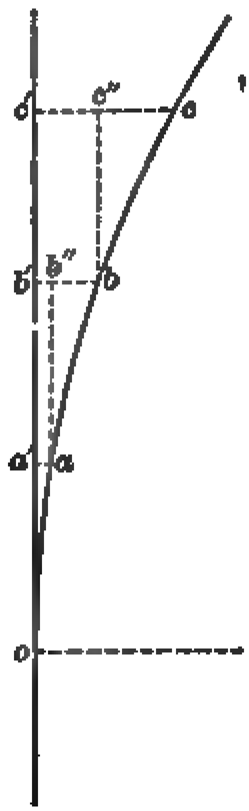


FIG. 17. FIG. 18.

the plates read 14° the telescope will point along the forward tangent.

The location of curves by deflection angles is the normal method. A few other methods, to be described, should be considered as exceptional.

57. Curve location by two transits. A curve might be located more or less on a swamp where accurate chaining would be exceedingly difficult if not impossible. The long chord AB (Fig. 17) may be determined by triangulation or otherwise, and the elements of the curve computed, including (possibly) subchords at each end. The deflection from A and B to each point may be computed. A rodman may then be sent (by whatever means) to locate long stakes at points determined by the simultaneous sightings of the two transits.

58. Curve location by tangential offsets. When a curve is very flat and no transit is at hand the following method may be used (see Fig. 18): Produce the back tangent as far forward as necessary. Compute the ordinates Oa', Ob', Oc', etc., and the abscissæ $a'a$, $b'b$, $c'c$, etc. If Oa is a full station (100 feet), then

$$\left.\begin{aligned}
Oa' &= Oa' &&= 100\cos\tfrac{1}{2}D,\ \text{also} = R\sin D;\\
Ob' &= Oa' + a'b' &&= 100\cos\tfrac{1}{2}D + 100\cos\tfrac{3}{2}D,\\
&&&\qquad \text{also} = R\sin 2D;\\
Oc' &= Oa' + a'b' + b'c' &&= 100(\cos\tfrac{1}{2}D + \cos\tfrac{3}{2}D + \cos\tfrac{5}{2}D),\\
&&&\qquad \text{also} = R\sin 3D;
\end{aligned}\right\}\quad (9)$$

etc.

$$\left.\begin{aligned}
a'a &= &&100\sin\tfrac{1}{2}D,\ \text{also} = R\operatorname{vers} D;\\
b'b &= a'a + b''b &&= 100\sin\tfrac{1}{2}D + 100\sin\tfrac{3}{2}D,\\
&&&\qquad \text{also} = R\operatorname{vers} 2D;\\
c'c &= b'b + c''c &&= 100(\sin\tfrac{1}{2}D + \sin\tfrac{3}{2}D + \sin\tfrac{5}{2}D),\\
&&&\qquad \text{also} = R\operatorname{vers} 3D;
\end{aligned}\right\}\quad (10)$$

etc.

The functions $\frac{1}{2}D$, $\frac{3}{2}D$, etc., may be more conveniently used *without* logarithms, by adding the several *natural* trigonometrical functions and pointing off two decimal places. It may also be noted that Ob' (for example) is one half of the long chord for four stations; also that $b'b$ is the middle ordinate for four stations. If the engineer is provided with tables giving the long chords and middle ordinates for various degrees of curvature, these quantities may be taken (perhaps by interpolation) from such tables.

If the curve begins or ends at a substation, the angles and terms will be correspondingly altered. The modifications may

be readily deduced on the same principles as above, and should be worked out as an exercise by the student.

In Table II are given the long chords for a 1° curve for various values of Δ. Dividing the value as given by the degree of the curve, we have an approximate value which is amply close for low degrees of curvature, especially for laying out curves without a transit. For example, given a 4° 30′ curve, required the ordinate Oc'. This is evidently one half of a chord of six stations, with $\Delta=27°$. Dividing 2675.1 (which is the long chord of a 1° curve with $\Delta=27°$) by 4.5 we have 594.47; one half of this is the required ordinate, $Oc'=297.23$. The exact value is 297.31, an excess of .08, or less than .03 of 1%. The true values are always slightly in excess of the value as computed from Table II.

Exercise. A 3° 40′ curve begins at sta. 18+70 and runs to sta. 23+60. Required the tangential offsets and their corresponding ordinates. The first ordinate $=30 \cos \frac{1}{2}(\frac{30}{100}\times 3° 40') =$ $30\times .99995=29.9985$; the offset $=30 \sin 0° 33'=30\times .0096=$ 0.288. For the second full station (sta. 20) the ordinate $=$ $\frac{1}{2}$ long chord for $\Delta=2(1° 06'+3° 40')$ with $D=3° 40'$. Dividing 476.12, from Table II, by $3\frac{2}{3}$, we have 129.85. Otherwise, by Eq. 9, the ordinate $=30\times \cos 0° 33'+100 \cos (1° 06'+1° 50')$ $=30.00+99.87=129.87$. The offset for sta: 20 $=30 \sin 0° 33'+$ $100 \sin (1° 06'+1° 50')=0.288+5.12=5.41$. Work out similarly the ordinates and offsets for sta. 21, 22, 23, and 23+60.

59. Curve location by middle ordinates. Take first the simpler case when the curve begins at an even station. If we consider (in Fig. 14) the curve produced back to z, the chord $za=$ $2\times 100 \cos \frac{1}{2}D$, $A'a=100 \cos \frac{1}{2}D$, and $A'A=am=zn=100 \sin \frac{1}{2}D$. Set off AA' perpendicular to the tangent and $A'a$ parallel to the tangent. $AA'=aa'=bb'=cc'$, etc. $=100 \sin \frac{1}{2}D$. Set off aa' perpendicular to $a'A$. Produce Aa' until $a'b=A'a$, thus determining b. Succeeding points of the curve may thus be determined indefinitely.

Suppose the curve begins with a subchord. As before $ra=Am'=c' \cos \frac{1}{2}d'$, and $rA=am'=c' \sin \frac{1}{2}d'$. Also $sz=An'=$ $c'' \cos \frac{1}{2}d''$, and $sA=zn'=c'' \sin \frac{1}{2}d''$, in which $(d'+d'')=D$. The points z and a being determined on the ground, aa' may be computed and set off as before and the curve continued in

full stations. A subchord at the end of the curve may be located by a similar process.

60. Curve location by offsets from the long chord. (Fig 21.) Consider at once the general case in which the curve commences with a subchord (curvature, d'), continues with one or more full

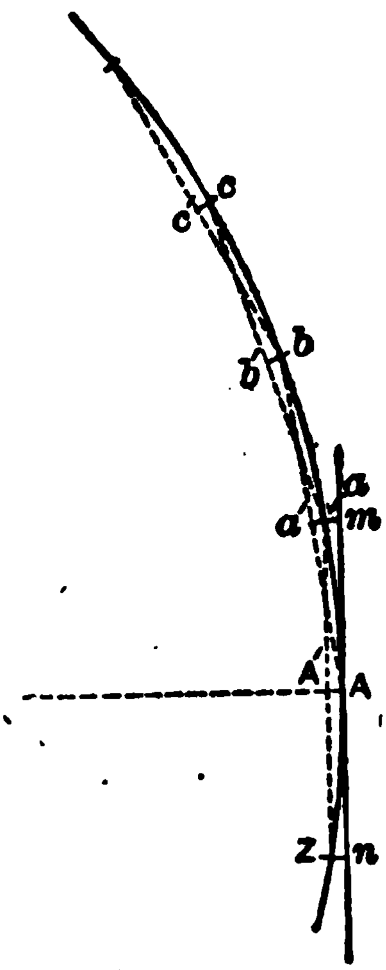


FIG. 19.

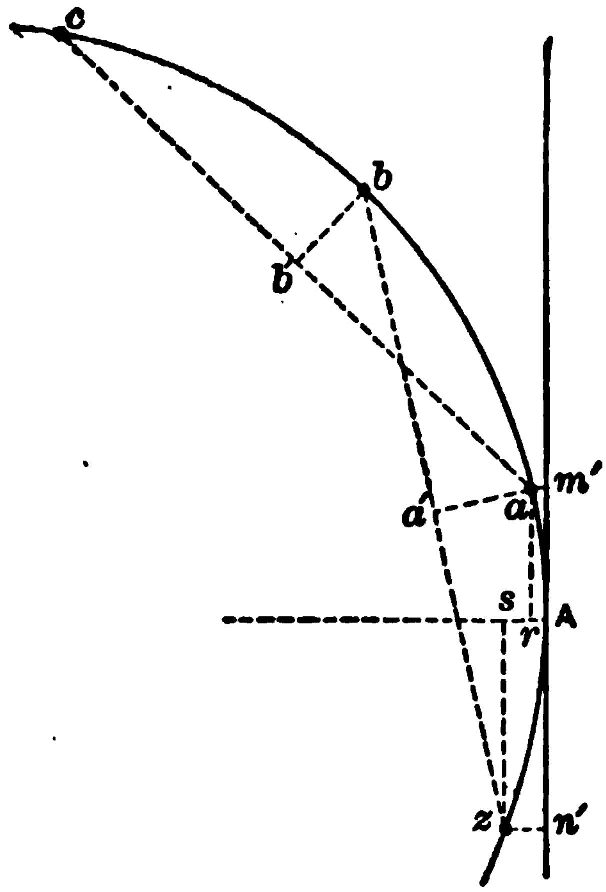


FIG. 20.

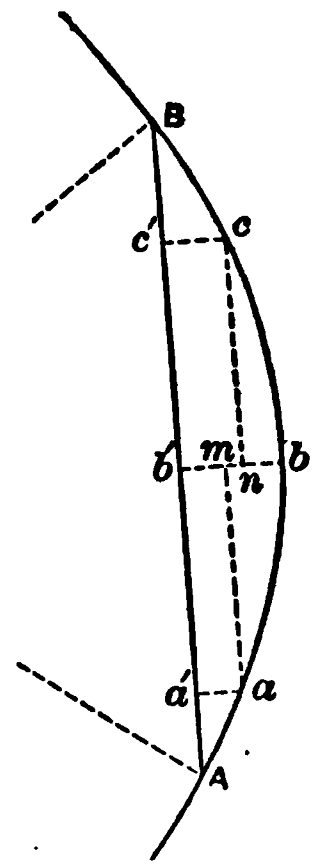


FIG. 21.

chords (curvature of each, D), and ends with a subchord with curvature d''. The numerical work consists in computing first AB, then the various abscissæ and ordinates. $AB = 2R \sin \frac{1}{2}\Delta$.

$$\left.\begin{aligned}
Aa' &= Aa' &&= c' \cos \tfrac{1}{2}(\Delta - d');\\
Ab' &= Aa' + a'b' &&= c' \cos \tfrac{1}{2}(\Delta - d') + 100 \cos \tfrac{1}{2}(\Delta - 2d' - D);\\
Ac' &= Aa' + a'b' + b'c' &&= c' \cos \tfrac{1}{2}(\Delta - d') + 100 \cos \tfrac{1}{2}(\Delta - 2d' - D)\\
&&&\qquad + 100 \cos \tfrac{1}{2}(\Delta - 2d'' - D);\\
&\text{also}\\
&= AB - Bc' &&= 2R \sin \tfrac{1}{2}\Delta - c'' \cos \tfrac{1}{2}(\Delta - d'').
\end{aligned}\right\} \quad (11)$$

$$\left.\begin{aligned}
a'a &= a'a &&= c' \sin \tfrac{1}{2}(\Delta - d');\\
b'b &= a'a + mb &&= c' \sin \tfrac{1}{2}(\Delta - d') + 100 \sin \tfrac{1}{2}(\Delta - 2d' - D);\\
c'c &= b'b - nb &&= c' \sin \tfrac{1}{2}(\Delta - d') + 100 \sin \tfrac{1}{2}(\Delta - 2d' - D)\\
&&&\qquad - 100 \sin \tfrac{1}{2}(\Delta - 2d'' - D);\\
&\text{also} &&= c'' \sin \tfrac{1}{2}(\Delta - d'').
\end{aligned}\right\} \quad (12)$$

The above formulæ are considerably simplified when the

curve begins and ends at even stations. When the curve is very long a regular law becomes very apparent in the formation of all terms between the first and last. There are too few terms in the above equations to show the law.

61. Use and value of the above methods. The chief value of the above methods lies in the possibility of doing the work without a transit. The same principles are sometimes employed, even when a transit is used, when obstacles prevent the use of the normal method (see § 62, *c*). If the terminal tangents have already been accurately determined, these methods are useful to locate points of the curve when rigid accuracy is not essential. Track foremen frequently use such methods to lay out unimportant sidings, especially when the engineer and his transit are not at hand. Location by tangential offsets (or by offsets from the long chord) is to be preferred when the curve is flat (i.e., has a small central angle Δ) and there is no obstruction along the tangent, or long chord. Location by middle ordinates may be employed regardless of the length of the curve, and in cases when both the tangents and the long chord are obstructed. The above methods are but samples of a large number of similar methods which have been devised. The choice of the particular method to be adopted must be determined by the local conditions.

62. Obstacles to location. In this section will be given only a few of the principles involved in this class of problems, with illustrations. The engineer must decide, in each case, which is the best method to use. It is frequently advisable to devise a special solution for some particular case.

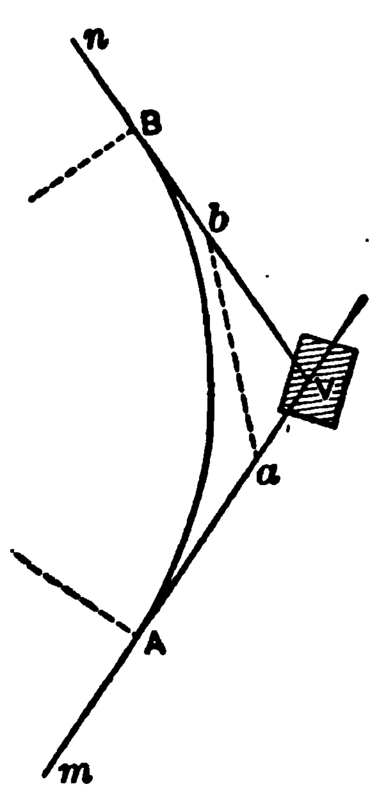


FIG. 22.

a. When the vertex is inaccessible. As shown in § 56, it is not absolutely essential that the vertex of a curve should be located on the ground. But it is very evident that the angle between the terminal tangents is determined with far less probable error if it is measured by a single measurement at the vertex rather than as the result of numerous angle measurements along the curve, involving several positions of the transit and comparatively short sights. Some-

times the location of the tangents is already determined on the ground (as by bn and am, Fig. 22), and it is required to join the tangents by a curve of given radius. *Method.* Measure ab and the angles Vba and baV. Δ is the sum of these angles. The distances bV and aV are computable from the above data. Given Δ and R, the tangent distances are computable, and then Bb and aA are found by subtracting bV and aV from the tangent distances. The curve may then be run from A, and the work may be checked by noting whether the curve as run ends at B—previously located from b.

Example. Assume $ab=546.82$; angle $a=15° 18'$; angle $b=18° 22'$; $D=3° 40'$; required aA and bB.

$\Delta=15° 18'+18° 22'=33° 40'$

Eq. (4)	R (3° 40′)	3.19392
	$\tan \frac{1}{2}\Delta=\tan 16° 50'$	9.48080
	$T=472.85$	2.67472
$aV=ab\,\dfrac{\sin 18° 22'}{\sin 33° 40'}$	ab	2.73784
	log sin 18° 22′	9.49844
	co-log sin 33° 40′	0.25621
	$aV=310.81$	2.49250
	$AV=472.85$	
	$aA=162.04$	
$bV=ab\,\dfrac{\sin 15° 18'}{\sin 33° 40'}$	ab	2.73784
	log sin 15° 18′	9.42139
	co-log sin 33° 40′	0.25621
	$bV=260.29$	2.41545
	$BV=472.85$	
	$bB=212.56$	

b. When the point of curve (or point of tangency) is inaccessible. At some distance (As, Fig. 23) an unobstructed line pn may be run parallel with AV. $nv=py=As=R$ vers a.

$$\therefore \text{vers } a=As\div R.$$

$$ns=ps=R\sin a.$$

At y, which is at a distance ps back from the *computed* position of A, make an offset sA to p. Run pn parallel to the tangent. A tangent to the curve at n makes an angle of a with np. From n the curve is run in as usual

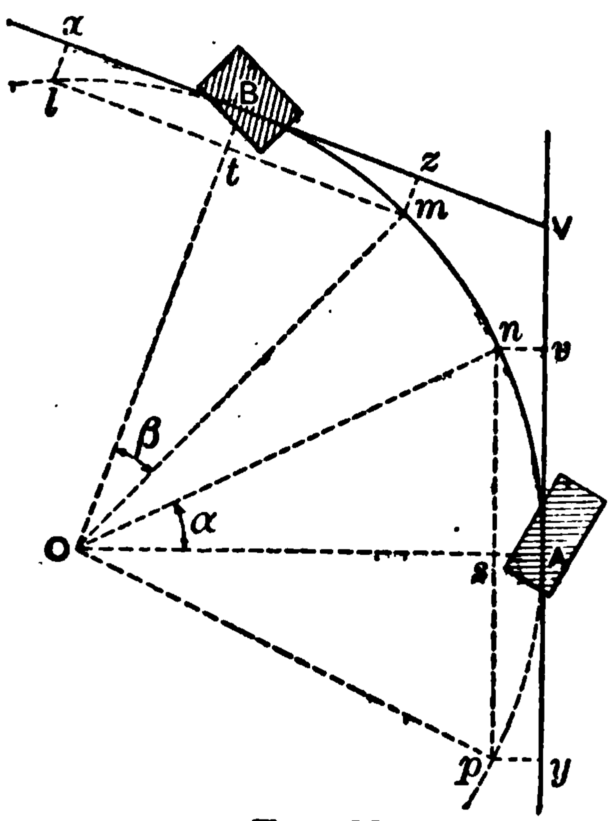


FIG. 23.

If the point of tangency is obstructed, a similar process, somewhat reversed, may be used. β is that portion of Δ still to be laid off when m is reached. $tm = tl = R \sin \beta$. $mz = tB = lx \doteq R$ vers β.

c. When the central part of the curve is obstructed. a is the central angle between two points of the curve between which a chord may be run. a may equal *any* angle, but it is preferable that a should be a multiple of D, the degree of curve, and that the points m and n should be on even stations. $mn = 2R \sin \frac{1}{2}a$. A point s may be located by an offset ks from the chord mn by a similar method to that outlined in § 60.

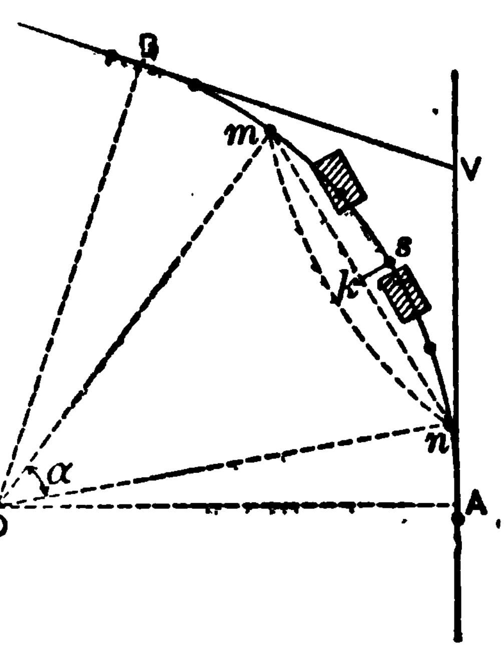


FIG. 24.

The device of introducing the dotted curve mn having the same radius of curvature as the other, although neither necessary nor advisable in the case shown in Fig. 24, is sometimes the best method of surveying around an obstacle. The offset from any point on the dotted curve to the corresponding point on the true curve is twice the "ordinate to the long cord," as computed in § 60.

63. Modifications of location. The following methods may be used in allowing for the discrepancies between the "paper location" based on a more or less rough preliminary survey and the more accurate instrumental location. (See § 18.) They are

also frequently used in locating new parallel tracks and modifying old tracks.

a. To move the forward tangent parallel to itself a distance x, the point of curve (A) remaining fixed. (Fig. 25.)

$$V'h = B'r = x'.$$

$$VV' = \frac{V'h}{\sin hVV'} = \frac{x}{\sin \Delta}. \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$AV' = AV + VV'.$$

The triangle BmB' is isosceles and $Bm = B'm$.

$$R' - R = O'O = mB = \frac{B'r}{\text{vers } B'mB} = \frac{x'}{\text{vers } \Delta}.$$

$$\therefore \; R' = R + \frac{x'}{\text{vers } \Delta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The solution is very similar in case the tangent is moved inward to $V''B''$. Note that this method necessarily changes the

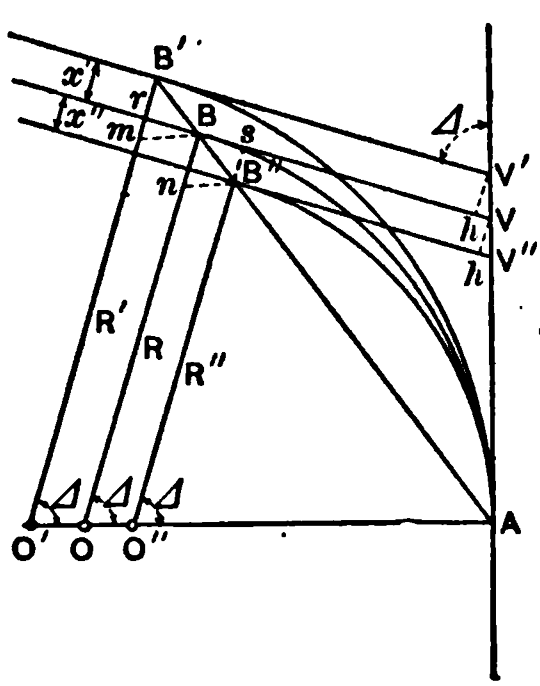


FIG. 25.

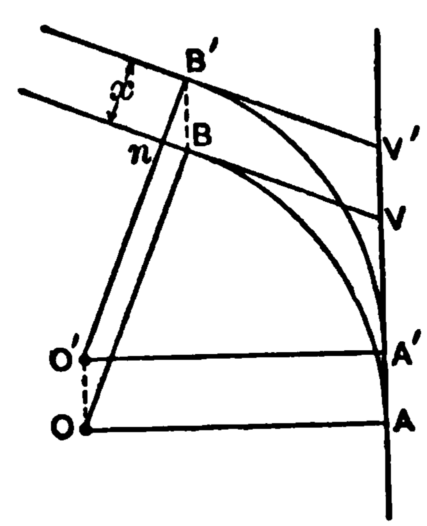


FIG. 26.

radius. If the radius is not to be changed, the point of curve must be altered as follows:

b. To move the forward tangent parallel to itself a distance x, the radius being unchanged. (Fig. 26.) In this case the whole

curve is moved bodily a distance $OO' = AA' = VV' = BB'$, and moved parallel to the first tangent AV

$$BB' = \frac{B'n}{\sin nBB'} = \frac{x}{\sin \Delta} = AA'. \quad . \quad . \quad . \quad (15)$$

c. To change the direction of the forward tangent at the point of tangency. (Fig. 27.) This problem involves a change (a) in the central angle and also requires a new radius. An error in the determination of the central angle furnishes an occasion for its use.

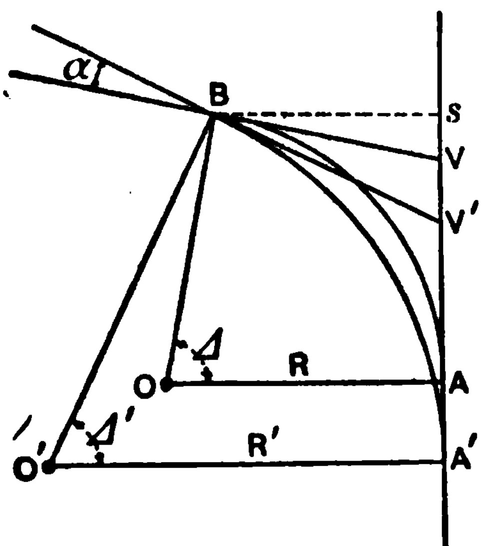


FIG. 27.

R, Δ, a, AV, and BV are known.

$$\Delta' = \Delta - a.$$

$$Bs = R \text{ vers } \Delta. \qquad Bs = R' \text{ vers } \Delta'.$$

$$\therefore R' = R \frac{\text{vers } \Delta}{\text{vers } (\Delta - a)}. \quad . \quad (16)$$

$$As = R \sin \Delta. \quad A's = R' \sin \Delta'.$$

$$\therefore AA' = A's - As = R' \sin \Delta' - R \sin \Delta. \quad . \quad . \quad (17)$$

The above solutions are given to illustrate a large class of problems which are constantly arising. All of the ordinary problems can be solved by the application of elementary geometry and trigonometry.

64. Limitations in location. It may be required to run a curve that shall join two given tangents and also pass through a given point. The point (P, Fig. 28) is assumed to be determined by its distance (VP) from the vertex and by the angle $AVP = \beta$.

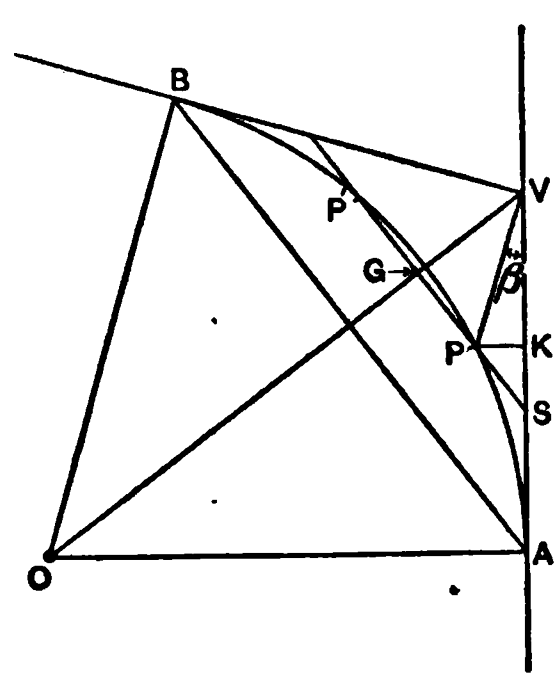


FIG. 28.

It is required to determine the radius (R) and the tangent distance (AV). Δ is known.

$$PVG = \tfrac{1}{2}(180° - \Delta) - \beta$$
$$= 90° - (\tfrac{1}{2}\Delta + \beta).$$
$$PP' = 2VP \sin PVG$$
$$= 2VP \cos (\tfrac{1}{2}\Delta + \beta).$$
$$PSV = \tfrac{1}{2}\Delta.$$
$$\therefore SP = VP \frac{\sin \beta}{\sin \tfrac{1}{2}\Delta}.$$

$$AS=\sqrt{SP\times SP'}=\sqrt{SP(SP+PP')}$$

$$=\sqrt{VP\frac{\sin\beta}{\sin\frac{1}{2}\Delta}\left[VP\frac{\sin\beta}{\sin\frac{1}{2}\Delta}+2VP\cos\left(\tfrac{1}{2}\Delta+\beta\right)\right]}$$

$$=VP\sqrt{\frac{\sin^2\beta}{\sin^2\frac{1}{2}\Delta}+\frac{2\sin\beta\cos\left(\frac{1}{2}\Delta+\beta\right)}{\sin\frac{1}{2}\Delta}}.$$

$$SV=VP\frac{\sin\left(\frac{1}{2}\Delta+\beta\right)}{\sin\frac{1}{2}\Delta}.$$

$$AV=AS+SV$$

$$=\frac{VP}{\sin\frac{1}{2}\Delta}\left[\sin\left(\tfrac{1}{2}\Delta+\beta\right)+\sqrt{\sin^2\beta+2\sin\beta\sin\tfrac{1}{2}\Delta\cos\left(\tfrac{1}{2}\Delta+\beta\right)}\right].\quad(18)$$

$$R=AV\cot\tfrac{1}{2}\Delta.$$

In the special case in which P is on the median line OV, $\beta=90°-\frac{1}{2}\Delta$, and $(\frac{1}{2}\Delta+\beta)=90°$. Eq. 18 then reduces to

$$AV=\frac{VP}{\sin\frac{1}{2}\Delta}(1+\cos\tfrac{1}{2}\Delta)=VP\cot\tfrac{1}{4}\Delta,$$

as might have been immediately derived from Eq. 8.

In case the point P is given by the offset PK and by the distance VK, the triangle PKV may be readily solved, giving the distance VP and the angle β, and the remainder of the solution will be as above.

65. Determination of the curvature of existing track. (a) *Using a transit.* Set up the transit at any point in the center of the track. Measure in each direction 100 feet to points also in the center of the track. Sight on one point with the plates at 0°. Plunge the telescope and sight at the other point. The angle between the chords equals the degree of curvature.

(b) *Using a tape and string.* Stretch a string (say 50 feet long) between two points on the inside of the head of the outer rail. Measure the ordinate (x) between the *middle* of the string and the head of the rail. Then

$$R=\frac{\text{chord}^2}{8x}\text{(very nearly)}.\quad .\quad .\quad .\quad .\quad(19)$$

For, in Fig. 29, since the triangles AOE and ADC are similar,

$AO : AE :: AD : DC$ or $R = \frac{1}{2}\overline{AD}^2 \div x$. When, as is usual, the arc is very short compared with the radius, $AD = \frac{1}{2}AB$, very nearly. Making this substitution we have Eq. 19. With a chord of 50 feet and a 10° curve, the resulting difference in x is .0025 of an inch—far within the possible accuracy of such a method. The above method gives the radius of the inner head of the outer rail. It should be diminished by $\frac{1}{2}g$ for the radius of the center of the track. With easy curvature, however, this will not affect the result by more than one or two tenths of one per cent.

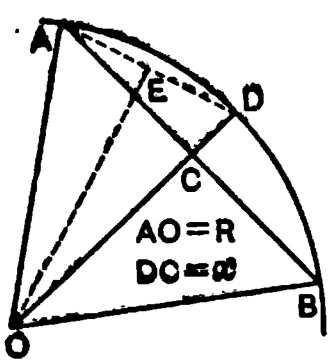


FIG. 29.

The inversion of this formula gives the required middle ordinate for a rail on a given curve. For example, the middle ordinate of a 30-foot rail, bent for a 6° curve, is

$$x = 900 \div (8 \times 955) = .118 \text{ foot} = 1.4 \text{ inches}.$$

Another much used rule is to require the foreman to have a string, knotted at the center, of such length that the middle ordinate, measured in inches, equals the degree of curve. To find that length, substitute (in Eq. 19) $5730 \div D$ for R and $D \div 12$ for x. Solving for *chord*, we obtain *chord* = 61.8 feet. The rule is not theoretically exact, but, considering the uncertain stretching of the string, the error is insignificant. In fact, the distance usually given is 62 feet, which is close enough for all purposes for which such a method should be used.

66. Problems. A systematic method of setting down the solution of a problem simplifies the work. Logarithms should always be used, and *all* the work should be so set down that a revision of the work to find a supposed error may be readily done. The value of such systematic work will become more apparent as the problems become more complicated. The two solutions given below will illustrate such work.

a. Given a 3° curve beginning at Sta. 27+60 and running to Sta. 32+45. Compute the ordinates and offsets used in locating the curve by tangential offsets.

b. With the same data as above, compute the distances to locate the curve by offsets from the long chord.

c. Assume that in Fig. 22 *ab* is measured as 217.6 feet, the

angle $abV=17° 42'$, and the angle $baV=21° 14'$. Join the tangents by a 4° 30′ curve. Determine bB and aA.

d. Assume that in a case similar to Fig. 23 it was noted that a distance (As) equal to 12 feet would clear the building. Assume that $\Delta=38° 20'$ and that $D=4° 40'$. Required the value of a and the position of n. *Solution:*

$\text{vers}\, a=As\div R$	$As=12$	$\log=1.07918$
	R (for 4° 40′ curve)	$\log=3.0892\bar{3}$
	$\underline{a=8° 01'}$	$\log \text{vers}\, a=7.9899\bar{4}$
$ns=R\sin a$		$\log \sin a=9.1444\bar{5}$
		$\log R=3.0892\bar{3}$
	$\underline{ns=171.27}$	$\log=2.23369$

e. Assume that the forward tangent of a 3° 20′ curve having a central angle of 16° 50′ must be moved 3.62 feet *inward*, without altering the $P.C.$ Required the change in radius.

f. Given two tangents making an angle of 36° 18′. It is required to pass a curve through a point 93.2 feet from the vertex, the line from the vertex to the point making an angle of 42° 21′ with the tangent. Required the radius and tangent distance. *Solution:* Applying Eq. 18, we have

	2	$\log=0.30103$
	$\beta=42° 21'$	$\log \sin=9.82844$
	$\frac{1}{2}\Delta=18° 09'$	$\log \sin=9.4934\bar{6}$
	$(\frac{1}{2}\Delta+\beta)=60° 30'$	$\log \cos=9.69234$
	.20667	$9.3152\bar{7}$
$\log \sin^2\beta=9.65688\ldots$	.45382	
$2\,\lvert 9.81987\ldots$	.66049	
$9.9099\bar{3}\ldots$	.81271	
nat. sin 60° 30′....	$.870\bar{3}$	
	$1.683\bar{0}$...........	$\log=0.22610$
$VP=93.2$..................		$\log=1.9694\bar{1}$
		$2.1955\bar{1}$
		$\log \sin \frac{1}{2}\Delta=9.4934\bar{6}$
$\underline{\text{Tang. dist. } AV=503.36}$..........		$\log=2.70205$
		$\log \cot \frac{1}{2}\Delta=10.48437$
$R=1536.1$..................		$\log=3.18642$
$\underline{D=3° 44'}$		

COMPOUND CURVES.

67. **Nature and use.** Compound curves are formed by a succession of two or more simple curves of different curvature. The curves must have a common tangent at the point of compound curvature (*P.C.C.*). In mountainous regions there is frequently a necessity for compound curves having several changes of curvature. Such curves may be located separately as a succession of simple curves, but a combination of two simple curves has special properties which are worth investigating and utilizing. In the following demonstrations R_2 always represents the *longer* radius and R_1 the *shorter*, no matter which succeeds the other. T_1 is the tangent adjacent to the curve of shorter radius (R_1), and is invariably the shorter tangent. Δ_1 is the central angle of the curve of radius R_1, but it may be greater or less than Δ_2

68. **Mutual relations of the parts of a compound curve having two branches.** In Fig. 30, AC and CB are the two branches of

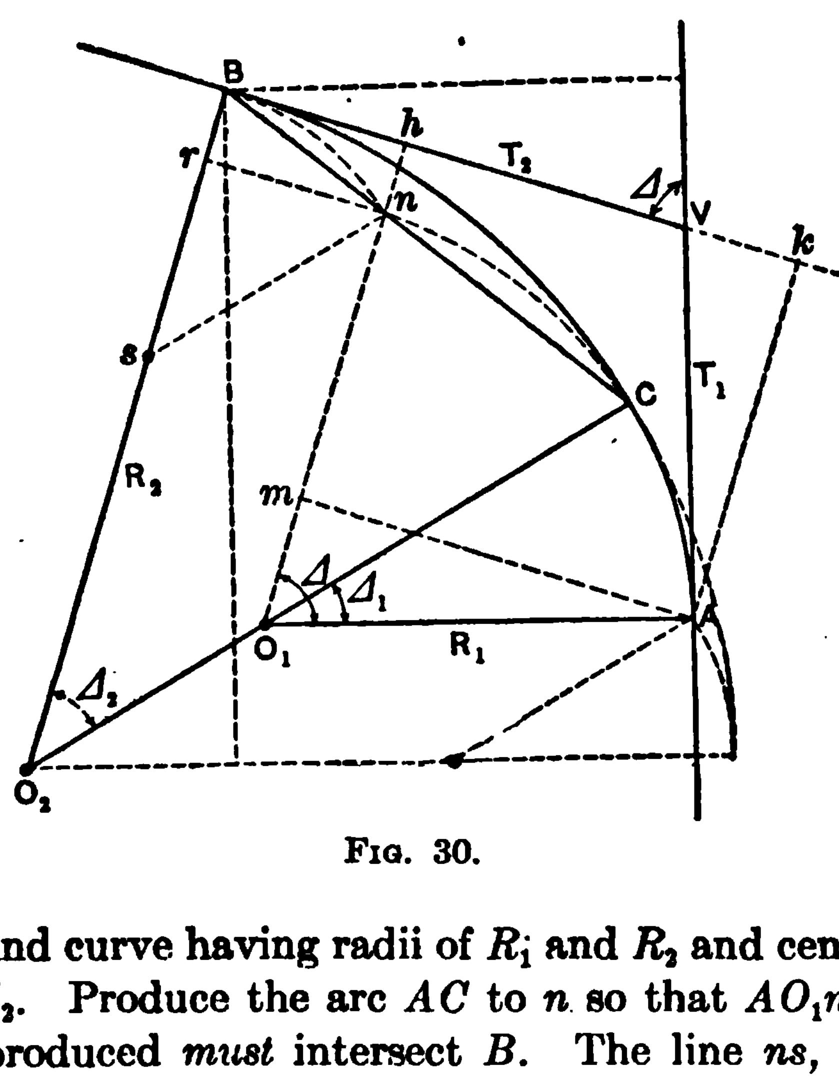


FIG. 30.

the compound curve having radii of R_1 and R_2 and central angles of Δ_1 and Δ_2. Produce the arc AC to n so that $AO_1n=\Delta$. The chord Cn produced *must* intersect B. The line ns, parallel to CO_2, will intersect BO_2 so that $Bs=sn=O_2O_1=R_2-R_1$. Draw Am perpendicular to O_1n. It will be parallel to hk.

$$\begin{aligned} Br &= sn \text{ vers } Bsn &&= (R_2 - R_1) \text{ vers } \Delta_2; \\ mn &= AO_1 \text{ vers } AO_1n &&= R_1 \text{ vers } \Delta; \\ Ak &= AV \sin AVk &&= T_1 \sin \Delta; \\ Ak &= hm = mn + nh = mn + Br. \end{aligned}$$

$$\therefore\ T_1 \sin \Delta = R_1 \text{ vers } \Delta + (R_2 - R_1) \text{ vers } \Delta_2. \quad . \quad . \quad (20)$$

Similarly it may be shown that

$$T_2 \sin \Delta = R_2 \text{ vers } \Delta - (R_2 - R_1) \text{ vers } \Delta_1. \quad . \quad . \quad (21)$$

The mutual relations of the elements of compound curves may be solved by these two equations. For example, assume the tangents as fixed (Δ therefore known) and that a curve of given radius R_1 shall start from a given point at a distance T_1 from the vertex, and that the curve shall continue through a given angle Δ_1. Required the other parts of the curve. From Eq. 20 we have

$$R_2 - R_1 = \frac{T_1 \sin \Delta - R_1 \text{ vers } \Delta}{\text{vers } \Delta_2}.$$

$$\therefore\ R_2 = R_1 + \frac{T_1 \sin \Delta - R_1 \text{ vers } \Delta}{\text{vers } (\Delta - \Delta_1)}. \quad . \quad . \quad . \quad . \quad . \quad (22)$$

T_2 may then be obtained from Eq. 21.

As another problem, given the location of the two tangents, with the two tangent distances (thereby locating the *PC* and *PT*), and the central angle of each curve; required the two radii. Solving Eq. 20 for R_1, we have

$$R_1 = \frac{T_1 \sin \Delta - R_2 \text{ vers } \Delta_2}{\text{vers } \Delta - \text{vers } \Delta_2}.$$

Similarly from Eq. 21 we may derive

$$R_1 = \frac{T_2 \sin \Delta - R_2(\text{vers } \Delta - \text{vers } \Delta_1)}{\text{vers } \Delta_1}.$$

Equating these, reducing, and solving for R_2, we have

$$R_2 = \frac{T_1 \sin \Delta \text{ vers } \Delta_1 - T_2 \sin \Delta \text{ (vers } \Delta - \text{vers } \Delta_2)}{\text{vers } \Delta_2 \text{ vers } \Delta_1 - (\text{vers } \Delta - \text{vers } \Delta_1)(\text{vers } \Delta - \text{vers } \Delta_2)}. \quad (23)$$

Although the various elements may be chosen as above with considerable freedom, there are limitations. For example, in Eq. 22, since R_2 is always greater than R_1, the term to be added to R_1 must be essentially positive—i.e., $T_1 \sin \Delta$ must be greater than R_1 vers Δ. This means that $T_1 > R_1 \frac{\text{vers } \Delta}{\sin \Delta}$, or that

$T_1 > R_1 \tan \frac{1}{2}\Delta$, or that T_1 is greater than the corresponding tangent on a simple curve. Similarly it may be shown that T_2 is *less* than $R_2 \tan \frac{1}{2}\Delta$ or *less* than the corresponding tangent on a simple curve. Nevertheless T_2 is always greater than T_1. In the limiting case when $R_2 = R_1$, $T_2 = T_1$, and $\Delta_2 = \Delta_1$.

69. Modifications of location. Some of these modifications may be solved by the methods used for simple curves. For example:

a. It is desired to move the tangent *VB*, Fig. 26, parallel to itself to *V'B'*. Run a new curve from the *P.C.C.* which shall reach the new tangent at *B'*, where the chord of the old curve

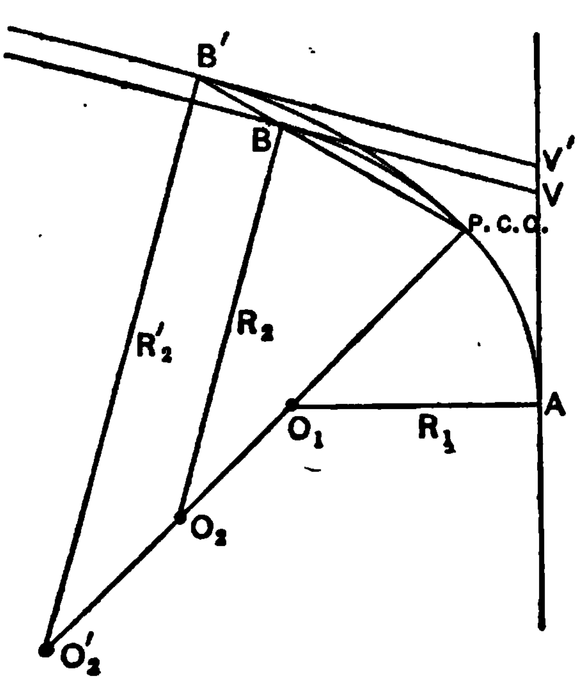


Fig. 31.

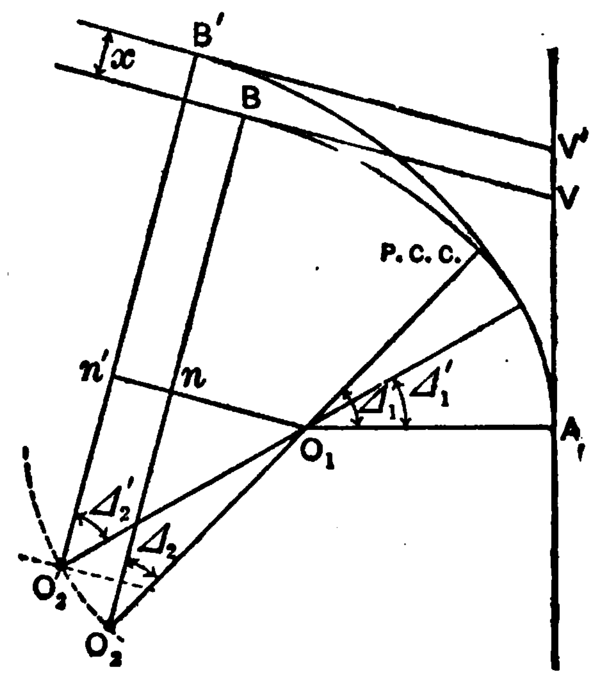


Fig. 32.

intersects the new tangent. The solution is almost identical with that in § 63, *a*.

b. Assume that it is desired to change the forward tangent (as above) but to retain the same radius. In Fig. 32

$$(R_2 - R_1)\cos \Delta_2 = O_2 n;$$
$$(R_2 - R_1)\cos \Delta_2' = O_2' n'.$$
$$x = O_2 n - O_2' n' = (R_2 - R_1)(\cos \Delta_2 - \cos \Delta_2').$$

$$\cos \Delta_2' = \cos \Delta_2 - \frac{x}{R_2 - R_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

The *P.C.C.* is moved *backward* along the sharper curve an angular distance of $\Delta_2' - \Delta_2 = \Delta_1 - \Delta_1'$.

In case the tangent is moved inward rather than outward, the solution will apply by transposing Δ_2 and Δ_2'. Then we shall have

$$\cos \Delta_2' = \cos \Delta_2 + \frac{x}{R_2 - R_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

The *P.C.C.* is then moved *forward*.

c. Assume the same case as (b) except that the larger radius comes first and that the tangent adjacent to the smaller radius is moved. In Fig. 33

$$(R_2-R_1)\cos\Delta_1=O_1n;$$
$$(R_2-R_1)\cos\Delta_1'=O_1'n'.$$

$$x=O_1'n'-O_1n$$
$$=(R_2-R_1)(\cos\Delta_1'-\cos\Delta_1).$$

$$\cos\Delta_1'=\cos\Delta_1+\frac{x}{R_2-R_1}. \qquad (26)$$

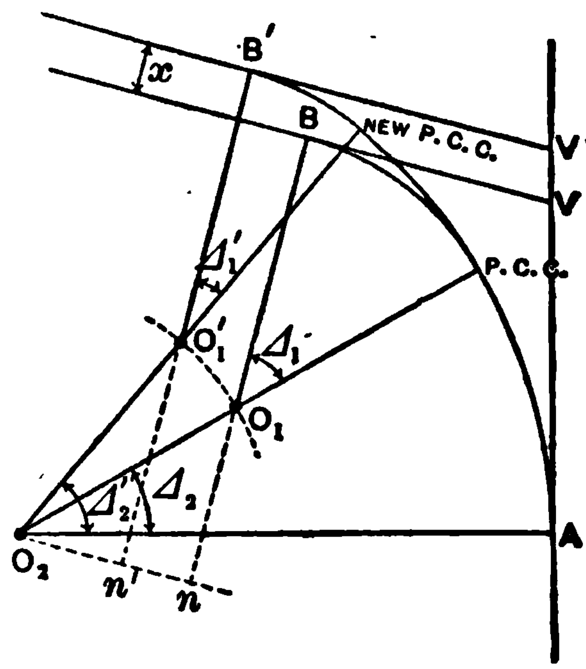


FIG. 33.

The *P.C.C.* is moved *forward* along the easier curve an angular distance of $\Delta_1'-\Delta_1=\Delta_2-\Delta_2'$.

In case the tangent is moved *inward*, transpose as before and we have

$$\cos\Delta_1'=\cos\Delta_1-\frac{x}{R_2-R_1}. \quad . \quad . \quad . \quad . \quad (27)$$

The *P.C.C.* is moved *backward*

d. Assume that the radius of one curve is to be altered without changing either tangent. Assume conditions as in Fig. 34. For the diagrammatic solution assume that R_2 is to be increased by O_2S. Then, since R_2' must pass through O_1 and extend beyond O_1 a distance O_1S, the locus of the new center must lie on the arc drawn about O_1 as center and with OS as radius. The locus of O_2' is also given by a line $O_2'p$ parallel to BV and at a distance of R_2' (equal to $S\ldots P.C.C.$) from it. The new center is therefore at the intersection O_2'. An arc with radius R_2' will therefore be tangent at B' and tangent to the old curve *produced* at NEW *P.C.C.* Draw O_1n' perpendicular to O_2B.

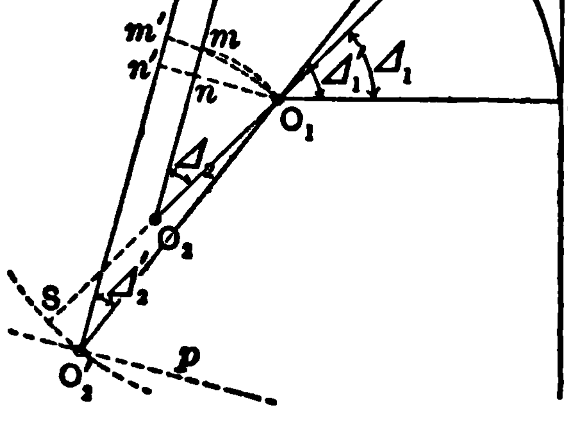


FIG. 34.

With O_2 as center draw the arc O_1m, and with O_2' as center draw the arc O_1m'. $mB = m'B' = R_1$.

$$\therefore\ mn = m'n' = (R_2' - R_1)\ \text{vers}\ \Delta_2' = (R_2 - R_1)\ \text{vers}\ \Delta_2.$$

$$\therefore\ \text{vers}\ \Delta_2' = \frac{(R_2 - R_1)}{(R_2' - R_1)}\ \text{vers}\ \Delta_2. \quad . \quad . \quad . \quad . \quad (28)$$

$$O_1n = (R_2 - R_1)\sin\Delta_2;$$

$$O_1n' = (R_2' - R_1)\sin\Delta_2'.$$

$$BB' = O_1n' - O_1n = (R_2' - R_1)\sin\Delta_2' - (R_2 - R_1)\sin\Delta_2. \qquad (29)$$

This problem may be further modified by assuming that the radius of the curve is decreased rather than increased, or that the smaller radius follows the larger. The solution is similar and is suggested as a profitable exercise.

It might also be assumed that, instead of making a given change in the radius R_2, a given change BB' is to be made. Δ_2' and R_2' are required. Eliminate R_2' from Eqs. 28 and 29 and solve the resulting equation for Δ_2'. Then determine R_2' by a suitable inversion of either Eq. 28 or 29.

As in §§ 62 and 63, the above problems are but a few, although perhaps the most common, of the problems the engineer may meet with in compound curves. All of the ordinary problems may be solved by these and similar methods.

70. Problems. *a.* Assume that the two tangents of a compound curve are to be 348 feet and 624 feet, and that $\Delta_1 = 22° 16'$ and $\Delta_2 = 28° 20'$. Required the radii.

[*Ans.* $R_1 = 326.92$; $R_2 = 1574.85$.]

b. A line crosses a valley by a compound curve which is first a 6° curve for 46° 30′ and then a 9° 30′ curve for 84° 16′. It is afterward decided that the last tangent should be 6 feet farther up the hill. What are the required changes? [*Note.* The second tangent is evidently moved *outward.* The solution corresponds to that in the first part of § 69, *c.* The *P.C.C.* is moved forward 16.39 feet. If it is desired to know how far the *P.T.* is moved in the direction of the tangent (i.e., the *projection* of BB', Fig. 33, on $V'B'$), it may be found by observing that it is equal to $nn' = (R_2 - R_1)(\sin\Delta_1 - \sin\Delta_1')$. In this case it equals 0.65 foot, which is very small because Δ_1 is nearly 90°. The value of Δ_2 (46° 30′) is not used, since the solution is independent of the value of Δ_2. The student should learn to recognize

which quantities are mutually related and therefore essential to a solution, and which are independent and non-essential.]

TRANSITION CURVES.

71. Superelevation of the outer rail on curves. When a mass is moved in a circular path it requires a centripetal force to keep it moving in that path. By the principles of mechanics we know that this force equals $Gv^2 \div gR$, in which G is the weight, v the velocity in feet per second, g the acceleration of gravity in feet per second in a second, and R the radius of curvature. If the two rails of a curved track were laid on a level (transversely), this centripetal force could only be furnished by the pressure of the wheel-flanges against the rails. As this is very objectionable, the outer rail is elevated so that the reaction of the rails against the wheels shall contain a horizontal component equal to the required centripetal force. In Fig. 35, if ob represents the reaction, oc will represent the weight G, and ao will represent the required centripetal force. From similar triangles we may write $sn : sm :: ao : oc$. Call $g = 32.17$. Call $R = 5730 \div D$, which is sufficiently accurate for this purpose (see § 48). Call $v = 5280V \div 3600$, in which V is the velocity in miles per hour. mn is the distance between rail centers, which, for an 80-lb. rail and standard gauge, is 4.916 feet sm is slightly less than this. As an average value we may call it 4.900, which is its exact value when the superelevation is 4¾ inches. Calling $sn = e$, measured in feet, we have

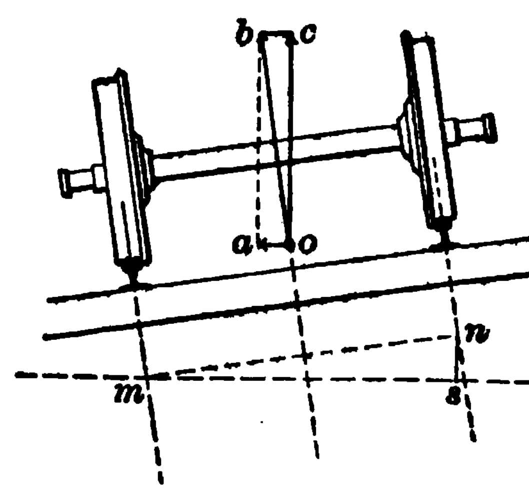


FIG. 35.

$$e = sm\frac{ao}{oc} = 4.9\frac{Gv^2}{gR}\frac{1}{G} = \frac{4.9 \times 5280^2 V^2 D}{32.17 \times 3600^2 \times 5730}.$$

$$e = .0000572V^2D. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

It should be noticed that, according to this formula, the required superelevation varies as the *square* of the velocity, which means that a change of velocity of only 10% would call for a change of superelevation of 21%. Since the velocities of trains over any road are extremely variable, it is impossible to adopt

any superelevation which will fit all velocities even approximately. The above fact also shows why any over-refinement in the calculations is useless and why the above approximations, which are really small, are amply justifiable. For example, the above formula contains the approximation that $R = 5730 \div D$. In the extreme case of a 10° curve the error involved would be about 1%. A change of about ½ of 1% in the velocity, or say from 40 to 40.2 miles per hour, would mean as much. The error in e due to the assumed constant value of sm is never more than a very small fraction of 1%. The rail-laying is not done closer than this. Table XIX is based on Eq. (30):

TABLE XIX. SUPERELEVATION OF THE OUTER RAIL (IN FEET) FOR VARIOUS VELOCITIES AND DEGREES OF CURVATURE.

Velocity in Miles per Hour.	Degree of Curve.									
	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
30	.05	.10	.15	.20	.26	.31	.36	.41	.46	.51
40	.09	.18	.27	.37	.46	.55	.64	.73	.82	
50	.14	.29	.43	.57	.71	.86				
60	.20	.41	.62	.82						

72. Practical rules for superelevation. A much used rule for superelevation is to "elevate one half an inch for each degree of curvature." The rule is rational in that e in Eq. 30 varies directly as D. The above rule therefore agrees with Eq. 30 when V is about 27 miles per hour. However applicable the rule may have been in the days of low velocities, the elevation thus computed is too small now. The rule to elevate one inch for each degree of curvature is also used and is precisely similar in its nature to the above rule. It agrees with Eq. 30 when the velocity is about 38 miles per hour, which is more nearly the average speed of trains.

Another (and better) rule is to "elevate for the speed of the fastest trains." This rule is further justified by the fact that a four-wheeled truck, having two parallel axles, will always tend to run to the outer rail and will require considerable flange pressure to guide it along the curve. The effect of an excess of superelevation on the slower trains will only be to relieve this flange pressure somewhat. This rule is coupled with the limitation

that the elevation should never exceed a limit of six inches—sometimes eight inches. This limitation implies that locomotive engineers must reduce the speed of fast trains around sharp curves until the speed does not exceed that for which the actual superelevation used is suitable. The heavy line in Table XIX shows the six-inch limitation.

Some roads furnish their track foremen with a list of the superelevations to be used on each curve in their sections. This method has the advantage that each location may be separately studied, and the proper velocity, as affected by local conditions (*e.g.*, proximity to a stopping-place for all trains), may be determined and applied.

Another method is to allow the foremen to determine the superelevation for each curve by a simple measurement taken at the curve. The rule is developed as follows: By an inversion of Eq. 19 we have

$$x = chord^2 \div 8R. \quad \dots\dots \quad (31)$$

Putting x equal to e in Eq. 30 and solving for "*chord*," we have

$$chord^2 = .0000572V^2D8R$$
$$= 2.621V^2.$$
$$chord = 1.62V. \quad \dots\dots\dots \quad (32)$$

To apply the rule, assume that 50 miles per hour is fixed as the velocity from which the superelevation is to be computed. Then $1.62V = 1.62 \times 50 = 81$ feet, which is the distance given to the trackmen. Stretch a tape (or even a string) with a length of 81 feet between two points on the concave side of the head of either the inner or the outer rail. The ordinate at the middle point then equals the superelevation. The values of this chord length for varying velocities are given in the accompanying tabular form.

Velocity in miles per hour...	20	25	30	35	40	45	50	55	60
Chord length in feet........	32.4	40.5	48.6	56.7	64.8	72.9	81.0	89.1	97.2

The following tabular form shows the standard (at one time) on the N. Y., N. H. & H. R. R. It should be noted that the elevations do not increase proportionately with the radius, and that they are higher for descending grades than for level or

ascending grades. This is on the basis that the velocity on curves and on ascending grades will be less than on descending grades. For example, the superelevation for a 0° 30′ curve on a descending grade corresponds to a velocity of about 54 miles per hour, while for a 4° curve on a level or ascending grade the superelevation corresponds to a velocity of only about 38 miles per hour.

TABLE OF THE SUPERELEVATION OF THE OUTER RAIL ON CURVES.
N. Y., N. H. & H. R. R.

Degree of curve.	Level or ascending grade.	Descending grade.
	inches.	inches.
0° 30′	$0\frac{3}{4}$	1
1 00	$1\frac{1}{2}$	$1\frac{3}{4}$
1 15	$1\frac{3}{4}$	2
1 30	2	$2\frac{1}{4}$
1 45	$2\frac{1}{4}$	$2\frac{1}{2}$
2 00	$2\frac{3}{8}$	$2\frac{3}{4}$
2 15	$2\frac{5}{8}$	3
2 30	$2\frac{7}{8}$	$3\frac{1}{4}$
2 45	3	$3\frac{3}{8}$
3 00	$3\frac{1}{8}$	$3\frac{5}{8}$
3 15	$3\frac{3}{8}$	$3\frac{7}{8}$
3 30	$3\frac{5}{8}$	4
3 45	$3\frac{7}{8}$	$4\frac{1}{8}$
4 00	4	$4\frac{1}{2}$

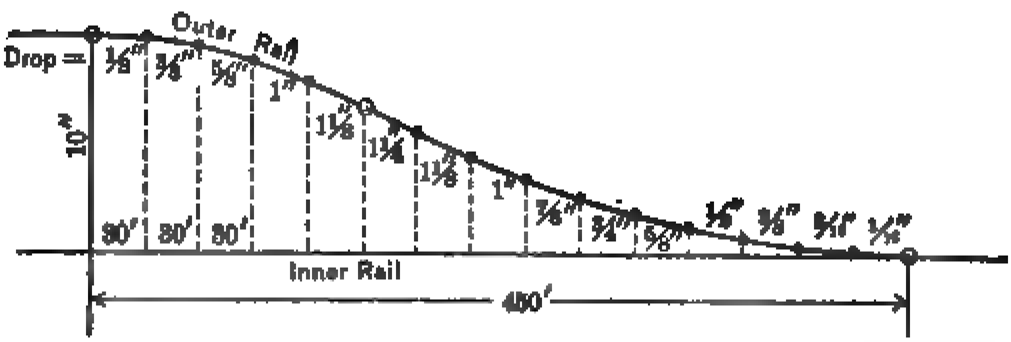
73. **Transition from level to inclined track.** On curves the track is inclined transversely; on tangents it is level. The transition from one condition to the other must be made gradually. If there is no transition curve, there must be either inclined track on the tangent or insufficiently inclined track on the curve or both. Sometimes the full superelevation is continued through the total length of the curve and the "run-off" (having a length of 100 to 400 feet) is located entirely on the tangents at each end. In other practice it is located partly on the tangent and partly on the curve. Whatever the method, the superelevation is correct at only one point of the run-off. At all other points it is too great or too small. This (and other causes) produces objectionable lurches and resistances when entering and leaving curves. The object of transition curves is to obviate these resistances.

On the Lehigh Valley R. R. the run-off is made in the form of a reversed vertical curve, as shown in the accompanying figure. According to this system the length of run-off varies

from 120 feet, for a superelevation of one inch, to 450 feet, for a superelevation of ten inches. Such a superelevation as ten inches is very unusual practice, but is successfully operated on that road. The curve is concave upward for two-thirds of its length and then reverses so that it is convex upward.

TABLE FOR RUN-OFF OF ELEVATION OF OUTER RAIL OF CURVES.

Drop in inches for each 30-foot rail commencing at theoretical point of curve.



The figure (and also the lower line of the tabulated form) shows the drop for each thirty-foot rail length. For shorter lengths of run-off, the drop for each 30 feet is shown by the corresponding lines in the tabular form. Note in each horizontal line that the sum of the drops, under which 30 is found, equals the total superelevation as found in the first column. For example, for 4 inches superelevation, length of curve 240 feet, the successive drops are $\frac{1}{4}''$, $\frac{1}{2}''$, $\frac{7}{8}''$, $\frac{7}{8}''$, $\frac{5}{8}''$, $\frac{1}{2}''$, $\frac{1}{4}''$, and $\frac{1}{8}''$ whose sum is 4 inches. Possibly the more convenient form would be to indicate for each 30-foot point the actual superelevation of the outer rail, which would be for the above case (running from the tangent to the curve) $\frac{1}{8}''$, $\frac{3}{8}''$, $\frac{7}{8}''$, $1\frac{1}{2}''$, $2\frac{3}{8}''$, $3\frac{1}{4}''$, $3\frac{3}{4}''$, $4''$.

74. Fundamental principle of transition curves. If a curve

has variable curvature, beginning at the tangent with a curve of infinite radius, and the curvature gradually sharpens until it equals the curvature of the required simple curve and there becomes tangent to it, the superelevation of such a transition curve may begin at zero at the tangent, gradually increase to the required superelevation for the simple curve, and yet have at every point the superelevation required by the curvature at that point. Since in Eq. (30) e is directly proportional to D, the required curve must be one in which the degree of curve increases directly as the distance along the curve.

75. Varieties of Transition Curves. A theoretically exact transition curve is very complicated and its mathematical solution very difficult. A committee of the Amer. Rwy. Eng. Assoc. investigated the many systems which have been proposed and reported that all of them seemed to be objectionable for one or more of the following reasons: "(1) If simple approximate formulas were used, they were not sufficiently accurate. (2) Accurate formulas were too complex. (3) The curve could not be expressed by formulas. (4) Formulas were of the endless series class. (5) Complex field methods were required to make the field-work agree with formulas with spirals of large angles." The committee then developed a method which gives results whose accuracy is beyond that of the most careful field-work and yet which is sufficiently simple for practical use. The mathematical development is so elaborate that it will not be detailed here, but the working formulas and a condensation of the table together with an explanation of their practical use and application, will be given, with numerical examples.

The general form of these curves, whatever their precise mathematical character, is shown in Fig. 36. AVB are two tangents, joined by the simple circular curve AMB, having the center O. Assume that the entire curve is moved in the direction MO a distance $OO'=MM'=BB'=AA'$. At some point TS on the tangent, the spiral begins and joins the circular curve tangentially at SC. The other spiral runs from CS to ST. The significance of these symbols may be readily remembered from the letters; T, S, and C signify tangent, spiral and circular curve; TS is the point of change from tangent to spiral, SC, the point of change from spiral to curve, etc. At the other end of the circular curve the letters are in reverse order, the station numbers increasing from A to B. The meaning of the various symbols is

indicated in Fig. 36. The student should appreciate the fact of the necessary distortion of the figure in order to make it plain. Based on the figures of the following numerical problem, the distance MM' is about fourteen times its proper amount. Another effect of the distortion is that the dimension U, instead of being

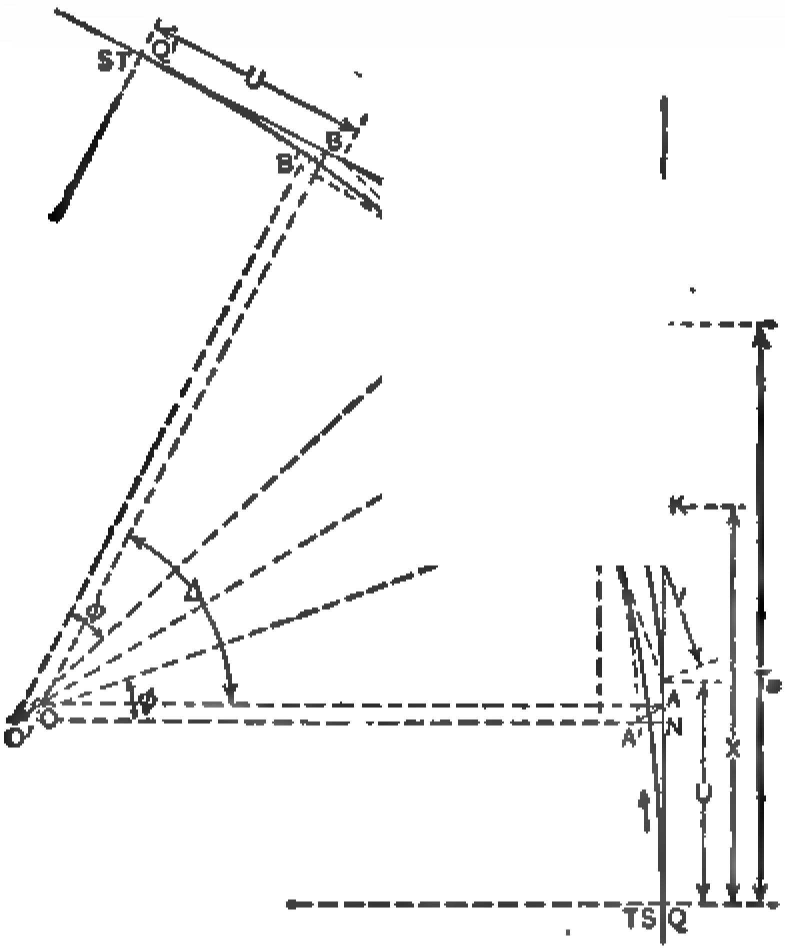


FIG. 36.

nearly twice V, which is usual, as given in Table IV, Part B, is only a little longer than V.

76. Proper length of spiral. This can only be computed on the basis of certain assumptions as to the desired rate of tipping the car, so as to avoid discomfort to passengers, and, of course, this depends on the expected velocity. There is also a maximum limitation, since the sum of the two spiral angles cannot exceed the total central angle of the curve. The *minimum* lengths recommended are as follows:

On curves which limit the speed:

6° and over, 240 feet;

Less than 6°, $5\frac{1}{3}\times$speed in m.p.h. for elevation of 8 inches.

On curves which do not limit the speed:

30 times elevation in inches, or

$\frac{2}{3}\times$ultimate speed in m.p.h.$\times$elevation in inches.

For example. (1) 5° curve which limits speed; speed limit 48 m.p.h. by interpolation in table, § 41; $48\times5\frac{1}{3}=256$ feet minimum length. (2) 3° curve; maximum operating speed 60 m.p.h.; superelevation, .62 feet $=7.44$ inches; $30\times7.44=223.2$ feet; or, $\frac{2}{3}\times60\times7.44=297.6$ feet. Of course the higher value should be used, or say 300 feet as the minimum length.

While it is generally true that the longer transition curves give easier riding, the spiral must not reach the center point of the curve. Since it is approximately true that the spiral extends for equal distances on each side of the original point of curve, it is nearly true that two spirals, each having the same length as the original curve, would just meet at the center. The length of a spiral should in general be very much less than the length of the original curve.

77. Symbols. Beside the symbols whose significance is clearly indicated in Fig. 36, the following are defined:

- a The angle between the tangent at the TS and the chord from the TS to *any* point on the spiral; a_1 is the angle to the *first* chord point.
- A The angle between the tangent at the TS and the chord from the TS to the SC.
- D The degree of the central circular curve.
- Δ The central angle of the original circular curve, or the angle between the tangents.
- ϕ The total central angle of the spiral.
- k The increase in degree of curve per station on the spiral.
- L The length of the spiral in feet from the TS to the SC.
- S The length of the spiral in stations from the TS to the SC.
- s The length of the spiral in stations from the TS to any given point.

78. Deflections. The field formulas for deflections are based on the following two equations:

$$a=10\ ks^2 \text{ minutes,}$$
$$A=10\ kS^2 \text{ minutes.}$$

The first deflection $a_1 = 10\ ks_1^2$ minutes. But k is the increase in degree of curve per station, and since the degree of curve increases as the length, $k = D \div S$, S being expressed in stations. For point 1, since $S = 10s$, $a_1 = 10\left(\frac{D}{10s_1}\right)s_1^2 = Ds_1$, which may be expressed as the degree of the curves times the length of the chord in stations. For example, if the spiral is 400 feet long (which means that $L = 400$ and $S = 4$) and runs on to a 5° curve (then $D = 5$), one chord is 40 feet long and $s = .4$ station. Then $a_1 = 5 \times 0.4 = 2$ minutes of arc for the deflection for the first chord point. And since the deflections are as the square of the number of stations, the deflections from TS to succeeding stations will be 4, 9, 16, 25, 36, 49, 64, 81, and 100 times 2 minutes, these factors being those given in the second vertical column of Part A of Table IV. The last deflection $= A = 100 \times 2' = 200' = 3°\ 20' = \frac{1}{3}\ (10°)$. $= \frac{1}{3}\phi$, ϕ being the total central angle of the spiral. Although it is always nearly true that $A = \frac{1}{3}\phi$, and the error is inappreciable for small angles, the error amounts to 30 seconds of arc when $\phi = 21°\ 30'$, an unusually large angle.

The deflection from any other point of the spiral to any other point, either forward or backward, may be found by multiplying the value of a_1 (in this case $2'$), by the coefficients in the proper vertical column of that table.

The spiral angle

$$\phi = \frac{kS^2}{2} = \frac{kL^2}{20000} = \frac{DL}{200} = \frac{5 \times 400}{200} = 10°.$$

Also,

$$\phi = \frac{kS^2}{2} = \frac{DS}{2} = \frac{5 \times 4}{2} = 10°$$

The values of the ratios $U \div L$ and $V \div L$ for even degrees, and for A, $C \div L$, $X \div L$, and $Y \div L$ for half degrees are given in Parts B and C of Table IV. When it is desired to temporarily omit locating the intermediate points of the spiral, the jump from the TS to the SC may be made by measuring the distance U from the TS along the tangent. At that point a deflection ϕ and a measured distance V will give not only the position of SC but also the direction of the tangent at the beginning of the circular curve. Another method of locating the SC without locating the intermediate points is to make the deflection A at the TS

and measure the long chord C. In the above numerical problem this equals $400 \times .998664 = 399.47$, a little over 6 inches short of the full 400 feet. By setting up the transit at the SC, back-sighting at the TS, and turning off the angle $(\phi - A)$, which in the above case is $10° - 3° \; 19' \; 57'' = 6° \; 20' \; 03''$, the direction of the tangent at the SC is obtained. In this case, the three seconds variation from the approximate value is utterly negligible. The other dimensions are easily determined from the tables if desired;

$$X = .996975 \times 400 = 398.79,$$

$$Y = .058053 \times 400 = 23.22,$$

$$U = .667742 \times 400 = 267.10$$

$$V = .334313 \times 400 = 133.73.$$

For greater convenience of notation, the points TS, SC, CS, and ST, in Fig. 36 are also indicated by the letters Q, Z, Z' and Q' respectively. The same letters are used for the corresponding points in Figs. 37 and 38.

79. **Location of spirals and circular curve with respect to tangents.** See Fig. 36. Let AV and BV be the tangents to be connected by a $D°$ curve, having a suitable spiral at each end. If no spirals were to be used, the problem would be solved as in simple curves giving the curve AMB. Introducing the spiral has the effect of throwing the curve away from the vertex a distance MM' and reducing the central angle of the $D°$ curve by 2ϕ. Continuing the curve beyond Z and Z' to A' and B', we will have $AA' = BB' = MM'$. $ZK =$ the Y ordinate and is therefore known. Call $MM' = m$. $A'N = Y - R \text{ vers } \phi$. Then

$$m = MM' = AA' = \frac{A'N}{\cos \frac{1}{2}\Delta} = \frac{Y - R \text{ vers } \phi}{\cos \frac{1}{2}\Delta}. \quad . \quad . \quad . \quad . \quad . \quad (33)$$

$$NA = AA' \sin \tfrac{1}{2}\Delta = (Y - R \text{ vers } \phi) \tan \tfrac{1}{2}\Delta.$$

$$\begin{aligned} VQ &= QK - KN + NA + AV \\ &= X - R \sin \phi + (Y - R \text{ vers } \phi) \tan \tfrac{1}{2}\Delta + R \tan \tfrac{1}{2}\Delta \\ &= X - R \sin \phi + Y \tan \tfrac{1}{2}\Delta + R \cos \phi \tan \tfrac{1}{2}\Delta. \quad . \quad . \quad . \quad (34) \end{aligned}$$

When $A'N$ has already been computed, it may be more convenient to write

$$VQ = X + R (\tan \tfrac{1}{2}\Delta - \sin \phi) + A'N \tan \tfrac{1}{2}\Delta. \quad . \quad . \quad . \quad . \quad (35)$$

$$
\begin{aligned}
VM' &= VM + MM' \\
&= R \text{ exsec } \tfrac{1}{2}\Delta + \frac{Y}{\cos \frac{1}{2}\Delta} - \frac{R \text{ vers } \phi}{\cos \frac{1}{2}\Delta}. \qquad (36)
\end{aligned}
$$

$$
\begin{aligned}
AQ &= VQ - AV \\
&= X - R \sin \phi + (Y - R \text{ vers } \phi) \tan \tfrac{1}{2}\Delta. \qquad (37)
\end{aligned}
$$

Example. To join two tangents making an angle of 34° 20′ by a 5° 40′ curve and suitable spirals. Assume that the spiral is 300 feet long. Then

$$\phi = \frac{DS}{2} = \frac{5.67 \times 3}{2} = 8.5^\circ = 8^\circ\ 30'.$$

Since, from Table IV, Part A, $Y \div L = .049374$ for $\phi = 8^\circ\ 30'$, $Y = 14.812$; similarly, we find $X = 299.344$ and $C = 299.71$.

[Eq. 33]			R	3.00497
			vers ϕ	8.04076
		11.110		1.04573
		Y = 14.812		
		$A'N$ = 3.702		0.56843
			$\cos \frac{1}{2}\Delta$	9.98021
		$m = MM' = AA'$ = 3.875		0.58822
[Eq. 36]			R	3.00497
			exsec $\frac{1}{2}\Delta$	8.66863
		VM = 47.164		1.67360
		m = 3.875		
		VM' = 51.039		
[Eq. 35]	X = 299.344	nat. tan $\frac{1}{2}\Delta$ =	.30891	
		nat. sin ϕ =	.14781	
			.16110	9.20709
			R	3.00497
	162.954			2.21206
		[See above]	$A'N$	0.56843
			$\tan \frac{1}{2}\Delta$	9.48984
	1.144		AN	0.05827
	VQ = 463.442			
[Eq. 37]			R	3.00497
			$\tan \frac{1}{2}\Delta$	9.48984
	312.471		AV	2.49481
	AQ = 150.971			

It should be noted that AQ is within a foot of equaling one-half the length of the spiral, which illustrates the general fact that a spiral begins at approximately one-half its length from the P.C. of the simple curve. All approximate systems of spirals assume this to be exactly true.

80. Field-work. When the spiral is designed during the original location, the tangent distance VQ should be computed and the point Q located. It is hardly necessary to locate all of the points of the spiral until the track is to be laid. The extremities should be located, and as there will usually be two or more full station points on the spiral, these should also be located. Z may be located by setting off $QK=X$ and $KZ=Y$, or else by the tabular deflection for Z from Q and the distance ZQ, which is the long chord c. Setting up the instrument at Z and sighting back at Q with the proper deflection, the tangent at Z may be found and the circular curve located as usual, its central angle being $\Delta-2\phi$. A similar operation will locate Q' from Z'.

To locate points on the spiral. Set up at Q, with the plates reading 0° when the telescope sights along VQ. Set off from Q the deflections computed from Table IV for the instrument at Q, using a chord length of $L\div 10$, the process being like the method for simple curves except that the deflections are variable. If a full station-point occurs within the spiral, interpolate between the deflections for the adjacent spiral-points. For example, a 400-foot spiral running on to a 3° 31′ curve begins at Sta. 56+15. The spiral points are 40 feet apart. Sta. 57 comes 5 feet beyond the second spiral point. The first deflection a_1 $=Ds=3.5\times.4=1.4$ min. The deflection to point 2 is 4×1.4 $=5.6$ min. and that to point 3 is $9\times1.4=12.6$ min. Then the deflection to Sta. 57 is $\frac{5}{40}\times(12.6-5.6)+5.6=6.47$ min.

This method is not theoretically accurate, but the error is small. Arriving at Z, the forward alinement may be obtained by sighting back at Q (or at any other point) with the proper deflection for that point from the station occupied. Then when the plates read 0° the telescope will be tangent to the spiral and to the succeeding curve. All rear points should be checked from Z. If it is necessary to occupy an intermediate station, use the deflections given for that station, orienting as just explained for Z, checking the back points and locating all forward points up to Z if possible.

After the center curve has been located and Z' is reached, the

other spiral must be located but *in reverse order*, i.e., the sharp curvature of the spiral is at Z' and the curvature decreases toward Q'.

81. To replace a simple curve by a curve with spirals. This *may* be done by the method of § 79, but it involves shifting the whole track a distance m, which in the given example equals 3.87 feet. Besides this the track is appreciably shortened,

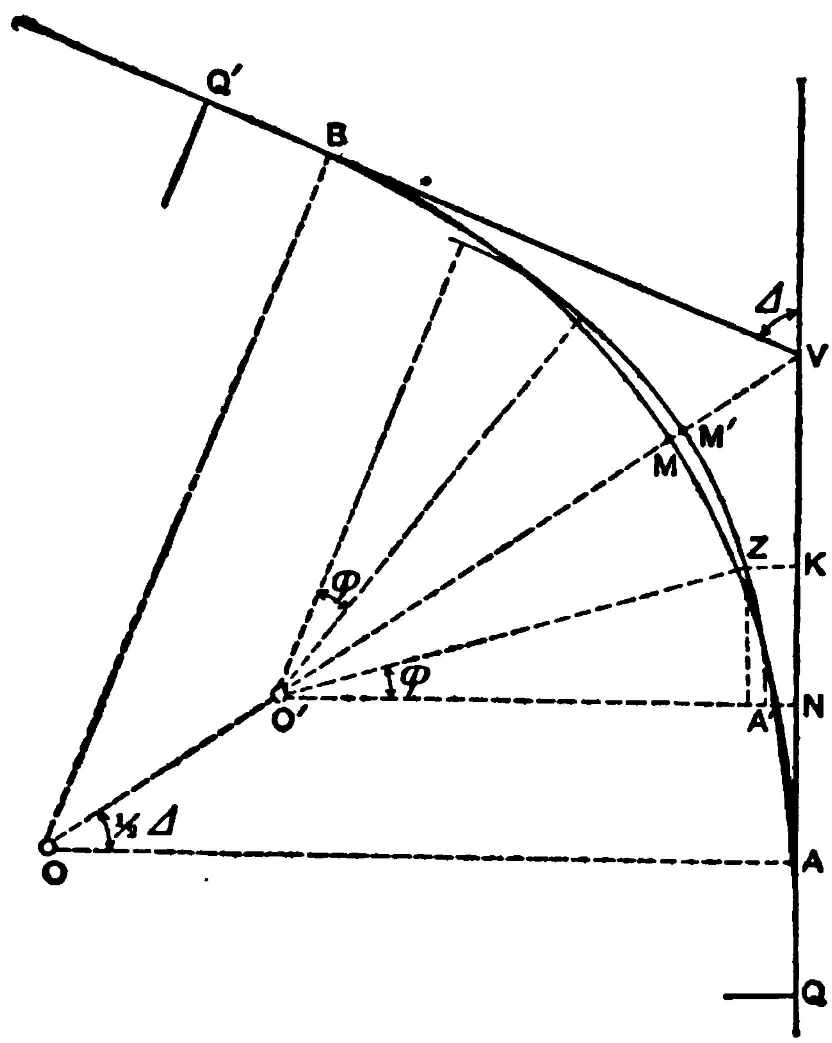


FIG. 37.

which would require rail-cutting. But the track may be kept at practically the same length and the lateral deviation from the old track may be made very small by slightly sharpening the curvature of the old track, moving the new curve so that it is wholly or partially *outside* of the old curve, the remainder of it with the spirals being *inside* of the old curve. It is found by experience that a decrease in radius of from 5% to 10% will answer the purpose. The larger the central angle the less the change. The solution is as indicated in Fig. 37.

$O'N = R' \cos \phi + Y.$

$O'V = O'N \sec \frac{1}{2}\Delta$

$\quad = R' \cos \phi \sec \frac{1}{2}\Delta + Y \sec \frac{1}{2}\Delta.$

$$
\begin{aligned}
m &= MM' = MV - M'V \\
&= R \text{ exsec } \tfrac{1}{2}\Delta - (O'V - R') \\
&= R \text{ exsec } \tfrac{1}{2}\Delta - R' \cos\phi \sec \tfrac{1}{2}\Delta - Y \sec \tfrac{1}{2}\Delta + R'. \qquad (38)
\end{aligned}
$$

$$
\begin{aligned}
AQ &= QK - KN + NV - VA \\
&= X - R' \sin\phi + (R' \cos\phi + Y) \tan \tfrac{1}{2}\Delta - R \tan \tfrac{1}{2}\Delta \\
&= X - R' \sin\phi + R' \cos\phi \tan \tfrac{1}{2}\Delta - (R - Y) \tan \tfrac{1}{2}\Delta. \qquad (39)
\end{aligned}
$$

The length of the old curve from Q to $Q' = 2AQ + 100\,\dfrac{\Delta}{D}$.

The length of the new curve from Q to $Q' = 2L + 100\,\dfrac{\Delta - 2\phi}{D'}$,

in which L is the length of each spiral.

Example. Suppose the old curve is a 7° 30′ curve with a central angle of 38° 40′. As a trial, compute the relative length of a new 8° 20′ curve with spirals 240 feet long. $\frac{1}{2}\Delta = 19° 20'$; R (for the 7° 30′ curve) $= 764.49$; R' (for the 8° 20′ curve) $= 688.16$; $\phi = 10° 0'$; $Y = 13.933$; $X = 239.274$.

[Eq. 38]			R	2.88337
			exsec ½Δ	8.77642
	45.687			1.65979
	R' = 688.16			
	733.847		R'	$2.8376\bar{8}$
			cos ϕ	9.99335
			sec ½Δ	0.02521
		718.200		$2.8562\bar{4}$
			Y	1.14405
			sec ½Δ	0.02521
		14.766		1.16926
	732.966	732.966		
	m = 0.881		R'	$2.8376\bar{8}$
[Eq. 39]	X = 239.274		sin ϕ	9.23967
		119.497		$2.0773\bar{5}$
			R'	2.83768
			cos ϕ	9.99335
			tan ½Δ	9.54512
	237.770			$2.3761\bar{5}$
			R = 764.49	
			Y = 13.93	
			750.56	$2.8753\bar{8}$
			tan ½Δ	9.54512
	477.044	263.333		$2.4205\bar{0}$
	382.830	382.830		
	AQ = 94.214			

The length of the old curve from Q to Q' is

$$100\frac{\Delta}{D}=100\frac{38.667}{7.5}= \quad \ldots\ldots\ldots \quad 515.556$$

$$2AQ=2\times 94.214= \quad \ldots\ldots\ldots \quad 188.428$$

$$703.984$$

New curve: $100\frac{\Delta-2\phi}{D'}=100\frac{38.667-20.000}{8.33}=224.000$

$$2L=2\times 240 \qquad =480.000$$

$$704.000 \qquad 704.000$$

Difference in length = 0.016

Considering that this difference may be divided among 21 joints (using 33-foot rails) no rail-cutting would be necessary. If the difference is too large, a slight variation in the value of the new radius R' will reduce the difference as much as necessary. A truer comparison of the lengths would be found by comparing the lengths of the arcs.

82. Application of transition curves to compound curves. Since compound curves are only employed when the location is limited by local conditions, the elements of the compound curve should be determined (as in §§ 68 and 69) regardless of the transition curves, depending on the fact that the lateral shifting of the curve when transition curves are introduced is very small. If the limitations are very close, an estimated allowance may be made for them.

Methods have been devised for inserting transition curves between the branches of a compound curve, but the device is complicated and usually needless, since when the train is once on a curve the wheels press against the outer rail steadily and a change in curvature will not produce a serious jar even though the superelevation is temporarily a little more or less than it should be.

If the easier curve of the compound curve is less than 3° or 4°, there may be no need for a transition curve off from that branch. This problem then has two cases according as transition curves are used at both ends or at one end only.

a. *With transition curves at both ends.* Adopting the method of § 79, calling $\Delta_1=\frac{1}{2}\Delta$, we may compute $m_1=MM_1'$. Similarly, calling $\Delta_2=\frac{1}{2}\Delta$, we may compute $m_2=MM_2'$. But M_1' and M_2' must be made to coincide. This may be done by moving the curve $Z'M_1'$ and its transition curve parallel to $Q'V$ a distance $M_1'M_3$, and the other curve parallel to QV a distance $M_2'M_3$.

In the triangle $M_1'M_3M_2'$, the angle at $M_1' = 90° - \Delta_1$, the angle at $M_2' = 90° - \Delta_2$, and the angle at $M_3 = \Delta$.

$$\left.\begin{aligned} \text{Then}\quad M_1'M_3 &= M_1'M_2'\frac{\sin(90° - \Delta_2)}{\sin \Delta} = (m_1 - m_2)\frac{\cos \Delta_2}{\sin \Delta}. \\ \text{Similarly}\quad M_2'M_3 &= M_1'M_2'\frac{\sin(90° - \Delta_1)}{\sin \Delta} = (m_1 - m_2)\frac{\cos \Delta_1}{\sin \Delta}. \end{aligned}\right\} \quad (40)$$

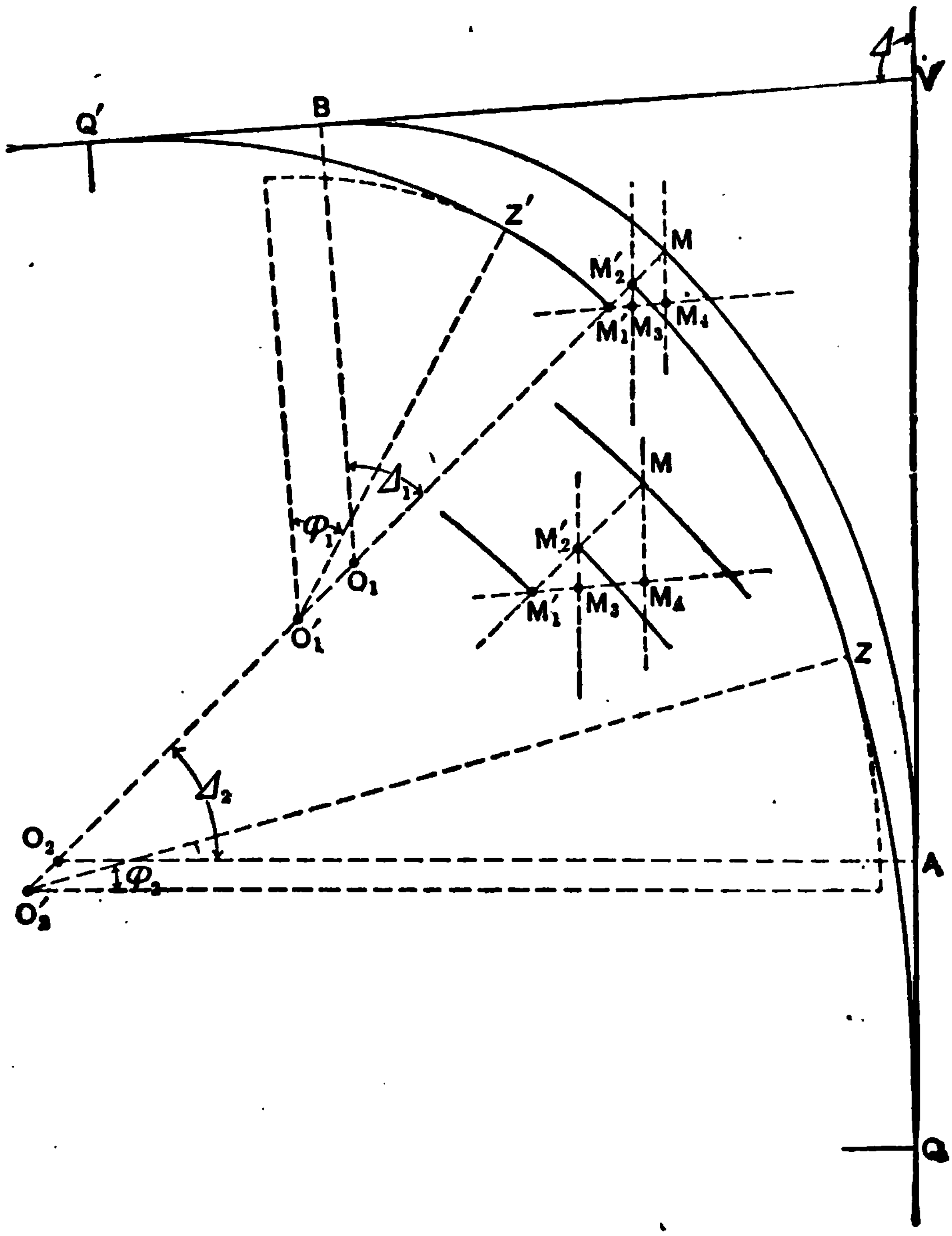


Fig. 38.

b. *With a transition curve on the sharper curve only.* Compute $m_1 = MM_1'$ as before; then move the curve Z_1M_1' parallel to $Q'V$ a distance of

$$M_1'M_4 = m_1\frac{\cos \Delta_2}{\sin \Delta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

The simple curve MA is moved parallel to VA a distance of

$$MM_4 = m_1 \frac{\cos \Delta_1}{\sin \Delta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (42)$$

If Δ_1 and Δ_2 are both small, $M_1'M_4$ and MM_4 may be more than m_1, but the lateral deviation of the new curve from the old will always be less than m_1.

83. To replace a compound curve by a curve with spirals. The numerical illustration given below employs another method. We first solve for m_1 for the sharper branch of the curve, placing $\Delta_1 = \frac{1}{2}\Delta$ in Eq. 38. A value for R_2' may be found whose corresponding value of m_2 will equal m_1. Solving Eq. 38 for R', we obtain

$$R' = \frac{R \text{ vers } \frac{1}{2}\Delta - m \cos \frac{1}{2}\Delta - Y}{\cos \phi - \cos \frac{1}{2}\Delta}. \quad . \quad . \quad . \qquad (43)$$

Substituting in this equation the known value of m_1 $(=m_2)$ and calling $R' = R_2'$, $R = R_2$, and $\Delta_2 = \frac{1}{2}\Delta$, solve for R_2'. Obtain the value of AQ for each branch of the curve separately by Eq. 39, and compare the lengths of the old and new lines.

Example. Assume a compound curve with $D_1 = 8°$, $D_2 = 4°$, $\Delta_1 = 36°$, and $\Delta_2 = 32°$. Use 240-foot spirals at each end. Assume that the sharper curve is sharpened from 8° 0′ to 8° 15′.

Eq. 38		R_1	$2.8553\bar{8}$
		exsec 36°	$9.3730\bar{3}$
169.21	. . . .		2.22842
695.09			
864.30	$\phi_1 = \frac{8.25 \times 240}{2}$	R_1' (8° 15′)	$2.8420\bar{4}$
		$\cos \phi_1$	$9.9934\bar{8}$
	$= 9.°9 = 9°54'$	$\sec \Delta_1$	0.09204
		846.39	2.92757
	$Y_1 = 240 \times .05747$	Y_1	1.13969
	$= 13.79$	$\sec \Delta_1$	0.09204
		17.05	1.23173
863.44		863.44	
$m_1 =$ 0.86			

[Eq. 43] $\phi_2 = \frac{4.05 \times 2.4}{2.}$ R_2 3.15615

vers 32° 9.18170

$= 4°.86 = 4°51'.6$

217.700 . . . . 2.33785

$Y_2 = .02826 \times 240.$ $m_1 = 0.86$ 9.93450

$= 6.782$ cos 32° 9.92842

0.729 . . . . . 9.86292

$Y_2 = 6.782$

7.511 7.511

210.189 . . . . 2.32261

nat. cos ϕ_2 = .99640

nat. cos Δ_2 = .84805

.14835 9.17129

$R_2' = 1416.84$ [4° 2′ 41″] 3.15132

Eq. 39] $X_1 = 239.286$ $X_1 = .997024 \times 240$

$= 239.286$ R_1' 2.84204

sin ϕ_1 9.23535

119.505 . . . . . 2.07739

R_1' 2.84204

cos ϕ_1 9.99348

tan $\frac{1}{2}\Delta[\Delta_1 = 36°]$ 9.86126

497.489 . . . . 2.69678

$R_1 = 716.78$

$Y_1 = 13.70$

703.08 2.84700

tan $\frac{1}{2}\Delta$ 9.86126

736.775

630.325 510.820 . . . . . 2.70826

$AQ_1 = 106.450$ 630.325

[Eq. 39] R_2' 3.15132

$X_2 = .999284 \times 240$ sin ϕ_2 8.92799

$= 239.828$ 120.035 . . . . 2.07931

R_2' 3.15132

cos ϕ_2 9.99843

tan $\frac{1}{2}\Delta(\Delta_2 = 32°)$ 9.79579

882.145 . . . . 2.94554

$R_2 = 1432.7$

$Y_2 = 6.8$

1425.9 3.15400

tan $\frac{1}{2}\Delta$ 9.79579

891.00 . . . . 2.94988

1121.973 1011.03

1011.03

$AQ_2 = 110.94$

For the length of the old track we have:

$$100\frac{\Delta_1}{D_1}=100\frac{36°}{8°}= 450.$$

$$100\frac{\Delta_2}{D_2}=100\frac{32°}{4°}= 800.$$

$$AQ_1 = 106.45$$

$$AQ_2 = \underline{110.94}$$

$$=1467.39$$

For the length of the new track we have:

$$100\frac{\Delta_1-\phi_1}{D'_1}=100\frac{26°.1}{8°.25} = 316.36$$

$$100\frac{\Delta_2-\phi_2}{D'_2}=100\frac{27.14}{4°.044} = 671.11$$

Spiral on 8° 15′ curve = 240.00

Spiral on 4° 02′ 41″ curve = 240.00

Length of new track = 1467.47

Length of old track = 1467.39

Excess in length of new track = 0.08 feet.

Since the new track is slightly longer than the old, it shows that the new track runs too far *outside* the old track at the *P.C.C.* On the other hand the offset m is only 0.86. The maximum amount by which the new track comes *inside* of the old track at two points, presumably not far from Z' and Z, is very difficult to determine exactly. Since it is desirable that the maximum offsets (inside and outside) should be made as nearly equal as possible, this feature should not be sacrificed to an effort to make the two lines of precisely equal length so that the rails need not be cut. Therefore, if it is found that the offsets inside the old track are nearly equal to m (0.86), the above figures should stand. Otherwise m may be diminished (and the above excess in length of track diminished) by *increasing* R_1' very slightly and making the necessary consequent changes.

VERTICAL CURVES

84. Necessity for their use. Whenever there is a change in the rate of grade, it is necessary to eliminate the angle that would be formed at the point of change and to connect the two grades by a curve. This is especially necessary at a sag between two grades, since the shock caused by abruptly forcing an upward motion to a rapidly moving heavy train is very severe both to the track and to the rolling stock. The necessity for vertical curves was even greater in the days when link couplers were in universal use and the "slack" in a long train was very great,

Under such circumstances, when a train was moving down a heavy grade the cars would crowd ahead against the engine. Reaching the sag, the engine would begin to pull out, rapidly taking out the slack. Six inches of slack on each car would amount to several feet on a long train, and the resulting jerk on the couplers, especially those near the rear of the train, has frequently resulted in broken couplers or even derailments. A vertical curve will practically eliminate this danger if the curve is made long enough.

85. Required length. Theoretically the length should depend on the change in the rate of grade and on the length of the longest train on the road. A sharp change in the rate of grade requires a long curve; a long train requires a long curve; but since the longest trains are found on roads with light grades and small changes of grade, the required length is thus somewhat equalized. The A.R.E.A. rule is: "On class A roads (see § 198) rates of change of 0.1 per cent per station on summits and 0.05 per cent per station in sags should not be exceeded. On minor roads 0.2 per cent per station on summits and 0.1 per cent per station in sags may be used." When changing from a down grade to an up grade (or vice versa) the change of grade equals the numerical sum of the two rates of grade. For example, if a 0.5 per cent down grade is followed by a 0.7 per cent up grade, the road being a "minor" road, then, by the above rule the length of the curve should be at least $[0.5-(-0.7)]\div 0.1=12$ stations or 1200 feet. Added length increases the amount of earthwork required both in cuts and fills, but the resulting saving in operating expenses will always justify a considerable increase.

86. Form of curve. In Fig. 39 assume that A and C, equi-

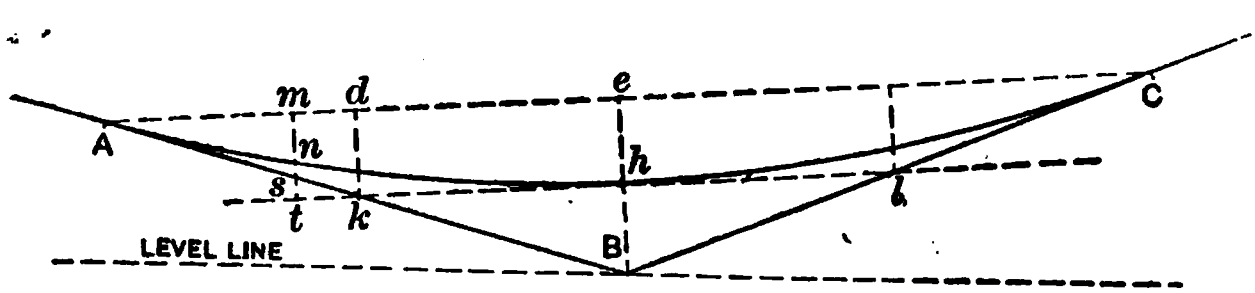


Fig. 39.

distant from B, are the extremities of the vertical curve. Bisect AC at e; draw Be and bisect it at h. Bisect AB and BC at k and l. The line kl will pass through h. A parabola may be drawn with its vertex at h which will be tangent to AB and BC at A and C. It may readily be shown * from the properties of a parabola that if an ordinate be drawn at *any* point (as at n) we will have

* See note at end of this chapter.

a parabola that if an ordinate be drawn at *any* point (as at n) we will have

$$sn : eh \text{ (or } hB) : : \overline{Am}^2 : \overline{Ae}^2$$

or

$$sn = eh\frac{\overline{Am}^2}{\overline{Ae}^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

In Fig. 39 the grades are necessarily exaggerated enormously. With the proportions found in practice we may assume that ordinates (such as mt, eB, etc.) are perpendicular to either grade, as may suit our convenience, without any appreciable error. In the numerical case given below, the variation of these ordinates from the vertical is 0° 07′, while the effect of this variation on the calculations in this case (as in the most extreme cases) is absolutely inappreciable. It may easily be shown that the angle CAB = half the *algebraic difference* of the rates of grade. Call the difference, expressed in per cent of grade, r; then $CAB = \frac{1}{2}r$. Let l = length (in "stations" of 100 feet) of the line AC, which is practically equal to the horizontal measurement. Since the angle CAB is one-half the total change of grade at B, it follows that $Be = \frac{1}{2}l \times \frac{1}{2}r$ Therefore

$$Bh = \tfrac{1}{8}lr. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

Since Bh (or eh) and Ae are constant for any one curve, the correction sn at any point (see Eq. 44) equals a constant times $\overline{Am}^2$.

87. Numerical example. Assume that B is located at Sta. 16+20; that the grade of AB is −0.5%, and of BC +0.7%; also that the elevation of B above the datum plane is 162.6. Then the algebraic difference of the grades, r, $=0.7-(-0.5)=1.2$; $l=12$. $Bh=\frac{1}{8}lr=\frac{1}{8}\times 12\times 1.2=1.8$. A is at Sta. 10+20 and its elevation is $162.6+(6\times 0.5)=165.6$; C is at Sta. 22+20 and its elevation is $162.6+(6\times 0.7)=166.8$. The elevation of Sta. 11 is found by adding sn to the elevation of s on the straight grade line. The constant $(eh \div \overline{Ae}^2)$ equals in this case $1.8 \div 600^2 = \frac{1}{200000}$. Therefore the curve elevations are

A, Sta.	10+20,	$162.6+(6.00\times 0.5)$	$=165.60$
	11	$165.6-(0.80\times 0.5)+\frac{1}{200000}\ 80^2$	$=165.23$
	12	$165.6-(1.80\times 0.5)+\frac{1}{200000}\ 180^2$	$=164.86$
	13	$165.6-(2.80\times 0.5)+\frac{1}{200000}\ 280^2$	$=164.59$
	14	$165.6-(3.80\times 0.5)+\frac{1}{200000}\ 380^2$	$=164.42$
	15	$165.6-(4.80\times 0.5)+\frac{1}{200000}\ 480^2$	$=164.35$
	16	$165.6-(5.80\times 0.5)+\frac{1}{200000}\ 580^2$	$=164.38$

B,	16+20,	162.6+ 1.80	=164.40
	17	$166.8-(5.20\times0.7)+\frac{1}{100000}520^2$	=164.51
	18	$166.8-(4.20\times0.7)+\frac{1}{200000}420^2$	=164.74
	19	$166.8-(3.20\times0.7)+\frac{1}{200000}320^2$	=165.07
	20	$166.8-(2.20\times0.7)+\frac{1}{100000}220^2$	=165.50
	21	$166.8-(1.20\times0.7)+\frac{1}{100000}120^2$	=166.03
	22	$166.8-(0.20\times0.7)+\frac{1}{200000}20^2$	=166.66
C,	22+20,	162.6+(6.00×0.7)	=166.80

DEMONSTRATION OF EQ. 44.

The general equation of a parabola passing through the point n (Fig. 36) may be written

$$y^2 + y_n{}^2 = 2p(x + x_n),$$

from which

$$x_n = \frac{y^2}{2p} + \frac{y_n{}^2}{2p} - x.$$

When $x = x_A$, $y = y_A$, and we have

$$x_n = \frac{y_A{}^2}{2p} + \frac{y_n{}^2}{2p} - x_A.$$

The general equation of a tangent passing through the point A may be written

$$yy_A = p(x + x_A),$$

from which

$$x = \frac{yy_A}{p} - x_A.$$

When $x = x_s$, $y = y_s[= y_n]$, and we have

$$x_s = \frac{y_n y_A}{p} - x_A.$$

$$\overline{sn} = x_n - x_s = \frac{y_A{}^2 + y_n{}^2 - 2y_n y_A}{2p}$$

$$= \frac{(y_A - y_n)^2}{2p} = \frac{\overline{Am}^2}{2p}$$

$$2p = \frac{y_A{}^2}{x_A} = \frac{\overline{Ae}^2}{\overline{eh}}.$$

$$\therefore\quad \overline{sn} = \overline{eh}\frac{\overline{Am}^2}{\overline{Ae}^2}.$$

This proves the general proposition that if secants are drawn parallel to the axis of x, intersecting a parabola and a tangent to it, the intercepts between the tangent and the parabola are proportional to the square of the distances (measured parallel to y) from the tangent point.

CHAPTER III.

EARTHWORK.

FORM OF EXCAVATIONS AND EMBANKMENTS.

88. Usual form of cross-section in cut or fill. The normal form of cross-section in cut is as shown in Fig. 40, in which $e \dots g$ represents the natural surface of the ground, no matter

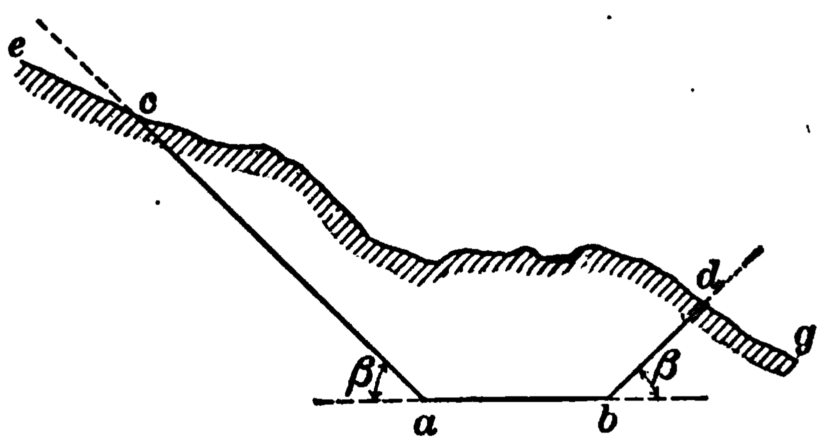


FIG. 40.

how irregular; *ab* represents the position and width of the required roadbed; *ac* and *bd* represent the "side slopes" which begin at *a* and *b* and which intersect the natural surface at such

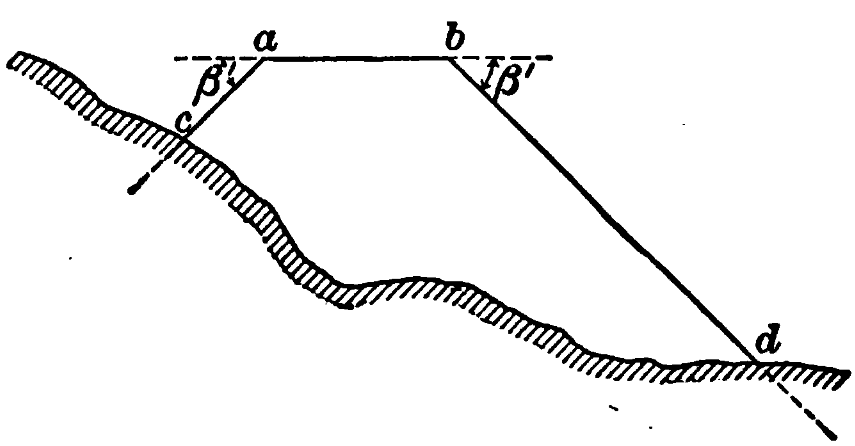


FIG. 41.

points (*c* and *d*) as will be determined by the required slope angle (β).

The normal section in fill is as shown in Fig. 41. The points *c* and *d* are likewise determined by the intersection of the re-

quired side slopes with the natural surface. In case the required roadbed (ab in Fig. 42) intersects the natural surface, both cut

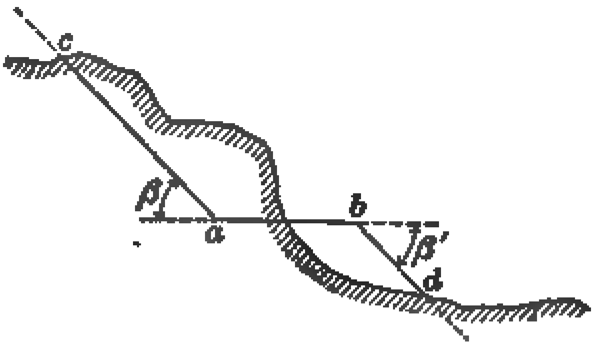


FIG. 42.

and fill are required, and the points c and d are determined as before. Note that β and β' are not necessarily equal. Their proper values will be discussed later.

89. **Terminal pyramids and wedges.** Fig. 43 illustrates the general form of cross-sections when there is a transition from cut to fill. $a \ldots g$ represents the grade line of the road which

FIG. 43.

passes from cut to fill at d. sdt represents the surface profile. A cross-section taken at the point where either side of the roadbed *first* cuts the surface (the point m in this case) will usually be triangular if the ground is regular. A similar cross-section should be taken at o, where the other side of the roadbed cuts the surface. In general the earthwork of cut and fill terminates

in two pyramids. In Fig. 43 the pyramid vertices are at *n* and *k*, and the bases are *lhm* and *opq*. The roadbed is generally wider in cut than in fill, and therefore the section *lhm* and the altitude *ln* are generally greater than the section *opq* and the altitude *pk*. When the line of intersection of the roadbed and natural surface (*nodkm*) becomes perpendicular to the axis of the roadbed (*ag*) the pyramids become wedges whose bases are the nearest convenient cross-sections.

90. Slopes. a. Cuttings. The required slopes for cuttings vary from perpendicular cuts, which may be used in hard rock which will not disintegrate by exposure, to a slope of perhaps 4 horizontal to 1 vertical in a soft material like quicksand or in a clayey soil which flows easily when saturated. For earthy materials a slope of 1 : 1 is the maximum allowable, and even this should only be used for firm material not easily affected by saturation. A slope of 1½ horizontal to 1 vertical is a safer slope for average earthwork. It is a frequent blunder that slopes in cuts are made too steep, and it results in excessive work in clearing out from the ditches the material that slides down, at a much higher cost per yard than it would have cost to take it out at first, to say nothing of the danger of accidents from possible landslides.

b. Embankments. The slopes of an embankment vary from 1 : 1 to 1.5 : 1. A rock fill will stand at 1 : 1, and if some care is taken to form the larger pieces on the outside into a rough dry wall, a much steeper slope can be allowed. This method is sometimes a necessity in steep side-hill work. Earthwork embankments generally require a slope of 1½ to 1. If made steeper at first, it generally results in the edges giving way, requiring repairs until the ultimate slope is nearly or quite 1½ : 1. The difficulty of incorporating the added material with the old embankment and preventing its sliding off frequently makes these repairs disproportionately costly.

91. Compound sections. When the cut consists partly of earth and partly of rock, a compound cross-section must be made. If borings have been made so that the contour of the rock surface is accurately known, then the true cross-section may be determined. The rock and earth should be calculated separately, and this will require an accurate knowledge of where the rock "runs out"—a difficult matter when it must be deter-

mined by boring. During construction the center part of the earth cut would be taken out first and the cut widened until a sufficient width of rock surface had been exposed so that the rock cut would have its proper width and side slopes. Then the earth slopes could be cut down at the proper angle. A "berm" of about three feet should be left on the edges of the rock cut as

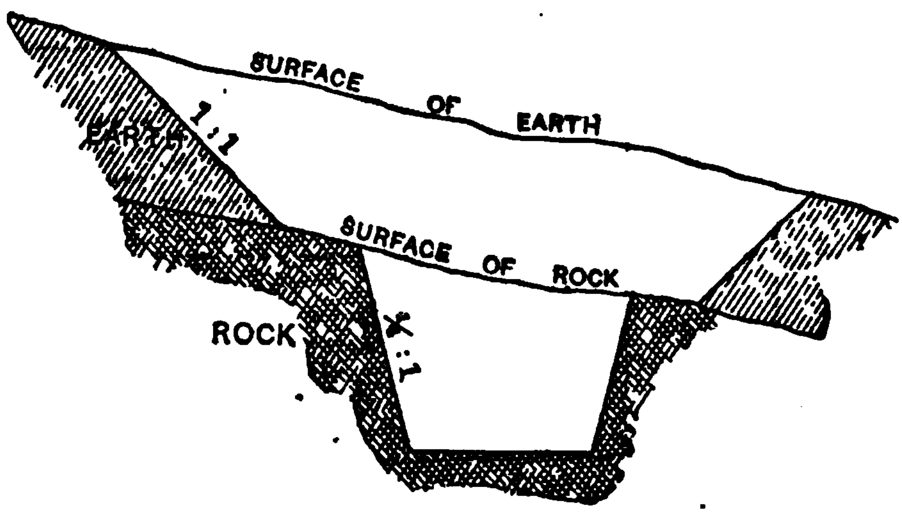


FIG. 44.

a margin of safety against a possible sliding of the earth slopes. After the work is done, the amount of excavation that has been made is readily computable, but accurate preliminary estimates are difficult. The area of the cross-section of earth in the figure must be determined by a method similar to that developed for borrow-pits (see § 120).

92. Width of roadbed. Owing to the large and often disproportionate addition to volume of cut or fill caused by the addition of even one foot to the width of roadbed, there is a natural tendency to reduce the width until embankments become unsafe and cuts are too narrow for proper drainage. The cost of maintenance of roadbed is so largely dependent on the drainage of the roadbed that there is true economy in making an ample allowance for it. The practice of some of the leading railroads of the country in this respect is given in the following table, in which are also given some data belonging more properly to the subject of superstructure.

It may be noted from the table that the average width for an *earthwork* cut, single track, is about 24.7 feet, with a minimum of 19 feet 2 inches. The widths of fills, single track, average over 18 feet, with numerous minimums of 16 feet. The widths for double track may be found by adding the distance between track centers, which is usually 13 feet.

WIDTH OF ROADBED FOR SINGLE AND DOUBLE TRACK—SLOPE RATIOS—DISTANCES BETWEEN TRACK CENTERS.

Road.	Single Track.		Double Track.		Slope Ratios.		Distance Between Track Centers.
	Cut.	Fill.	Cut.	Fill.	Cut.	Fill.	
A. T. & Santa Fé	28' earth	20	....	....	1 : 1	1.5 : 1	
....	22' rock	....	....	....	½ : 1		
Quincy ..	14+(2×5)*	16	28+(2×5)	30	1.5 : 1	1.5 : 1	14'
St. Paul...	18+(2×6)	20 to 24	31+(2×6)	33 to 37	1.5 : 1	1.5 : 1	13'
....	20+(2×4)	20	33+(2×4)	33	1 5 : 1	1.5 : 1	13'
....	32 5	18	....	....	1.5 · 1	1.5 : 1	
....	20' 8½"	20' 8½"	33' 8½"	33' 8½"	1.5 : 1	1.5 : 1	13'
....	14+(2×3 5)	16	27+(2×3.5)	30	1 : 1	1.5 : 1	13'
Southern.	....	....	33+(2×7.25)	32	1 5 . 1	1.5 : 1	13'
....	13+(2×4.5)	16	....	....	1 : 1	1.5 : 1	
....	....	....	(33+2×2 5)	33	1.5 : 1	1.5 : 1	13'
N. Y., N. H. & H.	....	....	30	30	1 5 1	1.5 : 1	12'
Norfolk & Western..........	21' 2" earth	17' 2"	34' 2" earth	30' 2"	1.5 : 1	1.5 : 1	13'
	16' rock	....	29' rock	....	½ : 1 ¼ : 1	....	13'
Pennsylvania..........	19' 2" light traffic	19' 2"					
	27' 2" heavy "	19' 2"	31' 4"+(2×4)	31' 4"	1.5 : 1	1.5 : 1	12' 2"
Union Pacific..........	14+(2×3.5)	16	....	....	1 : 1	1.5 : 1	

* (2×5) signifies two ditches each 5 feet wide, the following cases should be interpreted similarly.

Am. Rwy. Eng. Assoc. standard for Class A roads, 20 feet for single track fill, 20+width of ditches for cut: 16 feet for Class B roads and 14 feet for Class C roads. See § 234 for classification

93. Form of subgrade. Specifications (or the cross-section drawings) formerly required that the subgrade should have a curved form, convex upward, or that it should slope outward from a slight ridge in the center, with the evident purpose of draining to the sides all water which might percolate through the ballast. If the subsoil were hard and impenetrable by the ballast, the method might answer, but experience has shown that, with ordinary subsoils, the ballast immediately under each rail is forced a little deeper into the subsoil by the passage of each train. Periodical retamping of ballast under the ends of the ties, and little or no tamping under the center, only adds to the accumulation under each rail. A cross-section of a very old roadbed will frequently show twice as much depth of ballast under the rails as there is under the center. This method of tamping quickly obliterates the original line of demarcation between ballast and subsoil and any expected improvement in drainage due to sloping subsoil is not realized. Therefore the A.R.E.A. specifications call for *flat* subgrades.

94. Ditches. "The stability of the track depends upon the strength and permanence of the roadbed and structures upon which it rests; whatever will protect them from damage or prevent premature decay should be carefully observed. The worst enemy is WATER, and the further it can be kept away from the track, or the sooner it can be diverted from it, the better the track will be protected. Cold is damaging only by reason of the water which it freezes; therefore the first and most important provision for good track is drainage." (Rules of the Road Department, Illinois Central R. R.)

The form of ditch generally prescribed has a flat bottom 12″ to 24″ wide and with sides having a minimum slope, except in rock-work, of 1 : 1, more generally 1.5 : 1 and sometimes 2 : 1. Sometimes the ditches are made V-shaped, which is objectionable unless the slopes are low The best form is evidently that which will cause the greatest flow for a given slope, and this will evidently be the form in which the ratio of area to wetted perimeter is the largest. The semicircle fulfills this condition better than any other form, but the nearly vertical sides would be difficult to maintain. (See Fig. 45.) A ditch, with a flat bottom and such

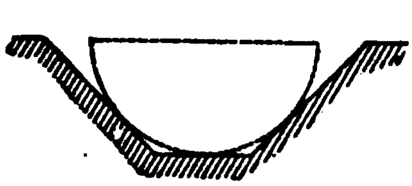


FIG. 45.

slopes as the soil requires, which approximates to the circular form will therefore be the best.

When the flow will probably be large and at times rapid it will be advisable to pave the ditches with stone, especially if the soil is easily washed away. Six-inch tile drains, placed 2′ under the ditches, are prescribed on some roads. (See Fig. 46.) No better method could be devised to insure a dry subsoil. The ditches through cuts should be led off at the end of the cut so that the adjacent embankment will not be injured.

Wherever there is danger that the drainage from the land above a cut will drain down into the cut, a ditch should be made near the edge of the cut to intercept this drainage, and this ditch should be continued, and paved if necessary, to a point where the outflow will be harmless. Neglect of these simple and inexpensive precautions frequently causes the soil to be loosened on the shoulders of the slopes during the progress of a heavy rain, and results in a landslide which will cost more to repair than the ditches which would have prevented it for all time.

Ditches should be formed along the bases of embankments; they facilitate the drainage of water from the embankment, and may prevent a costly slip and disintegration of the embankment.

95. Effect of sodding the slopes, etc. Engineers are unanimously in favor of rounding off the shoulders and toes of embankments and slopes, sodding the slopes, paving the ditches, and providing tile drains for subsurface drainage, all to be put in during original construction. (See Fig. 46.) Some of the highest grade specifications call for the removal of the top layer of vegetable soil from cuts and from under proposed fills to some convenient place, from which it may be afterwards spread on the slopes, thus facilitating the formation of sod from grass-seed. But while engineers favor these measures and their economic value may be readily demonstrated, it is generally impossible to obtain the authorization of such specifications from railroad directors and promoters. The addition to the original cost of the roadbed is considerable, but is by no means as great as the capitalized value of the extra cost of maintenance resulting from the usual practice. Fig. 46 is a copy of

designs * presented at a convention of the American Society of Civil Engineers by Mr. D. J. Whittemore, Past President of the Society and Chief Engineer of the Chi., Mil. & St. Paul

CUSTOMARY SECTION OF ROADBED ON EMBANKMENT.

FIG. 46.—"WHITTEMORE ON RAILWAY EXCAVATION AND EMBANKMENTS" Trans. Am. Soc. C. E., Sept. 1894.

R. R. The "customary sections" represent what is, with some variations of detail, the practice of many railroads. The "pro-

* Trans. Am. Soc. Civil Eng., Sept. 1894.

posed sections" elicited unanimous approval. They should be adopted when not prohibited by financial considerations.

EARTHWORK SURVEYS.

96. Relation of actual volume to the numerical result. It should be realized at the outset that the accuracy of the result of computations of the volume of any given mass of earthwork has but little relation to the accuracy of the mere numerical work. The process of obtaining the volume consists of two distinct parts. In the first place it is assumed that the volume of the earthwork may be represented by a more or less complicated geometrical form, and then, secondly, the volume of such a geometrical form is computed. A desire for simplicity (or a frank willingness to accept approximate results) will often cause the cross-section men to assume that the volume may be represented by a very simple geometrical form which is really only a very rough approximation to the true volume. In such a case, it is only a waste of time to compute the volume with minute numerical accuracy. One of the first lessons to be learned is that economy of time and effort requires that the accuracy of the numerical work should be kept proportional to the accuracy of the cross-sectioning work, and also that the accuracy of both should be proportional to the use to be made of the results. The subject is discussed further in § 125.

97. Prismoids. To compute the volume of earthwork, it is necessary to assume that it has some geometric form whose volume is readily determinable. The general method is to consider the volume as consisting of a series of *prismoids*, which are solids having parallel plane ends and bounded by surfaces which may be formed by lines moving continuously along the edges of the bases These surfaces may also be considered as the surfaces generated by lines moving along the edges joining the corresponding points of the bases, these edges being the directrices, and the lines being always parallel to either base, which is a plane director. The surfaces thus developed may or may not be planes. The volume of such a prismoid is readily determinable (as explained in § 110 *et seq.*), while its definition is so very general that it may be applied to very rough ground. The "two plane ends" are sections perpendicular to the axis of the road. The roadbed and side slopes (also plane) form three of

the side surfaces. The only approximation lies in the degree of accuracy with which the plane (or warped) surfaces coincide with the actual surface of the ground between these two sections. This accuracy will depend (*a*) on the number of points which are taken in each cross-section and the accuracy with which the lines joining these points coincide with the actual cross-sections; (*b*) on the skill shown in selecting places for the cross-sections so that the warped surfaces shall coincide as nearly as possible with the surface of the ground. In fairly smooth country, cross-sections every 100 feet, placed at the even stations, are sufficiently accurate, and such a method simplifies the computations greatly; but in rough country cross-sections must be interpolated as the surface demands. As will be explained later, carelessness or lack of judgment in cross-sectioning will introduce errors of such magnitude that all refinements in the computations are utterly wasted.

98. **Cross-sectioning.** The process of cross-sectioning consists in determining at any place the intersection by a vertical plane of the prism of earth lying between the roadbed, the side slopes, and the natural surface. The intersection with the road-

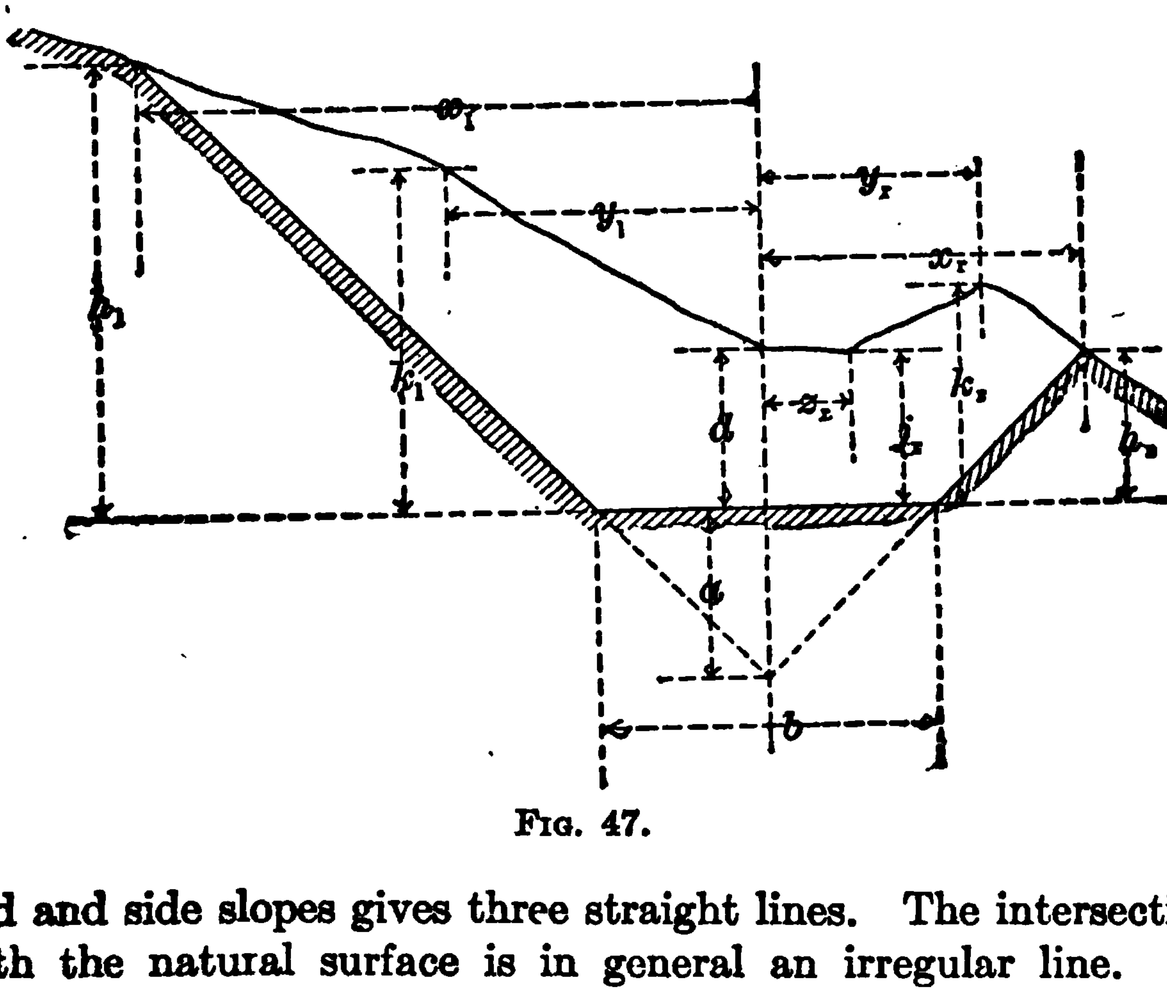


FIG. 47.

bed and side slopes gives three straight lines. The intersection with the natural surface is in general an irregular line. On smooth regular ground or when approximate results are acceptable this line is assumed to be straight. According to the irreg-

ularity of the ground and the accuracy desired more and more "intermediate points" are taken.

The distance (d in Fig. 47) of the roadbed below (or above) the natural surface at the center is known or determined from the profile or by the computed establishment of the grade line. The distances out from the center of all "breaks" are determined with a tape. To determine the elevations for a cut, set up a level at any convenient point so that the line of sight is higher than any point of the cross-section, and take a rod reading on the center point. This rod reading added to d gives the height of the instrument (H. I.) above the roadbed. Subtracting from H. I. the rod reading at any "break" gives the height of that point above the roadbed (h_l, k_l, h_r, etc.). This is true for all cases in excavation. For fill, the rod reading at center minus d equals the H. I., which may be positive or negative. When negative, add to the "H. I." the rod readings of the intermediate points to get their depths below "grade"; when positive, subtract the "H. I." from the rod readings.

The heights or depths of these intermediate points above or below grade need only be taken to the nearest tenth of a foot, and the distances out from the center will frequently be sufficiently exact when taken to the nearest foot. The roughness of the surface of farming land or woodland generally renders useless any attempt to compute the volume with any greater accuracy than these figures would imply unless the form of the ridges and hollows is especially well defined. The position of the slope-stake points is considered in the next section. Additional discussion regarding cross-sectioning is found in § 107.

99. Position of slope-stakes. The slope-stakes are set at the intersection of the required side slopes with the natural surface, which depends on the center cut or fill (d). The distance of the slope-stake from the center for the lower side is $x = \frac{1}{2}b + s(d+y)$; for the up-hill side it is $x' = \frac{1}{2}b + s(d-y')$. s is the "slope ratio" for the side slopes, the ratio of horizontal to vertical. In the above equation both x and y are unknown. Therefore some position must be found by trial which will satisfy the equation. As a preliminary, the value of x for the point $a = \frac{1}{2}b + sd$, which is the value of x for *level* cross-sections. In the case of fills on sloping ground the value of x on the *down-hill* side is *greater* than this; on the *up-hill* side it is *less*. The difference in distance is s times the difference of elevation. Take a

numerical case corresponding with Fig. 48. The rod reading on c is 2.9; $d=4.2$; therefore the telescope is $4.2-2.9=1.3$ *below* grade. $s=1.5:1$, $b=16$. Hence for the point a (or for level ground) $x=\frac{1}{2}\times16+1.5\times4.2=14.3$. At a distance out of 14.3 the ground is seen to be about 3 feet lower, which will not only require $1.5\times3=4.5$ more, but enough additional distance so that the added distance shall be 1.5 times the additional drop. As a first trial the rod may be held at 24 feet out and a reading of, say, 8.3 is obtained. $8.3+1.3=9.6$, the depth of the point below grade. The point on the slope line (n) which has this depth below grade is at a distance from the center

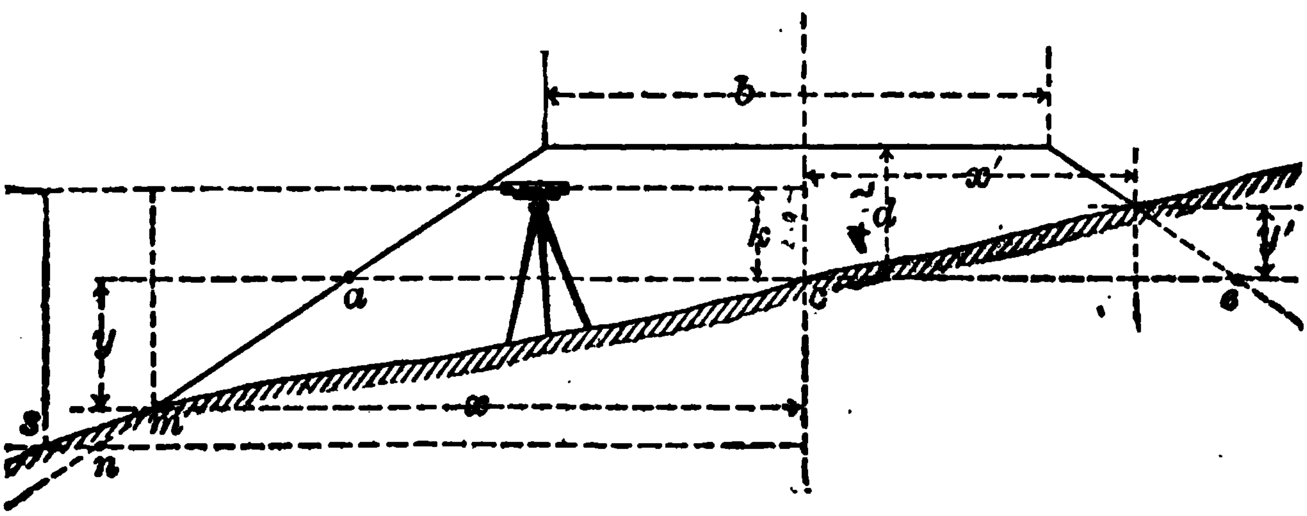


FIG. 48.

$x=8+1.5\times9.6=22.4$. The point on the surface (s) having that depth is 24 feet out. Therefore the true point (m) is nearer the center. A second trial at 20.5 feet out gives a rod reading of, say, 7.1 or a depth of 8.4 below grade. This corresponds to a distance out of 20.6. Since the natural soil (especially in farming lands or woods) is generally so rough that a difference of elevation of a tenth or so may be readily found by slightly varying the location of the rod (even though the distance from the center is the same), it is useless to attempt too much refinement, and so in a case like the above the combination of 8.4 below grade and 20.6 out from center may be taken to indicate the proper position of the slope-stake. This is usually indicated in the form of a fraction, the distance out being the denominator and the height above (or below) grade being the numerator; the fact of *cut* or *fill* may be indicated by *C* or *F*. Ordinarily a second trial will be sufficient to determine with sufficient accuracy the true position of the slope-stake. Experienced men will frequently estimate the required distance

out to within a few tenths at the first trial. The left-hand pages of the note-book should have the station number, surface elevation, grade elevation, center cut or fill, and rate of grade. The right-hand pages should be divided in the center and show the distances out and heights above grade of all points, as is illustrated in § 84. The notes should read UP the page, so that when looking ahead along the line the figures are in their proper relative position. The "fractions" farthest from the center line represent the slope-stake points.

100. Setting slope-stakes by means of "automatic" slope-stake rods. The equipment consists of a specially graduated tape and a specially constructed rod. The tape may readily be prepared by marking on the *back* side of an ordinary 50-foot tape which is graduated to feet and tenths. Mark "0" at "$\frac{1}{2}b$" from the tape-ring. Then graduate from the zero backward, at true scale, to the ring. Mark off "feet" and "tenths" on a scale proportionate to the slope ratio. For example, with the usual slope ratio of 1.5:1 each "foot" would measure 18 inches and each "tenth" in proportion.

The rod, 10 feet long, is shod at each end and has an endless tape passing within the shoes at each end and over pulleys—to reduce friction. The tape should be graduated in feet and tenths, from 0 to 20 feet—the 0 and 20 coinciding. By moving the tape so that 0 is at the bottom of the rod—or (practically) so that the 1-foot mark on the tape is one foot above the bottom of the shoe, an index mark may be placed on the back of the rod (say at 15—on the tape) and this readily indicates when the tape is "set at zero."

The method of use may best be explained from the figure and from the explicit rules as stated. The proof is given for two assumed positions of the level.

(1) Set up the level so that it is higher than the "center" and (if possible) higher than both slope-stakes, but not more than a rod-length higher. On very steep ground this may be impossible and each slope-stake must be set by separate positions of the level.

(2) Set the rod-tape at zero (i.e., so that the 15-foot mark on the *back* is at the index mark).

(3) Hold the rod at the center-stake (B) and note the reading (n_1 or n_2). Consider n to be always plus; consider d to be plus for cut and minus for fill.

(4) *Raise* the tape on the *face* side of the rod $(n+d)$. Applied literally (and algebraically), when the level is *below* the roadbed (only possible for fill), $(n+d)=(n_2+(-d_f))=n_2-d_f$. This being numerically negative, the tape is *lowered* (d_f-n_2). With level at (1), for fill, $(n+d)=(n_1+(-d_f))=(n_1-d_f)$; this being positive, the tape is raised. With level at (1), for cut, the tape is raised (n_1+d_c). In every case the effect is the same as if the telescope were set at the elevation of the roadbed.

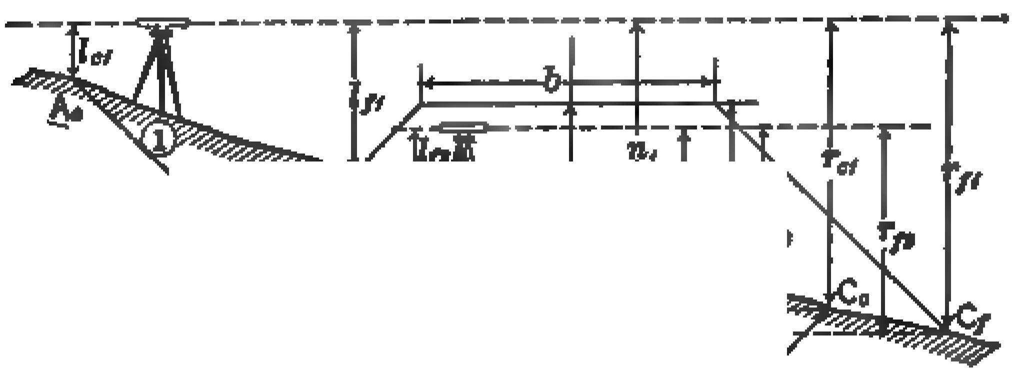


FIG. 49.

(5) With the special distance-tape, so held that its zero is $\frac{1}{2}b$ from the center, carry the rod out until the rod reading equals the reading indicated by the tape. Since in cut the tape is raised $(n+d)$, the zero of the rod-tape is always higher than the level (unless the rod is held at or below the elevation of the roadbed—which is only possible on side-hill work), and the reading at either slope-stake is necessarily *negative*. The reading for slope-stakes in fill is always positive.

(6) Record the rod-tape reading as the numerator of a fraction and the *actual distance* out (read directly from the *other* side of the distance-tape) as the denominator of the fraction.

Proof. Fill. Level at (1). Tape is raised (n_1-d_f). When rod is held at C_f, the rod reading is $+x$, which $=r_{f1}-(n_1-d_f)$. But the reading on the back side of the distance-tape is also x.

Fill. Level at (2). Tape is raised (n_2-d_f), i.e., it is *lowered* (d_f-n_2). When rod is held at C_f, the rod reading is $+x$, which similarly $=r_{f2}-(n_2-d_f)=r_{f2}+(d_f-n_2)$. Distance-tape as before.

Cut Level at (1). Tape is raised (n_1+d_c). When rod is held at C_c the rod reading is $-z$, which $= r_{c1}-(n_1+d_c)$, i.e., $z=(n_1+d_c)-r_{c1}$. The distance-tape will read z.

Side-hill work. It is easily demonstrated that the method, when followed literally, may be applied to side-hill work. although there is considerable chance for confusion and error, when, as is usual, $\frac{1}{2}b$ and the slope ratio are different for cut and for fill.

The method appears complicated at first, but it becomes mechanical and a time-saver when thoroughly learned. The advantages are especially great when the ground is fairly level transversely, but decrease when the difference of elevation of the center and the slope-stake is more than the rod length. By setting the rod-tape "at zero," the rod may always be used as an ordinary level rod and the regular method adopted, as in § 99. Many engineers who have thoroughly tested these rods are enthusiastic in their praise as a time-saver.

COMPUTATION OF VOLUME

§ 101. **Simple approximations.** The principles developed in §§ 96 and 97 show that, except where the ground is abnormally smooth and level, the earthwork to be excavated has a geometrical form whose volume cannot be *accurately* computed by any simple rule. The usual method is to consider that the volume is approximately measured by the product of the mean of the areas of two consecutive sections and the distance between those sections. When the ground is so regular that the error of such an approximation may be tolerated, or when only a rough approximation is necessary, such a computation may be accepted without correction. In any case, the "volume by averaging end areas" is computed as a first approximation and then correction is computed if desired. It should, therefore, be remembered that this approximate method, which is so common that it is often accepted without correction as the true volume, is never mathematically correct except under conditions which practically never exist. Whether a correction should be computed depends on the percentage of accuracy required, on the irregularity of the ground, and on the differences in the depth of adjacent center cuts—or fills. Experience gives the engineer such an idea of the probable amount of this correction under

any given conditions that he may judge when it is necessary to compute the correction in order to obtain the true volume with any desired degree of accuracy. The methods of computing this correction will be given later.

102. Approximate volume, level sections. When the country is very level or when only approximate preliminary results

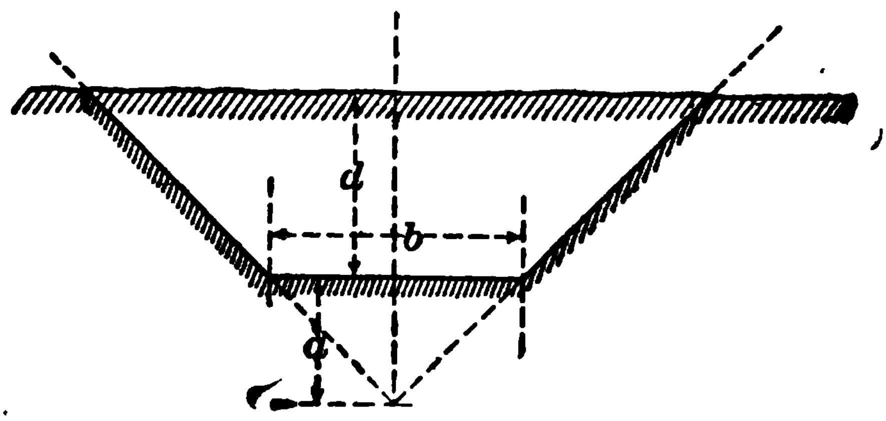


FIG. 50.

are required, it is sometimes assumed that the cross-sections are level. The area of the cross-section may be written

$$(a+d)^2 s-\frac{ab}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (46)$$

in which a, b and d are dimensions as indicated by the figure and s is the "slope ratio" or the ratio of the horizontal projection of the slope to the vertical. A table is very readily formed giving the area in square feet of a section of given depth and for any given width of roadbed and ratio of side slopes. Usually these tables give a number which equals that area times 100 and divided by 27, which is the volume in cubic yards of a prism 100 feet long and with that cross-sectional area. Table XVII is such a table.

The volume may also be readily determined (as illustrated in the following example), without the use of such a table; a table of squares will facilitate the work. Assuming the cross-sections at equal distances $(=l)$ apart, the total approximate volume for any distance will be

$$\frac{l}{2}[A_0+2(A_1+A_2+\ldots A_{n-1})+A_n]. \quad . \quad . \quad . \quad . \quad . \qquad (47)$$

103. Numerical example: level sections. Given the following center heights for the same number of consecutive stations 100 feet apart; width of roadbed 18 feet; slope $1\frac{1}{2}$ to 1.

The products in the fifth column may be obtained very readily and with sufficient accuracy by the use of the slide-rule described in § 106. The products should be considered as $(a+d)(a+d) \div \frac{1}{s}$. In this problem $s = 1\frac{1}{2}$, $\frac{1}{s} = .6667$. To apply the rule to the first case above, place 6667 on scale B over 89 on scale A, then opposite 89 on scale B will be found 118.8 on scale A. The position of the decimal point will be evident from an approximate mental solution of the problem.

Sta.	Center Height.	$a+d$	$(a+d)^2$	$(a+d)^2s$	Areas.
17	2.9	8.9	79.21	118.81	118.81
18	4.7	10.7	114.49	171.74	×2 = 343.48
19	6.8	12.8	163.84	245.76	×2 = 491.52
20	11.7	17.7	313.29	469.93	×2 = 939.86
21	4.2	10.2	104.04	156.06	×2 = 312.12
22	1.6	7.6	57.76	86.64	86.64

$\frac{ab}{2} = \frac{6 \times 18}{2} = 54$

2292.43
$10 \times 54 =$ 540
1752.43

$$\frac{1752.43 \times 100}{2 \times 27} = 3245 \text{ cub. yards} = \text{approx. vol.}$$

104. Equivalent sections. When sections are very irregular the following method may be used, especially if great accuracy

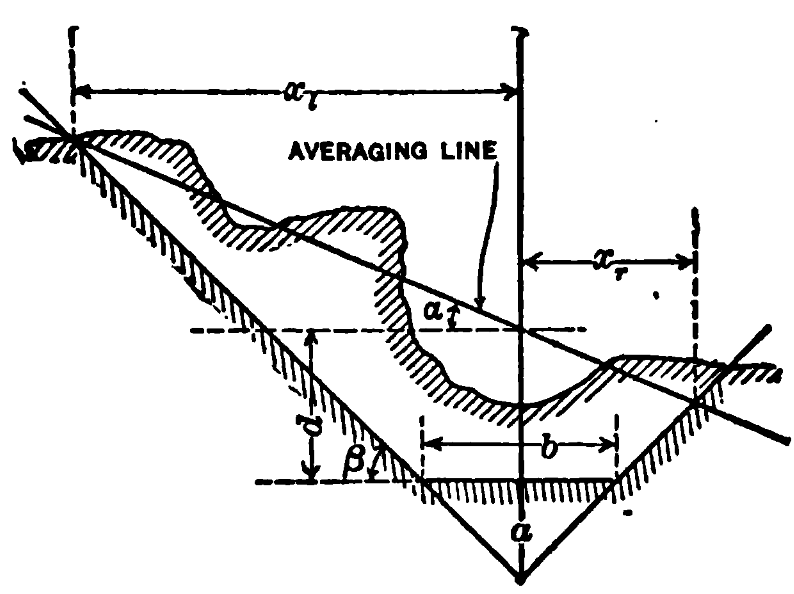


Fig. 51.—Equivalent Section.

is not required. The sections are plotted to scale and then a uniform slope line is obtained by stretching a thread so that the undulations are averaged and an *equivalent section* is obtained. Measure the distances (x_l and x_r) from the center. The area

may then be obtained independent of the center depth as follows: Let s = the slope ratio of the side slopes $=\cot\beta=\frac{b}{2a}$. (See Fig. 50.) Then the

$$\text{Area}=\frac{1}{2}\left(\frac{x_l+x_r}{s}\right)(x_l+x_r)-\frac{x_r}{s}\frac{x_r}{2}-\frac{x_l}{s}\frac{x_l}{2}-\frac{ab}{2}$$

$$=\frac{x_lx_r}{s}-\frac{ab}{2}. \qquad (48)$$

These approximate methods are particularly useful for rapidly making up monthly estimates, realizing that the inaccuracies, plus and minus, will be wiped out when the final computation is made by a more accurate method.

105. Three-level sections. The next method of cross-sectioning in the order of complexity, and therefore in the order of

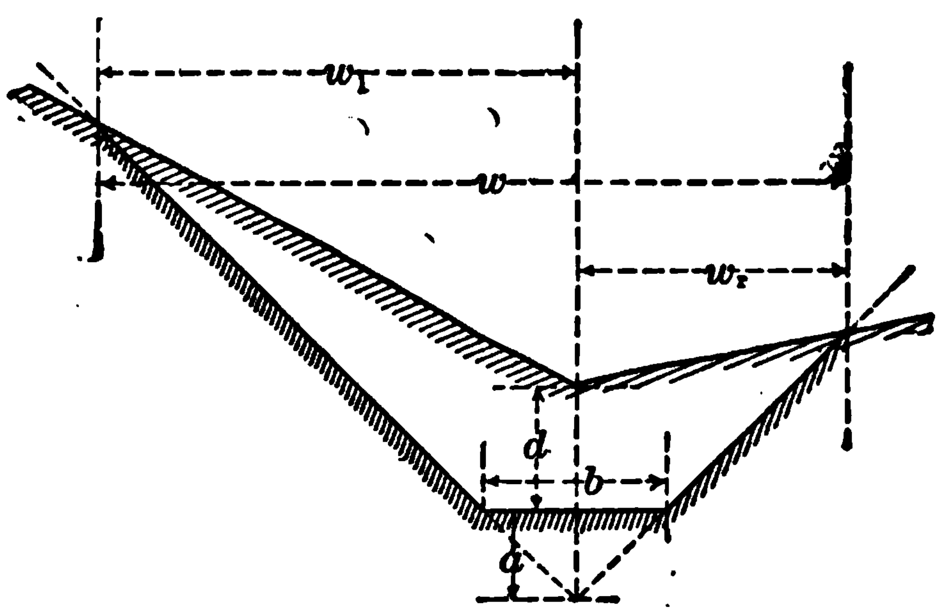


FIG. 52.

accuracy, is the method of three-level sections. The area of the section is $\frac{1}{2}(a+d)(w_r+w_l)-\frac{ab}{2}$, which may be written $\frac{1}{2}(a+d)w-\frac{ab}{2}$, in which $w=w_r+w_l$. If the volume is computed by averaging end areas, it will equal

$$\frac{l}{4}[(a+d')w'-ab+(a+d'')w''-ab]. \qquad (49)$$

Notes.				Approx. Volume.			Curvature Correction.†		
Station.	Center.	Left.	Right.	$a+d$	w	Yard	$x_l \sim x_r$	$\frac{V(x_l \sim x_r)}{3R}$	Curv. Corr.*
17	2.6F	$\frac{10.6F}{22.9}$	$\frac{0.8F}{8.2}$	7.3	31.1	210			
18	8.1F	$\frac{15.8F}{30.7}$	$\frac{3.4F}{12.1}$	12.8	42.8	507			+4
+40	10.7F	$\frac{20.2F}{37.3}$	$\frac{4.8F}{14.2}$	15.4	51.5	734			+4
19	6.4F	$\frac{14.0F}{23.0}$	$\frac{2.1F}{10.1}$	11.1	38.1	302			+5
20	3.7F	$\frac{5.8F}{15.7}$	$\frac{0.2F}{7.3}$	8.4	23.0	179			+3

Roadbed, 14′ wide in fill.
Slope 1½ to 1.

$a = \frac{b}{2s} = \frac{14}{3} = 4.7$

$\frac{25}{27}ab = 61.$

Approx. Vol. = 2094 —47 +16
* Pris. corr. = 47

True Vol. = 2047 (disregarding curv. corr.)†

* For the method of computing the prismoidal correction see § 114.

† For the derivation of the curvature correction, see § 124.

If we divide by 27 to reduce to cubic yards, we have, when $l=100$

$$\text{Vol}_{(\prime\ \ldots\ \prime\prime)}=\tfrac{25}{27}(a+d')w'-\tfrac{25}{27}ab+\tfrac{25}{27}(a+d'')w''-\tfrac{25}{27}ab.$$

For the next section

$$\text{Vol}_{(\prime\prime\ \ldots\ \prime\prime\prime)}=\tfrac{25}{27}(a+d'')w''-\tfrac{25}{27}ab+\tfrac{25}{27}(a+d''')w'''-\tfrac{25}{27}ab.$$

For a partial station length compute as usual and multiply result by $\dfrac{\text{length in feet}}{100}$.

The following example is given to illustrate the method of three-level sections.

In the first column of yards

$$210=\tfrac{25}{27}(a+d)w=\tfrac{25}{27}\times 7.3\times 31.1;$$

507, 734, etc., are found similarly;

$$595=210-61+507-61;$$

$$448=\tfrac{40}{100}(507-61+734-61);$$

$$602=\tfrac{60}{100}(734-61+392-61);$$

$$449=392-61+179-61.$$

The "*F*" in the columns of center heights, as well as the columns of "right" and "left" are inserted to indicate *fill* for all those points. Cut would be indicated by "*C*."

106. Computation of products. The quantities $\dfrac{25}{27}(a+d)w$ and $\dfrac{25}{27}ab$ represent in each case the product of two variable terms and a constant. These products are sometimes obtained from tables which are calculated for all ordinary ranges of the variable terms as arguments. A similar table computed for $\dfrac{25}{81}(d'-d'')(w''-w')$ will assist similarly in computing the prismoidal correction, see § 114. Prof. Charles L. Crandall, of Cornell University, is believed to be the first to prepare such a set of tables, which were first published in 1886 "Tables for the Computation of Railway and Other Earthwork." Another easy method of obtaining these products is by the use of a slide-rule. Any slide-rule, from which may be read directly three significant figures and from which the fourth may be read by estimation, can be utilized for this purpose. The Thacher or

the Stanley cylindrical rules are still more accurate. To illustrate its use, suppose $(a+d)=28.2$, and $w=62.4$; then

$$\frac{25}{27}(a+d)w=\frac{28.2\times 62.4}{1.08}.$$

Set 108 (which, being a constant of frequent use, may be specially marked) on the sliding scale (B) opposite 282 on the other scale (A), and then opposite 624 on scale B will be found 1629 on scale A, the 162 being read directly and the 9 read by estimation. Although strict rules may be followed for pointing off the final result, it only requires a very simple mental calculation to know that the result must be 1629 rather than 162.9 or 16290. For products less than 1000 cubic yards the result may be read directly from the scale; for products between 1000 and 5000 the result may be read directly to the nearest 10 yards, and the tenths of a division estimated. Between 5000 and 10,000 yards the result may be read directly to the nearest 20 yards, and the fraction estimated; but prisms of such volume will never be found as simple triangular prisms—at least, an assumption that any mass of ground was as regular as this would probably involve more error than would occur from faulty estimation of fractional parts. Facilities for reading as high as 10,000 cubic yards would not have been put on the scale except for the necessity of finding such products as $\frac{25}{27}(9.1\times 9.5)$, for example. This product would be read off from the same part of the rule as $\frac{25}{27}(91\times 95)$. In the first case the product (80.0) could be read directly to the nearest .2 of a cubic yard, which is unnecessarily accurate. In the other case, the product (8004) could only be obtained by estimating $\frac{4}{20}$ of a division.

The computation for the prismoidal correction (see § 114), may be made similarly except that the divisor is 3.24 instead of 1.08. For example, $\frac{25}{81}(5.5\times 11.7)=\frac{5.5\times 11.7}{3.24}$. Set the 324 on scale B (also specially marked like 108) opposite 55 on scale A, and proceed as before.

107. Approximate volume. Irregular sections. In cross-sectioning irregular sections, the distance from the center and the elevation above "grade" of every "break" in the cross-section must be observed. The area of the irregular section may be obtained by computing the area of the trapezoids (*five*, in Fig. 53) and subtracting the two external triangles. For Fig. 53 the area would be

$$\frac{h_l+k_l}{2}(x_l-y_l)+\frac{k_l+d}{2}y_l+\frac{d+j_r}{2}z_r+\frac{j_r+k_r}{2}(y_r-z_r)$$
$$+\frac{k_r+h_r}{2}(x_r-y_r)-\frac{h_l}{2}\left(x_l-\frac{b}{2}\right)-\frac{h_r}{2}\left(x_r-\frac{b}{2}\right).$$

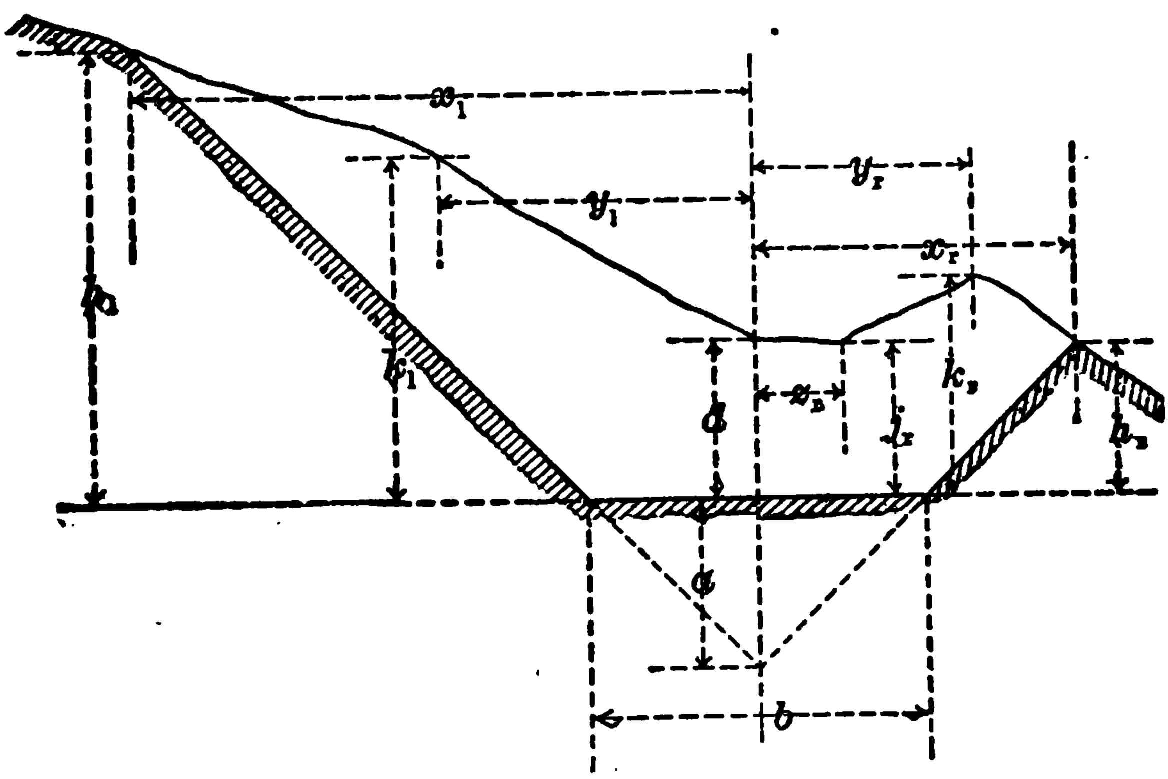


FIG. 53.

Expanding this and collecting terms, of which many will cancel, we obtain

$$\text{AREA}=\frac{1}{2}\Big[x_lk_l+y_l(d-h_l)+x_rk_r+y_r(j_r-h_r)$$
$$+z_r(d-k_r)+\frac{b}{2}(h_l+h_r)\Big]. \quad . \quad . \quad . \quad (50)$$

An examination of this formula will show a perfect regularity in its formation which will enable one to write out a similar formula for any section, no matter how irregular or how many points there are, without any of the preliminary work. The formula may be expressed in words as follows:

AREA equals one-half the sum of products obtained as follows:

the distance to each slope-stake times the height above grade of the point next inside the slope-stake;

the distance to each intermediate point in turn times the height of the point just inside minus the height of the point just outside;

finally, one-half the width of the roadbed times the sum of the slope-stake heights.

If one of the sides is perfectly regular from center to slope-stake, it is easy to show that the rule holds literally good. The "point next inside the slope-stake" in this case is the center; the intermediate terms for that side vanish. The *last term* must always be used. The rule holds good for three-level sections, in which case there are three terms, which may be reduced to two. Since these two terms are both variable quantities for each cross-section, the special method, given in § 105, in which one term ($\frac{1}{2}ab$) is a constant for all sections, is preferable for three-level sections. In the general method, each intermediate "break" adds another term.

108. Volume of an irregular prismoid. This is obtained by computing first the approximate volume by "averaging end areas" or by multiplying the length by the half sum of the end areas, as computed from Eq. (50). In other words, the Approx. volume $=\frac{100}{27}\times\frac{1}{2}$ (area$'$+area$''$). But since each area equals *one-half* the sum of products of width times height (see Eq. (50)) we may say that

$$\text{Approx. volume}=\frac{25}{27}\text{ (summation of } \textit{width} \text{ times } \textit{height}) \quad . \quad (51)$$

the terms of width times height being like those found within the bracket of Eq. (50).

As before, for partial station lengths, multiply the result by (length in feet $\div$ 100). There will be no constant subtractive term, $\frac{25}{27}ab$, as in § 105.

109. Numerical example; approximate volume; irregular sections. Assume the earthwork notes as given below where the roadbed is 18 feet wide in cut and the slope is 1½ to 1. Note that the stations read UP the page and that when the surveyor is looking ahead along the line the several combinations of *heights* and *distances out* have approximately the same relative position on the notebook as they have on the ground. For example, beginning at the bottom line (Sta. 16), the combination $\frac{8.9c}{21.4}$ means that the extreme left-hand point of that section (the "slope-stake") is 22.4 feet horizontally from the center and that it is 8.9 feet above the required roadbed. The cut (c) *would be* 8.9 feet to reach the roadbed, but of course the actual cutting is

zero at the slope stake. The next point is 12.0 feet horizontally from the center and 7.6 feet above the roadbed. The cut at the center is 6.8 feet. The combinations of dimensions on the right-hand side are to be interpreted similarly.

Sta.	Center {cut or fill.	Left.			Right.	
19	0.6c	$\frac{3.6c}{14.4}$			$\frac{0.1c}{4.2}$	$\frac{0.4c}{9.6}$
18	2.3c	$\frac{4.2c}{15.3}$	$\frac{6.8c}{8.4}$	$\frac{3.2c}{5.2}$		$\frac{1.2c}{10.8}$
17	7.6c	$\frac{8.2c}{21.3}$	$\frac{10.2c}{17.4}$	$\frac{8.0c}{6.1}$		$\frac{4.2c}{15.3}$
+42	10.2c	$\frac{12.2c}{27.3}$		$\frac{12.6c}{8.2}$	$\frac{6.2c}{7.5}$	$\frac{8.4c}{21.6}$
16	6.8c	$\frac{8.9c}{22.4}$		$\frac{7.6c}{12.0}$	$\frac{3.2c}{4.1}$	$\frac{2.6c}{12.9}$

The numerical computation is greatly facilitated by a systematic form as given below. For Sta. 16, the first term is "the distance to the left slope stake" (22.4) times "the height above grade of the point next inside" (the height being 7.6), and we place this pair of figures in the columns of "width" and "height." The "distance to the point next inside" is 12.0 and the "height of the point just inside (6.8) minus the height of the point just outside" (8.9) equals (−2.1) and these are the next pair of widths and heights. Taking $\frac{25}{27}$ of the product of each pair of numbers we have the numbers in the first column of "yards." The sum of all these numbers in the first and second groups multiplied by $\frac{42}{100}$ (that section being only 42 feet long) equals 378 cubic yards, the volume by averaging end areas. The determination of center heights and total widths and the application of Eq. (54), to obtain the approximate prismoidal correction (see § 114), is self-evident.

110. Prismoidal correction. The foregoing methods of calculation have been called approximate, although under many

VOLUME OF IRREGULAR PRISMOID, WITH APPROXIMATE PRISMOIDAL CORRECTION.

Sta.	W'th	H'ght	Yards.		Cen. Height.	Total width	$d'-d''$	$w''-w'$	Approx. pris. corr.
16	22.4 12.0 12.9 4.1 9.0	7.6 −2.1 3.2 4.2 11.5	158 −23 40 16 96		+6.8	35.3			
+42	27.3 8.2 21.6 7.5 9.0	12.6 −2.0 6.2 1.8 20.6	319 −15 124 13 172	378	+10.2	48.9	−3.4	+13.6	−14 (−6)
17	21.3 17.4 6.1 15.3 9.0	10.2 −0.2 −2.6 7.6 12.4	201 − 3 −14 107 103	584	+ 7.6	36.6	+2.6	−12.3	−10 (−6)
18	15.3 8.4 5.2 10.8 9.0	6.8 −1.0 −4.5 2.3 5.4	95 − 7 −22 23 45	528	+ 2.3	26.1	+5.3	−10.5	−17 (−17)
19	14.4 9.6 4.2 9 0	0.6 0.1 0.2 4.0	8 1 1 33	177	+ 0.6	24.0	+1.7	−2.1	−1 (−1)

Approx. volume = 1667 —30
Approx. pris. corr. = −30

Corrected volume = 1637 cubic yards

conditions such results are considered to be sufficiently accurate to serve as final. In any case the approximate result is first computed and then the "prismoidal correction" is computed if necessary. The mathematical necessity for a correction may be at once appreciated from the consideration that the volume of a prismoid having dissimilar and unequal ends is NOT equal to the length times the average of the end areas but is usually somewhat less. In an extreme case the correction is one-third of the approximate volume, or one-half of the true volume. The amount of the prismoidal correction for a triangular prism will be first determined and from that the correction for any kind of prism may be deduced.

Let Fig. 54 represent a triangular prismoid. The two triangles forming the ends lie in *parallel* planes, but since the angles of one triangle are not equal to the corresponding angles of the

other triangle, at least two of the surfaces must be *warped*. If a section, parallel to the bases, is made at any point at a dis-

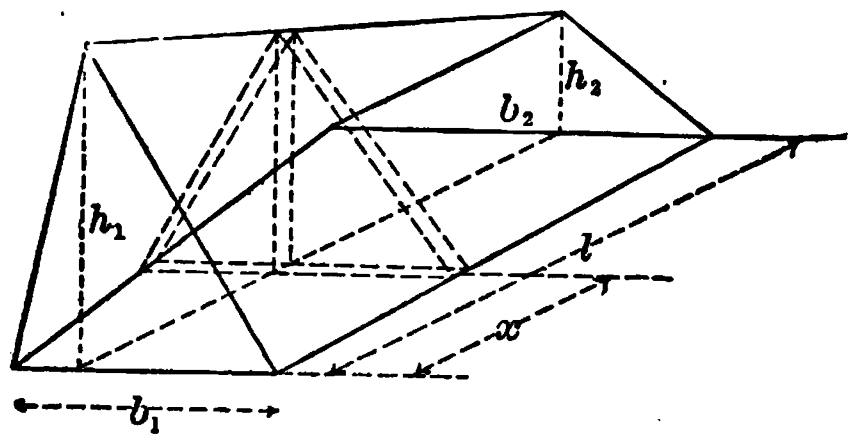


FIG. 54.

tance x from one end, the area of the section will evidently be

$$A_x = \tfrac{1}{2}b_xh_x = \tfrac{1}{2}\left[b_1 + (b_2 - b_1)\frac{x}{l}\right]\left[h_1 + (h_2 - h_1)\frac{x}{l}\right].$$

The volume of a section of infinitesimal length will be A_xdx, and the total volume of the prismoid will be *

$$\int_0^l A_xdx = \tfrac{1}{2}\int_0^l \left[b_1 + (b_2-b_1)\frac{x}{l}\right]\left[h_1 + (h_2-h_1)\frac{x}{l}\right]dx$$

$$= \tfrac{1}{2}\left[b_1h_1x + (b_2-b_1)h_1\frac{x^2}{2l} + b_1(h_2-h_1)\frac{x^2}{2l} \right.$$
$$\left. + (b_2-b_1)(h_2-h_1)\frac{x^2}{3l^2}\right]_0^l$$

$$= \tfrac{1}{2}\left\{b_1h_1l + [(b_2-b_1)h_1 + b_1(h_2-h_1)]\frac{l}{2} + (b_2-b_1)(h_2-h_1)\frac{l}{3}\right\}$$

$$= \frac{l}{2}[\tfrac{1}{3}b_1h_1 + \tfrac{1}{6}b_1h_2 + \tfrac{1}{6}b_2h_1 + \tfrac{1}{3}b_2h_2]$$

$$= \frac{l}{6}[\tfrac{1}{2}b_1h_1 + \tfrac{1}{2}b_1(h_1+h_2) + \tfrac{1}{2}b_2(h_1+h_2) + \tfrac{1}{2}b_2h_2]$$

$$= \frac{l}{6}\left[\tfrac{1}{2}b_1h_1 + 4\left(\frac{1}{2}\cdot\frac{b_1+b_2}{2}\cdot\frac{h_1+h_2}{2}\right) + \tfrac{1}{2}b_2h_2\right]$$

$$= \frac{l}{6}]A_1 + 4A_m + A_2], \qquad \qquad (52)$$

---

* Students unfamiliar with the Integral Calculus may take for granted the fundamental formulæ that $\int dx = x$, that $\int xdx = \frac{1}{2}x^2$, and that $\int x^2dx = \frac{1}{3}x^3$; also that in integrating between the limits of l and 0 (zero), the value of the integral may be found by simply substituting l for x after integration.

in which A_1, A_2, and A_m are the areas respectively of the two bases and of the middle section. Note that A_m is *not* the *mean* of A_1 and A_2, although it does not necessarily differ very greatly from it.

The above proof is absolutely independent of the values, absolute or relative, of b_1, b_2, h_1, or h_2. For example, h_2 may be zero and the second base reduces to a line and the prismoid becomes wedge-shaped; or b_2 and h_2 may both vanish, the second base becoming a point and the prismoid reduces to a pyramid. Since every prismoid (as defined in § 97) may be reduced to a combination of triangular prismoids, wedges, and pyramids, and since the formula is true for any one of them individually, it is true for all collectively; therefore it may be stated that *

The volume of a prismoid equals one sixth of the perpendicular distance between the bases multiplied by the sum of the areas of the two bases plus four times the area of the middle section.

While it is always possible to compute the volume of any prismoid by the above method, it becomes an extremely complicated and tedious operation to compute the true value of the middle section if the end sections are complicated in form. It therefore becomes a simpler operation to compute volumes by approximate formulæ and apply, if necessary, a correction. The most common methods are as follows:

111. **Correction for triangular prismoid.** The volume of the triangular prismoid (Fig. 54), computed by averaging end areas, is $\frac{l}{2}[\frac{1}{2}b_1h_1+\frac{1}{2}b_2h_2]$. Subtracting this from the true volume (as given in the equation above Eq. 52), we obtain the correction

$$\frac{l}{12}[(b_1-b_2)(h_2-h_1)]. \quad . \quad . \quad . \quad . \quad . \quad (53)$$

This shows that if either the h's or b's are equal, the correction vanishes; it also shows that if the bases are roughly similar and b varies roughly with h (which *usually* occurs, as will be seen later), the correction will be *negative*, which means that the method of averaging end areas *usually* gives *too large* results.

If the " base " at one end vanishes to a point, making a trian-

* The student should note that the derivation of equation (52) does not complete the proof, but that the statements in the following paragraph are logically necessary for a general proof.

gular pyramid, then b_1 and h_1 each equal zero and the correction reduces to

$$\frac{l}{12}[(-b_2)(h_2)] = -\frac{lb_2h_2}{12}.$$

But the volume of a triangular prismoid is one-third of the altitude times the area of the base or $\frac{1}{3}l(\frac{1}{2}b_2h_2) = \frac{1}{6}lb_2h_2$. The approximate volume, by averaging end areas, applying the rule strictly, is $\frac{1}{2}l(\frac{1}{2}b_2h_2 + 0) = \frac{1}{4}lb_2h_2$. The correction is therefore one-third of the approximate volume, or one-half of the true volume, in this extreme case. Therefore, when computing the volume of terminal pyramids and wedges (see § 89 and Fig. 43), by the method of averaging end areas, it must be remembered that, although the gross volume is comparatively small, the prismoidal correction is relatively very large.

112. Correction for level sections. Absolutely level sections are practically unknown, and the error involved in assuming any given sections as truly level will ordinarily be greater than the computed correction. If greater accuracy is required, more points should be obtained in the cross-sectioning, which will generally show that the sections are not truly level. But it may be easily computed that the correction equals

$$-\frac{l}{12}\frac{b}{a}\Sigma(d' \sim d'')^2.$$

The squares of the differences of center depth of consecutive sections are always positive, regardless of whether the differences are positive or negative. Therefore the correction is *always* negative, showing that the method of averaging end areas, when the sections are level, *always* gives too large results.

113. Prismoidal correction for "equivalent sections." It is a simple although tedious problem in mathematics to compute algebraically the true and approximate volumes of a prismoid when the areas are determined on the basis of "equivalent sections," § 104, and from thence to derive a formula for the prismoidal correction, but it is generally true that the errors due to such an approximate method of getting the area are so great that it is a needless refinement to compute the correction.

114. Prismoidal correction for three-level sections. The prismoidal correction may be obtained by applying Eq. 53 to each side in turn. For the left side we have

$$\frac{l}{12}[(a+d')-(a+d'')](w_l''-w_l'), \text{ which equals}$$

$$\frac{l}{12}(d'-d'')(w_l''-w_l').$$

For the right side we have, similarly,

$$\frac{l}{12}(d'-d'')(w_r''-w_r').$$

The total correction therefore equals

$$\frac{l}{12}(d'-d'')[(w_l''+w_r'')-(w_l'+w_r')]$$

$$=\frac{l}{12}(d'-d'')(w''-w').$$

Reduced to cubic yards, and with $l=100$,

$$\text{Pris. Corr.}=\tfrac{25}{81}(d'-d'')(w''-w'). \quad . \quad . \quad . \quad (54)$$

Applying this formula to the numerical problem worked out in § 105, the several values of $(d'-d'')$ and $w''-w')$ are computed as given in the first two columns under Prismoidal Correction. Then, for example,

$$-20=\tfrac{25}{81}(d'-d'')(w''-w')=\tfrac{25}{81}(2.6-8.1)(42.8—31.1)$$
$$=\tfrac{25}{81}(-5.5)(+11.7).$$

For the next line, $-3=\frac{40}{100}[\frac{25}{81}(-2.6)(+8.7)]$, and similarly for the rest. For this typical case, the correction is over 2% of the volume and is, as usual, negative, or in other words, the approximate method, if used without correction, allows a contractor in this case 2% too much.

115. Prismoidal correction; irregular sections. For reasons given in the next article, the correction is computed as if the sections were "three-level" sections. This method was used in the numerical problem worked out in § 109. Instead of considering the heights and widths of the separate triangles, the center height and total width for each section is recorded in two columns and the differences $(d'-d'')$ and $(w''-w')$ are computed. $(-3.4)\times(+13.6)\div 3.24=-14$, which would be the correction for a section 100 feet long. For 42 feet the correction is 42% of -14 or -6. Note that the total prismoidal correction for this stretch of 300 feet is negative, as is usual, and that it is a little less than 2%, about the same as the numerical problem of § 105.

116. Magnitude of the probable error of this method. In previous editions of this work, methods were given for computing the mathematically exact volume of a prismoid whose ends coincide with the "irregular sections" as measured, and whose upper surfaces are assumed to coincide with the actual surface of the ground. As in the previous methods, the "approximate volume" is computed by averaging end areas and then a correction is applied. If the end sections have the same number of intermediate points on each side, and if it can be assumed that the corresponding lines in each section are connected by plane or warped surfaces, which coincide with the surface of the ground, then the mathematically exact or "true" correction may be obtained by dividing the volume into elementary triangular prismoids, finding the correction for each and adding the results. Although such a method appears very complicated, it is readily possible to develop a law by means of which the true prismoidal correction may be written out (similarly to writing out the formula for the area, Eq. (50)) without any preliminary calculation. Such a law has a mathematical fascination, but it should be remembered that when the ground surface is so broken up that the cross-sections are "irregular" it is in general correspondingly rough and irregular between the cross-sections, especially when those sections are 100 feet apart. It is also true that the cross-sections do *not* usually have the same number of intermediate points on corresponding sides of the center. In such a case, unless the actual form of the ground between the cross-sections is observed and measured, the exact method cannot be used. An extra point in one cross-section implies an extra ridge (or hollow) which "runs out" or disappears by the time the adjoining section is reached. Theoretically a cross-section should be taken at the point where such a ridge or hollow runs out. In general this point will not be at an even 100-foot station. The attempt to compute the exact prismoidal correction usually gives merely a false appearance of extreme accuracy to the work which is not justified by the results. It should not be forgotten that it is readily possible to spend an amount of time on the surveying and computing which is worth more than the few cubic yards of earth which represents the additional accuracy of the more precise method. The accuracy of the office computation should be kept proportionate to the accuracy of the cross-sectioning

in the field. The discussion of the magnitude of the prismoidal correction in §§ 110–115 shows that it is small except when the two ends of the prismoid are very dissimilar. The dissimilarity between the two ends of a prismoid would be substantially the same whether the ends were actually "irregular" or had "three-level" sections, which for each end had the same slope stakes and center heights as the irregular sections. Experience proves that the approximate prismoidal correction, computed by considering the ground as three-level, is so nearly equal to the true prismoidal correction that the difference is perhaps no greater than the *probable* difference between the true volume of earth and the volume of the geometrical prismoid which is assumed to represent that volume. The experienced surveyor will take his cross-sections at such places and so close together that the warped surfaces joining the sections will lie very nearly in the surface or at least will so average the errors that they will substantially neutralize each other.

117. Numerical illustration of the accuracy of the approximate rule. The "true" prismoidal correction for the numerical case given in § 109 was computed by the method outlined above, and on the basis of certain figures as to the vanishing of the ridges and valleys found in one section and not found in the adjacent sections. The various quantities for the volumes between the cross-sections have been tabulated as shown.

	1	2	3	4	5	6	7
Sections.	Approx. vol. by averaging end areas.	True prismoidal correction.	True volume.	Approx. pris. corr. on basis of three-level ground.	Error: Col. 4 − col. 2.	Approx. vol. computed from center and side heights *only*.	Error: Col. 6 − col. 3.
16......16+42	378	− 5	373	− 6	−1	396	−23
16+42..17	584	− 3	581	− 6	−3	577	+ 4
17......18	528	−16	512	−17	−1	463	+49
18......19	177	− 3	174	− 1	+2	147	+27
	1667	−27	1640	−30	−3	1583	+57

There has also been shown in the last two columns the error involved if the "intermediate points" had been ignored in the cross-sectioning. From the tabular form we may learn that

1. The *differences* between the "true" and approximate

corrections is so small that it is *probably* swallowed up by errors resulting from inaccurate cross-sectioning.

2. The error which would have been involved in ignoring the intermediate points is so very large in comparison with the other corresponding errors that (although it proves nothing absolutely definite, being an individual case) the *probabilities* of the relative error from these sources are clearly indicated.

118. Cross-sectioning irregular sections. The slope stake should preferably be determined first, and then the "breaks" between the slope stake and the center. When, as is usual, the ground is not even between the cross-section just taken and the section at the next 100-foot station, a point should be selected for a cross-section such that the lines to the previous section should coincide with the actual surface of the ground as closely as the accuracy of the work demands. § 125 gives a numerical illustration of the magnitude of some of these errors. Although it is possible for a skillful surveyor to so choose his cross-sections in rough and irregular ground that the positive and negative errors will nearly balance, it requires exceptional skill. Frequently the work may be simplified by computing separately the volume of a mound or pit, the existence of which has been ignored in the regular cross-sectioning.

119. Side-hill work. When the natural slope cuts the roadbed there is a necessity for both cut and fill at the same cross-section.

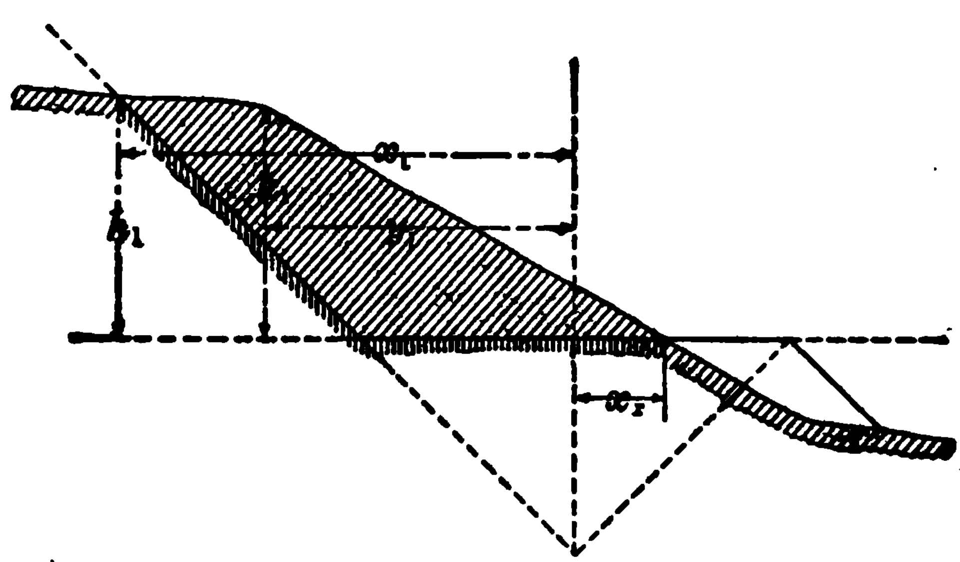


FIG. 55.

When this occurs the cross-sections of both cut and fill are often so nearly triangular that they may be considered as such without great error, and the volumes may be computed separately as triangular prismoids without adopting the more elaborate form

of computation so necessary for complicated irregular sections. When the ground is too irregular for this the best plan is to follow the uniform system. In computing the cut, as in Fig. 55, the left side would be as usual; there would be a small center cut and an ordinate of zero at a short distance to the right of the center. Then, *ignoring the fill*, and applying Eq. 56 strictly, we have two terms for the left side, one for the right, and the term involving $\frac{1}{2}b$, which will be $\frac{1}{2}bh_l$ in this case, since $h_r=0$, and the equation becomes

$$\text{Area}=\tfrac{1}{2}[x_lk_l+y_l(d-h_l)+x_rd+\tfrac{1}{2}bh_l].$$

The area for fill may also be computed by a strict application of Eq. 50, but for Fig. 56 all distances for the left side are zero and the elevation for the first point out is zero. d also must be

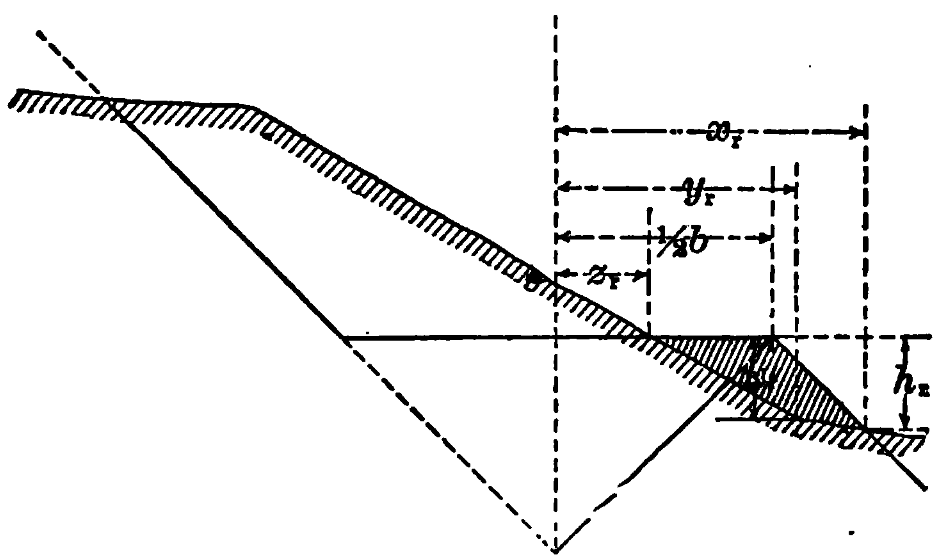


FIG. 56.

considered as zero. Following the rule, § 107, literally, the equation becomes

$$\text{Area}_{(\text{Fill})}=\tfrac{1}{2}[x_rk_r+y_r(o-h_r)+z_r(o-k_r)+\tfrac{1}{2}b(o+h_r)],$$

which reduces to

$$\tfrac{1}{2}[x_rk_r-y_rh_r-z_rk_r+\tfrac{1}{2}bh_r].$$

(*Note* that x_r, h_r, etc., have different significations and values in this and in the preceding paragraphs.) The "terminal pyramids" illustrated in Fig. 43 are instances of side-hill work for very short distances. Since side-hill work always implies *both* cut and fill at the same cross-section, whenever either the cut or fill disappears and the earthwork becomes wholly cut or wholly fill, that point marks the end of the "side-hill work," and a cross-section should be taken at this point.

120. Borrow-pits. The cross-sections of borrow-pits will vary not only on account of the undulations of the surface of the

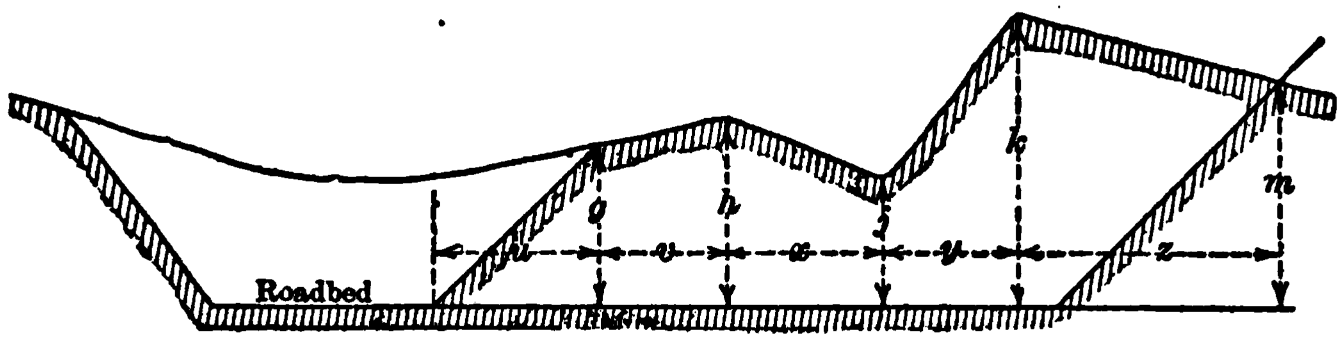


FIG. 57.

ground, but also on the sides, according to whether they are made by widening a convenient cut (as illustrated in Fig. 57) or simply by digging a pit. The sides should always be properly sloped and the cutting made cleanly, so as to avoid unsightly roughness. If the slope ratio on the right-hand side (Fig. 57) is s, the area of the triangle is $\frac{1}{2}sm^2$. The area of the section is $\frac{1}{2}[ug+(g+h)v+(h+j)x+(j+k)y+(k+m)z-sm^2]$. If all the horizontal measurements were referred to one side as an origin, a formula similar to Eq. 50 could readily be developed, but little or no advantage would be gained on account of any simplicity of computation. Since the *exact* volume of the earth borrowed is frequently necessary, the prismoidal correction should be computed; and since such a section as Fig. 57 does not even approximate to a three-level section, the method suggested in § 108 cannot be employed. It will then be necessary to employ the more exact method of dividing the volume into triangular prismoids and taking the summation of their correction, found according to the general method of § 110.

121. Correction for curvature. The volume of a solid, generated by revolving a plane area about an axis lying in the plane but outside of the area, equals the product of the given area times the length of the path of the center of gravity of the area. If the centers of gravity of all cross-sections lie in the center of the road, where the length of the road is measured, there is absolutely no necessary correction for curvature. If all the cross-sections in any given length were exactly the same and therefore had the same eccentricity, the correction for curvature would be very readily computed according to the above principle. But when both the areas and the eccentricities vary from point to point, as is generally the case, a theoretically exact

solution is quite complex, both in its derivation and application. Suppose, for simplicity, a curved section of the road, of uniform cross-sections and with the center of gravity of every cross-section at the same distance e from the center line of the road The length of the path of the center of gravity will be to the length of the center line as $R \pm e : R$. Therefore we have *True vol.*: *nominal vol.* $:: R \pm e : R$. $\therefore$ *True vol.* $= lA\frac{R \pm e}{R}$ for a volume of uniform area and eccentricity. For any other area and eccentricity we have, similarly, *True vol.'* $= lA'\frac{R \pm e'}{R}$. This shows that the effect of curvature is the same as increasing (or diminishing) the area by a quantity depending on the area and eccentricity, the increased (or diminished) area being found by multiplying the actual area by the ratio $\frac{R \pm e}{R}$. This being independent of the value of l, it is true for infinitesimal lengths. If the eccentricity is assumed to vary uniformly between two sections, the *equivalent area* of a cross-section located midway between the two end cross-sections would be $A_m\frac{\left(R \pm \frac{e'+e''}{2}\right)}{R}$. Therefore the volume of a solid which, when straight, would be $\frac{l}{6}(A' + 4A_m + A'')$, would then become

$$\textit{True vol.} = \frac{l}{6R}\left[A'(R \pm e') + 4\,A_m\left(R \pm \frac{e'+e''}{2}\right) + A''(R \pm e'')\right].$$

Subtracting the nominal volume (the true volume when the prismoid is straight), the

$$\textit{Correction} = \pm\frac{l}{6R}\left[(A' + 2A_m)e' + (2A_m + A'')e''\right]. \quad . \quad (55)$$

Another demonstration of the same result is given by Prof. C. L. Crandall in his "Tables for the Computation of Railway and other Earthwork," in which is obtained by calculus methods the summation of elementary volumes having variable areas with variable eccentricities. The exact application of Eq. 55 requires that A_m be known, which requires laborious computa-

tions, but no error worth considering is involved if the equation is written approximately

$$Curv.\,corr. = \frac{l}{2R}(A'e' + A''e''), \quad . \quad . \quad . \quad . \quad (56)$$

which is the equation generally used. The approximation consists in assuming that the difference between A' and A_m equals the difference between A_m and A'' but with opposite sign. The error due to the approximation is always utterly insignificant.

122. Eccentricity of the center of gravity. The determination of the true positions of the centers of gravity of a long series of irregular cross-sections would be a very laborious operation, but fortunately it is generally sufficiently accurate to consider the cross-sections as three-level ground, or, for side-hill work, to

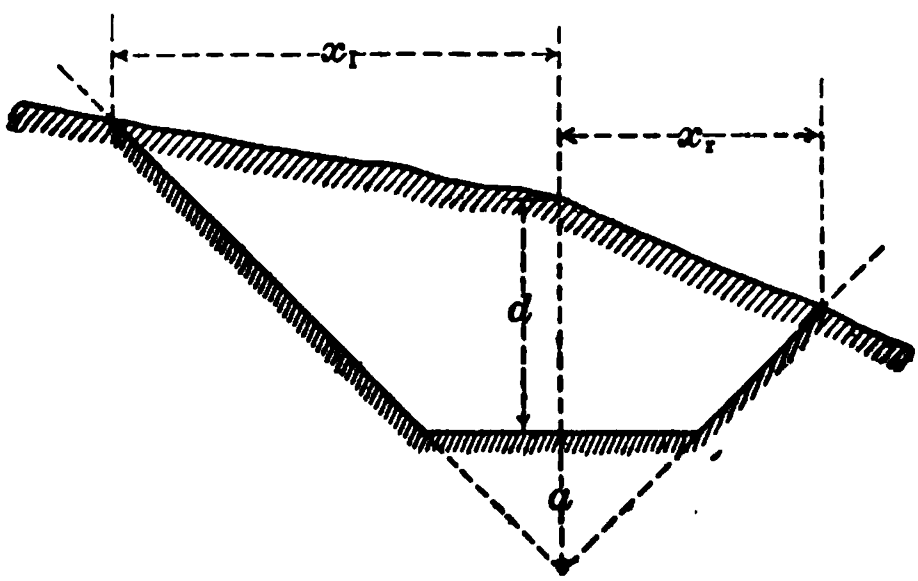


FIG. 58.

be triangular, *for the purpose of this correction.* The eccentricity of the cross-section of Fig. 58 (including the grade triangle) may be written

$$e = \frac{\frac{(a+d)x_l}{2}\frac{x_l}{3} - \frac{(a+d)x_r}{2}\frac{x_r}{3}}{\frac{(a+d)x_l}{2} + \frac{(a+d)x_r}{2}} = \frac{1}{3}\frac{x_l^2 - x_r^2}{x_l + x_r} = \frac{1}{3}(x_l - x_r). \quad . \quad . \quad (57)$$

The side toward x_l being considered positive in the above demonstration, if $x_r > x_l$, e would be negative, i.e., the center of gravity would be on the right side. Therefore, for three-level

ground, the correction for curvature (see Eq. 56) may be written

$$Correction = \frac{l}{6R}[A'(x_l' - x_r') + A''(x_l'' - x_r'')].$$

Since the approximate volume of the prismoid is

$$\frac{l}{2}(A + A') = \frac{l}{2}A' + \frac{l}{2}A'' = V' + V'',$$

in which V' and V'' represent the number of cubic yards corresponding to the area at each station, we may write

$$Corr.\ in\ cub.\ yds. = \frac{1}{3R}[V'(x_l' - x_r') + V''(x_l'' - x_r'')]. \qquad (58)$$

It should be noted that the value of e, derived in Eq. 57, is the eccentricity of the whole area including the triangle under the roadbed. The eccentricity of the true area is greater than this and equals

$$e \times \frac{\text{true area} + \frac{1}{2}ab}{\text{true area}} = e_1.$$

The required quantity ($A'e'$ of Eq. 56) equals *true area* $\times e_1$ which equals (*true area* $+ \frac{1}{2}ab) \times e$. Since the value of e is very simple, while the value of e_1 would, in general, be a complex quantity, it is easier to use the simple value of Eq. 57 and add $\frac{1}{2}ab$ to the area. Therefore, in the case of three-level ground the subtractive term $\frac{25}{27}ab$ (§ 105) should *not* be subtracted in computing this correction. For irregular ground, when computed by the method given in §§ 107 and 108, which does not involve the grade triangle, a term $\frac{25}{27}ab$ must be *added* at every station when computing the quantities V' and V'' for Eq. 58.

It should be noted that the factor $1 \div 3R$, which is constant for the length of the curve, may be computed with all necessary accuracy and without resorting to tables by remembering that

$$R = \frac{5730}{\text{degree of curve}}.$$

Since it is useless to attempt the computation of railroad earthwork closer than the nearest cubic yard, it will frequently

be possible to write out all curvature corrections by a simple mental process upon a mere inspection of the computation sheet. Eq. 58 shows that the correction for each station is of the form $\frac{V(x_l-x_r)}{3R}$. $3R$ is generally a large quantity—for a 6° curve it is 2865. (x_l-x_r) is generally small. It may frequently be seen by inspection that the product $V(x_l-x_r)$ is roughly twice or three times $3R$, or perhaps less than half of $3R$, so that the corrective term for that station may be written 2, 3, or 0 cubic yards, the fraction being disregarded. For much larger absolute amounts the correction must be computed with a correspondingly closer percentage of accuracy.

The algebraic sign of the curvature correction is best determined by noting that the center of gravity of the cross-section is on the right or left side of the center according as x_r is greater or less than x_l, and that the correction is *positive* if the center of gravity is on the *outside* of the curve, and *negative* if on the *inside*.

It is frequently found that x_l is uniformly greater (or uniformly less) than x_r throughout the length of the curve. Then the curvature correction for each station is uniformly positive or negative. But in irregular ground the center of gravity is apt

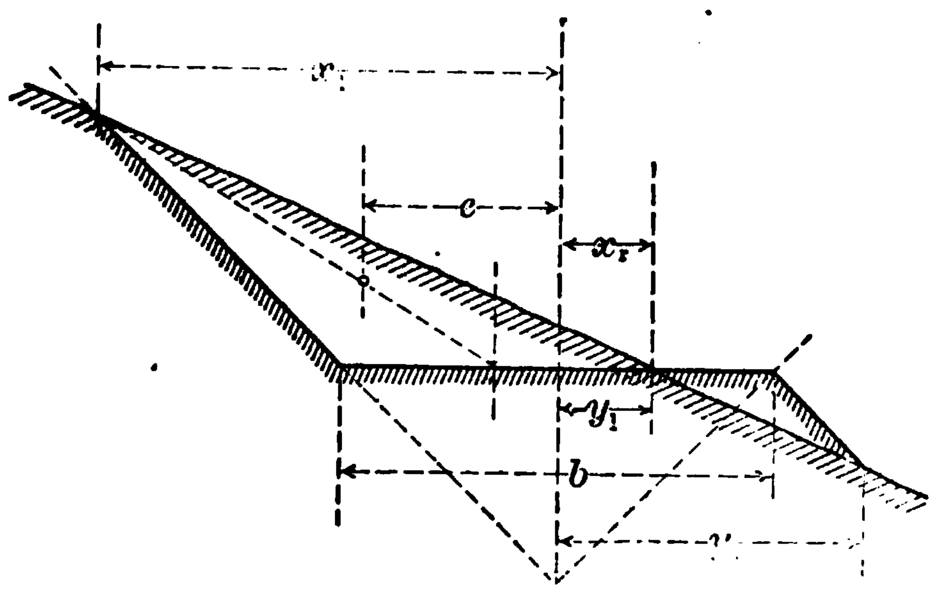


Fig. 59.

to be irregularly on the outside or on the inside of the curve, and the curvature correction will be correspondingly positive or negative. If the curve is to the *right*, the correction will be positive or negative according as (x_l-x_r) is positive or negative; if the curve is to the *left*, the correction will be positive or nega-

tive according as $(x_r - x_l)$ is positive or negative. Therefore when computing curves to the *right* use the form $(x_l - x_r)$ in Eqs. 58 and 60; when computing curves to the *left* use the form $(x_r - x_l)$ in these equations; the algebraic sign of the correction will then be strictly in accordance with the results thus obtained.

123. Center of gravity of side-hill sections. In computing the correction for side-hill work the cross-section would be treated as triangular unless the error involved would evidently be too great to be disregarded. The center of gravity of the triangle lies on the line joining the vertex with the middle of the base and at $\frac{1}{3}$ of the length of this line from the base. It is therefore equal to the distance from the center to the foot of this line plus $\frac{1}{3}$ of its horizontal projection. Therefore

$$e = \left[\frac{b}{2} - \frac{1}{2}\left(\frac{b}{2} + x_r\right)\right] + \frac{1}{3}\left[x_l - \left(\frac{b}{2} - \frac{1}{2}\left(\frac{b}{2} + x_r\right)\right)\right]$$

$$= \frac{b}{4} - \frac{x_r}{2} + \frac{x_l}{3} - \frac{b}{12} + \frac{x_r}{6}$$

$$= \frac{b}{6} + \frac{x_l}{3} - \frac{x_r}{3}$$

$$= \frac{1}{3}\left[\frac{b}{2} + (x_l - x_r)\right]. \qquad (59)$$

By the same process as that used in § 122 the correction equation may be written

$$\text{Corr. in cub. yds.} = \frac{1}{3R}\left[V'\left(\frac{b}{2} + (x_l' - x_r')\right) + V''\left(\frac{b}{2} + (x_l'' - x_r'')\right)\right]. \qquad (60)$$

It should be noted that since the grade triangle is not used in this computation the volume of the grade prism is *not* involved in computing the quantities V' and V''.

The eccentricities of cross-sections in side-hill work are *never* zero, and are frequently quite large. The total volume is generally quite small. It follows that the correction for curvature is generally a vastly larger proportion of the total volume than in ordinary three-level or irregular sections.

If the triangle is wholly to one side of the center, Eq. 59 can still be used. For example, to compute the eccentricity of the triangle of fill, Fig. 59, denote the two distances to the slope-

stakes by y_r and $-y_l$ (note the minus sign). Applying Eq. 59 literally (noting that $\frac{b}{2}$ must here be considered as negative in order to make the notation consistent) we obtain

$$e=\frac{1}{3}\left[-\frac{b}{2}+(-y_l-y_r)\right],$$

which reduces to

$$e=-\frac{1}{3}\left[\frac{b}{2}+y_l+y_r\right]. \quad . \quad . \quad . \quad . \quad . \qquad (61)$$

As the algebraic signs tend to create confusion in these formulæ, it is more simple to remember that for a triangle lying on *both* sides of the center e is always numerically equal to $\frac{1}{3}\left[\frac{b}{2}+(x_l \sim x_r)\right]$, and for a triangle entirely on one side, e is numerically equal to $\frac{1}{3}\left[\frac{b}{2}\right.$ + the numerical *sum* of the two distances out]. The algebraic sign of e is readily determinable as in § 122.

124. Example of curvature correction. Assume that the fill in § 105 occurred on a 6° curve to the *right*. $\frac{1}{3R}=\frac{1}{2865}$. The quantities 210, 507, etc., represent the quantities V', V'', etc., since they include in each case the 61 cubic yards due to the grade prism. Then

$$\frac{V(x_l \sim x_r)}{3R}=\frac{210(22.9-8.2)}{2865}=\frac{3101.7}{2865}=+1.$$

The sign is plus, since the center of gravity of the cross-section is on the left side of the center and the road curves to the right, thus making the true volume larger. For Sta. 18 the correction, computed similarly, is +3, and the correction for the whole section is $1+3=4$. For Sta. 18+40 the correction is computed as 6 yards. Therefore, for the 40 feet, the correction is $\frac{40}{100}(3+6)=3.6$, which is called 4. Computing the others similarly we obtain a total correction of +16 cubic yards.

125. Accuracy of earthwork computations. The preceding methods give the *precise volume* (except where approximations are distinctly admitted) of the prismoids which are *supposed* to represent the volume of the earthwork. To appreciate the accuracy necessary in cross-sectioning to obtain a given accuracy in volume, consider that a fifteen-foot length of the cross-section, which is assumed to be straight, really sags 0.1 foot, so that the cross-section is in error by a triangle 15 feet wide and 0.1 foot high. This sag 0.1 foot high would hardly be detected by the eye, but in a length of 100 feet in each direction it would make an error of volume of 1.4 cubic yards in *each* of the two prismoids, assuming that the sections at the other ends were perfect. If the cross-sections at both ends of a prismoid were in error by this same amount, the volume of that prismoid would be in error by 2.8 cubic yards if the errors of area were both plus or both minus. If one were plus and one minus, the errors would neutralize each other, and it is the compensating character of these errors which permits any confidence in the results as obtained by the usual methods of cross-sectioning. It demonstrates the utter futility of attempting any closer accuracy than the nearest cubic yard. It will thus be seen that if an error really exists at *any* cross-section it involves the prismoids on *both* sides of the section, even though all the other cross-sections are perfect. As a further illustration, suppose that cross-sections were taken by the three-level method (§ 105), and that a cross-section, assumed as uniform from center to side, sags 0.4 foot in a width of 20 feet. Assume an equal error (of same sign) at the other end of a 100-foot section. The error of volume for that one prismoid is 38 cubic yards.

The computations further assume that the warped surface, passing through the end sections, coincides with the surface of the ground. Suppose that the cross-sectioning had been done with mathematical perfection; and, to assume a simple case, suppose a sag of 0.5 foot between the sections, which causes an error equal to the volume of a pyramid having a base of 20 feet (in each cross-section) times 100 feet (between the cross-sections) and a height of 0.5 foot. The volume of this pyramid is $\frac{1}{3}(20\times100)\times0.5=333$ cub. ft. $=12$ cub. yds. And yet this sag or hump of 6 inches would generally be utterly unnoticed, or at least disregarded.

When the ground is very rough and broken it is sometimes

practically impossible, even with frequent cross-sections, to locate warped surfaces which will closely coincide with all the sudden irregularities of the ground. In such cases the computations are necessarily more or less approximate and dependence must be placed on the compensating character of the errors.

126. **Approximate computations from profiles.** When a "paper location" has been laid out on a topographical map having contours, it is possible to compute approximately the amount of earthwork required by some very simple and rapid calculations. A profile may be readily drawn by noting the intersections of the proposed center line with the various contours and plotting the surface line on profile paper. Drawing the grade-line on the profile, the depth of cut or fill may be scaled off at any point. When it is only desired to obtain

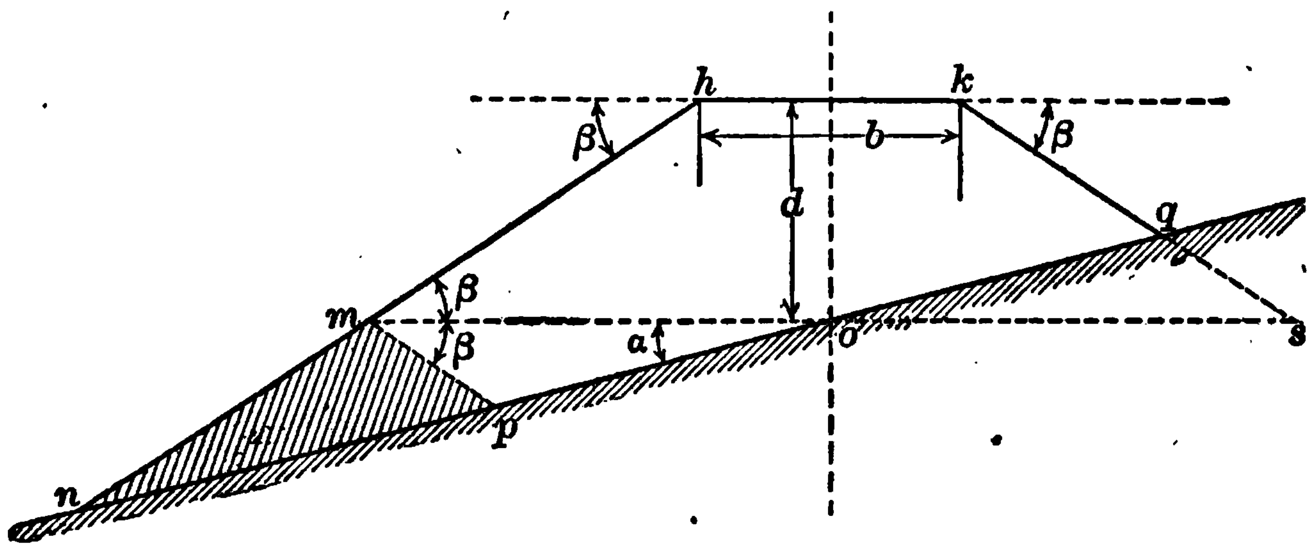


FIG. 60.

very quickly an approximate estimate of the amount of earthwork required on a suggested line, it may be done by the method described in § 103, or by the use of Table XVII. But the assumption that the surface of the ground at each cross-section is level invariably has the effect that the estimated volumes are not as large as those actually required. The difference between the "level section" *hkms* and the actual slope section *hknq* equals the difference between the triangles *mon* and *oqs*, and this difference equals the shaded area *mpn*. The excess volume is proportional to the area of the triangle *mpn*. This area may be expressed by the formula,

$$\text{Area } mpn = 2(\tfrac{1}{2}b + d\cot\beta)^2 \frac{\sin^2\alpha \sin\beta\cos\beta}{\cos 2\alpha - \cos 2\beta}.$$

The percentage of this excess area to the nominal area *hkms* therefore depends on the dimensions b and d and the angles α and β. A solution of this equation for ninety different combinations of various numerical values for these four variables is included in Table XVII for the purpose of making corrections. A study of this correction table points conclusively to the following laws, a thorough understanding of which will enable an engineer to appreciate the degree of accuracy which is attainable by this approximate method:

(*a*) Increasing the *width* of the roadbed (b), the other three factors remaining constant, *increases* the percentage of error, but the increase is comparatively small.

(*b*) Increasing the *depth* of cut or fill (d), *decreases* the percentage of error, but the decrease is almost insignificant.

(*c*) Increasing the angle of the side slopes (β) *decreases* the percentage of error, the decrease being very considerable.

(*d*) Increasing the angle of the slope of the ground (α), *increases* the percentage of error, the percentage rapidly increasing to infinity as the value of α approaches that of β. This is another method of stating the fact that α must always be less than β and, practically, must be considerably less, so that the slope stake shall be within a reasonable distance from the center.

Since the above value for the corrective area is a function of the angle α, which is usually variable and whose value is frequently known only approximately, it is useless to attempt to apply the correction with great precision, and the following rules will usually be found amply accurate, considering the probable lack of precision in the data used.

1. For embankments or cuts, having a slope of 1.5:1, and with a surface slope of 5° (nearly 9%) the excess of true area over nominal area is *about* 2%. There is only a slight variation from this value for all ordinary depths (d) and widths (b) of roadbed. Therefore the nominal volume would be about 2% too *small*. On the other hand, the effect of the prismoidal correction is such that, even with truly level sections, the nominal volume is too *large*. See §§ 103 and 104. The amount of the prismoidal correction depends on the differences between successive center depths. In the very ordinary numerical case given in § 104, the correction was nearly 3%, which more than neutralizes the error due to surface slope. Therefore in

many cases on slightly sloping ground the error due to the surface slope will so nearly neutralize the prismoidal correction that the quantities taken directly from the tables (without correction for either cause) will equal the true volume with as close an approach to accuracy as the precision of the surveying will permit.

2. For a cut with a slope of 1:1, and with a surface slope of 5° the error is about 1%. This will be neutralized by still smaller prismoidal corrections. Therefore, for surface slopes of 5° or less, no allowance should be made for this error unless the prismoidal correction is also considered.

3. When the surface slope is 10° (nearly 18%) the error for a 1.5:1 slope is from 7% to 10% and for a 1:1 slope from 3% to 5%.

4. For a 30° surface slope and 1.5:1 side slopes the excess volume is three or four times the nominal volume. Such a steep surface slope implies the probability of "side-hill work" to which the above corrective rules are not applicable. When the surface slopes are very steep careful work must be done to avoid excessive error. For a 1:1 side slope, the errors are from 50% to 80%.

A still closer approximation, especially for the steeper surface slopes, may be obtained by using, directly or by interpolation, figures from the corrective tabular form which forms part of Table XVII. Unless the surface slope angle is known accurately (especially when large) no great accuracy in the final result is possible. Close accuracy would also require the determination of the prismoidal correction. But if such close accuracy is deemed essential, it can be most easily obtained by accurate cross-sectioning at each station and the adoption of other methods of computation—such as are given in §§ 108 and 109.

When the contours have been drawn in for a sufficient distance on either side to include the position of both slope stakes at every station, as will usually be the case, cross-sections may be obtained by drawing lines on the map at each station perpendicular to the center line—see Fig. 4. The intersection of these lines with the contours will furnish the distances for drawing on cross-section paper the transverse profile at each station. Drawing on the same cross-section the lines representing the roadbed and the side slopes, the cross-section of

cut (or fill) is complete and its area may be obtained by scaling from the cross-section paper. If the contours have been located on the map with sufficient accuracy, such a method will determine the cross-sectional area very closely. When cross-sections have been taken with a wye- or hand-level, as described in § 12, the cross-sections as plotted will probably be more accurate than when the contours are run in from points determined by the stadia method. In fact this semi-graphical method is frequently used, in place of the purely numerical methods described in previous sections, to make final estimates of the volume of earthwork.

As a numerical example, an assumed location line was laid out on the contours given in Fig. 4. The volume of cut, as determined by Table XVII for a roadbed 20 feet wide, with side slopes of 1:1, was 5746 cubic yards. The surface slope varied from 3° to 11°. Computing the corrections by a careful interpolation from the corrective table, the total correction was found to be 128 cubic yards, or an average of a little over 2%. On the other hand the negative prismoidal correction amounts to 72 cubic yards, which leaves a net correction of 56 cubic yards—about 1%. It so happens that in this case a correction for curvature would tend further to wipe out this correction. These figures merely verify numerically the general conclusions stated above, although it should not be forgotten that in individual cases the figures taken from Table XVII require ample correction.

The following approximate rule, for which the author is indebted to Mr. W. H. Edinger, is exceedingly useful when it is desired to rapidly determine the approximate volume of earthwork between two points along the road. Its great merit lies in the fact that it only means the memorizing of a comparatively simple rule which will make it possible to make such computations in the field, without the use of tables. The rule is based on the fact that the area of any level section equals $bd+sd^2$; and therefore,

$$\Sigma(\text{vol.}) = (b\,\Sigma d + s\,\Sigma d^2)\frac{L}{27},$$

in which L is usually 100 feet. For strict accuracy this would only be the volume provided the total length was an even number of hundred feet, and the various values of d represented

the depths which were uniform for hundred foot sections. It makes no allowance for the comparatively large prismoidal error of the pyramidal and wedge-shaped sections usually found at each end of a cut or fill, but where an approximate estimate is desired, in which this inaccuracy may be neglected, the method is very useful. The method of applying this rule without tables may best be illustrated by a simple numerical example. Assume that the levels on a stretch of fairly level ground, which is about 500 feet long, have been taken, the depths being taken at points 100 feet apart, the first and last points being about 40 or 50 feet from the ends of the cut, or fill. The depths are as given in the first column in the tabular form below; the slope is 1.5:1, and the breadth (b) is 14 feet.

d	d^2
1.6	2.56
2.8	7.84
4.5	20.25
3.1	9.61
0.9	.81
$\Sigma d = 12.9$	$\Sigma d^2 = 41.07$
14	20.53
$b\Sigma d = 180.6$	$s\Sigma d^2 = 61.60$
61.60	
242.2	

$$24220 \div 27 = 897 \text{ cubic yards.}$$

The 180.6 is the $b\Sigma d$ and the 61.6 is $s\Sigma d^2$; adding these and moving the decimal point two places to multiply by 100, we only have to divide by 27 to obtain the value in cubic yards. Although the above rule requires more work than the employment of earthwork tables, yet it is a very convenient method of estimating the approximate volume of a short section of earthwork when no tables are at hand.

FORMATION OF EMBANKMENTS.

127. Shrinkage of earthwork. The statistical data indicating the amount of shrinkage is very conflicting, a fact which is probably due to the following causes:

1. The various kinds of earthy material act very differently as respects shrinkage. There is a great lack of uniformity in

the *classification of earths* in the tests and experiments which have been made.

2. Very much depends on the *method* of forming an embankment (as will be shown later). Different reports have been based on different methods—often without mention of the method.

3. An embankment requires considerable *time* to shrink to its final volume, and therefore much depends on the time elapsed between construction and the measurement of what is supposed to be the settled volume.

4. A soft subsoil will frequently settle under the weight of a high embankment and apparently indicate a far greater shrinkage than the actual reduction in volume.

5. An embankment of very soft material will sometimes "mush" or widen at the sides, with a consequent settling of the top, due to this cause alone.

This subject has called forth much discussion in the technical press and literature. Quotations can be made of figures covering a large range of values, but space will only permit the statement of the conclusions which may be drawn from the large mass of testimony which has been presented.

1. *Volume of loose material.* When material of any character is excavated and deposited loosely in a pile, its volume is always largely in excess of the volume of the excavation. Solid rock will occupy from 60% to 80% *more* space when broken up than when solid. A soft earth will have an excess volume of about 20% to 25%.

2. *Effect of method of depositing.* When material is deposited loosely, as from a trestle, the excess of volume when the embankment is just completed is very large. The time required for final settlement is also very great. When an embankment is formed by the wheelbarrow method, the initial expansion is about as great as when the material is merely dumped from cars. When the material is deposited in small increments from wagons and each layer is subjected to compression from horses' hoofs and from wheels, the contraction during construction is far greater and the additional shrinkage is comparatively small. Wheeled scrapers and drag scrapers will produce even more initial compression.

3. *Time required for final settlement.* This depends partly on the method of formation and also on the character of the

material. When a soft loamy soil is deposited loosely, the drying out of the soil during the first long dry season will develop large cracks. Subsequent rains will close these cracks by a general contraction of the whole mass. When the embankment is loosely formed it may take two years before additional settlement becomes inappreciable, but when the method of deposition ensures compression during construction the *subsequent* shrinkage is less in time as well as amount.

4. *Classification of soils with respect to shrinkage.* Loose vegetable surface soil will expand very greatly when excavated and first deposited, but will subsequently shrink to considerably less than its original volume. Clay soils are next in order and the sandy and gravelly soils come at the other end of the list of earthy materials. Rock expands very greatly when first broken up and deposited and there is no appreciable subsequent shrinkage.

128. Proper allowance for shrinkage. Specifications for the Mississippi River levees require that there shall be a 10% shrinkage allowance for embankments formed by team work and 25% allowance for wheelbarrow work. It is contended

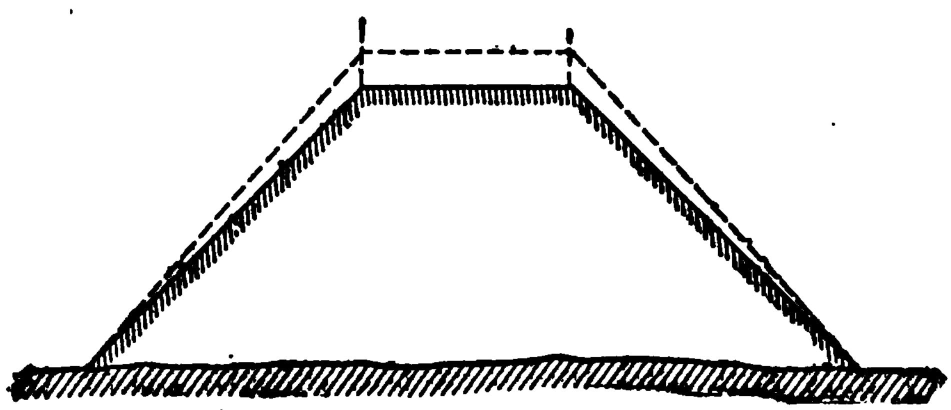


FIG. 61.

that such figures are only justified because the subsoil settles or because the embankments mush out at the sides, and that if these effects do not occur the levees are permanently higher than designed.

It is usual to require that embankments shall be constructed higher than their desired ultimate, as shown in Fig. 61. Since the base does not contract, the contraction may be said to be all vertical. Since a high embankment will unquestionably shrink a greater total amount than a low embankment (whatever the percentage), it follows that an embankment having

variable heights (as usual) should have an initial grade-line somewhat like the dotted line *adc* in Fig. 62. Although some such method is essential if there is to be no ultimate sag below the desired grade-line, the policy is sharply criticized. The grade *ad*, even though temporary, may prove objectionable from an operating standpoint. Frequently the allowance is made too great or the shrinkage is not as much as anticipated, and it becomes necessary to cut off the top of the bank. On the other hand, the expense of raising the track after the road is in operation and the inevitable loss of ballast is so great that the danger of being required to fill up a sag should be avoided if possible.

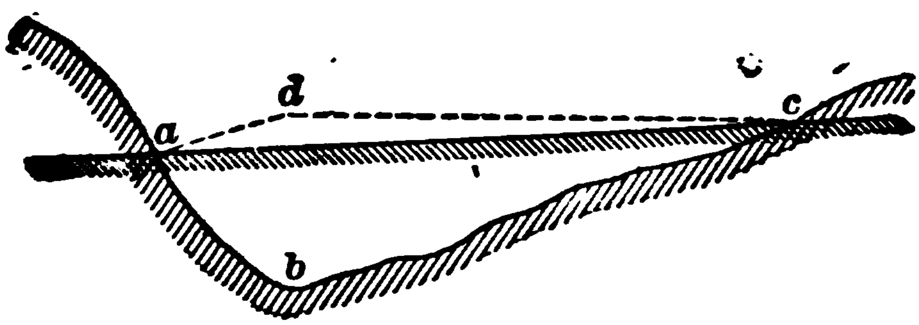


Fig. 62.

A sharp and clear distinction should be made between the *coefficient of extra height* of an embankment and the *coefficient of shrinkage* which determines how many cubic yards of settled embankment may be made from a definite volume of earth or rock measured in the excavation. The values quoted above for the Mississippi levees (from 10% to 25%) refers usually to a very soft soil and includes the effects other than actual contraction of volume. From 8% to 15% is usually quoted as the required extra height of embankments, although it is strenuously claimed by many that 3% or 2% is sufficient, or even that *no* allowance should be made.

The coefficients to determine the amount of settled embankment which may be made from a given volume of earth or rock measured in the excavation, are necessarily subject to variation on account of the method employed and the amount of compression and settlement which will take place during the progress of the work. The following figures have the weight of considerable authority but, if in error, the coefficients are probably high rather than low:

Gravel or sand	about	8%
Clay	"	10%
Loam	"	12%
Loose vegetable surface soil	"	15%

It may be noticed from the above table that the harder and cleaner the material the less is the contraction. Perfectly clean gravel or sand would not probably change volume appreciably. The above coefficients of shrinkage and expansion may be used to form the following convenient table:

Material.	To make 1000 cubic yards of embankment will require	1000 cubic yards measured in excavation will make
Gravel or sand	1087 cubic yards	920 cubic yards
Clay	1111 " "	900 " "
Loam....................	1136 " "	880 " "
Loose vegetable soil	1176 " "	850 " "
Rock, large pieces	714 " "	1400 " "
" small "	625 " "	1600 " "
	measured in excavation	of embankment.

Since writing the above the following values have been adopted by the American Railway Engineering Association as representing standard practice:

Coefficients of Shrinkage Allowance for Depositing Earthwork.

	Trestle filling.	Raising under traffic.
Black dirt	15%	5%
Clay	10%	5%
Sand	6%	5%

129. **Methods of forming embankments.** Embankments of moderate height are sometimes formed by scraping material with drag scrapers from ditches at the sides, especially if there is little or no cutting to be done in the immediate vicinity. Over a low level swampy stretch this method has the double advantage of building an embankment which is well above the general level and also provides generous drainage ditches which keep the embankment dry. Wheeled scrapers may be used economically up to a distance of 400 feet to excavate

cuts and deposit the material on low embankments. Such methods have the advantage of compacting the embankments during construction and reducing future shrinkage.

When carts are used, an embankment of any height may be formed by "dumping over the end" and building to the full height (or even higher to allow for shrinkage) as the embankment proceeds. The method is especially applicable when the material comes from a place as high as or higher than the grade-line, so that no up-hill hauling is necessary. Only a small contractor's plant is required for all of these methods.

Trestles capable of carrying carts, or even cars and locomotives, from which excavated material may be dropped, are found to be economical in spite of the fact that their cost is a construction expense. There is the disadvantage that such embankments require a long time to settle, but there are the advantages that the earth may be hauled by the train load from a distance of perhaps several miles, dumped from the

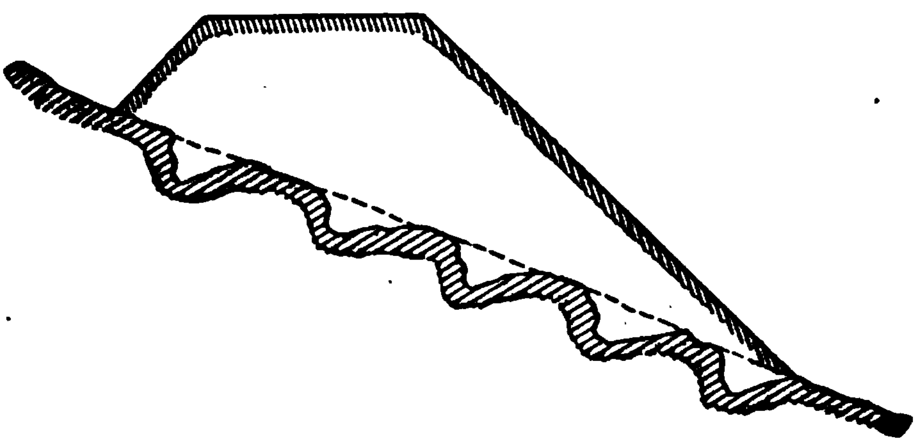


Fig. 63.

cars by train ploughs, or automatically dumped when the material is carried in patent dumping-cars, and all at a comparatively small cost per cubic yard. The disadvantages of slow settlement may be obviated, although at some additional cost, by making the trestle sufficiently strong to support regular traffic until the settlement is complete.

During recent years cableways have been utilized to fill comparatively narrow but deep ravines from material obtainable on either side of the ravine. This method obviates the construction of an excessively high trestle which might otherwise be considered necessary.

When an embankment is to be placed on a steep side hill which has a slippery clay surface, the embankment will some-

times slide down the hill, unless means are taken to prevent it. Some sort of bond between the old surface and the new material becomes necessary. This has sometimes been provided by cutting out steps somewhat as is illustrated in Fig. 63. It is possible that a deep ploughing of the surface would accomplish the result just as effectively and much cheaper. The tendency to slip is generally due not only to the nature of the soil but also to the usual accompanying characteristic that the soil is wet and springy. The sub-surface drainage of such a place with tile drains will still further prevent such slipping, which often proves very troublesome and costly.

COMPUTATION OF HAUL.

130. **Nature of subject.** As will be shown later when analyzing the cost of earthwork, the most variable item of cost is that depending on the distance hauled. As it is manifestly impracticable to calculate the exact distance to which every individual cartload of earth has been moved, it becomes necessary to devise a means which will give at least an equivalent of the haulage of all the earth moved. Evidently the *average* haul for any mass of earth moved is equal to the distance from the center of gravity of the excavation to the center of gravity of the embankment formed by the excavated material. As a rough approximation the center of gravity of a cut (or fill) may sometimes be considered to coincide with the center of gravity of that part of the profile representing it, but the error is frequently very large. The center of gravity may be determined by various methods, but the method of the "mass diagram" accomplishes the same ultimate purpose (the determination of the haul) with all-sufficient accuracy and also furnishes other valuable information.

131. **Mass diagram.** In Fig. 64 let $A'B' \ldots G'$ represent a profile and grade line drawn to the usual scales. Assume A' to be a point past which no earthwork will be hauled. Such a point is determined by natural conditions, as, for example, a river crossing, or one end of a long level stretch along which no grading is to be done except the formation of a low embankment from the material excavated from ample drainage ditches on each side. Above the profile draw an indefinite horizontal line (ACn in Fig. 64) which may be called the "zero line." Above every station point in the profile draw an ordinate (above or be-

low the zero line) which will represent the algebraic sum of the cubic yards of cut and fill (calling cut + and fill −) from the point A' to the point considered. The computations of these ordinates should first be made in tabular form as shown below. In doing this shrinkage must be allowed for by considering how much embankment would actually be made by so many cubic yards of excavation of such material. For example, it will be found that 1000 cubic yards of sand or gravel, measured in place (see § 128) will make about 920 cubic yards of embankment; therefore all cuttings in sand or gravel should be discounted in about this proportion. Excavations in rock should be increased in the proper ratio. In short, all excavations should be valued according to the amount of *settled* embankment that could be made from them. Place in the first column a list of the stations; in the second column, the number of cubic yards of cut or fill between each station and the preceding station; in the third and fourth columns, the kind of material and the proper shrinkage factor; in the fifth column, a repetition of the quantities in cubic yards, except that the excavations are diminished (or increased, in the case of rock) to the number of cubic yards of settled embankment which may be made from them. In the sixth column place the *algebraic sum* of the quantities in the fifth column (calling cuts + and fills −) from the starting-point to the station considered. These algebraic sums at each station will be the ordinates, drawn to some scale, of the mass curve. The scale to be used will depend somewhat on whether

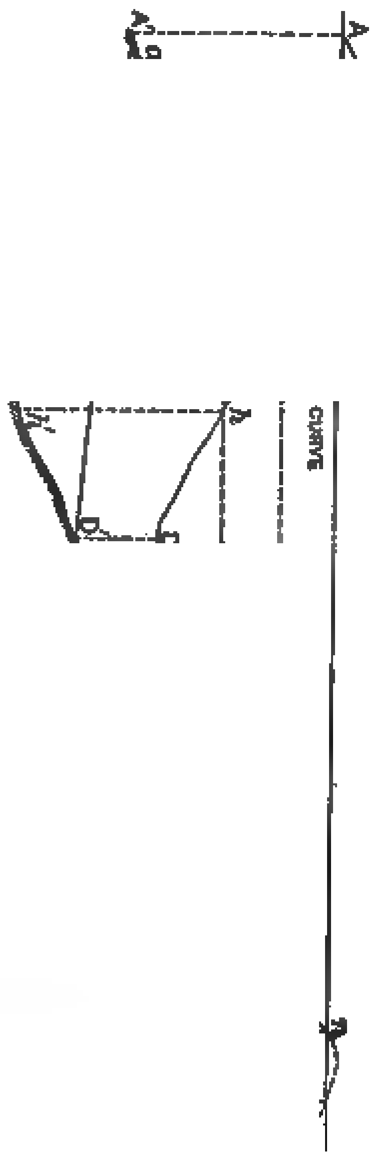


Fig. 64.—Mass Diagram.

the work is heavy or light, but for ordinary cases a scale of 5000 cubic yards per inch may be used. Drawing these ordinates to scale, a curve $A, B, \ldots G$ may be obtained by joining the extremities of the ordinates.

Sta.	Yards { cut + / fill −	Material.	Shrinkage factor.	Yards, reduced for shrinkage.	Ordinate in mass curve.
46 + 70	....	....	....	....	0
47	+ 195	Clayey soil	− 10 per cent	+ 175	+ 175
48	+ 1792	" "	− 10 "	+ 1613	+ 1788
+ 60	+ 614	" "	− 10 "	+ 553	+ 2341
49	− 143	....	....	− 143	+ 2198
50	− 906	....	....	− 906	+ 1292
51	− 1985	....	....	− 1985	− 693
52	− 1721	....	....	− 1721	− 2414
+ 30	− 112	....	....	− 112	− 2526
53	+ 177	Hard rock	+ 60 per cent	+ 283	− 2243
+ 70	+ 180	" "	+ 60 "	+ 289	− 1954
54	− 52	....	....	− 52	− 2006
+ 42	− 71	....	....	− 71	− 2077
55	+ 276	Clayey soil	− 10 per cent	+ 249	− 1828
56	+ 1242	" "	− 10 "	+ 1118	− 710
57	+ 1302	" "	− 10 "	+ 1172	+ 462

132. Properties of the mass curve.

1. The curve will be rising while over cuts and falling while over fills.

2. A tangent to the curve will be horizontal (as at B, D, E, F, and G) when passing from cut to fill or from fill to cut.

3. When the curve is *below* the "zero line" it shows that material must be drawn *backward* (to the left); and *vice versa*, when the curve is *above* the zero line it shows that material must be drawn *forward* (to the right).

4. When the curve crosses the zero line (as at A and C) it shows (in this instance) that the cut between A' and B' will just provide the material required for the fill between B' and C', and that no material should be hauled past C', or, in general, past any intersection of the mass curve and the zero line.

5. If any horizontal line be drawn (as ab), it indicates that the cut and fill between a' and b' will just balance.

6. When the center of gravity of a given volume of material is to be moved a given distance, it makes no difference (at least theoretically) how far each individual load may be hauled or how any individual load may be disposed of. The summation

of the products of each load times the distance hauled will be a constant, whatever the method, and will equal the total volume times the movement of the center of gravity. The *average haul*, which is the movement of the center of gravity, will therefore equal the summation of these products divided by the total volume. If we draw two horizontal parallel lines at an infinitesimal distance dx apart, as at ab, the small increment of cut dx at a' will fill the corresponding increment of fill at b', and this material must be hauled the distance ab. Therefore the product of ab and dx, which is the product of distance times volume, is represented by the area of the infinitesimal rectangle at ab, and the total area ABC represents the summation of volume times distance for all the earth movement between A' and C'. This summation of products divided by the total volume gives the average haul.

7. The horizontal line, tangent at E and cutting the curve at e, f, and g, shows that the cut and fill between e' and E' will just balance, and that a *possible* method of hauling (whether desirable or not) would be to "borrow" earth for the fill between C' and e', use the material between D' and E' for the fill between e' and D', and similarly balance cut and fill between E' and f' and also between f' and g'.

8. Similarly the horizontal line $hklm$ may be drawn cutting the curve, which will show another *possible* method of hauling. According to this plan, the fill between C' and h' would be made by borrowing; the cut and fill between h' and k' would balance; also that between k' and l' and between l' and m'. Since the area $ehDkE$ represents the measure of haul for the earth between e' and E', and the other areas measure the corresponding hauls similarly, it is evident that the sum of the areas $ehDkE$ and $ElFmf$, which is the measure of haul of all the material between e' and f', is largely in excess of the sum of the areas hDk, kEl, and lFm, plus the somewhat uncertain measures of haul due to borrowing material for $e'h'$ and wasting the material between m' and f'. Therefore to make the measure of haul a minimum a line should be drawn which will make the sum of the areas between it and the mass curve a minimum. Of course this is not necessarily the cheapest plan, as it implies more or less borrowing and wasting of material, which *may* cost more than the amount saved in haul. The comparison of the two methods is quite simple, however. Since the amount

of fill between e' and h' is represented by the *difference* of the ordinates at e and h, and similarly for m' and f', it follows that the amount to be borrowed between e' and h' will exactly equal the amount wasted between m' and f'. By the first of the above methods the haul is excessive, but is definitely known from the mass diagram, and all of the material is utilized; by the second method the haul is reduced to about one-half, but there is a known quantity in cubic yards wasted at one place and the same quantity borrowed at another. The length of haul necessary for the borrowed material would need to be ascertained; also the haul necessary to waste the other material at a place where it would be unobjectionable. Frequently this is best done by widening an embankment beyond its necessary width. The computation of the relative cost of the above methods will be discussed later (§ 148).

9. Suppose that it were deemed best, after drawing the mass curve, to introduce a trestle between s' and v', thus saving an amount in fill equal to tv. If such had been the original design, the mass curve would have been a straight horizontal line between s and t and would continue as a curve which would be at all points a distance tv above the curve $vFmzfGg$. If the line Ef is to be used as a zero line, its intersection with the new curve at x will show that the material between E' and x' will just balance if the trestle is used, and that the amount of haul will be measured by the area between the line Ex and the broken line $Estx$. The same computed result may be obtained without drawing the auxiliary curve txn . . . by drawing the horizontal line zy at a distance $xz(=tv)$ below Ex. The amount of the haul can then be obtained by adding the triangular area between Es and the horizontal line Ex, the rectangle between st and Ex, and the irregular area between vFz and y . . . z (which last is evidently equal to the area between tx and E . . . x). The disposal of the material at the right of z' would then be governed by the indications of the profile and mass diagram which would be found at the right of g'. In fact it is difficult to decide with the best of judgment as to the proper disposal of material without having a mass diagram extending to a considerable distance each side of that part of the road under immediate consideration.

133. Area of the mass curve. The area may be computed most readily by means of a planimeter, which is capable with reasonable care of measuring such areas with as great accuracy

as is necessary for this work. If no such instrument is obtainable, the area may be obtained by an application of "Simpson's rule." The ordinates will usually be spaced 100 feet apart. Select an *even* number of such spaces, leaving, if necessary, one or more triangles or trapezoids at the ends for separate and independent computation. Let $y_0 \ldots y_n$ be the ordinates, i.e., the number of cubic yards at each station of the mass curve, or the figures of "column six" referred to in § 131. Let the uniform distance between ordinates (=100 feet) be called 1, i.e., one *station.* Then the units of the resulting area will be cubic yards hauled one station. Then the

$$\text{Area} = \tfrac{1}{3}[y_0 + 4(y_1 + y_3 + \ldots y_{(n-1)}) + 2(y_2 + y_4 + \ldots y_{(n-2)}) + y_n]. \qquad (62)$$

When an ordinate occurs at a substation, the best plan is to ignore it at first and calculate the area as above. Then, if the difference involved is too great to be neglected, calculate the area of the triangle having the extremity of the ordinate at the substation as an apex, and the extremities of the ordinates at the adjacent stations as the ends of the base. This may be done by finding the ordinate at the substation that would be a proportional between the ordinates at the adjacent full stations. Subtract this from the real ordinate (or *vice versa*) and multiply the difference by $\frac{1}{2} \times 1$. An inspection will often show that the correction thus obtained would be too small to be worthy of consideration. If there is more than one substation between two full stations, the corrective area will consist of two triangles and one or more trapezoids which may be similarly computed, if necessary.

When the zero line (Fig. 64) is shifted to eE, the drop from AC (produced) to E is known in the same units, cubic yards. This constant may be subtracted from the numbers ("column 6," § 131) representing the ordinates, and will thus give, without any scaling from the diagram, the exact value of the modified ordinates,

134. Value of the mass diagram. The great value of the mass diagram lies in the readiness with which different plans for the disposal of material may be examined and compared. When the mass curve is once drawn, it will generally require only a shifting of the horizontal line to show the disposal of the material by any proposed method. The mass diagram also shows the

extreme length of haul that will be required by any proposed method of disposal of material. This brings into consideration the "limit of profitable haul," which will be fully discussed in § 148. For the present it may be said that with each method of carrying material there is some limit beyond which the expense of hauling will exceed the loss resulting from borrowing and wasting. With wheelbarrows and scrapers the limit of profitable haul is comparatively short, with carts and tram-cars it is much longer, while with locomotives and cars it may be several miles. If, in Fig. 64, *eE* or *Ef* exceeds the limit of profitable haul, it shows at once that some such line as *hklm* should be drawn and the material disposed of accordingly.

135. **Changing the grade line.** The formation of the mass curve and the resulting plans as to the disposal of material are based on the mutual relations of the grade line and the surface profile and the amounts of cut and fill which are thereby implied. If the grade line is altered, every cross-section is altered, the amount of cut and fill is altered, and the mass curve is also changed. At the farther limit of the actual change of the grade line the revised mass curve will have (in general) a different ordinate from the previous ordinate at that point. From that point on, the revised mass curve will be parallel to its former position, and the revised curve may be treated similarly to the case previously mentioned in which a trestle was introduced. Since it involves tedious calculations to determine accurately how much the volume of earthwork is altered by a change in grade line, especially through irregular country, the effect on the mass curve of a change in the grade line cannot therefore be readily determined except in an approximate way. Raising the grade line will evidently increase the fills and diminish the cuts, and *vice versa*. Therefore if the mass curve indicated, for example, either an excessively long haul or the necessity for borrowing material (implying a fill) and wasting material farther on (implying a cut), it would be *possible* to diminish the fill (and hence the amount of material to be borrowed) by lowering the grade line near that place, and diminish the cut (and hence the amount of material to be wasted) by raising the grade line at or near the place farther on. Whether the advantage thus gained would compensate for the possibly injurious effect of these changes on the grade line would require patient investigation. But the method outlined shows how the mass

curve might be used to indicate a possible change in grade line which might be demonstrated to be profitable.

136. Limit of free haul. It is sometimes specified in contracts for earthwork that *all* material shall be entitled to free haul up to some specified limit, say 500 or 1000 feet, and that all material drawn farther than that shall be entitled to an allowance on the *excess* of distance. It is manifestly impracticable to measure the excess for each load, as much so as to measure the actual haul of each load. The mass diagram also solves this problem very readily. Let Fig. 65 represent a pro-

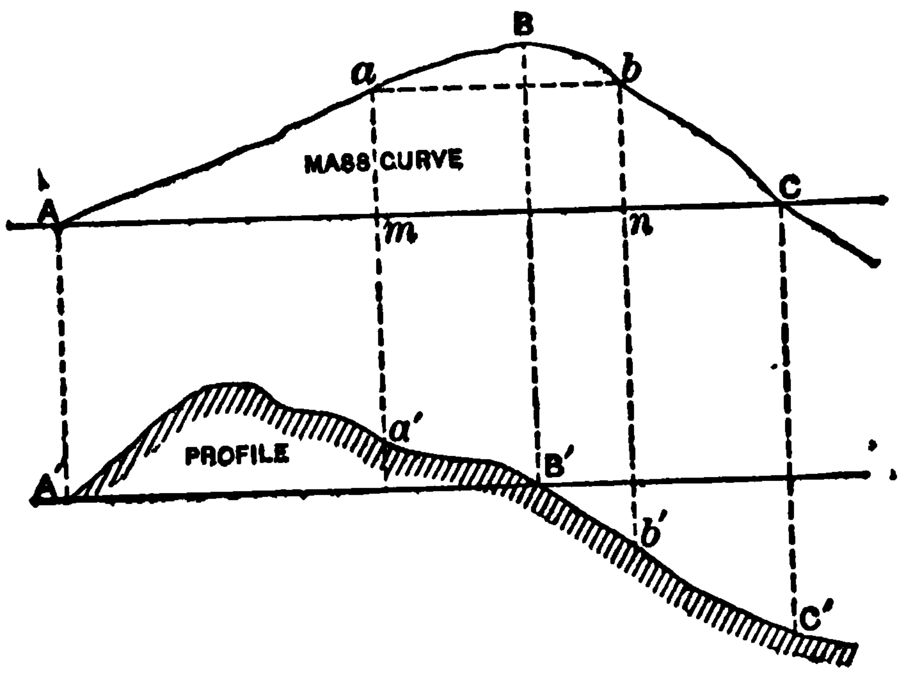


Fig. 65.

file and mass diagram of about 2000 feet of road, and suppose that 800 feet is taken as the limit of free haul. Find two points, a and b, in the mass curve *which are on the same horizontal line* and which are 800 feet apart. Project these points down to a' and b'. Then the cut and fill between a' and b' will just balance, and the cut between A' and a' will be needed for the fill between b' and C'. In the mass curve, the area between the horizontal line ab and the curve aBb represents the haulage of the material between a' and b', which is all free. The rectangle $abmn$ represents the haulage of the material in the cut $A'a'$ across the 800 feet from a' to b'. This is also free. The sum of the two areas Aam and bnC represents the haulage entitled to an allowance, since it is the summation of the products of cubic yards times the *excess* of distance hauled.

If the amount of cut and fill was symmetrical about the point

B', the mass curve would be a symmetrical curve about the vertical line through B, and the two limiting lines of free haul would be placed symmetrically about B and B'. In general there is no such symmetry, and frequently the difference is considerable The area $aBbnm$ will be materially changed according as the two vertical lines am and bn, always 800 feet apart, are shifted to the right or left. It is easy to show that the area $aBbnm$ is a *maximum* when ab is horizontal. The minimum value would be obtained either when m reached A or n reached C, depending on the exact form of the curve. Since the position for the minimum value is manifestly unfair, the best definite value obtainable is the maximum, which must be obtained as above described. Since $aBbnm$ is made maximum, the remainder of the area, which is the allowance for overhaul, becomes a minimum. The areas Aam and bCn may be obtained as in § 102. If the whole area $AaBbCA$ has been previously computed, it may be more convenient to compute the area $aBbnm$ and subtract it from the total area.

Since the intersections of the mass curve and the "zero line" mark limits past which no material is drawn, it follows that there will be no allowance for overhaul except where the distance between consecutive intersections of the zero line and mass curve exceeds the limit of free haul.

Frequently all allowances for overhaul are disregarded; the profiles, estimates of quantities, and the required disposal of material are shown to bidding contractors, and they must then make their own allowances and bid accordingly. This method has the advantage of avoiding possible disputes as to the amount of the overhaul allowance, and is popular with railroad companies on this account. On the other hand the facility with which different plans for the disposal of material may be studied and compared by the mass-curve method facilitates the adoption of the most economical plan, and the elimination of uncertainty will frequently lead to a safe reduction of the bid, and so the method is valuable to both the railroad company and the contractor.

ELEMENTS OF THE COST OF EARTHWORK.

137. Analysis of the total cost into items. The variation in the total cost of excavating earthwork, hauling it a greater or less distance, and forming with it an embankment of definite

form or wasting it on a spoil bank, is so great that the only possible method of estimating the cost under certain assumed conditions is to separate the total cost into elementary items. Ellwood Morris was perhaps the first to develop such a method —see *Journal of the Franklin Institute*, September and October, 1841. Trautwine used the same general method with some modifications. The following analysis will follow the same general plan, will quote some of the figures given by Morris and by Trautwine, but will also include facts and figures better adapted to modern conditions. Since every item of cost (except interest on cost of plant and its depreciation) is a direct function of the current price of common labor, all calculations will be based on the simple unit of $1 per day. Then the actual cost may be obtained by multiplying the calculated cost under the given conditions by the current price of day labor. When possible, figures will be quoted giving the cost of all items of work on a loose sandy soil which is the easiest to work and also for the cost of the heaviest soils, such as stiff clay and hard pan. These represent the extremes, excluding rock, which will be treated separately. The cost of intermediate grades may be interpolated between the extreme values according to the judgment of the engineer as to the character of the soil.

The possible division into items varies greatly according to the method adopted, but the differentiation into items given below (which is strictly applicable to the old fashioned simpler methods of work) can usually be applied to any other method by merely combining or eliminating some of the items. The items are

1. Loosening the natural soil.
2. Loading the soil into whatever carrier may be used.
3. Hauling excavated material from excavation to embankment or spoil bank.
4. Spreading or distributing the soil on the embankment.
5. Keeping roadways or tracks in good running order.
6. Trimming cuts to their proper cross-section (sometimes called "sandpapering").
7. Repairs, wear, depreciation, and interest on cost of plant.
8. Superintendence and incidentals.

138. Item 1. Loosening. (a) Ploughs. Very light sandy soils can frequently be shovelled without any previous loosening, but it is generally economical, even with very light material,

to use a plough. Morris quotes, as the results of experiments, that a three-horse plough would loosen from 250 to 800 cubic yards of earth per day, which at a valuation of $5 per day would make the cost per yard vary from 2 cents to 0.6 cent. Trautwine estimates the cost on the basis of two men handling a two-horse plough at a total cost of $3.87 per day, being $1 each for the men, 75 c. for each horse, and an allowance of 37 c. for the plough, harness, etc. From 200 to 600 cubic yards is estimated as a fair day's work, which makes a cost of 1.9 c. to 0.65 c. per yard, which is substantially the same estimate as above. Extremely heavy soils have sometimes been loosened by means of special ploughs operated by traction-engines.

Gillette estimates that "a two-horse team with a driver and a man holding the plough will loosen 25 cubic yards of fairly tough clay, or 35 cubic yards of gravel and loam per hour." For ten hours per day this would be 250 to 350 cubic yards per day. These values are neither as high nor as low as the extremes above noted. It is probably very seldom that a soil will be so light that a two-horse (or three-horse) plough can loosen as much as 600 (or 800) cubic yards per day.

It is sometimes necessary to plough up a macadamized street. This may be done by using as a plough a pointed steel bar which is fastened to a very strong plough frame. A preliminary hole must be made which will start the bar under the macadam shell. Then, as the plough is drawn ahead, the shell is ripped up. Four or six horses, or even a traction-engine, are used for such work. Gillette quotes two such cases where the cost of such loosening was 2 c. and 6 c. per cubic yard, with common labor at 15 c. per hour. Two-thirds of such figures will reduce them to the $1 per day basis. The cost for ploughing *on the* $1 *per day basis* may therefore be summarized as follows:

For very loose sandy soils...........	0.6 c. per cubic yard
" " heavy clay "	2.0 c. " " "
" hard pan and macadam, up to ...	4.0 c. " " "

(b) Picks. When picks are used for loosening the earth, as is frequently necessary and as is often done when ploughing would perhaps be really cheaper, an estimate* for a fair day's

* Trautwine.

work is from 14 to 60 cubic yards, the 14 yards being the estimate for stiff clay or cemented gravel, and the 60 yards the estimate for the lightest soil that would require loosening. At $1 per day this means about 7 c. to 1.7 c. per cubic yard, which is about three times the cost of ploughing. Five feet of the face is estimated * as the least width along the face of a bank that should be allowed to enable each laborer to work with freedom and hence economically.

(c) **Blasting.** Although some of the softer shaly rocks may be loosened with a pick for about 15 to 20 c. per yard, yet rock in general, frozen earth, and sometimes even compact clay are most economically loosened by blasting. The subject of blasting will be taken up later, §§ **140–155.**

(d) **Steam-shovels.** The items of loosening and loading merge together with this method, which will therefore be treated in the next section.

139. Item 2. LOADING. (a) **Hand-shovelling.** Much depends on proper management, so that the shovellers need not wait unduly either for material or carts. With the best of management considerable time is thus lost, and yet the intervals of rest need not be considered as entirely lost, as it enables the men to work, while actually loading, at a rate which it would be physically impossible for them to maintain for ten hours. Seven shovellers are sometimes allowed for each cart; otherwise there should be five, two on each side and one in the rear. Economy requires that the number of loads per cart per day should be made as large as possible, and it is therefore wise to employ as many shovellers as can work without mutual interference and without wasting time in waiting for material or carts. The figures obtainable for the cost of this item are unsatisfactory on account of their large disagreements. The following are quoted as the number of cubic yards that can be loaded into a cart by an average laborer in a working day of ten hours, the lower estimate referring to heavy soils, and the higher to light sandy soils: 10 to 14 cubic yards (Morris), 12 to 17 cubic yards (Haskoll), 18 to 22 cubic yards (Hurst), 17 to 24 cubic yards (Trautwine), 16 to 48 cubic yards (Ancelin). As these estimates are generally claimed to be based on actual experience, the discrepancies are probably due to differences of management. If the

* Hurst.

average of 15 to 25 cubic yards be accepted, it means, on the basis of $1 per day, 6.7 c. to 4 c. per cubic yard. These estimates apply only to earth. *Rockwork* costs more, not only because it is harder to handle, but because a cubic yard of solid rock, measured in place, occupies about 1.8 cubic yards when broken up, while a cubic yard of earth will occupy about 1.2 cubic yards. Rockwork will therefore require about 50% more loads to haul a given volume, *measured in place*, than will the same nominal volume of earthwork. The above authorities give estimates for loading rock varying from 6.9 c. to 10 c. per cubic yard. The above estimates apply only to the loading of carts or cars with shovels or by hand (loading masses of rock). The cost of loading wheelbarrows and the cost of scraper work will be treated under the item of hauling.

(b) **Steam-shovels.*** Whenever the magnitude of the work will warrant it there is great economy in the use of steam-shovels. These have a "bucket" or "dipper" on the end of a long beam, the bucket having a capacity varying from ¾ to 2½ cubic yards. Steam-shovels handle all kinds of material from the softest earth to shale rock, earthy material containing large boulders, tree-stumps, etc. The record of work done varies from 200 to 1000 cubic yards in 10 hours. They perform all the work of loosening and loading. Their economical working requires that the material shall be hauled away as fast as it can be loaded, which usually means that cars on a track, hauled by horses or mules, or still better by a locomotive, shall be used. The expenses for a steam-shovel, costing about $5000, will average about $1000 per month. Of this the engineer may get $100; the fireman $50; the cranesman $90; repairs perhaps $250 to $300; coal, from 15 to 25 tons, cost very variable on account of expensive hauling; water, a very uncertain amount, sometimes costing $100 per month; about five laborers and a foreman, the laborers getting $1.25 per day and the foreman $2.50 per day, which will amount to $227.50 per month. This gang of laborers is employed in shifting the shovel when necessary, taking up and relaying

* For a thorough treatment of the capabilities, cost, and management of steam-shovels the reader is referred to "Steam-shovels and Steam-shovel Work," by E. A. Hermann. D. Van Nostrand Co., New York.

This book is now out of print. "Earthwork and its Cost," by H. P. Gillette, to which the student is referred for a more elaborate exposition of the subject, has used many of Hermann's cuts.

tracks for the cars, shifting loaded and unloaded cars, etc. In shovelling through a deep cut, the shovel is operated so as to undermine the upper parts of the cut, which then fall down within reach of the shovel, thus increasing the amount of material handled for each new position of the shovel. If the material is too tough to fall down by its own weight, it is sometimes found economical to employ a gang of men to loosen it or even blast it rather than shift the shovel so frequently. Non-condensing engines of 50 horse-power use so much water that the cost of water-supply becomes a serious matter if water is not readily obtainable. The lack of water facilities will often justify the construction of a pipe line from some distant source and the installation of a steam-pump. Hence the seemingly large estimate of $100 per month for water-supply, although under favorable circumstances the cost may almost vanish. The larger steam-shovels will consume nearly a ton of coal per day of 10 hours. The expense of hauling this coal from the nearest railroad or canal to the location of the cut is often a very serious item of expense and may easily double the cost per ton. Some steam-shovels have been constructed to be operated by electricity obtained from a plant perhaps several miles away. Such a method is especially advantageous when fuel and water are difficult to obtain.

The following general requirements and specifications were recommended in 1907 by the American Railway Engineering Association:

Three important cardinal points should be given careful attention in the selection of a steam-shovel. These are in their order

(1) Care in the selection, inspection and acceptance of all material that enters into every part of the machine.

(2) Design for strength.

(3) Design for production.

GENERAL SPECIFICATIONS.

Weight of shovel: Seventy (70) tons.

Capacity of dipper: Two and one-half ($2\frac{1}{2}$) yards.

Steam pressure: One hundred and twenty (120) pounds.

Clear height above rail of shovel track at which dipper should unload: Sixteen (16) feet.

Depth below rail of shovel track at which dipper should dig Four (4) feet.

Number of movements of dipper per minute from time of entering bank to entering bank: Three (3).

Character of hoist: Cable.

Character of swing: Cable.

Character of housing: Permanent for all employes.

Capacity of tank: Two thousand (2000) gallons.

Capacity of coal-bunker: Four (4) tons.

Spread of jack arm: Eighteen (18) feet. A special short arm should be provided.

Form of steam-shovel track: "T" rails on ties.

Length of rails for ordinary work: Six (6) feet.

Form of rail joint: Strap.

Manufacturers of steam-shovels will sometimes "guarantee" that certain of their shovels will excavate, say 3000 cubic yards of earth per day of ten hours. Even if it were possible for a shovel to fill a car at the rate of 5 cubic yards per minute, it is always impracticable to maintain such a speed, since a shovel must always wait for the shifting of cars and for the frequent shifting of the shovel itself. There are also delays due to adjustments and minor breakdowns. The best shovel records are made when the cars are large—other things being equal. The item of interest and depreciation of the plant is very large in steam-shovel work. This will be discussed further later. The cost of loading alone will usually come to between 3 and 4 c. per cubic yard. The cost of shifting the cars so as to place them successively under the shovel, haul them to the dumping place, dump them and haul them back, will generally be as much more. Gillette quotes five jobs on one railroad where the total cost for loading and hauling varied from 5.9 c. to 11.4 c. per cubic yard. But as these figures are based on *car* measurement, the cost per cubic yard in *place* measurement must be increased about one-fourth, or from 7.4 c. to 14.2 c.

140. Item 3. Hauling. The cost of hauling depends on the number of round trips per day that can be made by each vehicle employed. As the cost of each vehicle is practically the same whether it makes many trips or few, it becomes important that the number of trips should be made a maximum, and to that end there should be as little delay as possible in loading and

unloading. Therefore devices for facilitating the passage of the vehicles have a real money value.

(a) **Carts.** The average speed of a horse hauling a two-wheeled cart has been found to be 200 feet per minute, a little slower when hauling a load and a little faster when returning empty. This figure has been repeatedly verified. It means an allowance of one minute for each 100 feet (or "station") of "lead—the lead being the distance the earth is hauled." The time lost in loading, dumping, waiting to load, etc., has been found to average 4 minutes per load. Representing the number of stations (100 feet) of lead by s, the number of loads handled in 10 hours (600 minutes) would be $600 \div (s+4)$. The number of loads per cubic yard, measured in the bank, is differentiated by Morris into three classes, viz.:

3 loads per cubic yard in descending hauling;
$3\frac{1}{2}$ " " " " " level hauling; and
4 " " " " " ascending hauling.

Attempts have been made to estimate the effect of the grade of the roadway by a theoretical consideration of its rate, and of the comparative strength of a horse on a level and on various grades. While such computations are always practicable on a railway (even on a temporary construction track), the traction on a temporary earth roadway is always very large and so very variable that any refinements are useless. On railroad earthwork the hauling is generally nearly level or it is *descending*—forming embankments on low ground with material from cuts in high ground. The only common exception occurs when an embankment is formed from borrow-pits on low ground. One method of allowing for ascending grade is to add to the horizontal distance 14 times the difference of elevation for work with carts and 24 times the difference of elevation for work with wheelbarrows, and use that as the lead. For example, using carts, if the lead is 300 feet and there is a difference of elevation of 20 feet, the lead would be considered equivalent to $300+(14\times20)=580$ feet on a level.

Trautwine assumes the average load for all classes of work to be $\frac{1}{3}$ cubic yard, which figure is justified by large experience. Using one figure for all classes of work simplifies the calculations and gives the number of cubic yards carried per day of 10 hours equal to $\frac{600}{3(s+4)}$. Dividing the cost of a cart per day by the

number of cubic yards carried gives the cost of hauling per yard. In computing the cost of a cart per day, Trautwine refers to the practice of having one driver manage four carts, thus making a charge of 25 c. per day for each cart for the driver. Although this might be an economical method when the haul is very long, it is not economical for short hauls. A safer estimate is to allow not more than two carts per driver and in many cases a driver for each cart. Some contractors employ a driver for each cart and then require that the drivers shall assist in loading. The policy to be adopted is sometimes dependent on labor union conditions, which may demand that drivers must not assist in loading. The supply of labor and the amount of work on hand have a great influence on the methods of work which a contractor may adopt, for a strike will often disarrange all plans.

The cost of a horse and cart must practically include a charge for the time of the horse on Sundays, rainy days and holidays. The cost of repairs of cart and harness is generally included in this item for simplicity, but, under a strict application of the analysis suggested in § 138, it should properly be included under Item 7, Repairs, etc.

Since the time required for loading loose rock is greater than for earthwork, less loads will be hauled per day. The time allowance for loading, etc., is estimated by Trautwine as 6 minutes instead of 4 as for earth. Considering the great expansion of rock when broken up (see § 128), one cubic yard of solid rock, measured in place, would furnish the equivalent of five loads of earthwork of $\frac{1}{3}$ cubic yard. Therefore, on the basis of five loads per cubic yard, the number of cubic yards handled per day per cart would be $\frac{600}{5(s+6)}$.

Let C represent the daily cost of a horse and cart and of the proportional cost of the driver (according to the number of carts handled by one driver), then the cost per cubic yard, *measured in the cut,* for hauling may be given by the formula:

$$\left.\begin{array}{l} \text{Cost per cu. yd. of hauling earth in carts} = \dfrac{C\times 3(s+4)}{600} \\ \text{"}\quad\text{"}\quad\text{"}\quad\text{"}\quad\text{"}\quad\text{"}\quad\text{rock}\quad\text{"}\quad\text{"} = \dfrac{C\times 5(s+6)}{600} \end{array}\right\} . \quad (63)$$

(b) **Wagons.** For longer leads (i.e., from $\frac{1}{4}$ to $\frac{3}{4}$ of a mile) wagons drawn by two (or three) horses are more economical. The old-style wagons (about 0.8 cu. yd.) have bottoms of loose thick narrow boards. Raising them individually deposits the load underneath. Modern dump wagons contain from 1.0 to 2.0 cu. yds. The daily cost may be estimated on the same principle as the cost of carts.

The number of wagon trips per 10 hours will depend somewhat on the management of the shovellers. Too many shovellers per wagon is not economical, measured in yards shovelled per man, although it may reduce the time consumed in loading any one wagon. At an average figure of 20 cubic yards, measured in place, per shoveller per 10 hours, seven shovellers would load 14 cubic yards per hour or one cubic yard in 4.3 minutes. This would be the allowance for a wagon with a capacity of about $1\frac{1}{3}$ yards of loose earth. Adding time for unloading, waiting to load and other possible "lost time," there is probably a total of six minutes. This figure will vary very considerably according to the number of shovellers per wagon, the capacity of the wagon, the type of wagon (whether self-dumping) and other details in the method of management. Adopting six minutes as the time used for loading, unloading, and other "lost time," the formula becomes.

$$\text{Cost per cubic yard of hauling in wagons} = \frac{C(s+6)}{600\,c}, \quad \ldots \quad (64)$$

in which C is the cost of the wagon, team and driver per day of 10 hours; s is the distance hauled in stations of 100 feet, and c is the capacity of the wagon in cubic yards, *place measurement,* which should be about three fourths of the nominal capacity of the wagon for earth and about sixty per cent when handling rock.

(c) **Wheelbarrows.** Gillette has computed from observations that a man will trundle a wheelbarrow at the rate of 250 feet per minute or 1.25 *stations* of lead per minute for the round trip. The time required for loading is estimated at $2\frac{1}{4}$ minutes and for unloading, adjusting wheeling planks, short rests, etc., $\frac{3}{4}$ minute, or a total of three minutes per trip for all purposes except hauling. Gillette allows for a load only 1/15 cubic yard,

measured in place, or about 1/11 yard, 2.5 cubic feet, on the wheelbarrow. With notation as before

$$\left.\begin{array}{l}\text{Cost per cubic yard of loading and}\\ \text{hauling earth in wheelbarrows}\end{array}\right\} = \frac{C \times 15(1.25s+3)}{600}. \qquad (65)$$

In this equation C is the cost of both loading and hauling, and usually includes the allowance (Item 7) for the cost, repairs and depreciation of the wheelbarrows, whose service is very short lived. Trautwine estimates this at five cents per day or a total of \$1.05 for labor and wheelbarrow.

The number of wheelbarrow loads required for a cubic yard of rock, measured in place, is about twenty-four. The time required for loading should also be increased about one fourth; the time required for all purposes except hauling is therefore about 3.75 minutes, and the corresponding equation becomes

$$\left.\begin{array}{l}\text{Cost per cubic yard of loading and}\\ \text{hauling rock in wheelbarrows}\end{array}\right\} = \frac{C \times 24(1.25s+3.75)}{600}. \qquad (66)$$

(d) **Scrapers.** These are made in three general ways, "buck" scrapers, "drag" scrapers and "wheeled" scrapers. The buck scraper in its original form consisted merely of a wide plank, shod with an iron strap on the lower edge and provided with a pole and a small platform on which the driver may stand to weight it down. The earth is not loaded on to any receptacle and carried, but is merely pushed over the ground. Notwithstanding the apparent inefficiency of the method, its extreme simplicity has caused its occasional adoption for the construction of canal embankments out of material from the bed of the canal. The occasions are rare when their use for railroad work would be practicable, and even then drag scrapers would probably be preferable.

A drag scraper is an immense "scoop shovel" about three feet long and three feet wide. There are usually two handles and a bail in front by which it is dragged by a team of horses. The nominal capacity varies from 7.5 cubic feet for the largest sizes, down to 3 cubic feet for the "one-horse" size, but these figures must be discounted by perhaps 40 or 50% for the actual average volume (as measured in the cut) loaded on during one scoop. The expansion of the earth during loosening is alone respons-

ible for a discount of 25%. These scrapers cost from $10 to $18.

A wheeled scraper is essentially an extra-large drag scraper which may be raised by a lever and carried on a pair of large wheels. Their nominal capacity ranges from 10 to 17 cubic feet, which should usually be liberally discounted when estimating output. They are loaded by dropping the scoop so that it scrapes up its load. The lever raises the scoop so that the load is carried on wheels instead of being dragged. At the dump the scoop is tipped so as to unload it. The movement of the scraper is practically continuous. They cost from $40 to $75. Their advantages over drag scrapers consist (1) in their greater capacity, (2) in the economy of transporting the load on wheels instead of by dragging, and (3) in the far greater length of haul over which the earth may be economically handled.

Morris estimated the speed of drag scrapers to be 140 feet per minute, or 70 feet of *lead* per minute. The "lead" should be here interpreted as the average distance from the center of the pit to the center of the dump. Gillette declares the speed to be 220 feet per minute. Some of this variation may be due to differences in the method of measuring the distance actually travelled, especially when the lead is very short, since the scraper teams must always travel a considerable extra distance at each end in order to turn around most easily. This extra distance is practically constant whether the lead is long or short. Gillette quotes an instance where the length of lead was actually about 20 feet, but the scraper teams travelled about 150 feet for each load carried. On this account Gillette adopts a minimum of 75 feet of lead no matter how short the lead actually may be. Of course the speed depends considerably on how strictly the men are kept to their work and also on the care which may be taken to obtain a full load for each scraper. As a compromise between Morris's and Gillette's estimates we may adopt the convenient rate of speed of 200 feet per minute, or 100 feet of lead per minute. There should also be allowed for the time lost in loading and unloading and for travelling the extra distance travelled by the teams in making the circuit, $1\frac{3}{4}$ minutes. Allowing the average value of seven loads per cubic yard and letting C represent the cost of scraper team and driver per day, we have for the cost as follows:

$$\left.\begin{array}{l}\text{Cost per cubic yard of loading and}\\ \text{hauling earth in drag scrapers}\end{array}\right\}=\frac{C\times 7\,(s+1\frac{1}{3})}{600}. \quad . \quad . \quad (67)$$

In this formula C should include the cost of not only the driver, team, and scraper, but also the proper proportion of the wages of an extra man, who assists each driver in loading his scraper, and whose wages should be divided among the two (or three) scrapers to which he is assigned. Scraper work nearly always implies ploughing, the cost of which should be computed as under Item 1.

When a low embankment is formed from borrow-pits on each side of the road, it may be done with scrapers, which move from one borrow-pit to the other, taking a load alternately from each side to the center and making but one half turn for each load carried. This reduces the time lost in turning by one third of a minute and reduces the constant in the numerator in Eq. (67) from $1\frac{1}{3}$ to 1. In this case the lead will usually be not greater than 75 feet, and therefore, if we consider this as a minimum value, s will ordinarily equal .75 and the quantity in the parenthesis will equal 1.75.

When using wheeled scrapers the catalogue capacity, which varies from 9 or 10 feet for a No. 1 scraper to 16 or 17 feet for a No. 3 scraper, must be reduced to 5 loads per cubic yard (place measurement) for a No. 1 scraper and to $2\frac{1}{2}$ loads per cubic yard for a No. 3, not only on account of the expansion of the earth during loosening, but also on account of the impracticability of loading these scrapers to their maximum nominal capacity. When the haul or lead for wheeled scrapers is 300 feet or over, it will be justifiable to employ shovellers to fill up the bowl of the shovel, especially when the soil is tough and when it is impracticable to fill the shovel even approximately full by the ordinary method. A snatch team to assist in loading the scrapers it also economical, especially with the larger scrapers. The proportionate number of snatch teams to the total number of scrapers of course depends on the length of haul. The cost of these extra shovellers and extra snatch teams must be divided proportionally among the number of scrapers assisted, in determining the value C in the formula given below. The extra time to be allowed on account of turning, loading, and dumping is about $1\frac{1}{2}$ minutes. The speed is considered one station of lead per minute as before. If we call C the average

daily cost of one scraper and n the capacity of the scraper, or the number of loads per cubic yard, we may write the following formula:

$$\left.\begin{array}{r}\text{Cost per cubic yard of loading and}\\ \text{hauling earth in wheeled scrapers}\end{array}\right\} = \frac{C \times n(s+1\frac{1}{2})}{600} \quad . \quad . \quad (68)$$

(e) **Cars and horses.** The items of cost by this method are (*a*) charge for horses employed, (*b*) charge for men employed strictly in hauling, (*c*) charge for shifting rails when necessary, (*d*) repairs, depreciation, and interest on cost of cars and track. Part of this cost should strictly be classified under items 5 and 7, mentioned in § 137, but it is perhaps more convenient to estimate them as follows:

The traction of a car on rails is so very small that grade resistance constitutes a very large part of the total resistance if the grade is 1% or more. For all ordinary grades it is sufficiently accurate to say that the grade resistance is to the gross weight as the rise is to the distance. If the distance is supposed to be measured along the slope, the proportion is strictly true; i.e., on a 1% grade the grade resistance is 1 lb. per 100 of weight or 20 lbs. per ton. If the resistance on a level at the usual velocity is $\frac{1}{120}$, a grade of 1:120 (0.83%) will exactly double it. If the material is hauled *down* a grade of 1:120, the cars will run by gravity after being started. The work of hauling will then consist practically of hauling the empty cars up the grade. The grade resistance depends only on the rate of grade and the weight, but the tractive resistance will be *greater per ton of weight* for the unloaded than for the loaded cars. The tractive power of a horse is less on a grade than on a level, not only because the horse raises his own weight in addition to the load, but is anatomically less capable of pulling on a grade than on a level. In general it will be possible to plan the work so that loaded cars need not be hauled *up* a grade, unless an embankment is to be formed from a low borrow-pit, in which case another method would probably be advisable. These computations are chiefly utilized in designing the method of work—the proportion of horses to cars. An example may be quoted from English practice (Hurst), in which the cars had a capacity of 3⅓ cubic yards, weighing 30 cwt. empty. Two horses took five "wagons" ¾ of a mile on a level

railroad and made 15 journeys per day of 10 hours, i.e., they handled 250 yards per day. In addition to those on the "straight road," another horse was employed to make up the train of loaded wagons. With a short lead the straight-road horses were employed for this purpose. In the above example the number of men required to handle these cars, shift the tracks, etc., is not given, and so the exact cost of the above work cannot be analyzed. It may be noticed that the two horses travelled 22½ miles per day, drawing in one direction a load, including the weight of the cars, of about 57,300 lbs., or 28.65 net tons. Allowing $\frac{1}{120}$ as the necessary tractive force, it would require a pull of 477.5 lbs., or 239 lbs. for each horse. With a velocity of 220 feet per minute this would amount to 1½ horse-power per horse, exerted for only a short time, however, and allowing considerable time for rest and for drawing only the empty cars. Gillette claims that the rolling-resistance for such cars on a contractor's track should be considered as 40 lbs. per ton (the equivalent of a 2% grade) and quotes many figures to support the assertion. Unquestionably the resistance on tracks with very light rails, light ties with wide spacing and no tamping, would be very great and might readily amount to 40 lbs. per ton. In the above case, the resistance could not have been much if any over $\frac{1}{120}$. A resistance of 40 lbs. per ton would have required each horse to pull about 573 lbs. for nearly five hours per day, beside pulling the empty cars the rest of the time. This is far greater exertion than any ordinary horse can maintain. The cars generally used in this country have a capacity of 1½ cubic yards and cost about $65 apiece. Besides the shovellers and dumping-gang, several men and a foreman will be required to keep the track in order and to make the constant shifts that are necessary. Two trains are generally used, one of which is loaded while the other is run to the dump. Some passing-place is necessary, but this is generally provided by having a switch at the cut and running the trains on each track alternately. This insures a train of cars always at the cut to keep the shovellers employed. The cost of hauling per cubic yard can only be computed when the number of laborers, cars, and horses employed are known, and these will depend on the lead, on the character of the excavation, on the grade, if any, etc., and must be so proportioned that the shovellers need not wait for cars to fill, nor the dumping-gang

for material to handle, nor the horses and drivers for cars to haul. Much skill is necessary to keep a large force in smooth running order.

(f) **Cars and locomotives.** 30-lb. rails are the lightest that should be used for this work, and 35- or 40-lb. rails are better. One or two narrow-gauge locomotives (depending on the length of haul), costing about $2500 each, will be necessary to handle two trains of about 15 cars each, the cars having a capacity of about 2 cubic yards and costing about $100 each. Some cars can be obtained as low as $70. A force of about five men and a foreman will be required to shift the tracks. The track-shifters, except the foreman, may be common laborers. The dumping-gang will require about seven men. Even when the material is all taken *down* grade the grades may be too steep for the safe hauling of loaded cars down the grade, or for hauling empty cars up the grade. Under such circumstances temporary trestles are necessary to reduce the grade. When these are used, the uprights and bracing are left in the embankment—only the stringers being removed. This is largely a necessity, but is partially compensated by the fact that the trestle forms a core to the embankment which prevents lateral shifting during settlement. The average speed of the trains may be taken as 10 miles per hour or 5 miles of lead per hour. The time lost in loading and unloading is estimated (Trautwine) as 9 minutes or .15 of an hour. The number of trips per day of 10 hours will equal $\dfrac{10}{\frac{1}{5}(\text{miles of lead}) + .15}$ or $\dfrac{50}{(\text{miles of lead}) + .75}$. Of course this quotient *must* be a whole number. Knowing the number of trains and their capacity, the total number of cubic yards handled is known, which, divided into the total daily cost of the trains, will give the cost of hauling per yard. The daily cost of a train will include

(*a*) Wages of engineer, who frequently fires his own engine;

(*b*) Fuel, about ½ to 1 ton of bituminous coal, depending on work done;

(*c*) Water, a very variable item, frequently costing $3 to $5 per day;

(*d*) Repairs, variable, frequently at rate of 50 to 60% per year;

(*e*) Interest on cost and depreciation, 16 to 40%.

To these must be added, to obtain the total cost of haul,

(*f*) Wages of the gang employed in shifting track.

The above calculation for the number of train loads depends on the assumption that 9 minutes is total time lost by a locomotive for each round trip. If the haul is very short it may readily happen that a steam-shovel cannot fill one train of cars before the locomotive has returned with a load of empties and is ready to haul a loaded train away. The estimation of the number of train loads is chiefly useful in planning the work so as to have every tool working at its highest efficiency. Usually the capacity of the steam-shovel or the ability to promptly "spot" the cars under the shovel is the real limiting agent which determines the daily output.

141. Choice of method of haul dependent on distance. In light side-hill work in which material need not be moved more than 12 or 15 feet, i.e., moved *laterally* across the roadbed, the earth may be moved most cheaply by mere shovelling. Beyond 12 feet scrapers are more economical. At about 100 feet drag-scrapers and wheelbarrows are equally economical. Between 100 and 200 feet wheelbarrows are generally cheaper than either carts or drag-scrapers, but wheeled scrapers are always cheaper than wheelbarrows. Beyond 500 feet two-wheeled carts become the most economical up to about 1700 feet; then four-wheeled wagons become more economical up to 3500 feet. Beyond this cars on rails, drawn by horses or by locomotives, become cheaper. The economy of cars on rails becomes evident for distances as small as 300 feet provided the volume of the excavation will justify the outlay. Locomotives will always be cheaper than horses and mules, providing the work to be done is of sufficient magnitude to justify the purchase of the necessary plant and risk the loss in selling the plant ultimately as second-hand equipment, or keeping the plant on hand and idle for an indefinite period waiting for other work. Horses will not be economical for distances much over a mile. For greater distances locomotives are more economical, but the question of "limit of profitable haul" (§ 148) must be closely studied, as the circumstances are certainly not common when it is advisable to haul material much over a mile.

142. Item 4. SPREADING. The cost of spreading varies with the method employed in dumping the load. When the earth

is tipped over the edge of an embankment there is little if any necessary work. Trautwine allows about ¼ c. per cubic yard for keeping the dumping-places clear and in order. This would represent the wages of one man at $1 per day attending to the unloading of 1200 two-wheeled carts each carrying ⅓ cubic yard. 1200 carts in 10 hours would mean an average of two per minute, which implies more rapid and efficient work than may be depended on. The allowance is probably too small. When the material is dumped in layers some levelling is required, for which Trautwine allows 50 to 100 cubic yards as a fair day's work, costing from 1 to 2 cents per cubic yard. The cost of spreading will not ordinarily exceed this and is frequently nothing—all depending on the method of unloading. It should be noted that Mr. Morris's examples and computations (Jour. Franklin Inst., Sept. 1841) disregard altogether any special charge for this item.

143. Item 5. KEEPING ROADWAYS IN ORDER. This feature is important as a measure of true economy, whatever the system of transportation, but it is often neglected. A petty saving in such matters will cost many times as much in increased labor in hauling and loss of time. With some methods of haul the cost is best combined with that of other items.

(a) **Wheelbarrows.** Wheelbarrows should generally be run on planks laid on the ground. The adjusting and shifting of these planks is done by the wheelers, and the time for it is allowed for in the "¾ minute for short rests, adjusting the wheeling plank, etc." The actual cost of the planks must be added, but it would evidently be a very small addition per cubic yard in a large contract. When the wheelbarrows are run on planks placed on "horses" or on trestles the cost is very appreciable; but the method is frequently used with great economy. The variations in the requirements render any general estimate of such cost impracticable.

(b) **Carts and wagons.** The cost of keeping roadways in order for carts and wagons is sometimes estimated merely as so much per cubic yard, but it is evidently a function of the *lead*. The work consists in draining off puddles, filling up ruts, picking up loose stones that may have fallen off the loads, and in general doing everything that will reduce the traction as much as possible. Temporary inclines, built to avoid excessive grade

at some one point, are often measures of true economy. Trautwine suggests $\frac{1}{10}$ c. per cubic yard per 100 feet of lead for earthwork and $\frac{2}{10}$ c. for rockwork, as an estimate for this item when carts are used.

(c) **Cars.** When cars are used a shifting-gang, consisting of a foreman and several men (say five), are constantly employed in shifting the track so that the material may be loaded and unloaded where it is desired. The average cost of this item may be estimated by dividing the total daily cost of this gang by the number of cubic yards handled in one day.

144. Item 6. **TRIMMING CUTS TO THEIR PROPER CROSS-SECTION.** This process, often called "sand-papering," must be treated as an expense, since the payment received for the very few cubic yards of earth excavated is wholly inadequate to pay for the work involved. Gillette quotes bids of 2 cents per *square* yard of surface trimmed, and from this argues that, for *average* excavations, it adds to the cost *four* cents per cubic yard of the total excavation. The shallower the cut the greater is the proportionate cost. Of course the actual cost to the contractor will depend largely on the accuracy of outline demanded by the engineer or inspector.

145. Item 7. **REPAIRS, WEAR, DEPRECIATION, AND INTEREST ON COST OF PLANT.** The amount of this item evidently depends upon the character of the soil—the harder the soil the worse the wear and depreciation. The *interest on cost* depends on the current borrowing value of money. The estimate for this item has already been included in the allowances for horses, carts, ploughs, harness, wheelbarrows, steam-shovels, etc. Trautwine estimates $\frac{1}{4}$ c. per cubic yard for picks and shovels. Depreciation is generally a large percentage of the cost of earth-working tools, the life of all being limited to a few years, and of many tools to a few months.

146. Item 8. **SUPERINTENDENCE AND INCIDENTALS.** The incidentals include the cost of water-boys, timekeepers, watchmen, blacksmiths, fences, and other precautions to protect the public from possible injury, cost of casualty insurance for workmen, etc. Although the cost of some of these sub-items may be definitely estimated, others are so uncertain that it is only possible to make a lump estimate and add say 5 to 7% of the sum of the previous items for this item.

147. Contractor's profit and contingencies. The word "contingencies" here refers to the abnormal expenses caused by freshets, continued wet weather, and "hard luck," as distinguished from mere incidentals which are really normal expenses. They are the expenses which literally cannot be foreseen, and on which the contractor must "take chances." They are therefore included with the expected profit. The allowance for these two elements combined is variously estimated up to 25% of the previously estimated cost of the work, according to the sharpness of the competition, the contractor's confidence in the accuracy of his estimates, and the possible uncertainty as to true cost owing to unfavorable circumstances. The contractor's real profit may vary considerably from this. He often pays clerks, boards and lodges the laborers in shanties built for the purpose, or keeps a supply-store, and has various other items both of profit and expense. His profit is largely dependent on skill in so handling the men that all can work effectively without interference or delays in waiting for others. An unusual season of bad weather will often affect the cost very seriously. It is a common occurrence to find that two contractors may be working on the same kind of material and under precisely similar conditions and at the same price, and yet one may be making money and the other losing it—all on account of difference of management.

148. Limit of profitable haul. As intimated in §§ 134 and 141, there is with every method of haul a limit of distance beyond which the expense for excessive hauling will exceed the loss resulting from borrowing and wasting. This distance is somewhat dependent on local conditions, thus requiring an independent solution for each particular case, but the general principles involved will be about as follows: Assume that it has been determined, as in Fig. 64, that the cut and fill will exactly balance between two points, as between e and x, assuming that, as indicated in § 132 (9), a trestle has been introduced between s and t, thus altering the mass curve to *Estxn* . . . Since there is a balance between A' and C', the material for the fill between C' and e' must be obtained either by "borrowing" in the immediate neighborhood or by transportation from the excavation between z' and n'. If cut and fill have been approximately

between z' and n'. If cut and fill have been approximately balanced in the selection of grade line, as is ordinarily done, borrowing material for the fill $C'e'$ implies a wastage of material at the cut $z'n'$. To compare the two methods, we may place against the plan of borrowing and wasting, (*a*) cost, if any, of extra right of way that may be needed from which to obtain earth for the fill $C'e'$; (*b*) cost of loosening, loading, hauling a distance equal to that between the centers of gravity of the borrow-pit and of the fill, and the other expenses incidental to borrowing M cubic yards for the fill $C'e'$; (*c*) cost of loosening, loading, hauling a distance equal to that between the centers of gravity of the cut $z'n'$ and of the spoil-bank, and the other expenses incidental to wasting M cubic yards at the cut $z'n'$: (*d*) cost, if any, of land needed for the spoil-bank. The cost of the other plan will be the cost of loosening, loading, hauling (the hauling being represented by the trapezoidal figure $Cexn$), and the other expenses incidental to making the fill $C'e'$ with the material from the cut $z'n'$, the amount of material being M cubic yards, which is represented in the figure by the vertical ordinate from e to the line Cn. The difference between these costs will be the cost, if any, of land for borrow-pit and spoil-bank *plus* the cost of loosening, loading, etc. (except hauling and roadways) of M cubic yards, *minus* the difference in cost of the excessive haul from Ce to xn and the comparatively short hauls from borrow-pit and to spoil-bank.

As an illustration, taking some of the estimates previously given for operating with average material, the cost of all items, except hauling and roadways, would be about as follows: loosening, with plough, 1.2 c., loading 5.0 c., spreading 1.5 c., wear, depreciation, etc., .25 c., superintendence, etc., 1.5 c.; total 8.95 c. Suppose that the haul for both borrowing and wasting averages 100 feet or 1 station. Then the cost of haul per yard, using carts, would be (§ 140, *a*) $[125\times3(1+4)]\div600=3.125$ c. The cost of roadways would be about 0.1 c. per yard, making a total of 3.225 c. per cubic yard. Assume $M=10000$ cubic yards and the area $Cexn=180000$ yards-stations or the equivalent of 10000 yards hauled 1800 feet. This haul would cost $[125\times3(18+4)]\div600=13.75$ c. per cubic yard. The cost of roadways will be $18\times.1$ or 1.8 c., making a total of 15.55 c. for hauling and roadways. The difference of cost of hauling and roadways will be $15.55-(2\times3.225)=9.10$ c. per yard or \$910

for the 10000 yards. Offsetting this is the cost of loosening, etc., 10000 yards, at 8.95 c., costing $895. These figures may be better compared as follows:

LONG HAUL.	Loosening, etc.,	10000 yards,		@ 8.95 c.	$ 895.
	Hauling,	" 10000 "		@ 15.55 c.	1555.
					$2450.
BORROWING AND WASTING.	Loosening, etc.,	10000 yards	(borrowed),	@ 8.95 c.	$895.
	" "	10000 "	(wasted),	@ 8.95 c.	895.
	Hauling, etc.,	10000 "	(borrowed),	@ 3.225 c.	322.50
	" "	10000 "	(wasted),	@ 3.225 c.	322.50
					$2435.00

These costs are practically balanced, but no allowance has been made for right of way. If any considerable amount had to be paid for that, it would decide this particular case in favor of the long haul. This shows that *under these conditions* 1800 feet is *about* the limit of profitable haul, the land costing nothing extra.

BLASTING.

149. **Explosives.** The effect of blasting is due to the extremely rapid expansion of a gas which is developed by the decomposition of a very small amount of solid matter. Blasting compounds may be divided into two general classes, (*a*) slow-burning and (*b*) detonating. Gunpowder is a type of the slow-burning compounds. These are generally ignited by heat; the ignition proceeds from grain to grain; the heat and pressure produced are comparatively low. Nitro-glycerine is a type of the detonating compounds. They are exploded by a shock which *instantaneously* explodes the whole mass. The heat and pressure developed are far in excess of that produced by the explosion of powder. Nitro-glycerine is so easily exploded that it is very dangerous to handle. It was discovered that if the nitro-glycerine was absorbed by a spongy material like infusorial earth, it was much less liable to explode, while its power when actually exploded was practically equal to that of the amount of pure nitro-glycerine contained in the dynamite, which is the name given to the mixture of nitro-glycerine and infusorial earth. Nitro-glycerine is expensive; many other explosive chemical compounds which properly belong to the slow-burning

class are comparatively cheap. It has been conclusively demonstrated that a mixture of nitro-glycerine and some of the cheaper chemicals has a greater explosive force than the sum of the strengths of the component parts when exploded separately. Whatever the reason, the fact seems established. The reason is possibly that the explosion of the nitro-glycerine is sufficiently powerful to produce a *detonation* of the other chemicals, which is impossible to produce by ordinary means, and that this explosion caused by detonation is more powerful than an ordinary explosion. The majority of the explosive compounds and "powders" on the market are of this character—a mixture of 20 to 60 per cent. of nitro-glycerine with variable proportions of one or more of a great variety of explosive chemicals.

The choice of the explosive depends on the character of the rock. A hard brittle rock is most effectively blasted by a detonating compound. The rapidity with which the full force of the explosive is developed has a shattering effect on a brittle substance. On the contrary, some of the softer tougher rocks and indurated clays are but little affected by dynamite. The result is but little more than an enlargement of the blast-hole. Quarrying must generally be done with blasting-powder, as the quicker explosives are too shattering. Although the results obtained by various experimenters are very variable, it may be said that pure nitro-glycerine is eight times as powerful as black powder, dynamite (75% nitro-glycerine) six times, and guncotton four to six times as powerful. For open work where time is not particularly valuable, black powder is by far the cheapest, but in tunnel-headings, whose progress determines the progress of the whole work, dynamite is so much more effective and so expedites the work that its use becomes economical.

150. Drilling. Although many very complicated forms of drill-bars have been devised, the best form (with slight modifications to suit circumstances) is as shown in Fig. 66 (*a*), and (*b*). The width should flare at the bottom (*a*) about 15 to 30%. For hard rock the curve of the edge should be somewhat flatter and for soft rock somewhat more curved than shown, Fig. 66, (*a*). Sometimes the angle of the two faces is varied from that given, Fig. 66, (*b*) and occasionally the edge is purposely blunted so as to give a crushing rather than a cutting effect. The drills will require sharpening for each 6 to 18 inches depth of hole, and will require a new edge to be worked every 2 to 4 days.

For drilling vertical holes the *churn-drill* is the most economical. The drill-bar is of iron, about 6 to 8 feet long, 1¼″ in diameter, weighs about 25 to 30 lbs., and is shod with a piece of steel welded on. The bar is lifted a few inches between each blow, turned partially around, and allowed to fall, the impact doing the work. From 5 to 15 feet of holes, depending on the character of the rock, is a fair day's work—10 hours. In very soft rocks even more than this may be done. This method is

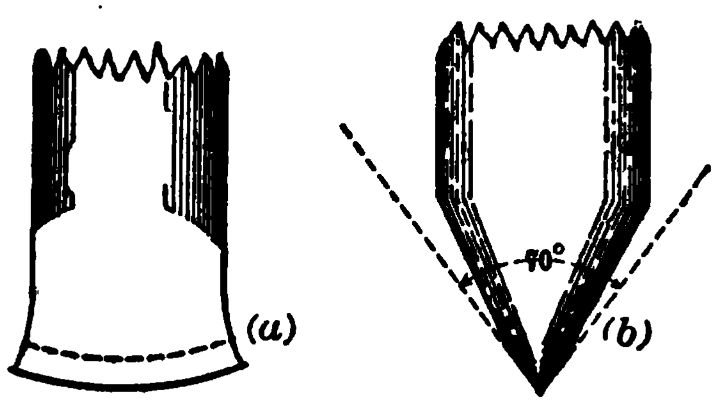


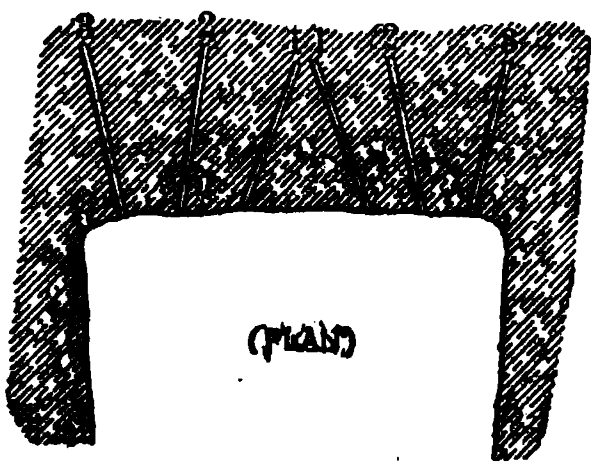
Fig. 66.

inapplicable for inclined holes or even for vertical holes in confined places, such as tunnel-headings. For such places the only practical *hand* method is to use hammers. This may be done by light drills and light hammers (one-man work), or by heavier drills held by one man and struck by one or two men with heavy hammers. The conclusion of an exhaustive investigation as to the relative economy of light or heavy hammers is that the light-hammer method is more economical for the softer rocks, the heavy-hammer method is more economical for the harder rocks, but that the light-hammer method is always more expeditious and hence to be preferred when time is important.

The subject of machine rock-drills is too vast to be treated here. The method is only practicable when the amount of work to be done is large, and especially when time is valuable. The machines are generally operated by compressed air for tunnel-work, thus doing the additional service of supplying fresh air to the tunnel-headings where it is most needed. The cost per foot of hole drilled is quite variable, but is usually somewhat less than that of hand-drilling—sometimes but a small fraction of it.

151. Position and direction of drill-holes. As the cost of drilling holes is the largest single item in the total cost of blasting, it is necessary that skill and judgment should be used in so

locating the holes that the blasts will be most effective. The greatest effect of a blast will evidently be in the direction of the "line of least resistance." In a strictly homogeneous material this will be the shortest line from the center of the explosive to the surface. The variations in homogeneity on account of laminations and seams require that each case shall be judged according to experience. In open-pit blasting it is generally easy to obtain two and sometimes three exposed faces to the rock, making it a simple matter to drill holes so that a blast will do effective work. When a solid face of rock must be broken into, as in a tunnel-heading, the work is necessarily ineffectual and expensive. A conical or wedge-shaped mass will first be blown out by simultaneous blasts in the holes marked 1, Fig. 67; blasts in the holes marked 2 and 3 will then complete the cross-section of the heading. A great saving in cost may often be secured by skilfully taking advantage of seams, breaks, and irregularities. When the work is economically done there is but little noise or throwing of rock, a covering of old timbers and branches of trees generally sufficing to confine the smaller pieces which would otherwise fly up.



DRILL HOLES IN TUNNEL HEADING

FIG. 67.

152. Amount of explosive. The amount of explosive required varies as the cube of the line of least resistance. The best results are obtained when the line of least resistance is ¾ of the depth of the hole; also when the powder fills about ½ of the hole. For average rock the amount of powder required is as follows:

Line of least resistance............	2 ft.	4 ft.	6 ft.	8 ft.
Weight of powder.................	¼ lb.	2 lbs.	6¾ lbs.	16 lbs.

Strict compliance with all of the above conditions would require that the diameter of the hole should vary for every case. While this is impracticable, there should evidently be some variation in the size of the hole, depending on the work to be done. For example, a 1″ hole, drilled 2′ 8″ deep, with its line of least resistance 2′, and loaded with ¼ lb. of powder, would

be filled to a depth of 9½", which is nearly ¼ of the depth. A 3" hole, drilled 8' deep, with its line of least resistance 6', and loaded with 6¾ lbs. of powder, would be filled to a depth of over 28", which is also nearly ¼ of the depth. One pound of blasting-powder will occupy about 28 cubic inches. Quarrying necessitates the use of numerous and sometimes repeated light charges of powder, as a heavy blast or a powerful explosive like dynamite is apt to shatter the rock. This requires more powder to the cubic yard than blasting for mere excavation, which may usually be done by the use of ¼ to ½ lb. of powder per cubic yard of easy open blasting. On account of the great resistance offered by rock when blasted in headings in tunnels, the powder used per cubic yard will run up to 2, 4, and even 6 lbs. per cubic yard. As before stated, nitro-glycerine is about eight times (and dynamite about six times) as powerful as the same *weight* of powder.

153. Tamping. Blasting-powder and the slow-burning explosives require thorough tamping. Clay is probably the best, but sand and fine powdered rock are also used. Wooden plugs, inverted expansive cones, etc., are periodically reinvented by enthusiastic inventors, only to be discarded for the simpler methods. Owing to the extreme rapidity of the development of the force of a nitro-glycerine or dynamite explosion, tamping is not so essential with these explosives, although it unquestionably adds to their effectiveness. Blasting under water has been effectively accomplished by merely pouring nitro-glycerine into the drilled holes through a tube and then exploding the charge without any tamping except that furnished by the superincumbent water. It has been found that air-spaces about a charge make a material reduction in the effectiveness of the explosion. It is therefore necessary to carefully ram the explosive into a solid mass. Of course the liquid nitro-glycerine needs no ramming, but dynamite should be rammed with a *wooden* rammer. Iron should be carefully avoided in ramming gunpowder. A copper bar is generally used.

154. Exploding the charge. Black powder is generally exploded by means of a fuse which is essentially a cord in which there is a thin vein of gunpowder, the cord being protected by tar, extra linings of hemp, cotton, or even gutta-percha. The fuse is inserted into the middle of the charge, and the tamping carefully packed around it so that it will not be injured. To

produce the detonation required to explode nitro-glycerine and dynamite, there must be an initial explosion of some easily ignited explosive. This is generally accomplished by means of caps containing fulminating-powder which are exploded by electricity. The electricity (in one class of caps) heats a very fine platinum wire to redness, thereby igniting the sensitive powder, or (in another class) a spark is caused to jump through the powder between the ends of two wires suitably separated. Dynamite can also be exploded by using a small cartridge of gunpowder which is itself exploded by an ordinary fuse.

155. Cost. Trautwine estimated the cost of blasting (for mere excavation) as averaging 45 cents per cubic yard, falling as low as 30 cents for easy but *brittle* rock, and running up to 60 cents and even $1 when the cutting is shallow, the rock especially tough, and the strata unfavorably placed. Increased costs of labor and material may add 50 to 100% to these estimates.

156. Classification of excavated material. The classification of excavated material is a fruitful source of dispute between contractors and railroad companies, owing mainly to the fact that the variation between the softest earth and the hardest rock is so gradual that it is very difficult to describe distinctions between different classifications which are unmistakable and indisputable. The classification frequently used is (*a*) earth, (*b*) loose rock, and (*c*) solid rock. As blasting is frequently used to loosen "loose rock" and even "earth" (if it is frozen), the fact that blasting is employed cannot be used as a criterion, especially as this would (if allowed) lead to unnecessary blasting for the sake of classifying material as rock.

Earth. This includes clay, sand, gravel, loam, decomposed rock and slate, boulders or loose stones not greater than 1 cubic foot (3 cubic feet, P. R. R.), and sometimes even "hard-pan." In general it will signify material which *can* be loosened by a plough with two horses, or with which one picker can keep one shoveller busy.

Loose rock. This includes boulders and loose stones of more than one cubic foot and less than one cubic yard; stratified rock, not more than six inches thick, separated by a stratum of clay; also all material (not classified as earth) which may be loosened by pick or bar and which "*can* be quarried without blasting, although blasting may occasionally be resorted to."

Solid rock includes all rock found in masses of over one cubic yard which cannot be removed except by blasting.

It is generally specified that the engineer of the railroad company shall be the judge of the classification of the material, but frequently an appeal is taken from his decisions to the courts.

157. Specifications for earthwork. The following specifications, issued by the Norfolk and Western R. R., represent the average requirements. It should be remembered that very strict specifications invariably increase the cost of the work, and frequently add to the cost more than is gained by improved quality of work.

1. The grading will be estimated and paid for by the cubic yard, and will include clearing and grubbing, and all open excavations, channels, and embankments required for the formation of the roadbed, and for turnouts and sidings; cutting all ditches or drains about or contiguous to the road; digging the foundation-pits of all culverts, bridges, or walls; reconstructing turnpikes or common roads in cases where they are destroyed or interfered with; changing the course or channel of streams; and all other excavations or embankments connected with or incident to the construction of said Railroad.

2. All grading, except where otherwise specified, whether for cuts or fills, will be measured in the excavations and will be classified under the following heads, viz.: Solid Rock, Loose Rock, Hard-pan, and Earth.

SOLID ROCK shall include all rock occurring in masses which, in the judgment of the said Engineer Maintenance of Way, may be best removed by blasting.

LOOSE ROCK shall include all kinds of shale, soapstone, and other rock which, in the judgment of the said Engineer Maintenance of Way, can be removed by pick and bar, and is soft and loose enough to be removed without blasting, although blasting may be occasionally resorted to; also, detached stone of less than one (1) cubic yard and more than one (1) cubic foot.

HARD-PAN shall consist of tough indurated clay or cemented gravel, which requires blasting or other equally expensive means for its removal, or which cannot be ploughed with less than four horses and a railroad plough, or which requires two pickers to a shoveller, the said Engineer Maintenance of Way to be the judge of these conditions.

EARTH shall include all material of an earthy nature, of whatever name or character, not unquestionably loose rock or hardpan as above defined.

POWDER. The use of powder in cuts will not be considered as a reason for any other classification than earth, unless the material in the cut is clearly other than earth under the above specifications.

3. Earth, gravel, and other materials taken from the excavations, except when otherwise directed by the said Engineer Maintenance of Way or his assistant, shall be deposited in the adjacent embankment; the cost of removing and depositing which, when the distance necessary to be hauled is not more than sixteen hundred (1600) feet, shall be included in the price paid for the excavation.

4. EXTRA HAUL will be estimated and paid for as follows: whenever material from excavations is necessarily hauled a greater distance than sixteen hundred (1600) feet, there shall be paid in addition to the price of excavation the price of extra haul per 100 feet, or part thereof, after the first 1600 feet; the necessary haul to be determined in each case by the said Engineer Maintenance of Way or his assistant, from the profile and cross-sections, and the estimates to be in accordance therewith.

5. All embankments shall be made in layers of such thickness and carried on in such manner as the said Engineer Maintenance of Way or his assistant may prescribe, the stone and heavy materials being placed in slopes and top. And in completing the fills to the proper grade such additional heights and fulness of slope shall be given them, to provide for their settlement, as the said Engineer Maintenance of Way, or his assistant, may direct. Embankments about masonry shall be built at such times and in such manner and of such materials as the said Engineer Maintenance of Way or his assistant may direct.

6. In procuring materials for embankments from without the line of the road, and in wasting materials from cuttings, the place and manner of doing it shall in each case be indicated by the Engineer Maintenance of Way or his assistant; and care must be taken to injure or disfigure the land as little as possible. Borrow-pits and spoil-banks must be left by the Contractor in regular and sightly shape.

7. The lands of the said Railroad Company shall be cleared to the extent required by the said Engineer Maintenance of

Way, or his assistant, of all trees, brushes, logs, and other perishable materials, which shall be destroyed by burning or deposited in heaps as the said Engineer Maintenance of Way, or his assistant, may direct. Large trees must be cut not more than two and one-half ($2\frac{1}{2}$) feet from the ground, and under embankments less than four (4) feet high they shall be cut close to the ground. All small trees and bushes shall be cut close to the ground.

8. Clearing shall be estimated and paid for by the acre or fraction of an acre.

9. All stumps, roots, logs, and other obstructions shall be grubbed out, and removed from all places where embankments occur less than two (2) feet in height; also, from all places where excavations occur and from such other places as the said Engineer Maintenance of Way or his assistant may direct.

10. Grubbing shall be estimated and paid for by the acre or fraction of an acre.

11. Contractors, when directed by the said Engineer Maintenance of Way or his assistant in charge of the work, will deposit on the side of the road, or at such convenient points as may be designated, any stone, rock, or other materials that they may excavate; and all materials excavated and deposited as above, together with all timber removed from the line of the road, will be considered the property of the Railroad Company, and the Contractors upon the respective sections will be responsible for its safe-keeping until removed by said Railroad Company, or until their work is finished.

12. Contractors will be accountable for the maintenance of safe and convenient places wherever public or private roads are in any way interfered with by them during the progress of the work. They will also be responsible for fences thrown down, and for gates and bars left open, and for all damages occasioned thereby.

13. Temporary bridges and trestles, erected to facilitate the progress of the work, in case of delays at masonry structures from any cause, or for other reasons, will be at the expense of the Contractor.

14. The line of road or the gradients may be changed in any manner, and at any time, if the said Engineer Maintenance of Way or his assistant shall consider such a change necessary or expedient; but no claim for an increase in prices of excavation

or embankment on the part of the Contractor will be allowed or considered unless made in writing before the work on that part of the section where the alteration has been made shall have been commenced. The said Engineer Maintenance of Way or his assistant may also, on the conditions last recited, increase or diminish the length of any section for the purpose of more nearly equalizing or balancing the excavations and embankments, or for any other reason.

15. The roadbed will be graded as directed by the said Engineer Maintenance of Way or his assistant, and in conformity with such breadths, depths, and slopes of cutting and filling as he may prescribe from time to time, and no part of the work will be finally accepted until it is properly completed and dressed off at the required grade.

CHAPTER IV.

TRESTLES.

158. Extent of use. Trestles constitute from 1 to 3% of the length of the average railroad. It was estimated in 1889 that there was then about 2400 miles of single-track railway trestle in the United States, divided among 150,000 structures and estimated to cost about $75,000,000. The annual charge for maintenance, estimated at $\frac{1}{8}$ of the cost, therefore amounted to about $9,500,000 and necessitated the annual use of perhaps 300,000,000 ft. B. M. of timber. The corresponding figures at the present time must be somewhat in excess of this. The magnitude of this use, which is causing the rapid disappearance of forests, has resulted in endeavors to limit the use of timber for this purpose. Trestles may be considered as justifiable under the following conditions:

a. Permanent trestles.

1. Those of *extreme* height — then called viaducts and frequently constructed of steel, as the Kinzua viaduct, 302 feet high.

2. Those across waterways—*e.g.*, that across Lake Pontchartrain, near New Orleans, 22 miles long.

3. Those across swamps of soft deep mud, or across a river-bottom, liable to occasional overflow.

b. Temporary trestles.

1. To open the road for traffic as quickly as possible—often a reason of great financial importance.

2. To quickly replace a more elaborate structure, destroyed by accident, on a road already in operation, so that the interruption to traffic shall be a minimum.

3. To form an earth embankment with earth brought from a distant point by the train-load, when such a measure would cost less than to borrow earth in the immediate neighborhood.

4. To bridge an opening temporarily and thus allow time to learn the regimen of a stream in order to better proportion the

size of the waterway and also to facilitate bringing *suitable* stone for masonry from a distance. In a new country there is always the double danger of either building a culvert too small, requiring expensive reconstruction, perhaps after a disastrous washout, or else wasting money by constructing the culvert unnecessarily large. Much masonry has been built of a very poor quality of stone because it could be conveniently obtained and because good stone was unobtainable except at a prohibitive cost for transportation. Opening the road for traffic by the use of temporary trestles obviates both of these difficulties.

159. **Trestles vs. embankments.** Low embankments are very much cheaper than low trestles both in first cost and maintenance. Very high embankments are very expensive to construct, but cost comparatively little to maintain. A trestle of equal height may cost much less to construct, but will be expensive to maintain—perhaps $\frac{1}{8}$ of its cost per year. To determine the height beyond which it will be cheaper to maintain a trestle rather than build an embankment, it will be necessary to allow for the cost of maintenance. The height will also depend on the relative cost of timber, labor, and earthwork. At the present average values, it will be found that for less heights than 25 feet the *first cost* of an embankment will *generally* be less than that of a trestle; this implies that a permanent trestle should never be constructed with a height less than 25 feet except for the reasons given in § 158. The height at which a permanent trestle is certainly cheaper than earthwork is more uncertain. A high grade line joining two hills will invariably imply at least a culvert if an embankment is used. If the culvert is built of masonry, the cost of the embankment will be so increased that the height at which a trestle becomes economical will be materially reduced. The cost of an embankment increases much more rapidly than the height—with very high embankments more nearly as the square of the height—while the cost of trestles does not increase as rapidly as the height. Although local circumstances may modify the application of any set rules, it is probably seldom that it will be cheaper to build an embankment 40 or 50 feet high than to permanently maintain a wooden trestle of that height. A steel viaduct would probably be the best solution of such a case. These are frequently used for permanent structures, especially when very high. The cost of maintenance is much less than that of wood, which makes the

use of steel preferable for permanent trestles unless wood is abnormally cheap. Neither the cost nor the construction of steel trestles will be considered in this chapter.

160. Two principal types. There are two principal types of wooden trestles—pile trestles and framed trestles. The great objection to pile trestles is the rapid rotting of the portion of the pile which is underground, and the difficulty of renewal. The maximum height of pile trestles is about 30 feet, and even this height is seldom reached. Framed trestles have been constructed to a height of considerably over 100 feet They are frequently built in such a manner that any injured piece may be readily taken out and renewed without interfering with traffic. Trestles consist of two parts—the supports called "bents," and the stringers and floor system. As the stringers and floor system are the same for both pile and framed trestles, the "bents" are all that need be considered separately.

PILE TRESTLES.

161. Pile bents. A pile bent consists generally of four piles driven into the ground deep enough to afford not only sufficient vertical resistance but also lateral resistance. On top of these piles is placed a horizontal "cap." The caps are fastened to the tops of the piles by methods illustrated in Fig. 68. The method of fastening shown in each case should not be considered as applicable only to the particular type of pile bent used to illustrate it. Fig. 68 (*a* and *d*) illustrates a mortise-joint with a hardwood pin about $1\frac{1}{4}''$ in diameter. The hole for the pin should be bored separately through the cap and the mortise, and the hole through the cap should be at a slightly higher level than that through the mortise, so that the cap will be drawn down tight when the pin is driven. Occasionally an iron dowel (an iron pin about $1\frac{1}{2}''$ in diameter and about $6''$ long) is inserted partly in the cap and partly in the pile. The use of drift-bolts, shown in Fig. 68 (*b*), is cheaper in first cost, but renders repairs and renewals very troublesome and expensive. "Split caps," shown in Fig. 68 (*c*), are formed by bolting two half-size strips on each side of a tenon on top of the pile. Repairs are very easily and cheaply made without interference with the traffic and without injuring other pieces of the bent. The smaller pieces are more easily obtainable in a sound condition; the

decay of one does not affect the other, and the first cost is but little if any greater than the method of using a single piece. For further discussion, see § 170.

For very light traffic and for a height of about 5 feet three vertical piles will suffice, as shown in Fig. 68 (*a*). Up to a height

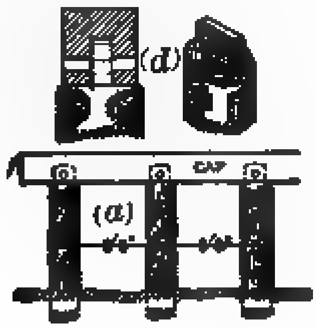


FIG. 68.

of 8 or 10 feet four piles may be used without sway-bracing, as in Fig. 68 (*b*), if the piles have a good bearing. For heights greater than 10 feet sway-bracing is generally necessary. The outside piles are frequently driven with a batter varying from 1 : 12 to 1 : 4.

Piles are made, if possible, from timber obtained in the vicinity of the work. Durability is the great requisite rather than strength, for almost any timber is strong enough (except as noted below) and will be suitable if it will resist rapid decay. The following list is quoted as being in the order of preference on account of durability·

1. Red cedar	5. White pine	9. White oak	12. Black oak
2. Red cypress	6. Redwood	10. Post-oak	13. Hemlock
3. Pitch-pine	7. Elm	11. Red oak	14. Tamarac
4. Yellow pine	8. Spruce		

Red-cedar piles are said to have an average life of 27 years with a possible maximum of 50 years, but the timber is rather

weak, and if exposed in a river to flowing ice or driftwood is apt to be injured. Under these circumstances oak is preferable, although its life may be only 13 to 18 years.

162. Methods of driving piles. The following are the principal methods of driving piles:

a. A hammer weighing 2000 to 3000 lbs. or more, sliding in guides, is drawn up by horse-power or a portable engine, and allowed to fall *freely*.

b. The same as above except that the hammer does not fall freely, but drags the rope and revolving drum as it falls and is thus quite materially retarded. The mechanism is a little more simple, but is less effective, and is sometimes made deliberately deceptive by a contractor by retarding the blow, in order to apparently indicate the requisite resistance on the part of the pile.

The above methods have the advantage that the mechanism is cheap and can be transported into a new country with comparative ease, but the work done is somewhat ineffective and costly compared with some of the more elaborate methods given below.

c. Gunpowder pile-drivers, which automatically explode a cartridge every time the hammer falls. The explosion not only forces the pile down, but throws up the hammer for the next blow. For a given height of fall the effect is therefore doubled. It has been shown by experience, however, that when it is attempted to use such a pile-driver rapidly the mechanism becomes so heated that the cartridges explode prematurely, and the method has therefore been abandoned.

d. Steam pile-drivers, in which the hammer is operated directly by steam. The hammer falls freely a height of about 40 inches and is raised again by steam. The effectiveness is largely due to the rapidity of the blows, which does not allow time between the blows for the ground to settle around the pile and increase the resistance, which does happen when the blows are infrequent. "The hammer-cylinder weighs 5500 lbs., and with 60 to 75 lbs. of steam gives 75 to 80 blows per minute. With 41 blows a large unpointed pile was driven 35 feet into a hard clay bottom in half a minute." Such a driver would cost about $800.

The above four methods are those usual for dry earth. In very soft wet or sandy soils, where an unlimited supply of water

is available, the *water-jet* is sometimes employed. A pipe is driven along the side of the pile and extends to the pile-point. If water is forced through the pipe, it loosens the sand around the point and, rising along the sides, decreases the side resistance so that the pile sinks by its own weight, aided perhaps by extra weights loaded on. This loading may be accomplished by connecting the top of the pile and the pile-driver by a block and tackle so that a portion of the weight of the pile-driver is continually thrown on the pile.

Excessive driving frequently fractures the pile below the surface and thereby greatly weakens its bearing power. To prevent excessive "brooming" of the top of the pile, owing to the action of the hammer, the top should be protected by an iron ring fitted to the top of the pile. The "brooming" not only renders the driving ineffective and hence uneconomical, but vitiates the value of any test of the bearing power of the pile by noting the sinking due to a given weight falling a given distance. If the pile is so soft that brooming is unavoidable, the top should be adzed off frequently, and especially should it be done just before the final blows which are to test its bearing-power.

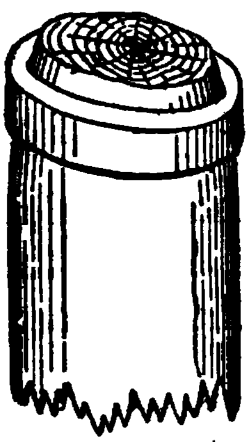


FIG. 69.

In a new country judgment and experience will be required to decide intelligently whether to employ a simple drop-hammer machine, operated by horse-power and easily transported but uneconomical in operation, or a more complicated machine working cheaply and effectively after being transported at greater expense.

163. Pile-driving formulæ. If R = the resistance of a pile, and s the set of the pile during the last blow, w the weight of the pile-hammer, and h the fall during the last blow, then we may state the approximate relation that $Rs = wh$, or $R = \frac{wh}{s}$. This is the basic principle of all rational formulæ, but the maximum weight which a pile will sustain after it has been driven some time is by no means the same as the resistance of the pile during the last blow. There are also many other modifying elements which have been variously allowed for in the many proposed formulæ. The formulæ range from the extreme of empirical simplicity to very complicated attempts to allow

properly for all modifying causes. As the simplest rule, the A. R. E. A. specifications require that the piles shall be driven until the pile will not sink more than $2\frac{1}{2}$ inches under five consecutive blows of a 3000-lb. hammer falling 15 feet. The "*Engineering News* formula" * gives the safe load as $\frac{2wh}{s+1}$, in which w= weight of hammer, h=fall in *feet*, s=set of pile in *inches* under the last blow. This formula is derived from the above basic formula by calling the safe load $\frac{1}{6}$ of the final resistance, and by adding (arbitrarily) 1 to the final set (s) as a compensation for the extra resistance caused by the settling of earth around the pile between each blow. This formula is used only for ordinary hammer-driving. When the piles are driven by a steam pile-driver the formula becomes safe load$=\frac{2wh}{s+0.1}$. For the "gunpowder pile-driver," since the explosion of the cartridge drives the pile in with the same force with which it throws the hammer upward, the effect is *twice* that of the fall of the hammer, and the formula becomes safe load$=\frac{4wh}{s+0.1}$. In these last two formulæ the constant in the denominator is changed from $s+1$ to $s+0.1$. The constant (1.0 or 0.1) is supposed to allow, as before stated, for the effect of the extra resistance caused by the earth settling around the pile between each blow. The more rapid the blows the less the opportunity to settle and the less the proper value of the constant.

The above formulæ have been given on account of their simplicity and their practical agreement with experience. Many other formulæ have been proposed, the majority of which are more complicated and attempt to take into account the weight of the pile, resistance of the guides, etc. While these elements, as well as many others, have their influence, their effect is so overshadowed by the indeterminable effect of other elements—as, for example, the effect of the settlement of earth around the pile between blows—that it is useless to attempt to employ anything but a purely empirical formula.

Examples. 1. A pile was driven with an ordinary hammer weighing 2500 pounds until the sinking under five consecutive blows was $15\frac{1}{2}$ inches. The fall of the hammer during the last

* *Engineering News*, Nov. 17, 1892.

blows was 24 feet. What was the safe bearing power of the pile?

$$\frac{2wh}{s+1}=\frac{2\times2500\times24}{(\frac{1}{5}\times15.5)+1}=\frac{120000}{4.1}=29300 \text{ pounds.}$$

2. Piles are being driven into a firm soil with a steam pile-driver until they show a *safe* bearing power of 20 tons. The hammer weighs 5500 pounds and its fall is 40 inches. What should be the sinking under the final blow?

$$40000\doteq\frac{2wh}{s+0.1}=\frac{2\times5500\times3.33}{s+0.1},$$

$$s=\frac{36667}{40000}-0.1=.81 \text{ inch.}$$

164. Pile-points and pile-shoes. Piles are generally sharpened to a blunt point. If the pile is liable to strike boulders, sunken logs, or other obstructions which are liable to turn the point, it is necessary to protect the point by some form of shoe. Several forms in cast iron have been used, also a wrought-iron shoe, having four "straps" radiating from the apex, the straps being nailed on to the pile, as shown in Fig. 70 (*b*). The cast-iron form shown in Fig. 70 (*a*) has a base cast around a drift-bolt. The recess on the top of the base receives the bottom of the pile and prevents a tendency to split the bottom of the pile or to force the shoe off laterally.

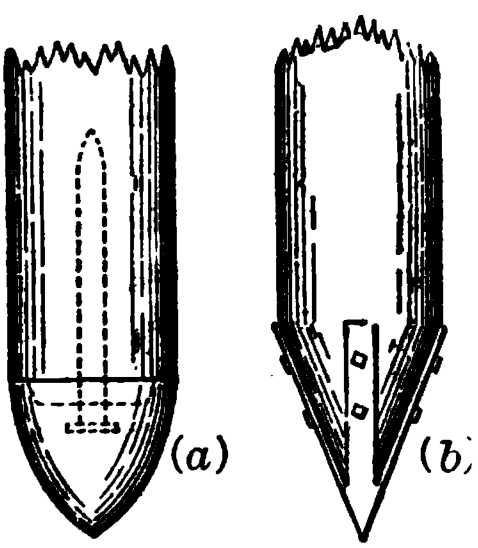


FIG. 70.

165. Details of design. No theoretical calculations of the strength of pile bents need be attempted on account of the extreme complication of the theoretical strains, the uncertainty as to the real strength of the timber used, the variability of that strength with time, and the insignificance of the economy that would be possible even if exact sizes could be computed. The caps are generally 14 feet long (for single track) with a cross-section 12″×12″ or 12″×14″. "Split caps" would consist

of two pieces 6″×12″. The sway-braces, never used for less heights than 6′, are made of 3″×12″ timber, and are spiked on with ⅝″ spikes 8″ long. The floor system will be the same as that described later for framed trestles.

166. Specifications for timber piles (Adopted 1909 by Amer. Rwy. Eng. Assoc.). 1. This grade [railroad heart grade] includes white, burr, and post oak; longleaf pine, Douglas fir, tamarack, Eastern white and red cedar, chestnut, Western cedar, redwood and cypress. 2. Piles shall be cut from sound trees; shall be close-grained and solid, free from defects, such as injurious ring shakes, large and unsound or loose knots, decay or other defects, which may materially impair their strength or durability. In Eastern red or white cedar a small amount of heart rot at the butt, which does not materially injure the strength of the pile, will be allowed. 3. Piles must be butt cut above the ground swell and have a uniform taper from butt to tip. Short bends will not be allowed. A line drawn from the center of the butt to the center of the tip shall lie within the body of the pile. 4. Unless otherwise allowed, piles must be cut when sap is down. Piles must be peeled soon after cutting. All knots shall be trimmed close to the body of the pile. 5. The minimum diameter at the tips of round piles shall be 9 inches for lengths not exceeding 30 feet; 8 inches for lengths over 30 feet but not exceeding 50 feet, and 7 inches for lengths over 50 feet The minimum diameter at one-quarter of the length from the butt shall be 12 inches and the maximum diameter at the butt 20 inches. 6. The minimum width of any side of the tip of a square pile shall be 9 inches for lengths not exceeding 30 feet; 8 inches for lengths over 30 feet but not exceeding 50 feet and 7 inches for lengths over 50 feet. The minimum width of any side at one-quarter of the length from the butt shall be 12 inches. 7. Square piles shall show at least 80% heart on each side at any cross-section of the stick, and all round piles shall show at least 10½ inches diameter of heart at the butt.

The second grade ("Railroad falsework grade") includes other woods which "will stand driving" and which cannot pass the specification for proportion of heart; also, they are usually not peeled.

167. Pile driving—principles of practice. As adopted by the Amer. Rwy. Eng. Assoc. 1911 and revised 1915.

1. A thorough exploration of the soil by borings, or preliminary

test piles, is the most important prerequisite to the design and construction of pile foundations.

2. Soil consisting wholly or chiefly of sand is most favorable to the use of the water-jet.

3. In harder soils containing gravel the use of the jet may be advantageous, if sufficient volume and pressure be provided.

4. In clay it may be economical to bore several holes in the soil with the aid of the jet before driving the pile, thus securing the accurate location of the pile, and its lubrication while being driven.

5. In general, the water-jet should not be attached to the pile, but handled separately.

6. Two jets will often succeed where one fails. In special cases a third jet extending a part of the depth aids materially in keeping loose the material around the pile.

7. Where the material is of such a porous character that the water from the jets may be dissipated and fail to come up in the immediate vicinity of the pile, the utility of the jet is uncertain, except for a part of the penetration.

8. A steam or drop hammer should be used in connection with the water-jet, and used to test the final rate of penetration.

9. The use of the water jet is one of the most effective means of avoiding injury to piles by overdriving.

10. There is danger from overdriving when the hammer begins to bounce. Overdriving is also indicated by the bending, kicking or staggering of the pile.

11. The brooming of the head of the pile dissipates a part, and in some cases all, of the energy due to the fall of the hammer.

12. The steam hammer is usually more effective than the drop hammer in securing the penetration of a wooden pile without injury, because of the shorter interval between blows.

13. Where shock to surrounding material is apt to prove detrimental to the structure, the steam hammer should always be used instead of the drop hammer. This is especially true in the case of sheet piling which is intended to prevent the passage of water. In some cases also the jet should not be used.

14. In general, the resistance of piles, penetrating soft material, depending solely upon skin friction, is materially increased after a period of rest. This period may be as short as fifteen minutes, and rarely exceeds twelve hours.

15. Where a pile penetrates muck or a soft yielding material and bears upon a hard stratum at its foot, its strength should be determined as a column or beam; omitting the resistance, if any, due to skin friction.

16. Unless the record of previous experience at the same site is available, the approximate bearing power may be obtained by loading test piles. The results of loading test piles should be used with caution, unless their condition is fairly comparable with that of the piles in the proposed foundation.

17. In case the piles in a foundation are expected to act as columns, the results of loading test piles should not be depended upon unless they are sufficient in number to insure their action in a similar manner; and unless they are stayed against lateral motion.

18. Before testing the penetration of a pile in a soft material where its bearing power depends principally, or wholly, upon skin friction, the pile should be allowed to rest for 24 hours after driving.

19. Where the resistance of piles depends mainly upon skin friction it is possible to diminish the combined strength, or bearing capacity, of a group of piles, by driving additional piles within the same area.

20. Where piles will foot in a hard stratum, investigation should be made to determine that this stratum is of sufficient depth and strength to carry the load.

21. Timber piles may be advantageously pointed, in some cases, to a 4-inch or 6-inch square at the end.

22. Piles should not be pointed when driven into soft material.

23. Shoes should be provided for piles when the driving is very hard, especially in riprap or shale. These shoes should be so constructed as to form an integral part of the pile.

24. The use of a cap is advantageous in distributing the impact of the hammer more uniformly over the head of the pile, as well as in holding it in position during driving.

168. Cost of pile trestles. The cost, per linear foot, of piling depends on the method of driving, the scarcity of suitable timber,

the price of labor, the length of the piles, and the amount of shifting of the pile-driver required. The cost of soft-wood piles varies from 8 to 15 cents per lineal foot, and the cost of oak piles varies from 10 to 30 cents per foot, according to the length, the longer piles costing more per foot. The total cost of putting the piles in place is so dependent on other items than the cost of driving, such as the cost of shifting the driver, getting the piles into the leaders, straightening and bracing them, leveling and nailing guide strips for sawing them off, and then the actual sawing, that there is a wide variation in the figures that are obtainable for the cost of such work. Of course the cost per pile of driving is also dependent on the total number of piles in the job. The cost per pile of placing a dozen piles for a single foundation would be far greater than the cost per pile for several hundred piles in one job. Among a large number of obtainable figures the average figure of $1.54 per pile for driving 1267 piles in 46 days is typical. Another quoted figure is $2.88 each, for driving 391 piles in 32 working days. On another job it cost $150 to drive thirty 30-foot piles, or an average of $5 each. In this case the piles cost $1.50 each or only 5 cents per lineal foot. The above cost figures are taken from Gillette's "Handbook of Cost Data" to which the student is referred for numerous examples of the cost of piles and pile-driving, as well as innumerable other cost analyses.

Specifications generally say that the piling will be paid for per lineal foot of piling *left in the work.* The wastage of the tops of piles sawed off is always something, and is frequently very large. Sometimes a small amount per foot of piling sawed off is allowed the contractor as compensation for his loss. This reduces the contractor's risk and possibly reduces his bid by an equal or greater amount than the extra amount actually paid him.

FRAMED TRESTLES

169. Typical design. A typical design for a framed trestle bent is given in Fig. 71. This represents, with slight variations of detail, the plan according to which a large part of the framed trestle bents of the country have been built—i.e., of those less than 20 or 30 feet in height, not requiring multiple story construction.

170. Joints. (a) **The mortise-and-tenon joint** is illustrated in

Fig. 71 and also in Fig. 68 (*a*). The tenon should be about

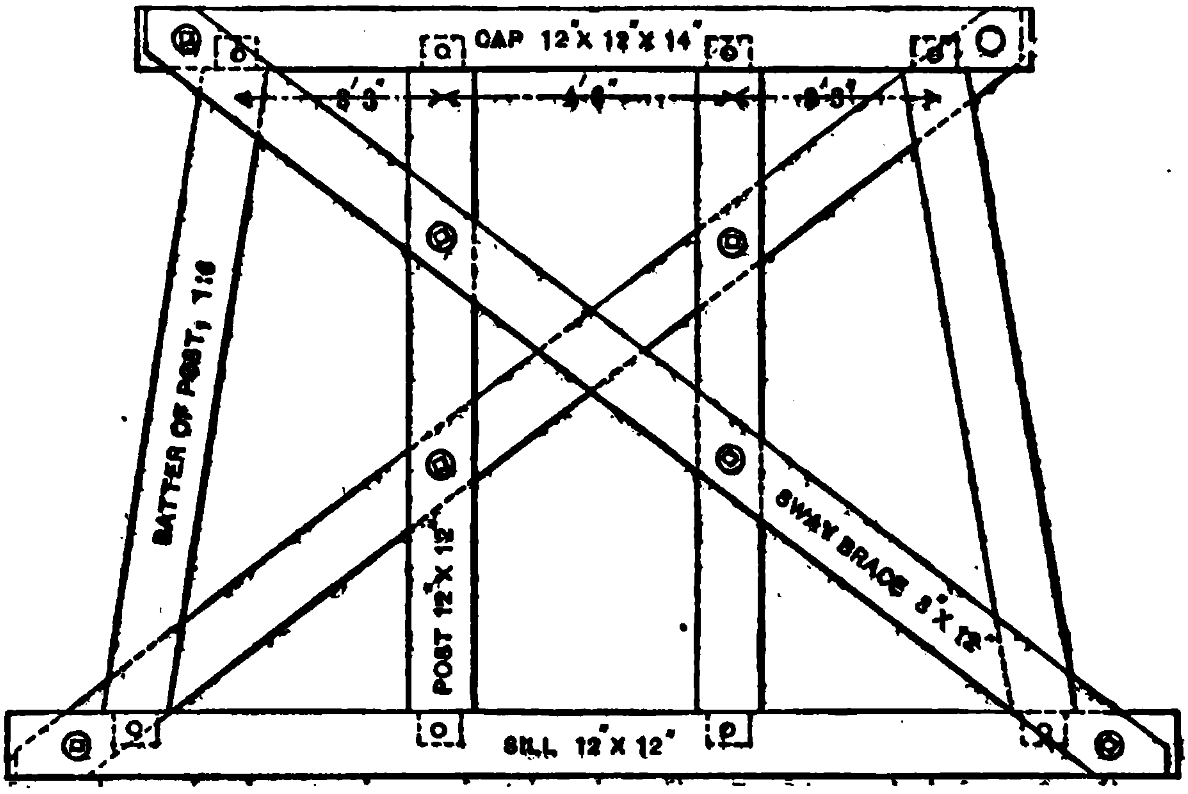


FIG. 71.

3" thick, 8" wide, and 5½" long. The mortise should be cut a little deeper than the tenon. "Drip-holes" from the mortise to the outside will assist in draining off water that may accumulate in the joint and thus prevent the rapid decay that would otherwise ensue. These joints are very troublesome if a single post decays and requires renewal. It is generally required that the mortise and tenon should be thoroughly daubed with paint before putting them together. This will tend to make the joint water-tight and prevent decay from the accumulation and retention of water in the joint.

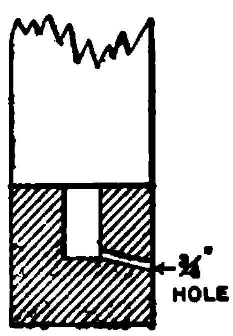


FIG. 72.

(*b*) **The plaster joint.** This joint is made by bolting and spiking a 3"×12" plank on both sides of the joint. The cap and sill should be notched to receive the posts. Repairs are greatly facilitated by the use of these joints. This method has been used by the Delaware and Hudson Canal Co. [R. R.].

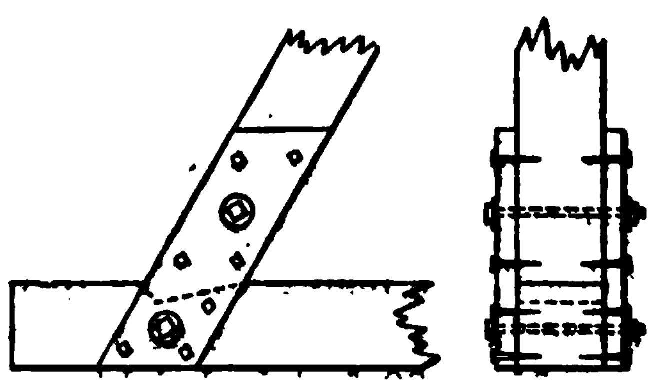


FIG. 73.

(*c*) **Iron plates.** An iron plate of the form shown in Fig. 74

(*b*) is bent and used as shown in Fig. 74 (*a*). Bolts passing through the bolt-holes shown secure the plates to the timbers and make a strong joint which may be readily loosened for repairs. By slight modifications in the design the method may be used for inclined posts and complicated joints.

Fig. 74.

(*d*) **Split caps and sills.** These are described in § 161. Their advantages apply with even greater force to framed trestles.

(*e*) **Dowels and drift-bolts.** These joints facilitate cheap and rapid construction, but renewals and repairs are very difficult, it being almost impossible to extract a drift-bolt, which has been driven its full length, without splitting open the pieces containing it. Notwithstanding this objection they are extensively used, especially for temporary work which is not expected to be used long enough to need repairs.

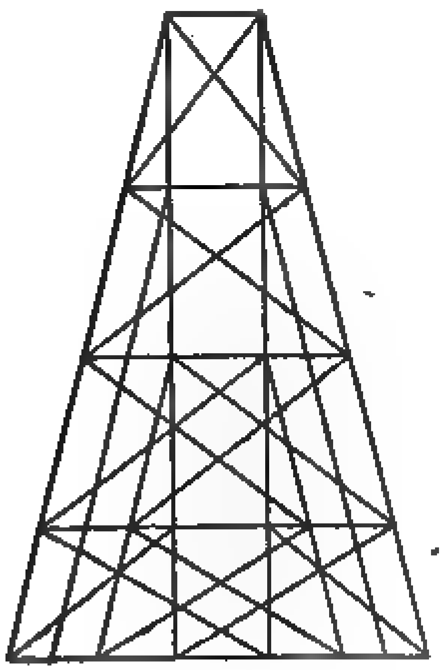


Fig. 75.

171. **Multiple-story construction.** Single-story framed trestle bents are used for heights up to 18 or 20 feet and exceptionally up to 30 feet. For greater heights some such construction as is illustrated in a skeleton design in Fig. 75 is used. By using split sills between each story and separate vertical and batter posts in each story, any piece may readily be removed and renewed if necessary. The height of these stories varies, in different designs, from 15 to 25 and even 30 feet. In some designs the structure of each story is independent of the stories above and below. This greatly facilitates both the original construction and subsequent repairs.

In other designs the verticals and batter-posts are made continuous through two consecutive stories. The structure is somewhat stiffer, but is much more difficult to repair.

Since the bents of any trestle are usually of variable height and those heights are not always an even multiple of the uniform height desired for the stories, it becomes necessary to make the

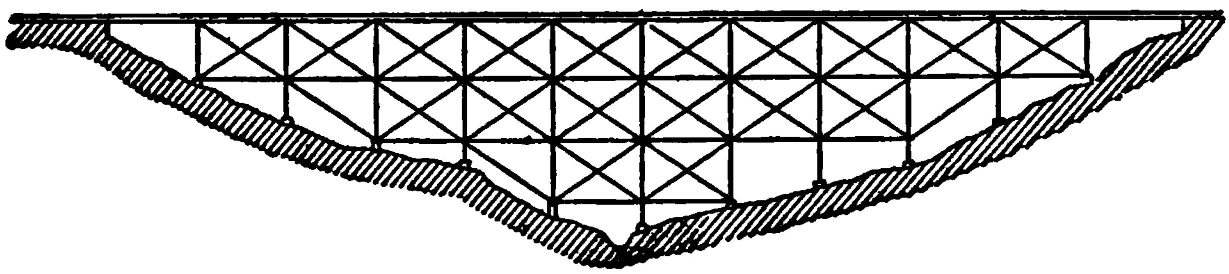


FIG. 76.

upper stories of uniform height and let the odd amount go to the lowest story, as shown in Figs. 75 and 76.

172. Span. The shorter the span the greater the number of trestle bents; the longer the span the greater the required strength of the stringers supporting the floor. Economy demands the adoption of a span that shall make the sum of these require-

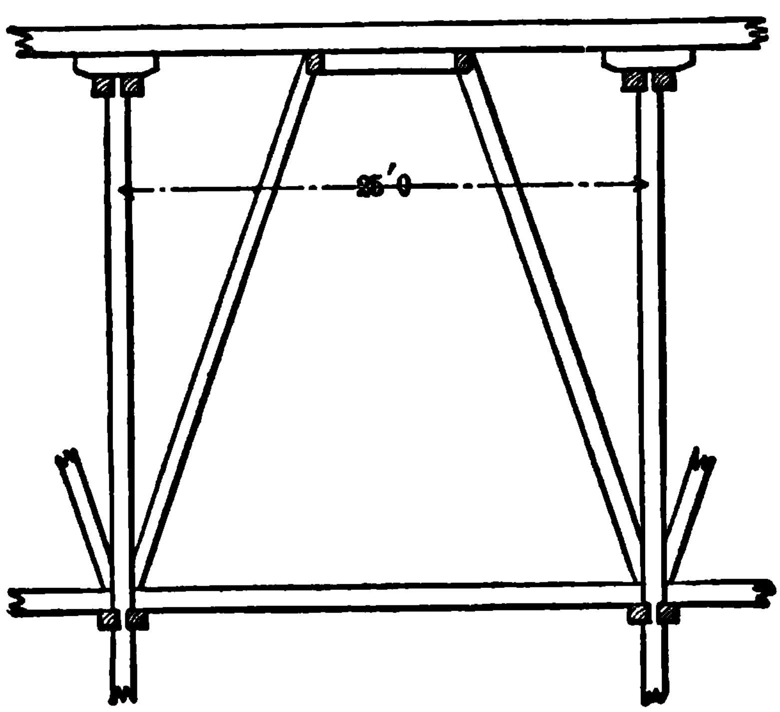


FIG. 77.

ments a minimum. The higher the trestle the greater the cost of each bent, and the greater the span that would be justifiable. Nearly all trestles have bents of variable height, but the advantage of employing uniform standard sizes is so great that many

roads use the same span and sizes of timber not only for the panels of any given trestle, but also for all trestles regardless of height. The spans generally used vary from 10 to 16 feet. The Norfolk and Western R. R. uses a span of 12′ 6″ for all single-story trestles, and a span of 25′ for all multiple-story trestles. The stringers are the same in both cases, but when the span is 25 feet, knee-braces are run from the sill of the first story below to near the middle of each set of stringers. These knee-braces are connected at the top by a "straining-beam" on which the stringers rest, thus supporting the stringer in the center and virtually reducing the span about one-half.

173. **Foundations.** (a) **Piles.** Piles are frequently used as a foundation, as in Fig. 78, particularly in soft ground, and also for temporary structures. These foundations are cheap, quickly constructed, and are particularly valuable when it is financially necessary to open the road for traffic as soon as possible and with the least expenditure of money; but there is the disadvantage of inevitable decay within a few years unless the piles are chemically treated, as will be discussed later. Chemical treatment, however, increases the cost so that such a foundation would often cost more than a foundation of stone. A pile should be driven under each post as shown in Fig. 78.

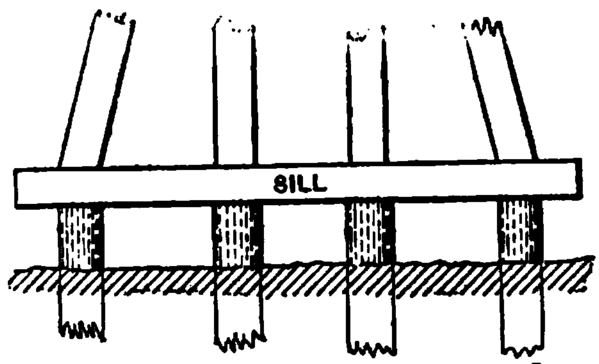


FIG. 78.

(b) **Mud-sills.** Fig. 79 illustrates the use of mud-sills as built by the Louisville and Nashville R. R. Eight blocks 12″×12″×6′ are used under each bent. When the ground is very soft, two additional timbers (12″×12″×length of bent-sill), as shown by the dotted lines, are placed underneath. The number required evidently depends on the nature of the ground.

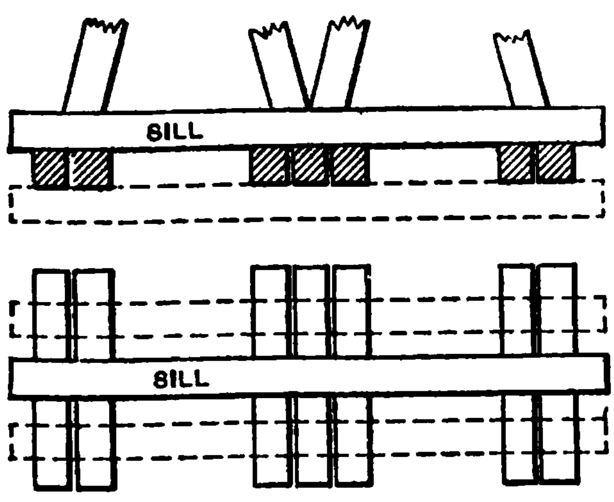


FIG. 79.

(c) **Stone foundations.** Stone foundations are the best and the most expensive. For very high trestles the Norfolk and

Western R. R. employs foundations as shown in Fig. 80, the walls being 4 feet thick. When the height of the trestle is 72 feet or less (the plans requiring for 72′ in height a foundation-wall 39′ 6″ long) the foundation is made continuous. The sill

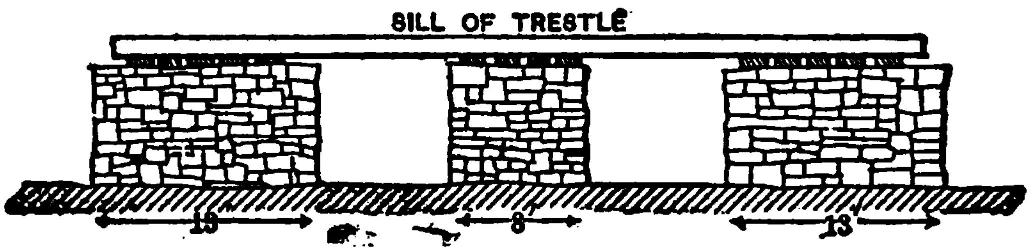


FIG. 80.

of the trestle should rest on several short lengths of 3″×12″ plank, laid transverse to the sill on top of the wall.

174. Longitudinal bracing. This is required to give the structure longitudinal stiffness and also to reduce the columnar length of the posts. This bracing generally consists of horizontal "waling-strips" and diagonal braces. Sometimes the braces are placed wholly on the outside posts unless the trestle is very high. For single-story trestles the P. R. R. employs the "laced" system, i.e., a line of posts joining the cap of one bent with the sill of the next, and the sill of that bent with the cap of the next. Some plans employ braces forming an × in alternate panels. Connecting these braces in the center more than doubles their columnar strength. Diagonal braces, when bolted to posts, should be fastened to them as near the ends of the posts as possible. The sizes employed vary largely, depending on the clear length and on whether they are expected to act by tension or compression. 3″×12″ planks are often used when the design would require tensile strength only, and 8″×8″ posts are often used when compression may be expected.

175. Lateral bracing. Several of the more recent designs of trestles employ diagonal lateral bracing between the caps of adjacent bents. It adds greatly to the stiffness of the trestle and better maintains its alignment. 6″×6″ posts, forming an × and connected at the center, will answer the purpose.

176. Abutments. When suitable stone for masonry is at hand and a suitable subsoil for a foundation is obtainable without too much excavation, a masonry abutment will be the best. Such an abutment would probably be used when masonry footings for trestle bents were employed (§ 173, *c*).

Another method is to construct a "crib" of 10″×12″ timber,

laid horizontally, drift-bolted together, securely braced and embedded into the ground. Except for temporary construction such a method is generally objectionable on account of rapid decay.

Another method, used most commonly for pile trestles, and for framed trestles having pile foundations (§ 173, *a*), is to use a pile bent at such a place that the natural surface on the *up-hill* side is not far below the cap, and the thrust of the material, filled in to bring the surface to grade, is insignificant. $3'' \times 12''$ planks are placed behind the piles, cap, and stringers to retain the filled material.

FIG. 31.

FLOOR SYSTEMS.

177. Stringers. The general practice is to use two, three, and even four stringers under each rail. Sometimes a stringer is placed under each guard-rail. Generally the stringers are made of two panel lengths and laid so that the joints alternate. A few roads use stringers of only one panel length, but this practice is strongly condemned by many engineers. The stringers should be separated to allow a circulation of air around them and prevent the decay which would occur if they were placed close together. This is sometimes done by means of 2″ planks, 6′ to 8′ long, which are placed over each trestle bent. Several bolts, passing through all the stringers forming a group and through the separators, bind them all into one solid construction. Cast-iron "spools" or washers, varying from 4″ to $\frac{3}{4}''$ in length (or thickness), are sometimes strung on each bolt so as to separate the stringers. Sometimes washers are used between the separating planks and the stringers, the object of the separating planks then being to bind the stringers, especially abutting stringers, and increase their stiffness.

The most common size for stringers is $8'' \times 16''$. The Pennsylvania Railroad varies the width, depth, and number of stringers under each rail according to the clear span. It may be noticed that, assuming a uniform load per running foot, both the pressure per square inch at the ends of the stringers (the

caps having a width of 12″) and also the stress due to transverse strain are kept *approximately* constant for the variable gross load on these varying spans.

Clear span.	No. of pieces under each rail.	Width.	Depth.
10 feet	2	8 inches	16 inches
12 "	2	10 "	16 "
14 "	3	10 "	16 "

178. Corbels. A corbel (in trestle-work) is a stick of timber (perhaps two placed side by side), about 3′ to 6′ long, placed underneath and along the stringers and resting on the cap. There are strong prejudices for and against their use, and a corresponding diversity in practice. They are bolted to the stringers and thus stiffen the joint. They certainly reduce the objectionable crushing of the fibers at each end of the stringer, but if the corbel is no wider than the stringers, as is generally the case, the area of pressure between the corbels and the cap is

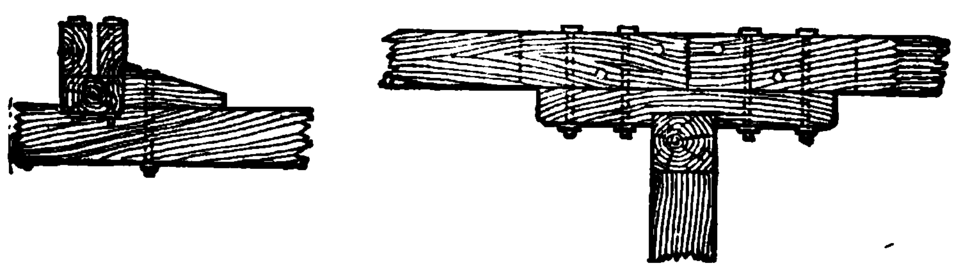


FIG. 82.

no greater and the pressure per square inch on the cap is no less than the pressure on the cap if no corbels were used. If the corbels and cap are made of hard wood, as is recommended by some, the danger of crushing is lessened, but the extra cost and the frequent scarcity of hard wood, and also the extra cost and labor of using corbels, may often neutralize the advantages obtained by their use.

179. Guard-rails. These are frequently made of 5″×8″ stuff, notched 1″ for each tie. The sizes vary up to 8″×8″, and the depth of notch from ¾″ to 1½″. They are generally bolted to every third or fourth tie. It is frequently specified that they shall be made of oak, white pine, or yellow pine. The joints are made over a tie, by halving each piece, as illustrated in Fig. 83. The joints on opposite sides of the trestle should be "stag-

gered." Some roads fasten every tie to the guard-rail, using a bolt, a spike, or a lag-screw.

Guard-rails were originally used with the idea of preventing the wheels of a derailed truck from running off the ends of the ties. But it has been found that an outer guard-rail alone (without an inner guard-rail) becomes an actual element of danger, since it has frequently happened that a derailed wheel has caught on the outer guard-rail, thus causing the truck to slew around

FIG. 83.

and so produce a dangerous accident. The true function of the *outside* guard-rail is thus changed to that of a tie-spacer, which keeps the ties from spreading when a derailment occurs. The inside guard-rail generally consists of an ordinary steel rail spiked about 10 inches inside of the running rail. These inner guard-rails should be bent inward to a point in the center of the track about 50 feet beyond the end of the bridge or trestle. If the inner guard-rails are placed with a clear space of 10 inches inside the running rail, the outer guard-rails should be *at least* 6′ 10″ apart. They are generally much farther apart than this.

180. Ties on trestles. If a car is derailed on a bridge or trestle, the heavily loaded wheels are apt to force their way between the ties by displacing them unless the ties are closely spaced and fastened. The clear space between ties is generally equal to or less than their width. Occasionally it is a little more than their width. 6″×8″ ties, spaced 14″ to 16″ from center to center, are most frequently used. The length varies from 9′ to 12′ for single track. They are generally notched ½″ deep on the under side where they rest on the stringers. Oak ties are generally required even when cheaper ties are used on the other sections of the road. Usually every third or fourth tie is bolted to the stringers. When stringers are placed underneath the guard-rails, bolts are run from the top of the guard-rail to the under side of the stringer. The guard-rails thus hold down the whole system of ties, and no direct fastening of the ties to the stringers is needed.

181. Superelevation of the outer rail on curves. The location of curves on trestles should be avoided if possible, especially when the trestle is high. Serious additional strains are intro-

duced especially when the curvature is sharp or the speed high. Since such curves are sometimes practically unavoidable, it is necessary to design the trestle accordingly. If a train is stopped on a curved trestle, the action of the train on the trestle is evidently vertical. If the train is moving with a considerable velocity, the resultant of the weight and the centrifugal action is a force somewhat inclined from the vertical. Both of these conditions may be expected to exist at times. If the *axis* of the system of posts is vertical (as illustrated in methods *a*, *b*, *c*, *d*, and *e*), any lateral force, such as would be produced by a moving train, will tend to rack the trestle bent. If the stringers are set vertically, a centrifugal force likewise tends to tip them sidewise. If the axis of the system of posts (or of the stringers) is inclined so as to coincide with the pressure of the train on the trestle when the train is moving at its normal velocity, there is no tendency to rack the trestle when the train is moving at that velocity, but there will be a tendency to rack the trestle or twist the stringers when the train is stationary. Since a moving train is usually the normal condition of affairs, as well as the condition which produces the maximum stress, an inclined axis is evidently preferable from a theoretical standpoint; but whatever design is adopted, the trestle should evidently be sufficiently cross-braced for either a moving or a stationary load, and any proposed design must be studied as to the effect of *both* of these conditions. Some of the various methods of securing the requisite superelevation may be described as follows:

(a) Framing the outer posts longer than the inner posts, so that the cap is inclined at the proper angle; axis of posts vertical. (Fig 84.) The method requires more work in framing the trestle, but simplifies subsequent track-laying and maintenance, unless it should be found that the superelevation adopted is unsuitable, in which case it could be corrected by one of the other methods given below. The stringers tend to twist when the train is stationary.

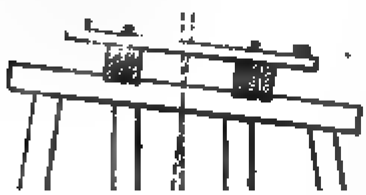


FIG. 84.

(b) Notching the cap so that the stringers are at a different

elevation. (Fig. 85.) This weakens the cap and requires that all ties shall be notched to a bevelled surface to fit the stringers, which also weakens the ties. A centrifugal force will tend to twist the stringers and rack the trestle.

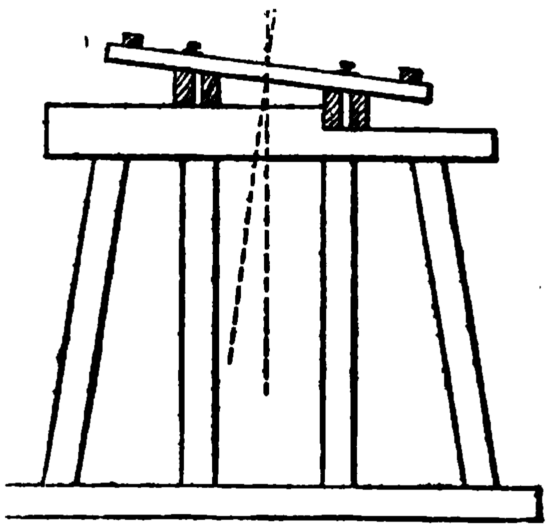


FIG. 85.

(c) **Placing wedges underneath the ties at each stringer.** These wedges are fastened with two bolts. Two or more wedges will be required for each tie. The additional number of pieces required for a long curve will be immense, and the work of inspection and keeping the nuts tight will greatly increase the cost of maintenance.

(d) **Placing a wedge under the outer rail at each tie.** This requires but one extra piece per tie. There is no need of a wedge under the inner tie in order to make the rail normal to the tread. The resulting inward inclination is substantially that produced by some forms of rail-chairs or tie-plates. The spikes (a little longer than usual) are driven through the wedge into the tie. Sometimes "lag-screws" are used instead of spikes. If experience proves that the superelevation is too much or too little, it may be changed by this method with less work than by any other.

(e) **Corbels of different heights.** When corbels are used (see § 178) the required inclination of the floor system may be obtained by varying the depth of the corbels.

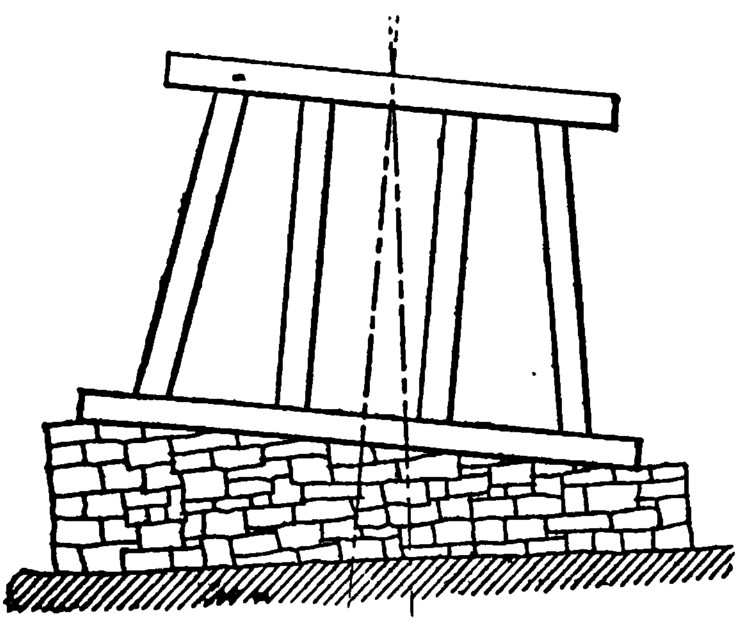


FIG. 86.

(f) **Tipping the whole trestle.** This is done by placing the trestle on an inclined foundation. If very much inclined, the trestle bent must be secured against the possibility of slipping sidewise, for the slope would be considerable with a sharp curve, and the

vibration of a moving train would reduce the coefficient of friction to a comparatively small quantity.

(g) **Framing the outer posts longer.** This case is identical with case (*a*) except that the axis of the system of posts is inclined, as in case (*f*), but the sill is horizontal.

The above-described plans will suggest a great variety of methods which are possible and which differ from the above only in minor details.

182. Protection from fire. Trestles are peculiarly subject to fire, from passing locomotives, which may not only destroy the trestle, but perhaps cause a terrible disaster. This danger is sometimes reduced by placing a strip of galvanized iron along the top of each set of stringers and also along the tops of the caps. Still greater protection was given on a long trestle on the Louisville and Nashville R. R. by making a solid flooring of timber, covered with a layer of ballast on which the ties and rails were laid as usual.

Barrels of water should be provided and kept near all trestles, and on very long trestles barrels of water should be placed every two or three hundred feet along its length. A place for the barrels may be provided by using a few ties which have an extra length of about four feet, thus forming a small platform, which should be surrounded by a railing. The track-walker should be held accountable for the maintenance of a supply of water in these barrels, renewals being frequently necessary on account of evaporation. Such platforms should also be provided as REFUGE-BAYS for track-walkers and trackmen working on the trestle. On very long trestles such a platform is sometimes provided with sufficient capacity for a hand-car.

183. Timber. Any strong durable timber may be used when the choice is limited, but oak, pine, or cypress are preferred when obtainable. When all of these are readily obtainable, the various parts of the trestle will be constructed of different kinds of wood—the stringers of long-leaf pine, the posts and braces of pine or red cypress, and the caps, sills, and corbels (if used) of white oak. The use of oak (or a similar hard wood) for caps, sills, and corbels is desirable because of its greater strength in resisting crushing across the grain, which is the critical test for these parts. There is no physiological basis to the objection, sometimes made, that different species of timber, in contact with each other, will rot quicker than if only one

TWO CAPPED

(H + 0.4) + 1' 1" = LENGTH OF SILL.
H — 1' 11" = LENGTH OF 12" X 12" TIMBER REQUIRED FOR PLUMB POSTS, SINGLE BENTS
H — 21' 11" = " " " " " " " " IN LOWER STORY, DOUBLE "
H — 3' 6" = " " PLUMB POSTS BETWEEN SHOULDERS, SINGLE "
H — 22' 11" = " " " " " " " " " DOUBLE "
(LENGTH OF PLUMB POST X 1.031) + 5" = LENGTH OF BATTER POST, EXCEPT IN 6" X 12" INTERMEDIATE BATTER POSTS WHERE ADD 3" INSTEAD OF 5"
WITH ALTERNATIVE ARRANGEMENT OF STRINGERS, ADD 2" TO LENGTHS GIVEN BY ABOVE FORMULAS.

(*To face page* 216.)

PLATE II.

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kind of timber is used. When a very extensive trestle is to be built at a place where suitable growing timber is at hand but there is no convenient sawmill, it will pay to transport a portable sawmill and engine and cut up the timber as desired.

184. Cost of framed timber trestles. The cost varies widely on account of the great variation in the cost of timber. When a railroad is first penetrating a new and undeveloped region, the cost of timber is frequently small, and when it is obtainable from the company's right-of-way the only expense is felling and sawing. The work per M, B. M., is small, considering that a single stick 12″×12″×25′ contains 300 feet, B. M., and that sometimes two hours' work, worth perhaps $1, will finish all the work required on it. Smaller pieces will of course require more work per foot, B. M. Long-leaf pine can be purchased from the mills at from $27 to $45 per M feet, B. M., according to the dimensions. To this must be added the freight and labor of erection. The cartage from the nearest railroad to the trestle may often be a considerable item. Wrought iron will cost about 3 cents per pound and cast iron 2 cents, although the prices are often lower than these. The amount of iron used depends on the detailed design, but, as an average, will amount to $1.50 to $2 per 1000 feet, B. M., of timber. A large part of the trestling of the country has been built at a contract price of about $30 per 1000 feet, B. M., erected. While the cost will frequently rise to $50 and even $60 when timber is scarce, it will drop to $13 (cost quoted) when timber is cheap.

DESIGN OF WOODEN TRESTLES.

185. Common practice. A great deal of trestling has been constructed without any rational design except that custom and experience have shown that certain sizes and designs are *probably* safe. This method has resulted occasionally in failures but more frequently in a very large waste of timber. Many railroads employ a uniform size for all posts, caps, and sills, and a uniform size for stringers, all regardless of the height or span of the trestle. For repair work there are practical reasons favoring this. "To attempt to run a large lot of sizes would be more wasteful in the end than to maintain a few stock sizes only. Lumber can be bought more cheaply by giving a general order for 'the run of the mill for the season,' or 'a cargo lot,' specify-

ing approximate percentages of standard stringer size, of 12×12-inch stuff, 10×10-inch stuff, etc., and a liberal proportion of 3- or 4-inch plank, all lengths thrown in. The 12×12-inch stuff, etc., is ordered all lengths, from a certain specified length up. In case of a wreck, washout, burn-out, or sudden call for a trestle to be completed in a stated time, it is much more economical and practical to order a certain number of carloads of 'trestle stuff' to the ground and there to select piece after piece as fast as needed, dependent only upon the length of stick required. When there is time to make the necessary surveys of the ground and calculations of strength, and to wait for a special bill of timber to be cut and delivered, the use of different sizes for posts in a structure would be warranted to a certain extent." * For new construction, when there is generally sufficient time to design and order the proper sizes, such wastefulness is less excusable, and under any conditions it is both safer and more economical to prepare *standard designs* which can be made applicable to varying conditions and which will at the same time utilize as much of the strength of the timber as can be depended on. In the following sections will be given the elements of the preparation of such standard designs, which will utilize uniform sizes with as little waste of timber as possible. It is *not* to be understood that special designs should be made for each individual trestle.

186. Required elements of strength. The *stringers* of trestles are subject to transverse strains, to crushing across the grain at the ends, and to shearing along the neutral axis. The strength of the timber must therefore be computed for all these kinds of stress. *Caps* and *sills* will fail, if at all, by crushing across the grain; although subject to other forms of stress, these could hardly cause failure in the sizes usually employed. There is an apparent exception to this: if piles are improperly driven and an uneven settlement subsequently occurs, it may have the effect of transferring practically all of the weight to two or three piles, while the *cap* is subjected to a severe transverse strain which may cause its failure. Since such action is caused generally by avoidable errors of construction it may be considered as abnormal, and since such a failure will generally occur by a *gradual* settlement, all danger may be avoided by reasonable

* From "Economical Designing of Timber Trestle Bridges."

care in inspection. *Posts* must be tested for their columnar strength. These parts form the bulk of the trestle and are the parts which can be definitely designed from known stresses. The stresses in the bracing are more indefinite, depending on indeterminate forces, since the inclined posts take up an unknown proportion of the lateral stresses, and the design of the bracing may be left to what experience has shown to be safe, without involving any large waste of timber.

187. Strength of timber. Until recently tests of the strength of timber have generally been made by testing small, selected, well-seasoned sticks of "clear stuff," free from knots or imperfections. Such tests would give results so much higher than the vaguely known strength of large unseasoned "commercial" timber that very large factors of safety were recommended—factors so large as to detract from any confidence in the whole theoretical design. Recently the U. S. Government has been making a thoroughly scientific test of the strength of full-size timber under various conditions as to seasoning, etc. The work has been so extensive and thorough as to render possible the economical designing of timber structures.

One important result of the investigation is the determination of the great influence of the moisture in the timber and the law of its effect on the strength. It has been also shown that timber soaked with water has substantially the same strength as green timber, even though the timber had once been thoroughly seasoned. Since trestles are exposed to the weather they should be designed on the basis of using green timber. It has been shown that the strength of green timber is very regularly about 55 to 60% of the strength of timber in which the moisture is 12% of the dry weight, 12% being the proportion of moisture usually found in timber that is protected from the weather but not heated, as, *e.g.*, the timber in a barn. Since the moduli of rupture have all been reduced to this standard of moisture (12%), if we take *one-eighth* of the rupture values, it still allows a factor of safety of about five, even on green timber. In Table XX there are quoted the values taken from the U. S. Government reports on the strength of timber, the tests probably being the most thorough and reliable that were ever made.

In Table XXI are given the "working unit stresses for structural timber, expressed in pounds per square inch," as recommended by the committee on "Wooden Bridges and Trestles,"

of the American Railway Engineering Association. The report was presented at their tenth annual convention, held in Chicago, in March, 1909.

TABLE XX. MODULI OF RUPTURE FOR VARIOUS TIMBERS.
[12% moisture.]
(Condensed from U. S. Forestry Circular, No. 15.)

No.	Species.	Weight per cubic foot.	Cross-bending. Ultimate Strength.	Cross-bending. Modulus of Elasticity.	Crushing endwise.	Crushing across the grain.	Shearing along the grain.
1	Long-leaf pine....	38	12 600	2 070 000	8000	1180	700
2	Cuban "	39	13 600	2 370 000	8700	1220	700
3	Short-leaf "	32	10 100	1 680 000	6500	960	700
4	Loblolly "	33	11 300	2 050 000	7400	1150	700
5	White "	24	7 900	1 390 000	5400	700	400
6	Red "	31	9 100	1 620 000	6700	1000	500
7	Spruce "	39	10 000	1 640 000	7300	1200	800
8	Bald cypress......	29	7 900	1 290 000	6000	800	500
9	White cedar.......	23	6 300	910 000	5200	700	400
10	Douglas spruce....	32	7 900	1 680 000	5700	800	500
11	White oak.......	50	13 100	2 090 000	8500	2200	1000
12	Overcup "	46	11 300	1 620 000	7300	1900	1000
13	Post "	50	12 300	2 030 000	7100	3000	1100
14	Cow "	46	11 500	1 610 000	7400	1900	900
15	Red "	45	11 400	1 970 000	7200	2300	1100
16	Texan "	46	13 100	1 860 000	8100	2000	900
19	Willow "	45	10 400	1 750 000	7200	1600	900
20	Spanish "	46	12 000	1 930 000	7700	1800	900
21	Shagbark hickory..	51	16 000	2 390 000	9500	2700	1100
27	Pignut " ..	56	18 700	2 730 000	10900	3200	1200
28	White elm........	34	10 300	1 540 000	6500	1200	800
29	Cedar "	46	13 500	1 700 000	8000	2100	1300
30	White ash........	39	10 800	1 640 000	7200	1900	1100

188. Loading. As shown in § 172, the span of trestles is always small, is generally 14 feet, and is never greater than 18 feet except when supported by knee-braces. The greatest load that will ever come on any one span will be the concentrated loading of the drivers of a consolidation locomotive. With spans of 14 feet or less it is impossible for even the four pairs of drivers to be on the same span at once. The weight of the rails, ties, and guard-rails should be added to obtain the total load on the stringers, and the weight of these, plus the weight of the stringers, should be added to obtain the pressure on the caps or corbels.

TABLE XXI. WORKING UNIT STRESSES FOR STRUCTURAL TIMBER EXPRESSED IN LBS. PER SQ. IN. RECOMMENDED BY THE COMMITTEE ON WOODEN BRIDGES AND TRESTLES AMER. RWY. ENG. ASSOC., 1909.

Kind of Timber.	Bending.			Shearing.				Compression.						Ratio of length of stringer to depth.
	Extreme fiber stress.		Modulus of elasticity.	Parallel to grain.		Longitudinal shear in beams		Perpendicular to grain.		Parallel to grain.		For colmns under 15 diams. safe stress.	Formulas for working stress in long column over 15 diams.	
	Aver. ultimate.	W'rking stress	Average.	Aver. ultimate.	W'rking stress	Aver. ultimate.	Working stress.	Elastic limit.	Working stress.	Aver. ultimate.	Working stress.			
Douglas fir.....	6100	1200	1,510,000	690	170	270	110	630	310	3600	1200	900	$1200\left(1-\frac{L}{60D}\right)$	10
Long-leaf pine...	6500	1300	1,610,000	720	180	300	120	520	260	3800	1300	980	$1300\left(1-\frac{L}{60D}\right)$	10
Short-leaf pine..	5600	1100	1,480,000	710	170	330	130	340	170	3400	1100	830	$1100\left(1-\frac{L}{60D}\right)$	10
White pine.....	4400	900	1,130,000	400	100	180	70	290	150	3000	1000	750	$1000\left(1-\frac{L}{60D}\right)$	10
Spruce.........	4800	1000	1,310,000	600	150	170	70	370	180	3200	1100	830	$1100\left(1-\frac{L}{60D}\right)$	......
Norway pine....	4200	800	1,190,000	590	130	250	100	......	150	2600*	800	600	$800\left(1-\frac{L}{60D}\right)$	......
Tamarack......	4600	900	1,220,000	670	170	260	100	......	220	3200*	1000	750	$1000\left(1-\frac{L}{60D}\right)$	......
Western hemlock	5800	1100	1,480,000	630	160	270*	100	440	220	3500	1200	900	$1200\left(1-\frac{L}{60D}\right)$	......
Redwood.......	5000	900	800,000	300	80	......	......	400	150	3300	900	680	$900\left(1-\frac{L}{60D}\right)$	......
Bald cypress....	4800	900	1,150,000	500	120	......	......	340	170	3900	1100	830	$1100\left(1-\frac{L}{60D}\right)$	......
Red cedar......	4200	800	860,000	......	......	......	......	470	230	2800	900	680	$900\left(1-\frac{L}{60D}\right)$	......
White oak......	5700	1100	1,150,000	840	210	270	110	920	450	3500	1300	980	$1300\left(1-\frac{L}{60D}\right)$	12

Note.—These unit stresses are for a green condition of timber and are to be used without increasing the live load stresses for impact. * Partially air-dry.

L = length in inches. D = least side in ins.

These working stresses are for railroad bridges and trestles. For highway bridges and trestles increase the figures by 25 per cent. For buildings, etc., when protected from weather and free from impact, increase them 50 per cent. To compute deflection, under long-continued loading, use 50 per cent of modulus of elasticity.

This dead load is almost insignificant compared with the live load and may be included with it. The weight of rails, ties, etc., may be estimated at 240 pounds per foot. To obtain the weight on the caps the weight of the stringers must be added, which depends on the design and on the weight per cubic foot of the wood employed. But as the weight of the stringers is comparatively small, a considerable percentage of variation in weight will have but an insignificant effect on the result. Disregarding all refinements as to actual dimensions, the ordinary maximum loading for standard-gauge railroads may be taken as that due to four driving-axles, spaced 5′ 0″ apart and giving a pressure of 40000 pounds per axle. This should be increased to 54000 pounds per axle (same spacing) for the heaviest traffic. On the basis of 40000 pounds per axle or 20000 pounds per wheel the following results have been computed: This loading is assumed to allow for impact.

STRESSES ON VARIOUS SPANS DUE TO MOVING LOADS OF 20000 POUNDS, SPACED 5′ 0″ APART, WITH 120 POUNDS PER FOOT OF DEAD LOAD.

Span in feet.	Max. moment, ft. lbs.	Max. shear.	Max. load on one cap under one rail.
10	51 500	30 600	41 200
12	82 160	35 720	49 440
14	112 940	39 410	57 680
16	123 840	43 460	65 920
18	164 860	47 747	75 160

Although the dead load does not vary in proportion to the live load, yet, considering the very small influence of the dead load, there will be no appreciable error in assuming the corresponding values, for a load of 54000 lbs. per axle, to be $\frac{54}{40}$ of those given in the above tabulation.

189. Factors of safety. The most valuable result of the government tests is the knowledge that under given moisture conditions the strength of various species of sound timber is not the variable uncertain quantity it was once supposed to be, but that its strength can be relied on to a comparatively close percentage. This confidence in values permits the employment of lower factors of safety than have heretofore been permissible. Stresses, which when excessive would result in immediate destruction, such as cross-breaking and columnar stresses, should be allowed a higher factor of safety—say 6 or 8 for green timber. Other stresses, such as crushing across the grain and shearing along the

neutral axis, which will be apparent to inspection before it is dangerous, may be allowed lower factors—say 3 to 5.

190. Design of stringers. The strength of rectangular beams of equal width varies as the square of the depth; therefore deep beams are the strongest. On the other hand, when any cross-sectional dimension of timber much exceeds 12″ the cost is much higher per M, B. M , and it is correspondingly difficult to obtain thoroughly sound sticks, free from wind-shakes, etc. Wind-shakes especially affect the shearing strength. Also, if the required transverse strength is obtained by using high narrow stringers, the area of pressure between the stringers and the cap may become so small as to induce crushing across the grain. This is a very common defect in trestle design. As already indicated in § 172, the span should vary roughly with the average height of the trestle, the longer spans being employed when the trestle bents are very high, although it is usual to employ the same span throughout any one trestle.

To illustrate, if we select a span of 14 feet, the load on one cap will be 57680 lbs. If the stringers and cap are made of long-leaf yellow pine, the allowable value, according to Table XXI, for "compression across the grain" is 260 pounds per square inch; this will require 222 square inches of surface. If the cap is 12″ wide, this will require a width of 18.5 inches, or say 2 stringers under each rail, each 9 inches wide. For rectangular beams.

$$\text{Moment} = \tfrac{1}{6}R'bh^2.$$

Using for R' the safe value 1300 lbs. per square inch, we have

$$112940 \times 12 = \tfrac{1}{6} \times 1300 \times 18 \times h^2,$$

from which $h = 18''.7$. If desired, the width may be increased to 10″ and the depth correspondingly reduced, which will give similarly $h = 17''.7$ or say 18″. This shows that two beams, 10″×18″, under each rail will stand the transverse bending and have more than enough area for crushing.

The shear per square inch will equal

$$\frac{3}{2}\,\frac{\text{total shear}}{\text{cross-section}} = \frac{3}{2}\,\frac{39410}{2 \times 10 \times 18} = 164 \text{ lbs. per sq. inch.}$$

This is higher than the recommended working value. The combination suggested in § 177, viz., 3 beams 10″×16″ for 14 feet span, gives a far safer value. Considering that wooden beams,

tested to destruction, usually fail by shearing, the three-beam combination is safer.

The deflection should be computed to see if it exceeds the somewhat arbitrary standard of $\frac{1}{200}$ of the span. The deflection for *uniform loading* is

$$\Delta=\frac{5Wl^3}{32bh^3E},$$

in which l = length in inches;

W = total load, assumed as uniform = 57680;
E = modulus of elasticity, given as 1610000 lbs.

per sq. in. for long-leaf pine, according to Table XXI. Then

$$\Delta=\frac{5\times57680\times168^3}{32\times30\times16^3\times1610000}=0''.216$$

$$\frac{1}{200}\times168''=0''.84,$$

so that the calculated deflection is well within the limit. Of course the loading is not strictly uniform, but even with a liberal allowance the deflection is still safe.

For the heaviest practice (54000 lbs. per axle) these stringer dimensions must be correspondingly increased.

191. Design of posts. Four posts are generally used for single-track work. The inner posts are usually braced by the cross-braces, so that their columnar strength is largely increased; but as they are apt to get more than their share of work, the advantage is compensated and they should be treated as unsupported columns for the total distance between cap and sill in simple bents, or for the height of stories in multiple-story construction. The caps and sills are assumed to have a width of 12″. It facilitates the application of bracing to have the columns of the same width and vary the other dimension as required.

Unfortunately the experimental work of the U. S. Government on timber testing has not yet progressed far enough to establish unquestionably a general relation between the strength of long columns and the crushing strength of short blocks. The

following formula has been suggested, but it cannot be considered as established:

$$f=F\times\frac{700+15c}{700+15c+c^2}, \quad \text{in which}$$

f = allowable working stress per sq. in for long columns;
F = " " " " " " " short blocks;
$c=\frac{l}{d}$;
l = length of column in inches;
d = least cross-sectional dimensions in inches.

The formula recommended by the A. R. E. A. is found in Table XXI. For all columns of which the length is less than 15 times the least diameter, a uniform unit stress is recommended. For longer columns, a unit stress is multiplied by the factor $(1-l\div 60d)$, which is always less than unity. For the above case, $l=240$ and $d=12$, and the factor $=.667$, which, multiplied by 1300, gives a unit stress of 867 lbs. per square inch for a long-leaf yellow pine column of these dimensions.

$867\times 144=124848$ lbs., the *working load* for *each* post. This is more than the total load on one trestle bent and illustrates the usual great waste of timber. Making the post $8''\times 12''$ and calculating similarly, we have $f=650$, and the working load per column is $650\times 96=62400$ lbs. As considerable must be allowed for "weathering," which destroys the strength of the outer layers of the wood, and also for the dynamic effect of the live load, $8''\times 12''$ may not be too great, but it is certainly a safe dimension, considered as a column. One method of allowing for weathering is to disregard the outer half-inch on all sides of the post, i.e., to calculate the strength of a post one inch smaller in each dimension than the post actually employed. On this basis an $8''\times 12''\times 20'$ post, computed as a $7''\times 11''$ post, would have a *safe* columnar strength of 556 lbs. per square inch. With an area of 77 square inches, this gives a working load of 42812 lbs. for *each post*, or 171248 lbs. for the four posts. Considering that 115360 lbs. is the maximum load on one cap (14 feet span), the great excess of strength is apparent.

192. Design of caps and sills. The stresses in caps and sills are very indefinite, except as to crushing across the grain. As

the stringers are placed almost directly over the inner posts, and as the sills are supported just under the posts, the transverse stresses are almost insignificant. In the above case four posts have an area of $4 \times 12'' \times 8'' = 384$ sq. in. The total load 115360 lbs. will then give a pressure of 300 pounds per square inch, which is more than the allowable limit. This one feature will require the use of $12'' \times 12''$ (or at least $10'' \times 12''$) posts rather than $8'' \times 12''$ posts, for the smaller posts, although probably strong enough as posts, would produce an objectionably high pressure.

193. **Bracing.** Although some idea of the stresses in the bracing could be found from certain assumptions as to wind-pressure, etc., yet it would probably not be found wise to decrease, for the sake of economy, the dimensions which practice has shown to be sufficient for the work. The economy that would be possible would be too insignificant to justify any risk. Therefore the usual dimensions, given in §§ 173 and 174, should be employed.

CHAPTER V.

TUNNELS.

SURVEYING.

194. Surface surveys. As tunnels are always dug from each end and frequently from one or more intermediate shafts, it is necessary that an accurate surface survey should be made between the two ends. As the natural surface in a locality where a tunnel is necessary is almost invariably very steep and rough, it requires the employment of unusually refined methods of work to avoid inaccuracies. It is usual to run a line on the surface that will be at every point vertically over the center line of the tunnel. Tunnels are generally made straight unless curves are absolutely necessary, as curves add greatly to the cost. Fig. 87 represents roughly a longitudinal section of the

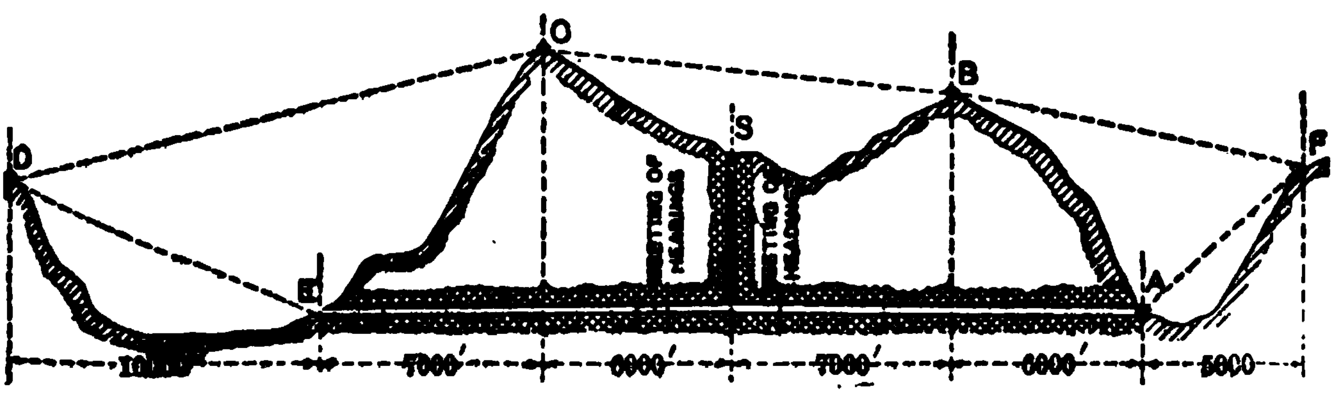


FIG. 87.—SKETCH OF SECTION OF THE HOOSAC TUNNEL.

Hoosac Tunnel. Permanent stations were located at *A*, *B*, *C*, *D*, *E*, and *F*, and stone houses were built at *A*, *B*, *C*, and *D*. These were located with ordinary field transits at first, and then all the points were placed as nearly as possible in one vertical plane by repeated trials and minute corrections, using a very large specially constructed transit. The stations *D* and *F* were necessary because *E* and *A* were invisible from *C* and *B*. The alinement at *A* and *E* having been determined with great accuracy, the true alinement was easily carried into the tunnel.

The relative elevations of A and E were determined with great accuracy. Steep slopes render necessary many settings of the level per unit of horizontal distance and require that the work be unusually accurate to obtain even fair accuracy per unit of distance. The levels are usually re-run many times until the probable error is a very small quantity

The exact horizontal distance between the two ends of the tunnel must also be known, especially if the tunnel is on a grade. The usual steep slopes and rough topography likewise render accurate horizontal measurements very difficult. Frequently when the slope is steep the measurement is best obtained by measuring along the slope and allowing for grade. This may be very accurately done by employing two tripods (level or transit tripods serve the purpose very well), setting them up slightly less than one tape-length apart and measuring between horizontal needles set in wooden blocks inserted in the top of each tripod. The elevation of each needle is also observed. The true horizontal distance between two successive positions of the needles then equals the square root of the difference of the squares of the inclined distance and the difference of elevation. Such measurements will probably be more accurate than those made by attempting to hold the tape horizontal and plumbing down with plumb-bobs, because (1) it is practically difficult to hold both ends of the tape truly horizontal; (2) on steep slopes it is impossible to hold the down-hill end of a 100-foot tape (or even a 25-foot length) on a level with the other end, and the great increase in the number of applications of the unit of measurement very greatly increases the probable error of the whole measurement; (3) the vibrations of a plumb-bob introduce a large probability of error in transferring the measurement from the elevated end of the tape to the ground, and the increased number of such applications of the unit of measurement still further increases the probable error.

195. Surveying down a shaft. If a shaft is sunk, as at S, Fig. 87, and it is desired to dig out the tunnel in both directions from the foot of the shaft so as to meet the headings from the outside, it is necessary to know, when at the bottom of the shaft, the elevation, alinement, and horizontal distance from each end of the tunnel.

The elevation is generally carried down a shaft by means of a steel tape. This method involves the least number of appli-

cations of the unit of measurement and greatly increases the accuracy of the final result.

The *horizontal distance from each end* may be easily transferred down the shaft by means of a plumb-bob, using some of the precautions described in the next paragraph.

To transfer the *alinement* from the surface to the bottom of a shaft requires the highest skill because the shaft is always small, and to produce a line perhaps several thousand feet long in a direction given by two points 6 or 8 feet apart requires that the two points must be determined with extreme accuracy. The eminently successful method adopted in the Hoosac Tunnel will be briefly described: Two beams were securely fastened across the top of the shaft (1030 feet deep), the beams being placed transversely to the direction of the tunnel and as far apart as possible and yet allow plumb-lines, hung from the intersection of each beam with the tunnel center line, to swing freely at the bottom of the shaft. These intersections of the beams with the center line were determined by averaging the results of a large number of careful observations for alinement. Two fine parallel wires, spaced about $\frac{1}{16}''$ apart, were then stretched between the beams so that the center line of the tunnel bisected at all points the space between the wires. Plumb-bobs, weighing 15 pounds, were suspended by fine wires beside each cross-beam, the wires passing between the two parallel alinement wires and bisecting the space. The plumb-bobs were allowed to swing in pails of water at the bottom. Drafts of air up the shaft required the construction of boxes surrounding the wires. Even these precautions did not suffice to absolutely prevent vibration of the wire at the bottom through a very small arc. The mean point of these vibrations in each case was then located on a rigid cross-beam suitably placed at the bottom of the shaft and at about the level of the roof of the tunnel. Short plumb-lines were then suspended from these points whenever desired; a transit was set (by trial) so that its line of collimation passed through both plumb-lines and the line at the bottom could thus be prolonged.

Some recent experience in the "Tamarack" shaft, 4250 feet deep, shows that the accuracy of the results may be affected by air-currents to an unsuspected extent. Two 50-lb. cast-iron plumb-bobs were suspended with No. 24 piano-wire in this shaft. The carefully measured distances between the wires

at top and bottom were 16.32 and 16.43 feet respectively. After considerable experimenting to determine the cause of the variation, it was finally concluded that air-currents were alone responsible. The variation of the bobs from a true vertical plane passing through the wires at the top was of course an unknown quantity, but since the variation in *one* direction amounted to 0.11 foot, the accuracy in other directions was very questionable. This shows that a careful comparative measurement between the wires at top and bottom should always be made as a test of their parallelism.

196. Underground surveys. Survey marks are frequently placed on the timbering, but they are apt to prove unreliable on account of the shifting of the timbering due to settlement of the surrounding material. They should never be placed at the bottom of the tunnel on account of the danger of being disturbed or covered up. Frequently holes are drilled in the roof and filled with wooden plugs in which a hook is screwed exactly on line. Although this is probably the safest method, even these plugs are not always undisturbed, as the material, unless very hard, will often settle slightly as the excavation proceeds. When a tunnel is perfectly straight and not too long, alinement-points may be given as frequently as desired from permanent stations located *outside* the tunnel where they are not liable to disturbance. This has been accomplished by running the alinement through the upper part of the cross-section, at one side of the center, where it is out of the way of the piles of masonry material, débris, etc., which are so apt to choke up the lower part of the cross-section. The position of this line relative to the cross-section being fixed, the alignment of any required point of the cross-section is readily found by means of a light frame or template with a fixed target located where this line would intersect the frame when properly placed. A level-bubble on the frame will assist in setting the frame in its proper position.

FIG. 88.

In all tunnel surveying the cross-wires must be illuminated by a lantern, and the object sighted at must also be illuminated. A powerful dark-lantern with the opening covered with *ground glass* has been found useful. This may be used to illuminate a plumb-bob string or a very fine rod, or to place behind a brass plate having a narrow slit in it, the axis of the slit and plate being coincident with the plumb-bob string by which it is hung.

On account of the interference to the surveying caused by the work of construction and also by the smoke and dust in the air resulting from the blasting, it is generally necessary to make the surveys at times when construction is temporarily suspended.

197. Accuracy of tunnel surveying. Apart from the very natural desire to do surveying which shall check well, there is an important financial side to accurate tunnel surveying. If the survey lines do not meet as desired when the headings come together, it may be found necessary, if the error is of appreciable size, to introduce a slight curve, perhaps even a reversed curve, into the alinement, and it is even conceivable that the tunnel section would need to be enlarged somewhat to allow for these curves. The cost of these changes and the perpetual annoyance due to an enforced and undesirable alteration of the original design will justify a considerable increase in the expenses of the survey. Considering that the cost of surveys is usually but a small fraction of the total cost of the work, an increase of 10 or even 20% in the cost of the surveys will mean an insignificant addition to the total cost and frequent'y, if not generally, it will result in a saving of many times the increased cost. The accuracy actually attained in two noted American tunnels is given as follows: The Musconetcong tunnel is about 5000 feet long, bored through a mountain 400 feet high. The error of alinement at the meeting of the headings was 0'.04, error of levels 0'.015, error of distance 0'.52. The Hoosac tunnel is over 25000 feet long. The heading from the east end met the heading from the central shaft at a point 11274 feet from the east end and 1563 feet from the shaft. The error in alinement was $\frac{5}{16}$ of an inch, that of levels "a few hundredths," error of distance "trifling." The alinement, corrected at the shaft, was carried on through and met the heading from the west end at a point 10138 feet from the west end and 2056 feet from the shaft. Here the error of alinement was $\frac{9}{16}''$ and that of levels 0.134 foot.

DESIGN

198. Cross-section. Nearly all tunnels have cross-sections peculiar to themselves—all varying at least in the details. The general form of a great many tunnels is that of a rectangle surmounted by a semi-circle or semi-ellipse. In very soft material an inverted arch is necessary along the bottom. In such cases the sides will generally be arched instead of vertical. The sides are frequently battered. In very long tunnels, several forms of cross-section will often be used in the same tunnel, owing to differences in the material encountered. In solid rock, which will not disintegrate upon exposure, no lining is required, and the cross-section will be the irregular section left by the blasting, the only requirement being that no rock shall be left within the required cross-sectional figure. Farther on, in the same tunnel, when passing through some very soft treacherous material, it may be necessary to put in a full arch lining—top, sides, and bottom—which will be nearly circular in cross-section. For an illustration of this see Figs. 89 and 90.

The cross-section recommended by the A. R. E. A. for single track is a rectangle 16 feet wide by 16 feet 6 inches high, surmounted by a semi-circle with a radius of 8 feet. The top of the tie is to be 2 feet above the bottom which is at sub-grade. If the surrounding material is yielding and exerts great pressure, the sides should be battered inward 1 foot at the bottom. For a double track tunnel the design is similar, except that the width is increased by the standard spacing between double tracks and the top is a compound curve made up of two 8-foot-radius curves at the sides which compound into a curve over the center which will give a clear height of 22 feet 6 inches over the center of each tie. The base of the roof curve is 13 feet 6 inches above the top of the ties. The bottom slopes to a central gutter which is 6 inches below the side corners, which are at sub-grade. Six-inch cast-iron pipes should be spaced as needed and run from each side to the central gutter. The width of both single and double track tunnels should be increased, if the tunnel is on a curve, and the track centers should also be displaced, so that the clearance on each side is as great as on a tangent. Figs. 89, 90 and 91,* show some typical cross-sections.

199. Grade. A grade of at least 0.2% is needed for drainage. If the tunnel is at the summit of two grades, the tunnel grade

* Drinker's "Tunneling."

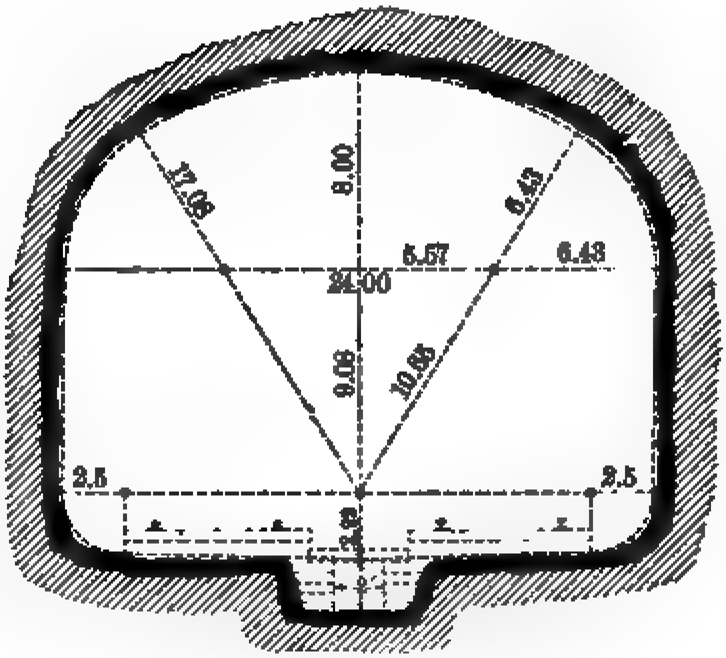


FIG. 89.—HOOSAC TUNNEL. SECTION THROUGH SOLID ROCK.

FIG. 90.—HOOSAC TUNNEL. SECTION THROUGH SOFT GROUND.

should be practically level, with an allowance for drainage, the actual summit being at either end but *not* in the center. When the tunnel forms part of a long ascending grade, it is advisable to reduce the grade through the tunnel unless the tunnel is very short. The additional atmospheric resistance and the decreased adhesion of the driver wheels on the damp rails in a tunnel will cause an engine to work very hard and still more rapidly vitiate the atmosphere until the accumulation of poisonous gases becomes a source of actual danger to the engineer and

FIG. 91. – ST. CLOUD TUNNEL.

fireman of the locomotive and of extreme discomfort to the passengers. If the nominal ruling grade of the road were maintained through a tunnel, the maximum resistance would be found in the tunnel. This would probably cause trains to stall there, which would be objectionable and perhaps dangerous.

200. Lining. It is a characteristic of many kinds of rock and of all earthy material that, although they may be self-sustaining when first exposed to the atmosphere, they rapidly disintegrate and require that the top and perhaps the sides and even the bottom shall be lined to prevent caving in. In this country, when timber was cheap, it was formerly framed as an arch and used as the *permanent* lining, but masonry is always

to be preferred. Frequently the cross-section is made extra large so that a masonry lining may subsequently be placed inside the wooden lining and thus postpone a large expense until the road is better able to pay for the work. In very soft unstable material, like quicksand, an arch of cut stone voussoirs may be necessary to withstand the pressure. A good quality of brick is occasionally used for lining, as they are easily handled and make good masonry if the pressure is not excessive. Only the best of cement mortar should be used, economy in this feature being the worst of folly. Of course the excavation must include the outside line of the lining. Any excavation which is made outside of this line (by the fall of earth or loose rock or by excessive blasting) must be refilled with stone well packed in. Occasionally it is necessary to fill these spaces with concrete. Of course it is not necessary that the lining be uniform throughout the tunnel.

201. Shafts. Shafts are variously made with square, rectangular, elliptical, and circular cross-sections. The rectangular

FIG. 92.—CONNECTION WITH SHAFT, CHURCH HILL TUNNEL.

cross-section, with the longer axis parallel with the tunnel, is most usually employed. Generally the shaft is directly over the center of the tunnel, but that always implies a complicated connection between the linings of the tunnel and shaft, provided

such linings are necessary. It is easier to sink a shaft near to one side of the tunnel and make an opening through the nearly vertical side of the tunnel. Such a method was employed in the Church Hill Tunnel, illustrated in Fig. 92.* Fig. 93 † shows a cross-section for a large main shaft. Many shafts have been built with the idea of being left open permanently for ventilation and have therefore been elaborately lined with masonry.

FIG. 93.—CROSS-SECTION. LARGE MAIN SHAFT.

The general consensus of opinion now appears to be that shafts are worse than useless for ventilation; that the quick passage of a train through the tunnel is the most effective ventilator; and that shafts only tend to produce cross-currents and are ineffective to clear the air. In consequence, many of these elaborately lined shafts have been permanently closed, and the more recent practice is to close up a shaft as soon as the tunnel is completed. Shafts always form drainage-wells for the material they pass through, and sometimes to such an extent that it is a serious matter to dispose of the water that collects at the bottom, requiring the construction of large and expensive drains.

202. Drains. A tunnel will almost invariably strike veins of water which will promptly begin to drain into the tunnel and not only cause considerable trouble and expense during construction, but necessitate the provision of permanent drains for its perpetual disposal. These drains must frequently be so large as

* Drinker's "Tunneling."

† Rziha, "Lehrbuch der Gesammten Tunnelbaukunst."

to appreciably increase the required cross-section of the tunnel. Generally a small open gutter on each side will suffice for this purpose, but in double-track tunnels a large covered drain is often built between the tracks. It is sometimes necessary to thoroughly grout the outside of the lining so that water will not force its way through the masonry and perhaps injure it, but may freely drain down the sides and pass through openings in the side walls near their base into the gutters.

CONSTRUCTION.

203. Headings. The methods of all tunnel excavation depend on the general principle that all earthy material, except the softest of liquid mud and quicksand, will be self-sustaining over a greater or less area and for a greater or less time after excavation is made, and the work consists in excavating some material and immediately propping up the exposed surface by timbering and poling-boards. The excavation of the cross-section begins with cutting out a "heading," which is a small horizontal drift whose breast is constantly kept 15 feet or more in advance of the full cross-sectional excavation. In solid self-sustaining rock, which will not decompose upon exposure to air, it becomes simply a matter of excavating the rock with the least possible expenditure of time and energy. In soft ground the heading must be heavily timbered, and as the heading is gradually enlarged the timbering must be gradually extended and perhaps replaced, according to some regular system, so that when the full cross-section has been excavated it is supported by such timbering as is intended for it. The heading is sometimes made on the center line near the top; with other plans, on the center line near the bottom; and sometimes two simultaneous headings are run in the two lower corners. Headings near the bottom serve the purpose of draining the material above it and facilitating the excavation. The simplest case of heading timbering is that shown in Fig. 94, in which cross-timbers are placed at intervals just under the roof, set in notches cut in the side walls and supporting poling-boards which sus-

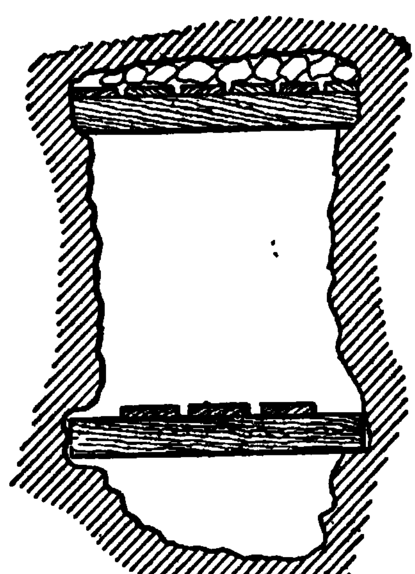


FIG. 94.

tain whatever pressure may come on them. Cross-timbers near the bottom support a flooring on which vehicles for transporting material may be run and under which the drainage may freely escape. As the necessity for timbering becomes greater, side timbers and even bottom timbers must be added, these timbers supporting poling-boards, and even the breast of the heading must be protected by boards suitably braced,

FIG. 95.—TIMBERING FOR TUNNEL HEADING.

as shown in Fig. 95. The supporting timbers are framed into collars in such a manner that added pressure only increases their rigidity.

204. **Enlargement.** Enlargement is accomplished by removing the poling-boards, one at a time, excavating a greater or less amount of material, and immediately supporting the exposed material with poling-boards suitably braced. (See Figs. 95 and 96.) This work being systematically done, space is thereby obtained in which the framing for the full cross-section may be gradually introduced. The framing is constructed with a cross-

section so large that the masonry lining may be constructed within it.

205. Distinctive features of various methods of construction. There are six general systems, known as the English, German, Belgian, French, Austrian, and American. They are so named

Fig. 96.

from the origin of the methods, although their use is not confined to the countries named. Fig. 97 shows by numbers (1 to 5) the order of the excavation within the cross-sections. The English, Austrian, and American systems are alike in excavating the entire cross-section before beginning the construction of the masonry lining. The German method leaves a solid core (5) until practically the whole of the lining is complete. This has the disadvantage of extremely cramped quarters for work, poor ventilation, etc. The Belgian and French methods agree in excavating the upper part of the section, building the arch at once, and supporting it temporarily until the side walls are built. The Belgian method then takes out the core (3), removes very short sections of the sides (4) immediately underpinning the arch with short sections of the side walls and thus gradually constructing the whole side wall. The French method digs out the sides (3), supporting the arch temporarily with timbers and then replacing the timbers with masonry; the core (4) is taken out last. The French method has the same disadvantage as the German—working in a cramped space. The Belgian and French systems have the disadvantage that the arch, supported temporarily on timber, is very apt to be strained and cracked by the slight settlement that so frequently occurs in soft material. The English, Austrian, and American methods differ mainly in the

design of the timbering. The English support the roof by lines of very heavy *longitudinal* timbers which are supported at comparatively wide intervals by a heavy framework occupying the

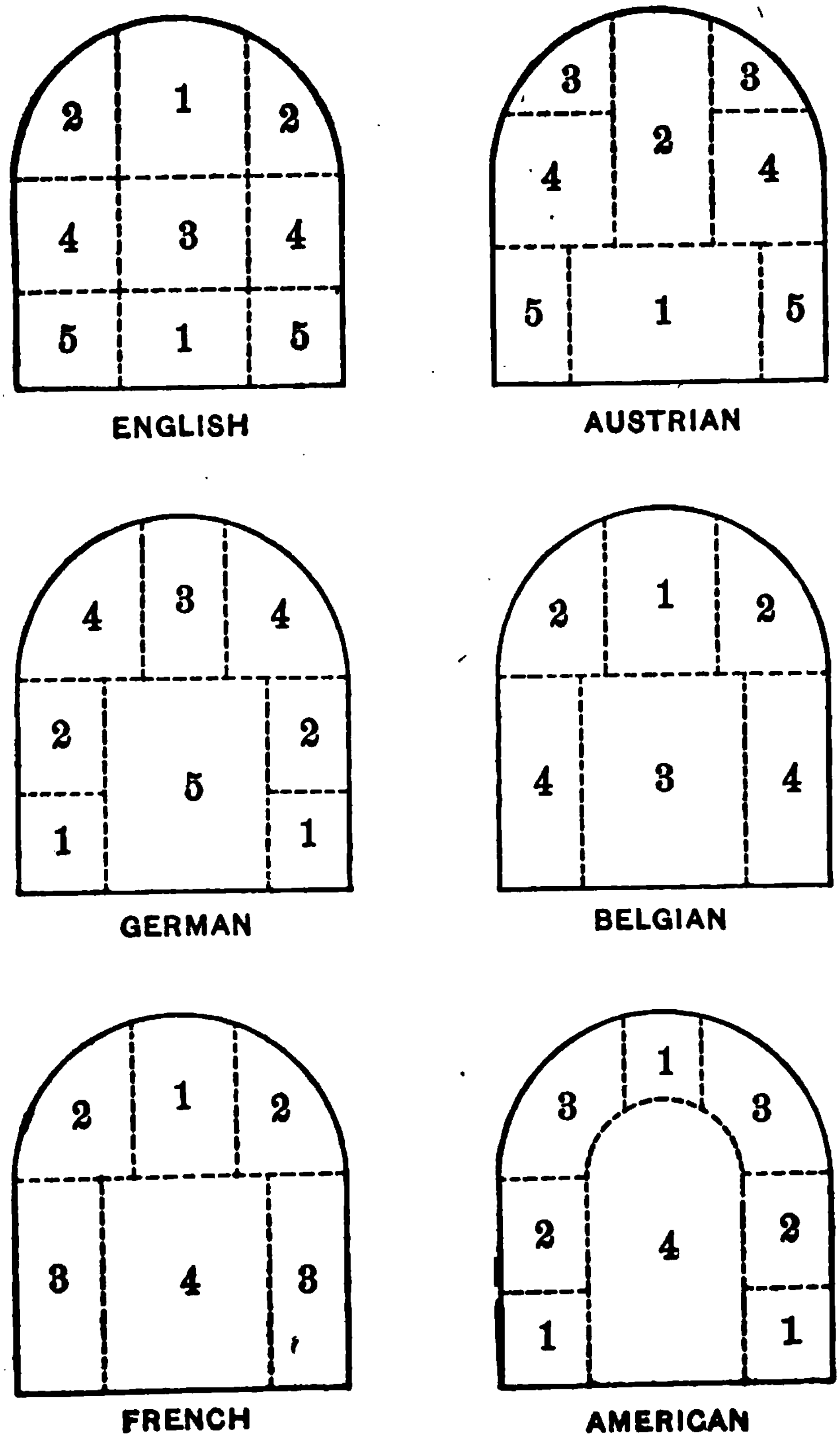


FIG. 97.—ORDER OF WORKING BY THE VARIOUS SYSTEMS.

whole cross-section. The Austrian system uses such frequent cross-frames of timber-work that poling-boards will suffice to support the material between the frames. The American system agrees with the Austrian in using frequent cross-frames

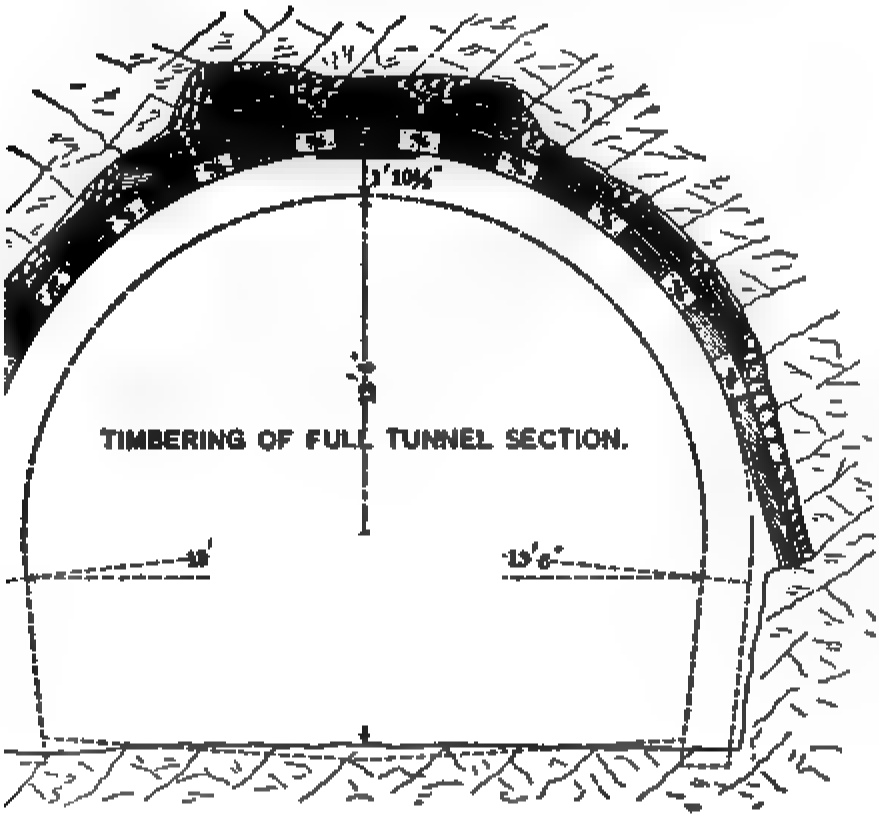
supporting poling-boards, but differs from it in that the "cross-frames" consist simply of arches of 3 to 15 wooden voussoirs, the voussoirs being blocks of 12"×12" timber about 2 to 8 feet long and cut with joints normal to the arch. These arches are put together on a centering which is removed as soon as the arch is keyed up and thus immediately opens up the full cross-section, so that the center core (4) may be immediately dug out and the masonry constructed in a large open space. The American system has been used successfully in very soft ground, but its advantages are greater in loose rock, when it is much cheaper than the other methods which employ more timber. Fig. 92 and Plate III illustrate the use of the American system. Fig. 92 shows the wooden arch in place. The masonry arch may be placed when convenient, since it is *possible* to lay the track and commence traffic as soon as the wooden arch is in place. The student is referred to Drinker's "Tunneling" and to Rziha's "Lehrbuch der Gesammten Tunnelbaukunst" for numerous illustrations of European methods of tunnel timbering.

206. Ventilation during construction. Tunnels of any great length must be artificially ventilated during construction. If the excavated material is rock so that blasting is necessary, the need for ventilation becomes still more imperative. The invention of compressed-air drills simultaneously solved two difficulties. It introduced a motive power which is unobjectionable in its application (as gas would be), and it also furnished at the same time a supply of just what is needed—pure air. If no blasting is done (and frequently even when there is blasting), air must be supplied by direct pumping. The cooling effect of the sudden expansion of compressed air only reduces the otherwise objectionably high temperature sometimes found in tunnels. Since pure air is being continually pumped in, the foul air is thereby forced out.

207. Excavation for the portals. Under normal conditions there is always a greater or less amount of open cut preceding and following a tunnel. Since all tunnel methods depend (to some slight degree at least) on the capacity of the exposed material to act as an arch, there is implied a considerable thickness of material above the tunnel. This thickness is reduced to nearly zero over the tunnel portals and therefore requires special treatment, particularly when the material is very soft. Fig. 98 *

* Rziha, "Lehrbuch der Gesammten Tunnelbaukunst."

PLATE III



TIMBERING OF FULL TUNNEL SECTION.

LONGITUDINAL SECTION OF PORTAL.

siderations and annual maintenance charges directly or indirectly connected with it. Even when an open cut may be constructed at the same cost as a tunnel (or perhaps a little cheaper) the tunnel may be preferable under the following conditions:

1. When the soil indicates that the open cut would be liable to landslides.

2. When the open cut would be subject to excessive snow-drifts or avalanches.

3. When land is especially costly or it is desired to run under existing costly or valuable buildings or monuments. When running through cities, tunnels are sometimes constructed as open cuts and then arched over.

These cases apply to tunnels *vs.* open cuts when the alinement is fixed by other considerations than the mere topography. The broader question of excavating tunnels to avoid excessive grades or to save distance or curvature, and similar problems, are hardly susceptible of general analysis except as questions of railway economics and must be treated individually.

209. Cost of tunneling. The cost of any construction which involves such uncertainties as tunneling is very variable. It depends on the material encountered, the amount and kind of timbering required, on the size of the cross-section, on the price of labor, and especially on the *reconstruction* that *may* be necessary on account of mishaps.

Headings generally cost $4 to $5 per cubic yard for excavation, while the remainder of the cross-section in the same tunnel may cost about half as much. The average cost of a large number of tunnels in this country may be seen from the following table:*

Material.	Cost per cubic yard.				Cost per lineal foot.	
	Excavation.		Masonry.			
	Single.	Double.	Single.	Double.	Single.	Double.
Hard rock.....	$5.89	$5.45	$12.00	$8.25	$69.76	$142.82
Loose rock.....	3.12	3.48	9.07	10.41	80.61	119.26
Soft ground....	3.62	4.64	15.00	10.50	135.31	174.42

* Figures derived from Drinker's "Tunneling."

A considerable variation from these figures may be found in individual cases, due sometimes to unusual skill (or the lack of it) in prosecuting the work, but the figures will generally be sufficiently accurate for preliminary estimates or for the comparison of two proposed routes.

CHAPTER VI.

CULVERTS AND MINOR BRIDGES.

210. Definition and object. Although a variable percentage of the rain falling on any section of country soaks into the ground and does not immediately reappear, yet a very large percentage flows over the surface, always seeking and following the lowest channels. The roadbed of a railroad is constantly intersecting these channels, which frequently are normally dry. In order to prevent injury to railroad embankments by the impounding of such rainfall, it is necessary to construct waterways through the embankment through which such rainflow may freely pass. Such waterways, called culverts, are also applicable for the bridging of very small although perennial streams, and therefore in this work the term culvert will be applied to all water-channels passing through a railroad embankment which are not of sufficient magnitude to require a special structural design, such as is necessary for a large masonry arch or a truss bridge.

211. Elements of the design. A well-designed culvert must afford such free passage to the water that it will not "back up" over the adjoining land nor cause any injury to the embankment or culvert. The ability of the culvert to discharge freely all the water that comes to it evidently depends chiefly on the area of the waterway, but also on the form, length, slope, and materials of construction of the culvert and the nature of the approach and outfall. When the embankment is very low and the amount of water to be discharged very great, it sometimes becomes necessary to allow the water to discharge "under a head," i.e., with the surface of the water above the top of the culvert. Safety then requires a much stronger construction than would otherwise be necessary to avoid injury to the culvert or embankment by washing. The necessity for such construction should be avoided if possible.

AREA OF THE WATERWAY.

212. **Elements involved.** The determination of the required area of the waterway involves such a multiplicity of indeterminate elements that any close determination of its value from purely theoretical considerations is a practical impossibility. The principal elements involved are:

a. **Rainfall.** The real test of the culvert is its capacity to discharge without injury the flow resulting from the extraordinary rainfalls and "cloud bursts" that may occur once in many years. Therefore, while a knowledge of the average annual rainfall is of very little value, a record of the maximum rainfall during heavy storms for a long term of years may give a relative idea of the maximum demand on the culvert.

b. **Area of watershed.** This signifies the total area of country draining into the channel considered. When the drainage area is very small it is sometimes included within the area surveyed by the preliminary survey. When larger it is frequently possible to obtain its area from other maps with a percentage of accuracy sufficient for the purpose. Sometimes a special survey for the purpose is considered justifiable.

c. **Character of soil and vegetation.** This has a large influence on the rapidity with which the rainflow from a given area will reach the culvert. If the soil is hard and impermeable and the vegetation scant, a heavy rain will run off suddenly, taxing the capacity of the culvert for a short time, while a spongy soil and dense vegetation will retard the flow, making it more nearly uniform and the maximum flow at any one time much less.

d. **Shape and slope of watershed.** If the watershed is very long and narrow (other things being equal), the water from the remoter parts will require so much longer time to reach the culvert that the flow will be comparatively uniform, especially when the slope of the whole watershed is very low. When the slope of the remoter portions is quite steep it may result in the nearly simultaneous arrival of a storm-flow from all parts of the watershed, thus taxing the capacity of the culvert.

e. **Effect of design of culvert.** The principles of hydraulics show that the slope of the culvert, its length, the form of the cross-section, the nature of the surface, and the form of the

approach and discharge all have a considerable influence on the area of cross-section required to discharge a given volume of water in a given time, but unfortunately the combined hydraulic effect of these various details is still a very uncertain quantity.

213. **Methods of computation of area.** There are three possible methods of computation.

(a) **Theoretical.** As shown above it is a practical impossibility to estimate correctly the combined effect of the great multiplicity of elements which influence the final result. The nearest approach to it is to estimate by the use of empirical formulæ the amount of water which will be presented at the upper end of the culvert in a given time and then to compute, from the principles of hydraulics, the rate of flow through a culvert of given construction, but (as shown in § 212, *e*) such methods are still very unreliable, owing to lack of experimental knowledge. This method has *apparently* greater scientific accuracy than other methods, but a little study will show that the elements of uncertainty are as great and the final result no more reliable. The method is most reliable for streams of uniform flow, but it is under these conditions that method (*c*) is most useful. The theoretical method will not therefore be considered further.

(b) **Empirical.** As illustrated in § 214, some formulæ make the area of waterway a function of the drainage area, the formula being affected by a coefficient the value of which is estimated between limits according to the judgment Assuming that the formulæ are sound, their use only narrows the limits of error, the final determination depending on experience and judgment.

(c) **From observation.** This method, considered by far the best for permanent work, consists in observing the high-water marks on contracted channel-openings which are on the same stream and as near as possible to the proposed culvert. If the country is new and there are no such openings, the wisest plan is to bridge the opening by a temporary structure in wood which has an ample waterway (see § 158, *b*, 4) and carefully observe all high-water marks on that opening during the 6 to 10 years which is ordinarily the minimum life of such a structure. As shown later, such observations may be utilized for a close computation of the required waterway. Method (*b*) may be utilized for an approximate calculation for the required area for the tem-

porary structure, using a value which is intentionally excessive, so that a permanent structure of sufficient capacity may subsequently be constructed *within* the temporary structure.

214. Empirical formulæ. Two of the best known empirical formulæ for area of the waterway are the following:

(a) Myer's formula:

Area of waterway in square feet $= C \times \sqrt{\text{drainage area in acres}}$, where C is a coefficient varying from 1 for flat country to 4 for mountainous country and rocky ground. As an illustration, if the drainage area is 100 acres, the waterway area should be from 10 to 40 square feet, according to the value of the coefficient chosen. It should be noted that this formula does not regard the great variations in rainfall in various parts of the world nor the design of the culvert, and also that the final result depends largely on the choice of the coefficient.

(b) Talbot's formula:

Area of waterway in square feet $= C \times \sqrt[4]{(\text{drainage area in acres})^3}$.

"For steep and rocky ground C varies from $\frac{2}{3}$ to 1. For rolling agricultural country subject to floods at times of melting snow, and with the length of the valley three or four times its width, C is about $\frac{1}{3}$; and if the stream is longer in proportion to the area, decrease C. In districts not affected by accumulated snow, and where the length of the valley is several times the width, $\frac{1}{5}$ or $\frac{1}{6}$, or even less, may be used. C should be increased for steep side slopes, especially if the upper part of the valley has a much greater fall than the channel at the culvert." * As an illustration, if the drainage area is 100 acres the area of waterway should be $C \times 31.6$. The area should then vary from 5 to 31 square feet, according to the character of the country. Like the previous estimate, the result depends on the choice of a coefficient and disregards local variations in rainfall, except as they may be arbitrarily allowed for in choosing the coefficient.

215. Value of empirical formulæ. The fact that these formulæ, as well as many others of similar nature that have been suggested, depend so largely upon the choice of the coefficient shows that they are valuable "more as a guide to the judgment than as a working rule," as Prof. Talbot explicitly declares in

* Prof. A. N. Talbot, "Selected Papers of the Civil Engineers' Club of the Univ. of Illinois."

commenting on his own formula. In short, they are chiefly valuable in indicating a probable maximum and minimum between which the true result probably lies.

216. Results based on observation. As already indicated in § 213, observation of the stream in question gives the most reliable results. If the country is new and no records of the flow of the stream during heavy storms has been taken, even the life of a temporary wooden structure may not be long enough to include one of the unusually severe storms which must be allowed for, but there will usually be some high-water mark which will indicate how much opening will be required. The following quotation illustrates this: "A tidal estuary may generally be safely narrowed considerably from the extreme water lines if stone revetments are used to protect the bank from wash. Above the true estuary, where the stream cuts through the marsh, we generally find nearly vertical banks, and we are safe if the faces of abutments are placed even with the banks. In level sections of the country, where the current is sluggish, it is usually safe to encroach somewhat on the general width of the stream, but in rapid streams among the hills the width that the stream has cut for itself through the soil should not be lessened, and in ravines carrying mountain torrents the openings must be left very much larger than the ordinary appearance of the banks of the stream would seem to make necessary." *

As an illustration of an observation of a storm-flow through a temporary trestle, the following is quoted: "Having the flood height and velocity, it is an easy matter to determine the volume of water to be taken care of. I have one ten-bent pile trestle 135 feet long and 24 feet high over a spring branch that ordinarily runs about six cubic inches per second. Last summer during one of our heavy rainstorms (four inches in less than three hours) I visited this place and found by float observations the surface velocity at the highest stage to be 1.9 feet per second. I made a high-water mark, and after the flood-water receded found the width of stream to be 12 feet and an average depth of $2\frac{1}{2}$ feet. This, with a surface velocity of 1.9 feet per second, would give approximately a discharge of 50

* J. P. Snow, Boston & Maine Railway. From Report to Association of Railway Superintendents of Bridges and Buildings, 1897.

cubic feet, or 375 gallons, per second. Having this information it is easy to determine size of opening required." *

217. Degree of accuracy required. The advantages resulting from the use of standard designs for culverts (as well as other structures) have led to the adoptión of a comparatively small number of designs. The practical use made of a computation of required waterway area is to determine which one of several standard designs will most nearly fulfill the requirements. For example, if a 24-inch iron pipe, having an area of 3.14 square feet, is considered to be a little small, the next size (30-inch) would be adopted; but a 30-inch pipe has an area of 4.92 square feet, which is 56% larger. A similar result, except that the percentage of difference might not be quite so marked, will be found by comparing the areas of consecutive standard designs for stone box culverts.

The advisability of designing a culvert to withstand any storm-flow that may *ever* occur is considered doubtful. Several years ago a record-breaking storm in New England carried away a very large number of bridges, etc., hitherto supposed to be safe. It was not afterward considered that the design of those bridges was faulty, because the extra cost of constructing bridges capable of withstanding such a flood, added to interest for a long period of years, would be enormously greater than the cost of repairing the damages of such a storm once or twice in a century. Of course the element of danger has some weight, but not enough to justify a great additional expenditure, for common prudence would prompt unusual precautions during or immediately after such an extraordinary storm.

PIPE CULVERTS.

218. Advantages. Pipe culverts, made of cast iron or earthenware, are very durable, readily constructed, moderately cheap, will pass a larger volume of water in proportion to the area than many other designs on account of the smoothness of the surface, and (when using iron pipe) may be used very close to the track when a low opening of large capacity is required. Another advantage lies in the ease with which they may be inserted through a somewhat larger opening that has been

* A. J. Kelley, Kansas City Belt Railway. From Report to Association of Railway Superintendents of Bridges and Buildings. 1897.

temporarily lined with wood, without disturbing the roadbed or track

219. Construction. Permanency requires that the foundation shall be firm and secure against being washed out. To accomplish this, the soil of the trench should be hollowed out to fit the lower half of the pipe, making suitable recesses for the bells. In very soft treacherous soil a foundation-block of concrete is sometimes placed under each joint, or even throughout the whole length. When pipes are laid through a slightly larger timber culvert great care should be taken that the pipes are properly supported, so that there will be no settling nor development of unusual strains when the timber finally decays and gives way. To prevent the washing away of material around the pipe the ends should be protected by a bulkhead. This is best constructed of masonry (see Fig. 99), although wood is sometimes used for cheap and minor constructions. The joints should be calked, especially when the culvert is liable to run full or when the outflow is impeded and the culvert is liable to be partly or wholly filled during freezing weather. The cost of a calking of clay or even hydraulic cement is insignificant compared with the value of the additional safety afforded. When the grade of the pipe is perfectly uniform, a very low rate of grade will suffice to drain a pipe culvert, but since some unevenness of grade is inevitable through uneven settlement or imperfect construction, a grade of 1 in 20 should preferably be required, although much less is often used. The length of a pipe culvert is approximately determined as follows:

$$Length = 2s\ (depth\ of\ embankment) + (width\ of\ roadbed),$$

in which s is the slope ratio (horizontal to vertical) of the banks. In practice an even number of lengths should be used which will equal or exceed the length given by this formula.

220. Iron-pipe culverts. Simple cast-iron pipes are used in sizes from 12″ to 48″ diameter. These are usually made in lengths of 12 feet with a few lengths of 6 feet, so that any required length may be more nearly obtained. The lightest pipes made are sufficiently strong for the purpose, and even those which would be rejected because of incapacity to withstand considerable internal pressure may be utilized for this work. In Fig. 99 are shown the standard plans used on the C. C. C. & St. L. Ry., which may be considered as typical plans.

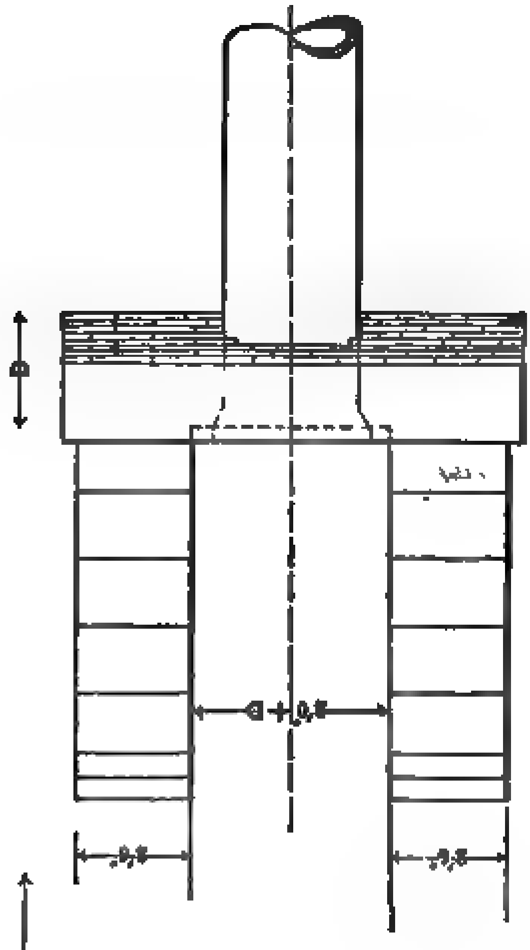


FIG. 99.—STANDARD CAST-IRON PIPE CULVERT. C. C. C. & ST. L. RY. (MAY 1893.)

Pipes formed of cast-iron segments have been used up to 12 feet diameter. The shell is then made comparatively thin, but is stiffened by ribs and flanges on the outside. The segments break joints and are bolted together through the flanges. The joints are made tight by the use of a tarred rope, together with neat cement.

221. Tile-pipe culverts. The pipes used for this purpose vary from 12″ to 30″ in diameter. When a larger capacity is required two or more pipes may be laid side by side, but in such a case another design might be preferable. It is frequently specified that "double-strength" or "extra-heavy" pipe shall be used, evidently with the idea that the stresses on a culvert-pipe are greater than on a sewer-pipe. But it has been conclusively demonstrated that, no matter how deep the embankment, the pressure cannot exceed a somewhat uncertain maximum, also that the greatest danger consists in placing the pipe so near the ties that shocks may be directly transferred to the pipe without the cushioning effect of the earth and ballast. When the pipes are well bedded in *clear* earth and there is a

on the "Plant system." Tile pipe is much cheaper than iron pipe, but is made in much shorter lengths and requires much more work in laying and especially to obtain a uniform grade.

Concrete pipes, factory made, both plain and with metal reinforcement, 12" to 48" in diameter, have come into use in recent years. They are stronger and more dependable than tile and there is no deterioration.

BOX CULVERTS.

222. Wooden box culverts. This form serves the purpose of a cheap temporary construction which allows the use of a ballasted roadbed. As in all temporary constructions, the area should be made considerably larger than the calculated area (§§ 213–216), not only for safety but also in order that, if the smaller area is demonstrated to be sufficiently large, the permanent construction (probably pipe) may be placed inside without disturbing the embankment. All designs agree in using heavy timbers (12"×12", 10"×12", or 8"×12") for the side walls, cross-timbers for the roof, every fifth or sixth timber being notched down so as to take up the thrust of the side walls, and planks for the flooring. Fig. 101 shows some of the standard designs as used by the C., M. & St. P. Ry.

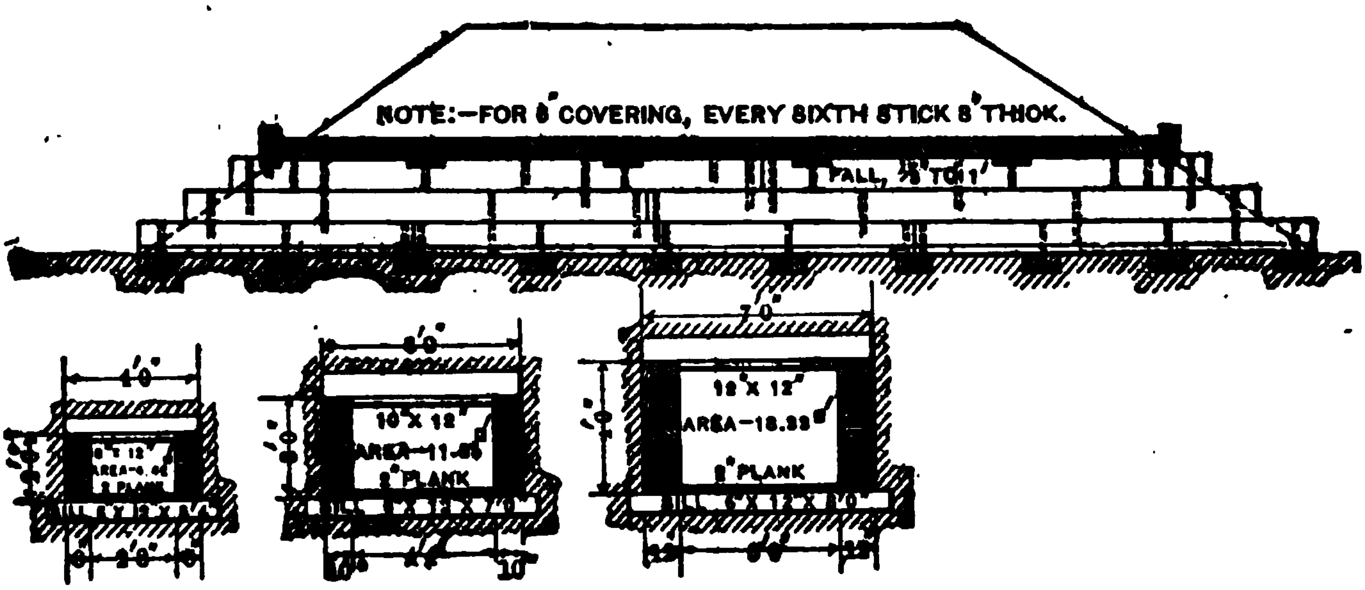
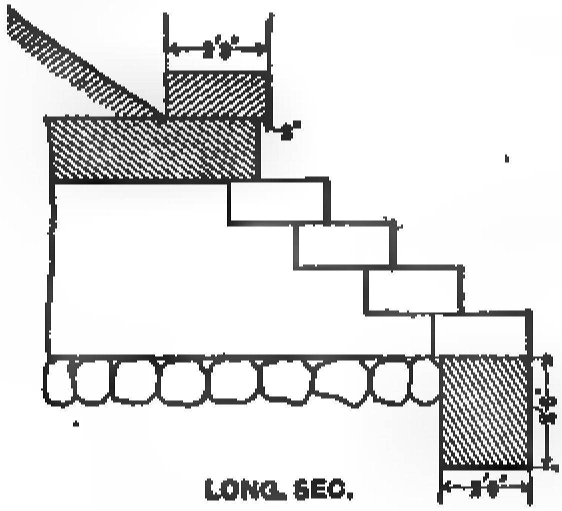


FIG. 101.—STANDARD TIMBER BOX CULVERT. C., M. & ST. P. RY. (Feb. 1889.)

223. Stone box culverts. In localities where a good quality of stone is cheap, stone box culverts are the cheapest form of permanent construction for culverts of medium capacity, but their use is decreasing owing to the frequent difficulty in obtaining really suitable stone within a reasonable distance of the culvert. The clear span of the cover-stones varies from 2 to 4 feet. The required thickness of the cover-stones is sometimes

calculated by the theory of transverse strains on the basis of certain assumptions of loading—as a function of the height of the embankment and the unit strength of the stone used. Such a method is simply another illustration of a class of calculations

utiful, but which are worse than
account of the hopeless uncer-



s Culvert (3'×4'). N. & W. R.R.
1890.)

certain quantities which must be
. the first place the true value of
e is such an uncertain and variable

quantity that calculations based on any assumed value for it are of small reliability. In the second place the weight of the prism of earth lying directly above the stone, plus an allowance for live load, is by no means a measure of the load on the stone nor of the forces that tend to fracture it. All earthwork will tend to

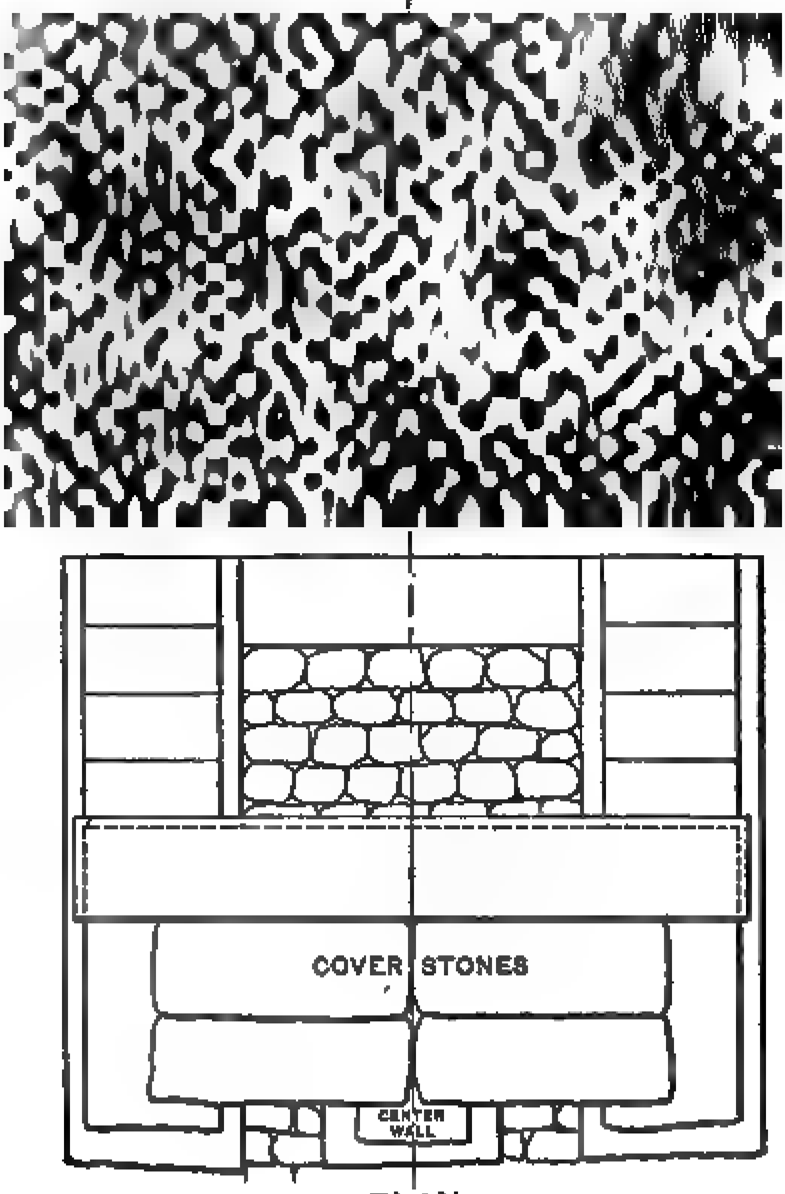


FIG. 103.—STANDARD DOUBLE STONE CULVERT (3'×4'). N. & W. R. R. (1890.)

form an arch above any cavity and thus relieve an uncertain and probably variable proportion of the pressure that might otherwise exist. The higher the embankment the *less* the *pro-*

portionate loading, until at some uncertain height an increase in height will not increase the load on the cover-stones. The effect of frost is likewise large, but uncertain and not computable. The usual practice is therefore to make the thickness such as experience has shown to be safe with a good quality of stone, i.e., about 10 or 12 inches for 2 feet span and up to 16 or 18 inches for 4 feet span. The side walls should be carried down deep enough to prevent their being undermined by scour or heaved by frost. The use of cement mortar is also an important feature of first-class work, especially when there is a rapid scouring current or a liability that the culvert will run under a head. In Figs. 102 and 103 are shown standard plans for single and double stone box culverts as used on the Norfolk & Western R.R.

224. Old-rail culverts. It sometimes happens (although very rarely) that it is necessary to bring the grade line within 3 or 4 feet of the bottom of a stream and yet allow an area of 10 or 12 square feet. A single large pipe of sufficient area could not be used in this case. The use of several smaller pipes side by side would be both expensive and inefficient. For similar reasons neither wooden nor stone box culverts could be used. In such cases, as well as in many others where the head-room is not so limited, the plan illustrated in Fig. 104 is a very satisfactory

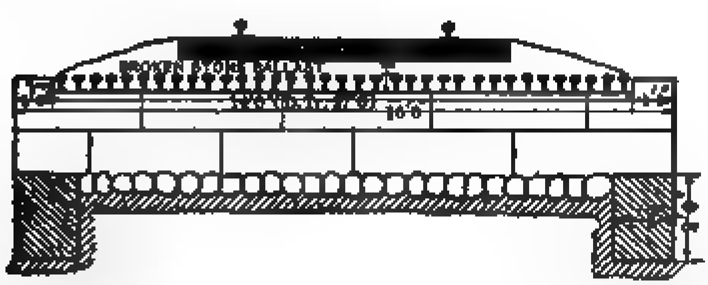


FIG. 104.—STANDARD OLD-RAIL CULVERT. N. & W. R.R. (1895.)

solution of the problem. The old rails, having a length of 8 or 9 feet, are laid close together across a 6-foot opening. Sometimes the rails are held together by long bolts passing through

the sides of the walls. In the plan shown the walls are con by low end walls on each abutment. This plan requires 15 inches between the base of the rail and the top of the cul channel. It also gives a continuous ballasted roadbed.

225. Reinforced Concrete Culverts. The development reinforced concrete as a structural material is illustrated i extensive adoption for arches and also for culverts. One of special types which has been adopted is that of a box cul which has a concrete bottom. Since this bottom can be m so that it will withstand an upward transverse stress, it furni a broad foundation for the whole culvert and thus enti eliminates the necessity for extensive footing to the side walls the culvert, such as are necessary in soft ground with an ordin stone culvert. Another advantage is that the inside of the c vert may be made perfectly smooth and thus offer less resista to the passage of water through it. As may be noticed fr Fig. 105, such a culvert is provided with flaring head walls, s sunken end walls, so that the water may not scour underneat the culvert, and other features common to other types. N attempt will here be made to discuss the design of reinforc concrete, except to say that all four sides of such a box culve are designed to withstand a computed bursting pressure whic tends to crush the flat sides inward. In Fig. 105 is shown on illustration of the many types of culverts which have bee designed of reinforced concrete.

ARCH CULVERTS.

191. Influence of design on flow. The variations in the design of arch culverts have a very marked influence on the cost and efficiency. To combine the least cost with the greatest efficiency, due weight should be given to the following elements: (a) amount of masonry, (b) the simplicity of the constructive work, (c) the design of the wing walls, (d) the design of the junction of the wing walls with the barrel and faces of the arch, and (e) the safety and permanency of the construction. These elements are more or less antagonistic to each other, and the defects of most designs are due to a lack of proper proportion in the design of these opposing interests. The simplest construction (satisfying elements *b* and *c*) is the straight barrel arch

[illegible]

the webs of the rails. In the plan shown the rails are confined by low end walls on each abutment. This plan requires only 15 inches between the base of the rail and the top of the culvert channel. It also gives a continuous ballasted roadbed.

225. Reinforced Concrete Culverts. The development of reinforced concrete as a structural material is illustrated in its extensive adoption for arches and also for culverts. One of the special types which has been adopted is that of a box culvert which has a concrete bottom. Since this bottom can be made so that it will withstand an upward transverse stress, it furnishes a broad foundation for the whole culvert, and thus entirely eliminates the necessity for extensive footing to the side walls of the culvert, such as are necessary in soft ground with an ordinary stone culvert. Another advantage is that the inside of the culvert may be made perfectly smooth and thus offer less resistance to the passage of water through it. As may be noticed from Fig. 105, such a culvert is provided with flaring head walls, and sunken end walls, so that the water may not scour underneath the culvert, and other features common to other types. No attempt will here be made to discuss the design of reinforced concrete, except to say that all four sides of such a box culvert are designed to withstand a computed bursting pressure which tends to crush the flat sides inward. In Fig. 105 is shown one illustration of the many types of culverts which have been designed of reinforced concrete.

ARCH CULVERTS.

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STANDARD ARCH CULVERT

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PLATE IV

PLAN

Fig. 105.—Reinforced Concrete Box Culvert.

between two parallel vertical head walls, as sketched in Fig. 106, *a*. From a hydraulic standpoint the design is poor, as the water eddies around the corners, causing a great resistance which decreases the flow. Fig. 106, *b*, shows a much better

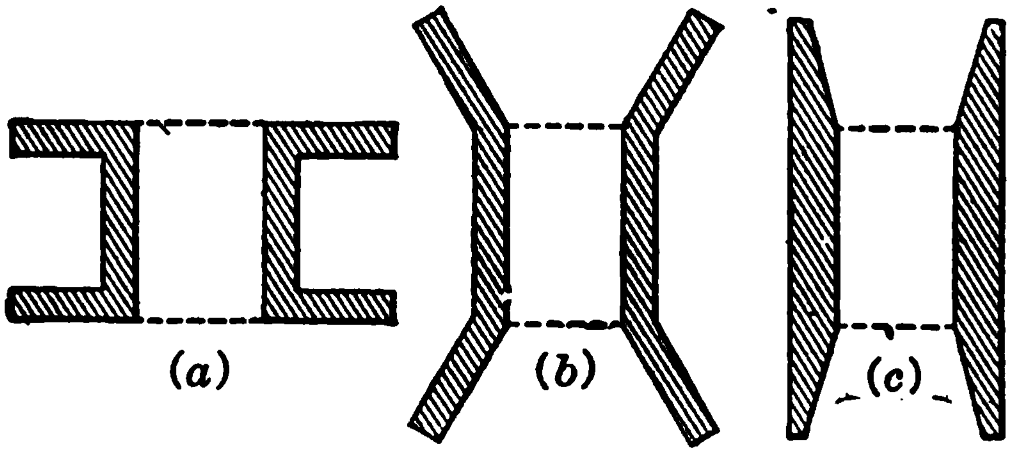


FIG. 106.—TYPES OF CULVERTS.

design in many respects, but much depends on the details of the design as indicated in elements (*b*) and (*d*). As a general thing a good hydraulic design requires complicated and expensive masonry construction, i.e., elements (*b*) and (*d*) are opposed. Design 106, *c*, is sometimes inapplicable because the water is liable to work in behind the masonry during floods and perhaps cause scour. This design uses less masonry than (*a*) or (*b*).

227. Example of arch culvert design. In Plate IV is shown the design for an 8-foot arch culvert according to the standard of the Norfolk and Western R. R. Note that the plan uses the flaring wing walls (Fig. 106, *b*) on the up-stream side (thus protecting the abutments from scour) and straight wing walls (similar to Fig. 106, *c*) on the down-stream end. This economizes masonry and also simplifies the constructive work. Note also the simplicity of the junction of the wing walls with the barrel of the arch, there being no re-entrant angles below the springing line of the arch. The design here shown is but one of a set of designs for arches varying in span from 6′ to 30′.

MINOR OPENINGS.

228. Cattle-guards. **(a) Pit guards.** Cattle-guards will be considered under the head of minor openings, since the old-fashioned plan of pit guards, which are even now defended and

preferred by some railroad men, requires a break in the continuity of the roadbed. A pit about three feet deep, five feet

stone (sometimes with wood), and the rails are supported on heavy timbers laid longitudinally with the rails. The break in the continuity of the roadbed produces a disturbance in the elastic wave running through the rails, the effect of which is noticeable at high velocities. The greatest objection, however, lies in the dangerous consequences of a derailment or a failure of the timbers owing to unobserved decay or destruction by fire—caused perhaps by sparks and cinders from passing locomotives. The very insignificance of the structure often leads to careless inspection. But if a single pair of wheels gets off the rails and drops into the pit, a costly wreck is inevitable.

(b) **Surface cattle-guards.** These are fastened on top of the ties; the continuity of the roadbed is absolutely unbroken and thus is avoided much of the danger of a bad wreck owing to a possible derailment. The device consists essentially of overlaying the ties (both inside and outside the rails) with a surface on

which cattle will not walk. The multitudinous designs for such a surface are variously effective in this respect. An objection,

FIG. 108.—SHEFFIELD CATTLE-GUARD.

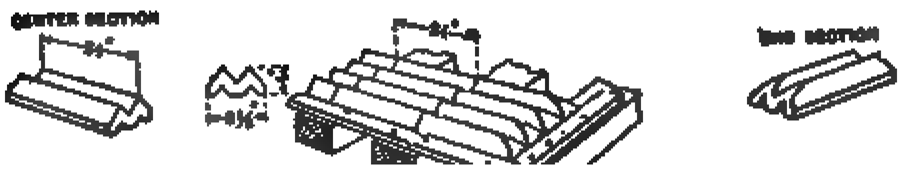


FIG. 109.—CLIMAX CATTLE-GUARD (TILE).

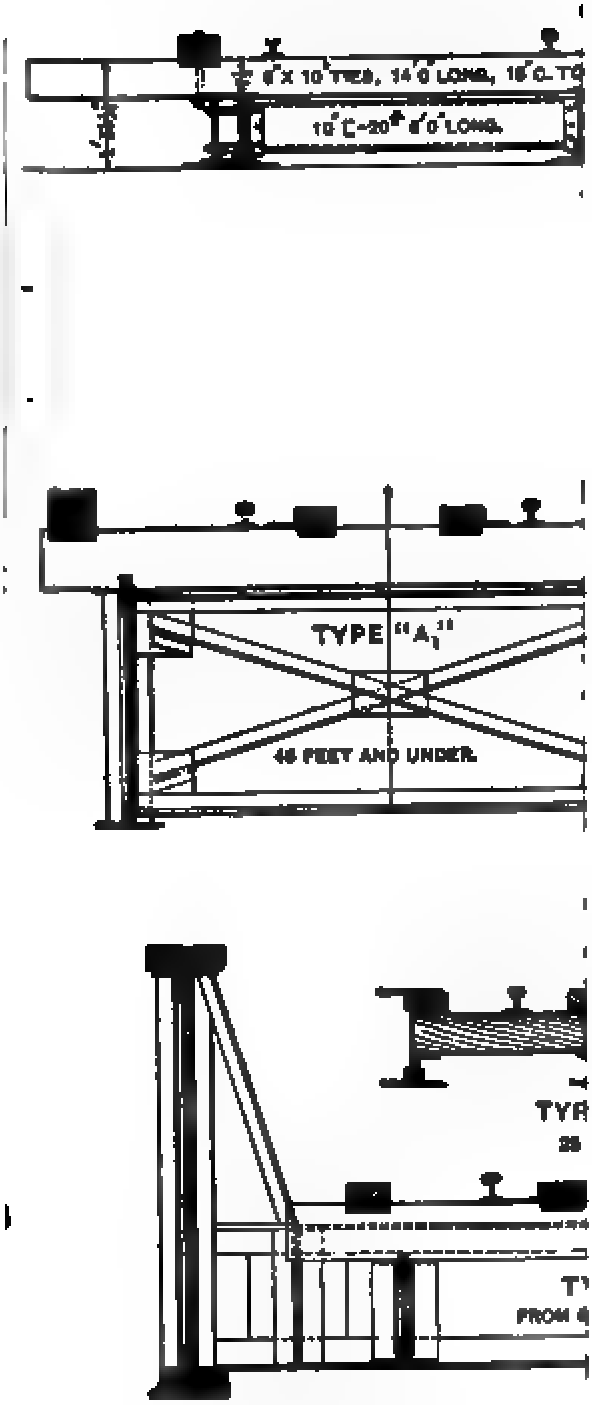
which is often urged indiscriminately against all such designs, is the liability that a brake-chain which may happen to be dragging *may* catch in the rough bars which are used. The bars

are sometimes "home-made," of wood, as shown in Fig. 107. Steel guards may be made as shown in Fig. 108. The general construction is the same as for the wooden bars. The metal bars have far greater durability, and it is claimed that they are more effective in discouraging cattle from attempting to cross.

229. Cattle-passes. Frequently when a railroad crosses a farm on an embankment, cutting the farm into two parts, the railroad company is obliged to agree to make a passageway through the embankment sufficient for the passage of cattle and perhaps even farm-wagons. If the embankment is high enough so that a stone arch is practicable, the initial cost is the only great objection to such a construction; but if an open wooden structure is necessary, all the objections against the old-fashioned cattle-guards apply with equal force here. The avoidance of a grade crossing which would otherwise be necessary is one of the great compensations for the expense of the construction and maintenance of these structures. The construction is sometimes made by placing two pile trestle bents about 6 to 8 feet apart, supporting the rails by stringers in the usual way, the special feature of this construction being that the embankments are filled in behind the trestle bents, and the thrust of the embankments is mutually taken up through the stringers, which are notched at the ends or otherwise constructed so that they may take up such a thrust. The designs for old-rail culverts and arch culverts are also utilized for cattle-passes when suitable and convenient, as well as the designs illustrated in the following section, and the reinforced concrete design of § 225.

230. Standard stringer and I-beam bridges. The advantages of standard designs apply even to the covering of short spans with wooden stringers or with I beams—especially since the methods do not require much vertical space between the rails and the upper side of the clear opening, a feature which is often of prime importance. These designs are chiefly used for culverts or cattle-passes and for crossing *over* highways—providing such a narrow opening would be tolerated. The plans all imply stone abutments, or at least abutments of sufficient stability to withstand all thrust of the embankments. Some of the designs are illustrated in Plate V. The preparation of these standard designs should be attacked by the same general methods as already illustrated in § 190. When computing the required

transverse strength, due allowance should be made for lateral bracing, which should be amply provided for. Note particularly the methods of bracing illustrated in Plate V. The designs calling for iron (or steel) stringers may be classed as permanent constructions, which are cheap, safe, easily inspected and maintained, and therefore a desirable method of construction.



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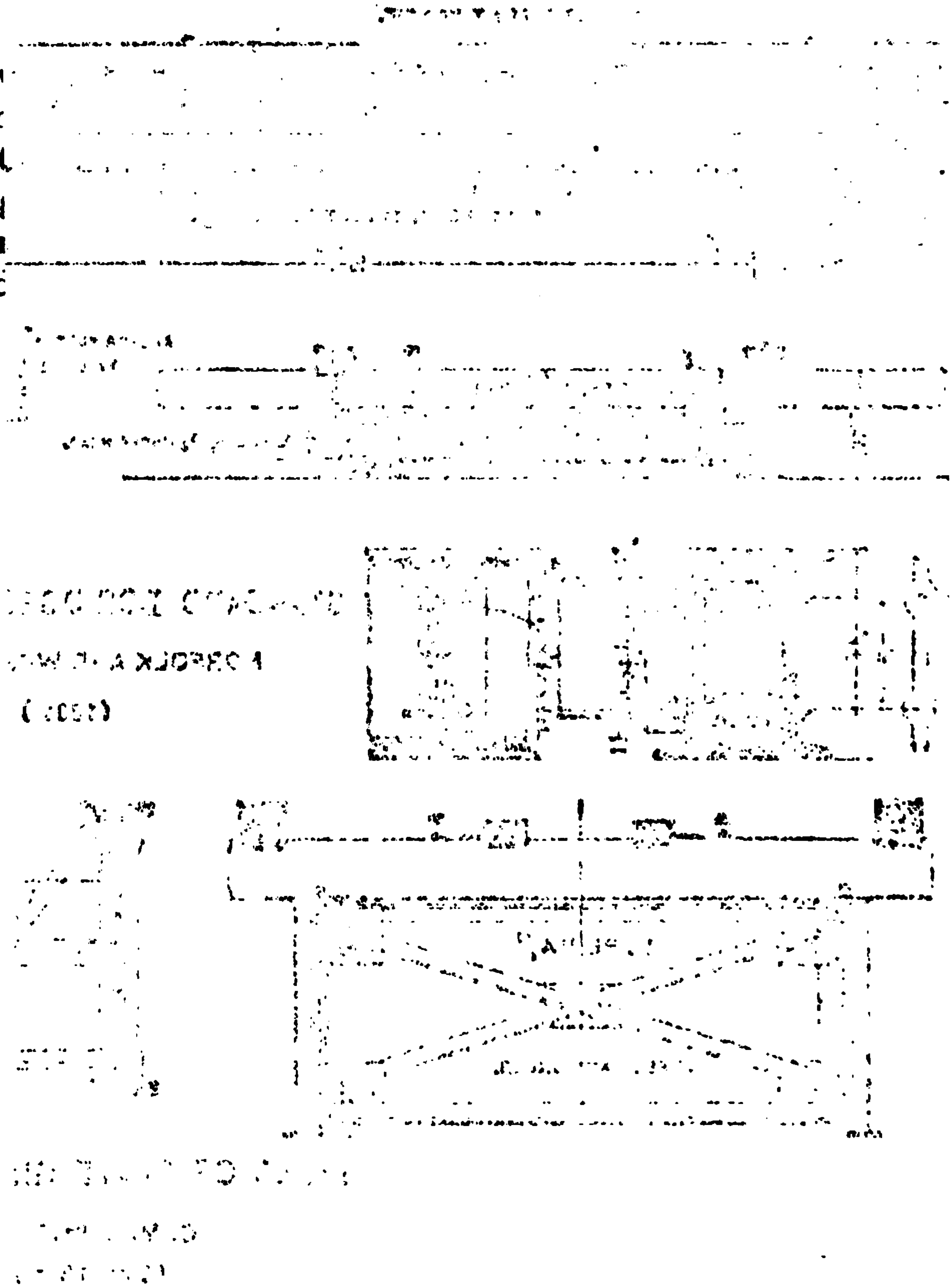


CHAPTER VII.

BALLAST.

231. **Purpose and requirements.** "The object of the ballast is to transfer the applied load over a large surface; to hold the timber work in place horizontally; to carry off the rain-water from the superstructure and to prevent freezing up in winter; to afford means of keeping the ties truly up to the grade line; and to give elasticity to the roadbed." This extremely condensed statement is a description of an ideally perfect ballast. The value of any given kind of ballast is proportional to the extent to which it fulfills these requirements. The ideally perfect ballast is not necessarily the most economical ballast for all roads. Light traffic generally justifies something cheaper, but a very common error is to use a very cheap ballast when a small additional expenditure would procure a much better ballast, which would be much more economical in the long run.

232. **Materials.** The materials most commonly employed are gravel and broken stone. In many sections of the country other materials which more or less perfectly fulfill the requirements as given above, are used. The various materials including some of these special types have been defined by the American Railway Engineering Association as follows:

DEFINITIONS.

Ballast. Selected material placed on the roadbed for the purpose of holding the track in line and surface.

Stone ballast. Stone broken by artificial means into small fragments of specified sizes.

Burnt clay. A clay or gumbo which has been burned into material for ballast.

Chats. Tailings from mills in which zinc, lead, silver and other ores are separated from the rocks in which they occur.

Chert. An impure flint or hornstone occurring in beds.

Cinders. The residue from the coal used in locomotives and other furnaces.

Gravel. Worn fragments of rock, occurring in natural deposits, that will pass through a 2½-inch ring and be retained upon a No. 10 screen.

Gumbo. A term commonly used for a peculiarly tenacious clay, containing no sand.

Sand. Any hard, granular, comminuted rock which will pass through a No. 10 screen and be retained upon a No. 50 screen.

Slag. The waste product, in a more or less vitrified form, of furnaces for reduction of ore. Usually the product of a blast-furnace.

There is still another classification which may or may not be considered as ballast. It is perhaps hardly correct to speak of the natural soils as ballast, yet many miles of cheap railways are "ballasted" with the natural soil, which is then called Mud ballast.

Broken or crushed stone. Rock ballast is generally specified to be that which may all be passed through a 1½ inch (or 2 inch) ring, but which cannot pass through a ¾-inch mesh. It is most easily handled with forks. This method also has the advantage that when it is being rehandled the fine chips which would interefere with effectual drainage will be screened out. Rock ballast is more expensive in first cost and is also more troublesome to handle, but in heavy traffic especially, the track will be kept in better surface and will require less work for maintenance after the ties have become thoroughly bedded.

Burnt clay. This material has been used in many sections of the country where broken stone or gravel are unobtainable except at a prohibitive cost, and where a suitable quality of clay is readily obtained. This clay should be of "gumbo" variety and contain no gravel. It is sometimes burnt in a kiln, or it is sometimes burnt by piling the clay in long heaps over a mass of fuel, the pile being formed in such a way that a temporary but effectual kiln is made. It is necessary that a clear, clean fuel shall be used and that the firing shall be done by a man who is experienced in maintaining such a fire until the burning is completed. Such ballast may be burned very hard and it will last from four to six years. The cost of

burning varies from 30 to 60 cents per cubic yard, according to the circumstances.

Chats. This is a form of ballast which is peculiar to Southwestern Missouri and Southeastern Kansas. When this material was first used it was obtained from the refuse piles of the mills which treated the zinc and lead ores mined in those regions. With the processes then employed the material was obtained in lumps as large as broken stone, and they were considered to be as valuable as broken stone for ballast. Improvements in the processes of treating the ores have resulted in making this by-product very much smaller grained and of less value as ballast, although it is still considered a desirable form of ballast where it may readily be obtained. It should be noted that it is classed with gravel and cinders in the forms of cross-section shown later.

Chert. This is a form of flint or hornstone which occurs in nodules of a size that is suitable for ballast, and is a very good type of ballast wherever it is found, but its occurrence is comparatively infrequent. It is classed with cemented gravel in the design of cross-sections of ballast.

Cinders. This is one of the most universal forms of ballast, since it is a by-product of every road which uses coal as fuel. The advantages consist in the fairly good drainage, the ease of handling and the cheapness—after the road is in operation. One of the greatest disadvantages is the fact that the cinders are readily reduced to dust, which in dry weather becomes very objectionable. Cinders are usually considered preferable to gravel in yards.

Gravel. This is one of the most common forms of good ballast. There are comparatively few railroads which cannot find, at some place along their line, a gravel pit which will afford a suitable supply of gravel for ballast. Sometimes it is unnecessary to screen it, but usually it is better to screen the gravel over a screen having a ½-inch mesh so as to screen out all the dirt and the finer stones.

Sand. Railroads which run along the coast are frequently ballasted merely with the sand obtained in the immediate neighborhood. One great advantage lies in the almost perfect drainage which is obtained.

Slag. When slag is readily obtainable it furnishes an excellent ballast which is free from dust and perfect in drainage

qualities. Slag is classified with crushed rock in the cross-sections shown below, but it should be noted that this only applies to the best qualities of slag, since its quality is quite variable.

Mud ballast. When the natural soil is gravelly so that rain will drain through it quickly, it will make a fair roadbed for light traffic, but for heavy traffic, and for the greater part of the length of most roads, the natural soil is a very poor material for ballast; for, no matter how suitable the soil might be along limited sections of the road, it would practically never happen that the soil would be uniformly good throughout the whole length of the road. Considering that a heavy rain will in one day spoil the results of weeks of patient "surfacing" with mud ballast, it is seldom economical to use "mud" if there is a gravel-bed or other source of ballast anywhere on the line of the road.

233. **Cross-sections.** The required depth of the cross-section to the sub-soil depends largely on the weight of the rolling stock which is to pass over the track. A careful examination of a roadbed to determine the changes which take place under the ties and also an examination of the track and ties during the passage of a heavy train shows that the heavy loads which are now common on railroad tracks force the tie into the ballast with the passage of every wheel load. The effect on the ballast is a greater or less amount of crushing of the ballast. Even the very hardest grades of broken stone are more or less crushed by grinding against each other during the passage of a train. The softer and weaker forms of ballast are ground up much more quickly. One result is the formation of a fine dust which interferes with the proper drainage of water through the ballast. A second result is the compression of the ballast immediately under the tie into the sub-soil. In a comparatively short time a hole is formed under the tie which acts virtually like a pump. With every rise and fall of the tie under each wheel load, the tie actually pumps the water from the surrounding ballast and sub-soil into these various holes. When the ballast is of such a character that the water does not drain through it easily, the water will settle in these holes long enough to seriously deteriorate the ties. When the track becomes so much out of line or level, or so loose that it needs to be tamped up, the process of tamping has practically the effect of deepen-

ing the amount of ballast immediately under the tie, while the sub-soil is forced up between the ties. A longitudinal section of the sub-soil of a track which has been frequently tamped generally has a saw-tooth appearance, and the sub-soil, instead of being a uniform line, has a high spot between each tie, while the ballast is considerably below its normal level immediately under the tie.

234. Classification of Railroads. The American Railway Engineering Association has divided railroads into three classes with respect to the standards of construction which should be adopted for ballasting, as well as other details of construction. The three classes are as follows (quoted from the Association Manual):

"Class 'A' shall include all districts of a railway having more than one main track, or those districts of a railway having a single main track with a traffic that equals or exceeds the following:

Freight-car mileage passing over districts per year per mile	150000
or,	
Passenger-car mileage per annum per mile of district...	10000

with maximum speed of passenger-trains of 50 miles per hour.

"Class 'B' shall include all districts of a railway having a single main track with a traffic that is less than the minimum prescribed for Class 'A' and that equals or exceeds the following:

Freight-car mileage passing over districts per year per mile	50000
or,	
Passenger-car mileage per annum per mile of district...	5000

with maximum speed of passenger-trains of 40 miles per hour.

"Class 'C' shall include all districts of a railway not meeting the traffic requirements of Classes 'A' or 'B.'"

The classification was adopted on the consideration that *quality* of traffic as well as mere tonnage should determine the classification of a railroad. For example, it is considered that a road which operates a train at a speed of 50 miles an hour should adopt the first class or Class "A" standards, even though there is but one train per day on that railroad. It likewise means that any road whose traffic makes necessary the

construction of a regular double track should adopt the first class specifications.

235. Recommended sections for the several classifications. In Fig. 110 are shown a series of cross-sections which were

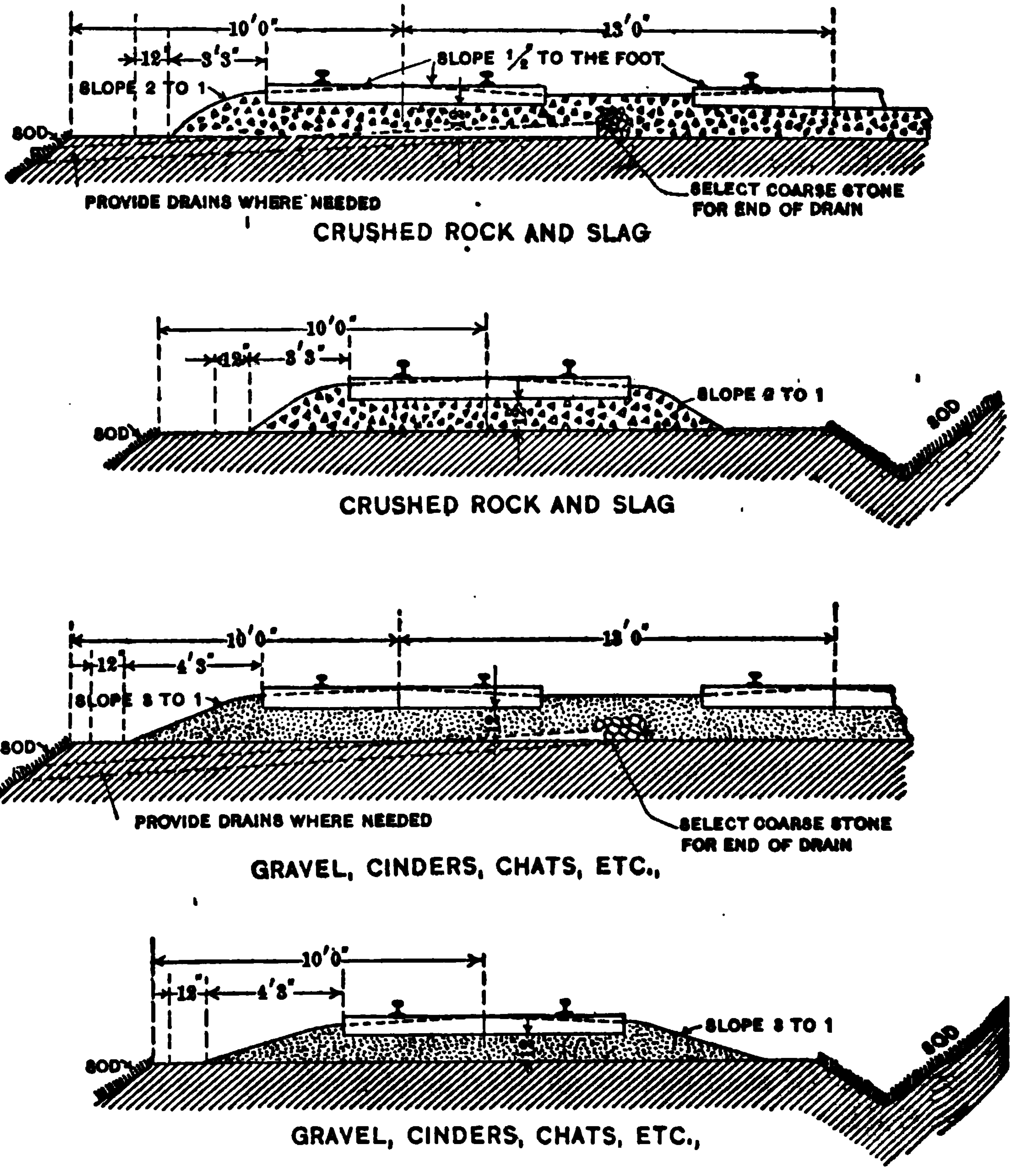


FIG. 110.—CROSS-SECTIONS OF BALLAST FOR CLASS "A" ROADS.

recommended by that association for Class "A" traffic. It should be noticed that in each case the cross-section of the roadbed from shoulder to shoulder of the roadbed is 20 feet plus the space between track centers for double track if any.

The width of side ditches is merely added to that of the roadbed. The clear thickness of the ballast underneath the ties is made 12 inches, but even this should be considered as the *minimum* depth and is recommended for use only on the firmest, most substantial and well-drained subgrades. The slope of ½ inch to the foot from the center of the track to the end of the tie, which is common to all the cross-sections, is designed with the idea of allowing a clear space of 1 inch underneath the rail. The ballast is then rounded off on a curve of 4 feet radius and finally reaches the subsoil on a slope which is 2 : 1 for broken stone, and 3:1 for all other materials. The flat slope adopted for gravel, etc., which adds considerably to the required width of roadbed, has been so designed in order that the considerable mass of material at the ends of the ties shall be better able to hold the track in place laterally. The sod on the embankment over the shoulder of the roadbed up to within 12 inches of the edge of the ballast is strongly recommended on account of the protection it affords to the shoulder of the roadbed. It should be noticed that the latest decision of that association regarding the form of subgrade is that the subgrade should be made level and not crowned, as suggested and discussed in § 93.

In Fig. 111 are shown a series of cross-sections for various classes of ballast for railroads that belong to Class "B." It may be noted that the thickness of the ballast under the tie is 9 inches for this class. The width of roadbed between the shoulders, recommended for Class "B" is 16 feet. As before, the width of the ditches is supposed to be added to this width. It should be noted that when using cementing gravel and chert the slope of 3 : 1 is made to begin at the bottom of the tie instead of at a point about 2 inches below the top of the tie. This is done in order to prevent water from accumulating around the end of the tie in a material which is less permeable than the other forms of ballast.

In Fig. 112 are shown two cross-sections for ballast for roads belonging to Class "C." On roads of this class it is assumed that crushed rock will not be used for ballast. The width of roadbed between shoulders is 14 feet, while the depth of ballast underneath the tie is 6 inches.

It should be noticed that the above sections issued by the association do not include any cross-section which is recom-

mended when no special ballast is used other than the natural soil. In such a case a cross-section very similar to the sections shown for cementing gravel and chert should be used. The essential feature of such a section is that the soil, which is probably not readily permeable, should be kept away from

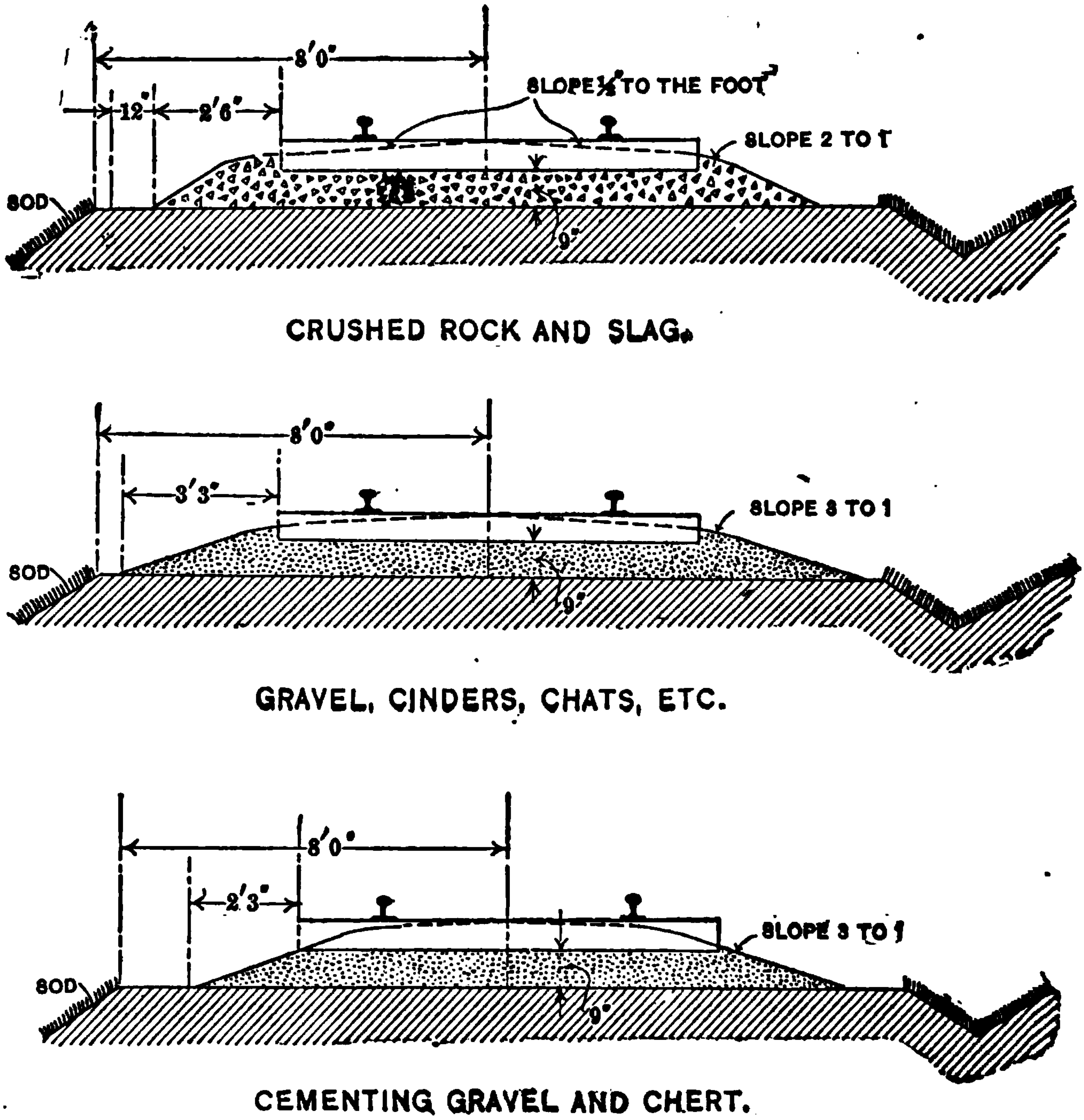
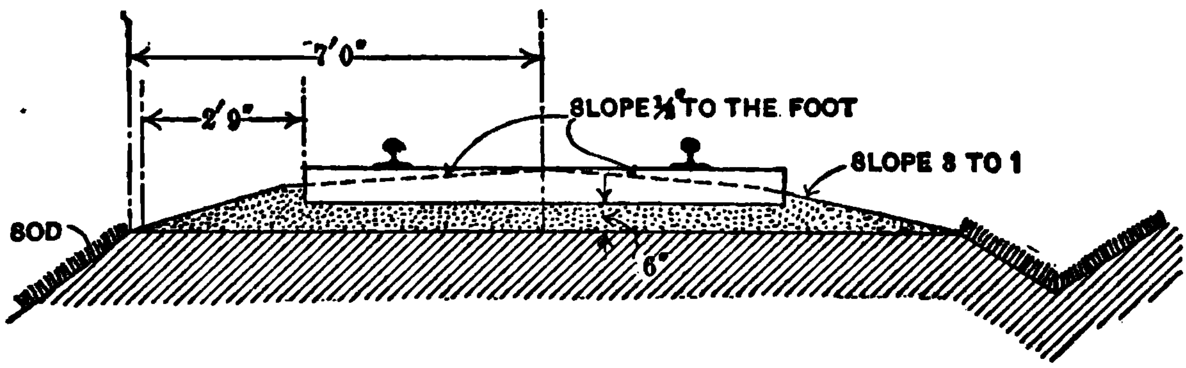


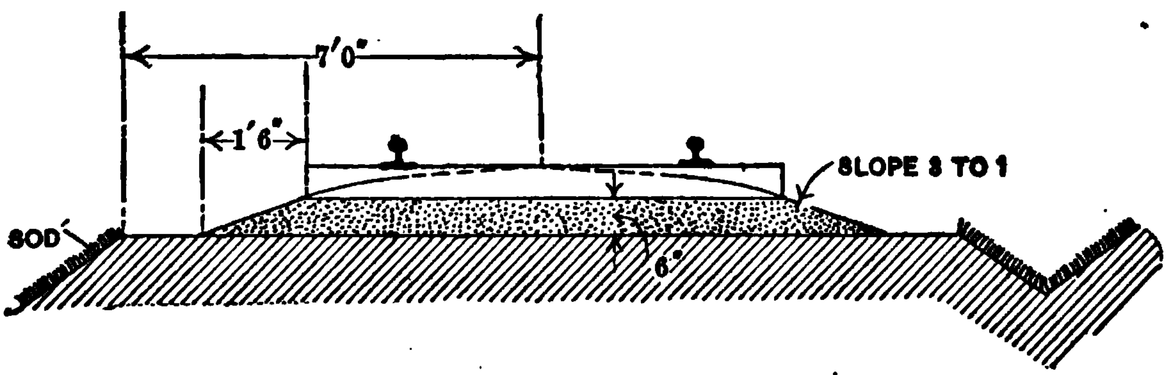
FIG. 111.—CROSS-SECTIONS OF BALLAST FOR CLASS "B" ROADS.

the ends of the ties. Specifications for the placing of mud ballast, as well as other forms of ballast, have frequently specified that the ballast should be crowned about 1 inch above the level of the tops of the ties in the center of the track. This feature of any cross-section, although proposed, was rejected by the association, in spite of the fact that when a tie is so imbedded it certainly will have a somewhat greater holding power in the ballast.

236. Proper depth of ballast. The *depth of ballast* is officially defined by the A. R. E. A. as "the distance from the bottom of the tie to the top of the subgrade." In the recommended sections (Figs. 110 to 112) the depth shown varies from 6 inches to 12 inches. But the Ballast Committee reported in 1915 as a recommended conclusion that "From the data available, it is concluded that with ties 7 in. by 9 in. by 8½ ft., spaced approximately 24 in. to 25.5 ins., center to center, a depth of 24 inches of stone ballast is necessary to produce uniform pressure on the subgrade, and a combination of a lower layer of gravel or cinder



GRAVEL, CINDERS, CHATS, ETC.,



CEMENTING GRAVEL AND CHERT.

FIG. 112.—CROSS-SECTIONS OF BALLAST FOR CLASS "C" ROADS.

ballast, 18 inches to 14 inches, and an upper layer of stone ballast, 6 inches to 10 inches, approximately 24 inches deep in the aggregate, with the same spacing of the ties, will produce nearly the same results." New sections for Class "A" roads which would conform with the above were also recommended. These were not adopted, but the adoption (substantially) was probably only postponed. Future specifications will probably require that a sub-ballast of less expensive material shall be laid under the ballast which immediately supports the rails for all Class "A," and perhaps Class "B," roads. As previously stated, old track generally has a depth of ballast under the tie which is greater than the 2 feet recommended—often 3 or 4 feet.

237. Methods of laying ballast. The cheapest method of laying ballast on new roads is to lay ties and rails directly on the prepared subgrade and run a construction train over the track to distribute the ballast. Then the track is lifted up until sufficient ballast is worked under the ties and the track is properly surfaced. This method, although cheap, is apt to injure the rails by causing bends and kinks, due to the passage of loaded construction trains when the ties are very unevenly and roughly supported, and the method is therefore condemned and prohibited in some specifications. The best method is to draw in carts (or on a contractor's temporary track) the ballast that is required under the level of the *bottom* of the ties. Spread this ballast carefully to the required surface. Then lay the ties and rails, which will then have a very fair surface and uniform support. A construction train can then be run on the rails and distribute sufficient additional ballast to pack around and between the ties and make the required cross-section.

The necessity for constructing some lines at an absolute minimum of cost and of opening them for traffic as soon as possible has often led to the policy of starting traffic when there is little or no ballast—perhaps nothing more than a mere tamping of the natural soil under the ties. When this is done ballast may subsequently be drawn where required by the train-load on flat cars and unloaded at a minimum of cost by means of a "plough." The plough has the same width as the cars and is guided either by a ridge along the center of each car or by short posts set up at the sides of the cars. It is drawn from one end of the train to the other by means of a cable. The cable is sometimes operated by means of a small hoisting-engine carried on a car at one end of the train. Sometimes the locomotive is detached temporarily from the train and is run ahead with the cable attached to it.

238. Cost. The cost of ballast *in the track* is quite a variable item for different roads, since it depends (*a*) on the first cost of the material as it comes to the road, (*b*) on the distance from the source of supply to the place where it is used, and (*c*) on the method of handling. The first cost of cinder or slag is frequently insignificant. A gravel-pit may cost nothing except the price of a little additional land beyond the usual limits of the right of way. Broken stone will usually cost $1 or more per cubic yard. If suitable stone is obtainable on the com-

pany's land, the cost of blasting and breaking should be somewhat less than this. The cost of hauling will depend on the distance hauled, and also, to a considerable extent, on the limitations on the operation of the train due to the necessity of keeping out of the way of regular trains. There is often a needless waste in this way. The "mud train" is considered a pariah and entitled to no rights whatever, regardless of the large daily cost of such a train and of the necessary gang of men. The cost of broken-stone ballast *in the track* is estimated at $1.25 per cubic yard. The cost of gravel ballast is estimated at 60 c. per cubic yard in the track. The cost of placing and tamping gravel ballast is estimated at 20 c. to 24 c. per cubic yard, for cinders 12 c. to 15 c. per cubic yard. The cost of loading gravel on cars, using a steam-shovel, is estimated at 6 c. to 10 c. per cubic yard.*

* Report Roadmasters' Association, 1885.

CHAPTER VIII.

TIES,

AND OTHER FORMS OF RAIL SUPPORT.

239. Various methods of supporting rails. It is necessary that the rails shall be sufficiently supported and braced, so that the gauge shall be kept constant and that the rails shall not be subjected to excessive transverse stress. It is also preferable that the rail support shall be neither rigid (as if on solid rock) nor too yielding, but shall have a *uniform* elasticity throughout. These requirements are more or less fulfilled by the following methods.

(a) **Longitudinals.** Supporting the rails throughout their entire length. This method is very seldom used in this country except occasionally on bridges and in terminals when the longitudinals are supported on cross-ties. In § 264 will be described a system of rails, used to some extent in Europe, having such broad bases that they are self-supporting on the ballast and are only connected by tie-rods to maintain the gauge.

(b) **Cast-iron "bowls" or "pots."** These are castings resembling large inverted bowls or pots, having suitable chairs on top for holding and supporting the rails, and tied together with tie-rods. They will be described more fully later (§ 263).

(c) **Cross-ties of metal or wood.** These will be discussed in the following sections.

240. Economics of ties. The true cost of ties depends on the relative total cost of maintenance for long periods of time. The first cost of the ties delivered to the road is but one item in the economics of the question. Cheap ties require frequent renewals, which cost for the *labor* of each renewal practically the same whether the tie is of oak or of hemlock. Cheap ties make a poor roadbed which will require more track labor to keep even in tolerable condition. The roadbed will require to be disturbed so frequently on account of renewals that the ties never get an opportunity to get settled and to form a smooth roadbed for any length of time. Irregularity in width, thickness, or length of ties is especially detrimental in causing the ballast to act and wear unevenly. The life of ties has thus a more or less direct influence on the life of the rails, on the wear of rolling stock, and on the speed of trains. These last items are not so readily reducible to dollars and cents, but when it can be shown that

the total cost, for a long period of time, of several renewals of cheap ties, with all the extra track labor involved, is as great as or greater than that of a few renewals of durable ties, then there is no question as to the real economy. In the following discussions of the merits of untreated ties (either cheap or costly), chemically treated ties, or metal ties, the true question is therefore of the ultimate cost of maintaining any particular kind of ties for an indefinite period, the cost including the first cost of the ties, the labor of placing them and maintaining them to surface, and the somewhat uncertain (but not therefore non-existent) effect of frequent renewals on repairs of rolling stock, on possible speed, etc.

WOODEN TIES.

241. Choice of wood. This naturally depends, for any particular section of country, on the supply of wood which is most readily available. The woods most commonly used, especially in this country, are oak and pine, oak being the most durable and generally the most expensive. Redwood is used very extensively in California and proves to be extremely durable, so far as decay is concerned, but it is very soft and is much injured by "rail-cutting." This defect is being partly remedied by the use of tie-plates, as will be explained later. Cedar, chestnut, hemlock, and tamarack are frequently used in this country. In tropical countries very durable ties are frequently obtained from the hard woods peculiar to those countries.

TABLE XXII.—NUMBER AND VALUE OF CROSS-TIES USED ON STEAM AND STREET RAILWAYS IN UNITED STATES IN 1906.

(U. S. Dept. Agric.—Forestry Service, No. 124.)

Kind of wood.	Number of ties.	Per cent.	Total value.	Aver. value
Oaks	[illegible]	44 1	$23,278,052	$0 51
Southern pines	[illegible]	18.3	9,567,745	.51
Douglas fir	[illegible]	7 1	3,010,392	.42
Cedar	[illegible]	6 9	3 310,116	.47
Chestnut	[illegible]	6 4	2,995,942	.49
Cypress	[illegible]	5 0	1,862,135	.36
Western pine	[illegible]	3 9	1,698,027	.43
Tamarack	[illegible]	2.5	889,561	.35
Hemlock	[illegible]	2.0	582,968	.28
Redwood	[illegible]	1.2	536,172	.43
Lodgepole pine	[illegible]	0.5	210,818	.38
White pine	[illegible]	0 3	151,052	40
All others	[illegible]	1.8	726,144	.40
Total	102,834,042	100 0	$48,819,124	$0 47

The limitations of timber supply have somewhat diminished the use of oak and increased the use of the softer woods in recent years.

242. Durability. The durability of ties depends on the climate; the drainage of the ballast; the volume, weight, and speed of the traffic; the curvature, if any; the use of tie-plates; the time of year of cutting the timber; the age of the timber and the degree of its seasoning before placing in the track; the nature of the soil in which the timber is grown; and, chiefly, on the species of wood employed. The variability in these items will account for the discrepancies in the reports on the life of various woods used for ties.

White oak is credited with a life of 5 to 12 years, depending principally on the traffic. It is both hard and durable, the hardness enabling it to withstand the cutting tendency of the rail-flanges, and the durability enabling it to resist decay. *Pine* and *redwood* resist decay very well, but are so soft that they are badly cut by the rail-flanges and do not hold the spikes very well, necessitating frequent respiking. Since the spikes must be driven within certain very limited areas on the face of each tie, it does not require many spike-holes to "spike-kill" the tie. On sharp curves, especially with heavy traffic, the wheel-flange pressure produces a side pressure on the rail tending to overturn it, which tendency is resisted by the spike, aided sometimes by rail-braces. Whenever the pressure becomes too great the spike will yield somewhat and will be slightly withdrawn. The resistance is then somewhat less and the spike is soon so loose that it must be redriven in a new hole. If this occurs very often, the tie may need to be replaced long before any decay has set in. When the traffic is very light, the wood very durable, and the climate favorable, ties have been known to last 25 years.

243. Dimensions. The usual dimensions for the best roads (standard gauge) are 8′ to 9′ long, 6″ to 7″ thick, and 8″ to 10″ wide on top and bottom (if they are hewed) or 8″ to 9″ wide if they are sawed. For cheap roads and light traffic the length is shortened sometimes to 7′ and the cross-section also reduced. On the other hand a very few roads use ties 9′ 6″ long.

Two objections are urged against sawed ties: first, that the grain is torn by the saw, leaving a woolly surface which induces decay; and secondly, that, since timber is not perfectly straight-

grained, some of the fibers are cut obliquely, exposing their ends, which are thus liable to decay. The use of a "planer-saw" obviates the first difficulty. Chemical treatment of ties obviates both of these difficulties. Sawed ties are more convenient to handle, are a necessity on bridges and trestles, and it is even claimed, although against commonly received opinion, that actual trial has demonstrated that they are more durable than hewed ties.

244. Spacing. The spacing is usually 14 to 16 ties to a 30-foot rail. This number is sometimes reduced to 12 and even 10, and on the other hand occasionally increased to 18 or 20 by employing narrower ties. There is no economy in reducing the number of ties very much, since for any required stiffness of track it is more economical to increase the number of supports than to increase the weight of the rail. The decreasing cost of rails and the increasing cost of ties have materially changed the relation between number of ties and weight of rail to produce a given stiffness at minimum cost, but many roads have found it economical to employ a large number of ties rather than increase the weight of the rail. On the other hand there is a practical limit to the number that may be employed, on account of the necessary space between the ties that is required for proper tamping. This width is ordinarily about twice the width of the tie. At this rate, with light ties 6″ wide and with 12″ clear space, there would be 20 ties per 30-foot rail, or 3520 per mile. The smaller ties can generally be bought much cheaper (proportionately) than the larger sizes, and hence the economy.

Track instructions to foremen generally require that the spacing of ties shall *not* be uniform along the length of any rail. Since the joint is generally the weakest part of the rail structure, the joint requires more support than the center of the rail. Therefore the ties are placed with but 8″ or 10″ clear space between them at the joints, this applying to 3 or 4 ties at each joint; the remaining ties, required for each rail length, are equally spaced along the remaining distance.

245. Specifications. The specifications for ties are apt to include the items of size, kind of wood, and method of construction, besides other minor directions about time of cutting, seasoning, delivery, quality of timber, etc.

(a) Size. The particular size or sizes required will be somewhat as indicated in § 243.

(b) **Kind of wood.** When the kind or kinds of wood are specified, the most suitable kinds that are available in that section of country are usually required.

(c) **Method of construction.** It is generally specified that the ties shall be hewed on two sides; that the two faces thus made shall be parallel planes and that the bark shall be removed. It is sometimes required that the ends shall be sawed off square; that the timber shall be cut in the winter (when the sap is down); and that the ties shall be seasoned for six months These last specifications are not required or lived up to as much as their importance deserves. It is sometimes required that the ties shall be delivered on the right of way, neatly piled in rows, the alternate rows at right angles, piled if possible on ground not lower than the rails and at least ten feet away from the nearest rail, the lower row of ties resting on two ties which are themselves supported so as to be clear of the ground.

(d) **Quality of timber.** The usual specifications for sound timber are required, except that they are not so rigid as for a better class of timber work The ties must be sound, reasonably straight-grained, and not very crooked—one test being that a line joining the center of one end with the center of the middle shall not pass outside of the other end. Splits or shakes, especially if severe, should cause rejection.

Specifications sometimes require that the ties shall be cut from small trees, making what is known as "pole ties" and definitely condemning those which are cut or split from larger trunks, giving two "slab ties" or four "quarter ties" for each cross-section, as is illustrated in Fig. 113. Even if pole ties are better, their exclusive use means the rapid destruction of forests of young trees.

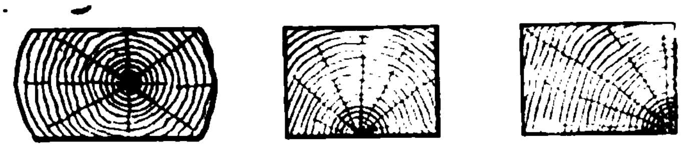


FIG. 113.—METHODS OF CUTTING TIES.

246. Regulations for laying and renewing ties. The regulations issued by railroad companies to their track foremen will generally include the following, in addition to directions regarding dimensions, spacing, and specifications given in §§ 242–245. When hewn ties of somewhat variable size are used, as is frequently the case, the largest and best are to be selected for use as joint ties. If the upper surface of a tie is found to be warped (contrary to the usual specifications) so that one or both rails do

not get a full bearing across the whole width of the tie, it must be adzed to a true surface along its whole length and not merely notched for a rail-seat. When respiking is necessary and spikes have been pulled out, the holes should be immediately plugged with "wooden spikes," which are supplied to the foreman for that express purpose, so as to fill up the holes and prevent the decay which would otherwise take place when the hole becomes filled with rain-water. Ties should always be laid at right angles to the rails and never obliquely Minute regulations to prevent premature rejection and renewal of ties are frequently made. It is generally required that the requisitions for renewals shall be made by the actual count of the individual ties to be renewed instead of by any wholesale estimates. It is unwise to have ties of widely variable size, hardness, or durability adjacent to each other in the track, for the uniform elasticity, so necessary for smooth riding, will be unobtainable under those circumstances.

After a considerable discussion of the two policies of tie renewals over long continuous stretches of track or of single tie renewals where individually needed, the A. R. E. A. has decided in favor of single tie renewals, as being most economical and producing least track disturbance.

247. Dating nails. These are made of iron or steel, galvanized with zinc. They should be $2\frac{1}{2}$ inches long, $\frac{1}{4}$ inch in diameter, with $\frac{5}{8}$-inch head, which has two figures $\frac{3}{16}$ inch high, denoting the year, which are stamped, by depression, into the head. They should be driven into the upper side of all treated ties, 10 inches inside the rail, on the line side of the track. The use of such dates gives definite knowledge of the life of the tie when it is renewed and a means of studying the effectiveness of the tie treatment.

248. Cost of ties. When railroads can obtain ties cut by farmers from woodlands in the immediate neighborhood, the price will frequently be as low as 35 cents for the smaller sizes, running up to 60 cents for the larger sizes and better qualities, especially when the timber is not very plentiful. Sometimes if a railroad cannot procure suitable ties from its immediate neighborhood, it will find that adjacent railroads control all adjacent sources of supply for their own use and that ties can only be procured from a considerable distance, with a considerable added cost for transportation. First-class oak ties cost about 80 to 90 cents and frequently much more.

PRESERVATIVE PROCESSES FOR WOODEN TIES.

249. General principles. Wood has a fibrous cellular structure, the cells being filled with sap or air. The woody fiber is but little subject to decay unless the sap undergoes fermentation. Preservative processes generally aim at removing as much of the water and sap as possible and filling up the pores of the wood with an antiseptic compound. The most common methods all agree in this general process and only differ in the method employed to get rid of the sap and in the antiseptic chemical with which the fibers are filled. One valuable feature of these processes lies in the fact that the softer cheaper woods are more readily treated than are the harder woods and from them a tie can be made which will be as durable as the best (from the standpoint of decay), and, if protected from mechanical wear by tie-plates, will have a very long life. The following woods may be used without preservative treatment: White oak family, long-leaf strict heart yellow pine, cypress, excepting the white cypress, redwood, white cedar, chestnut, catalpa, locust, except the honey locust, walnut and black cherry. The following woods should preferably not be used without preservative treatment: Red oak family, beech, elm, maple, gum, loblolly, short-leaf, Western yellow pine, Norway, North Carolina pine and other sap pines, red fir, spruce, hemlock, and tamarack. It is better to use an excess of chemical rather than not enough. Ties should be grouped before treatment; for example, green ties should not be mixed with seasoned ties, since the treatment should be different. Ties should be air-seasoned before being treated. When there is time to air-season them at the plant before treatment, they should be piled in groups having the same degree of seasoning, so that they rest on seasoned stringers, the lowest ties at least 6 inches from the ground, which should be thoroughly drained and cleared from weeds, high grass and decaying matter. The ties should not be allowed to over-season or deteriorate. Ties which show signs of checking should be secured with S-irons or bolts to prevent further checking. When ties are to be adzed or bored for the use of tie-plates or screw spikes, the adzing or boring should be done before chemical treatment. When it is necessary to treat unseasoned or only partially seasoned ties, they should be steamed to remove the sap.

To do the work, long cylinders, which may be opened at the ends, are necessary. Usually the timbers are run in and out on iron carriages running on rails fastened to braces on the inside of the cylinder. When the load has been run in, the ends of the cylinder are fastened on. The water and air in the pores of the wood are drawn out by subjecting the wood alternately to steam-pressure and to the action of a vacuum-pump. Live steam should be admitted so that a pressure of 20 lbs. is produced within 30 to 50 minutes. This pressure may be maintained from 1 to 5 hours, depending on the condition of the wood, but the pressure should never exceed 20 lbs. A vent should be provided to allow the escape of air and condensed water. After steaming, a vacuum of not less than 24 inches of mercury at sea-level (or correspondingly less for higher altitudes), shall be produced and maintained for half an hour. Then, without breaking the vacuum, the chemical shall be admitted.

250. Creosoting. This process consists in impregnating the wood with creosote oil, a product obtained from coal-gas tar or coke oven tar which shall be free from any tar, including coal-gas tar, oil or residue obtained from petroleum or any other source. The pure creosote oil is strongly recommended by the A.R.E.A., but they recognize that the practice of using other coal tar distillates, when the available supply of creosote is inadequate, is firmly established, and have made specifications accordingly.

It would require about 35 to 50 lbs of creosote to completely fill the pores of a cubic foot of wood. But it would be impossible to force such an amount into the wood, nor is it necessary or desirable. About 10 lbs. per cubic foot, or about 35 lbs. per tie, is all that is necessary. For piling placed in salt water about 18 to 20 lbs. per cubic foot is used, and the timber is then perfectly protected against the ravages of the *teredo navalis*. After one of the vacuum periods, the cylinder is filled with creosote oil at a temperature of about 170° F. The pumps are kept at work until the pressure is about 80 to 100 lbs. per square inch, and is maintained at this pressure from one to two hours according to the size of the timber. The oil is then withdrawn, the cylinders opened, the train pulled out and another load made up in 40 to 60 minutes. The average time required for treating a load is about 18 or 20 hours, the absorption about 10 or 11 lbs. of oil per cubic foot, and the cost (1894) from $12.50 to $14.50 per thousand feet B. M.

251. Burnettizing (chloride-of-zinc process). This process is very similar to the creosoting process except that the chemical is chloride of zinc. The chemical is heated to 140° F. before using. The preliminary treatment of the wood to alternate vacuum and pressure is not continued for quite so long a period as in the creosoting process. Care must be taken, in using this process, that the ties are of as uniform quality as possible, for seasoned ties will absorb much more zinc chloride than unseasoned (in the same time), and the product will lack uniformity unless the seasoning is uniform. The amount of solution injected shall be equivalent to $\frac{1}{2}$ lb. of dry soluble zinc-chloride per cubic foot of timber. The solution shall be as weak as can be used and still obtain the desired absorption of zinc-chloride, and shall not be stronger than 5%. If the cylinders are provided with steam coils, steam pressure shall be maintained in these coils during treatment. One great objection to burnettized ties is the fact that the chemical is somewhat easily washed out, when the wood again becomes subject to decay. Another objection, which is more forcible with respect to timber subject to great stresses as in trestles, than to ties, is the fact that when the solution of zinc chloride is made strong (over 3%) the timber is made very brittle and its strength is reduced. The reduction in strength has been shown by tests to amount to $\frac{1}{4}$ to $\frac{1}{10}$ of the ultimate strength, and that the elastic limit has been reduced by about $\frac{1}{7}$.

252. Kyanizing (bichloride-of-mercury or corrosive-sublimate process). This is a process of "steeping." It requires a much longer time than the previously described processes, but does not require such an expensive plant. Wooden tanks of sufficient size for the timber are all that is necessary. The corrosive sublimate is first made into a concentrated solution of one part of chemical to six parts of *hot* water. When used in the tanks this solution is weakened to 1 part in 100 or 150. The wood will absorb about 5 to 6.5 pounds of the bichloride per 100 cubic feet, or about one pound for each 4 to 6 ties. The timber is allowed to soak in the tanks for several days, the general rule being about one day for each inch of least thickness and one day over—which means seven days for 6-inch ties, or thirteen (to fifteen) days for 12-inch timber (least dimension). The process is somewhat objectionable on account of the chemical being such a virulent poison, workmen sometimes being sickened by the fumes

arising from the tanks. On the Baden Railway (Germany) kyanized ties last 20 to 30 years. On this railway the wood is always air-dried for two weeks after impregnation and before being used, which is thought to have an important effect on its durability. The solubility of the chemical and the liability of the chemical washing out and leaving the wood unprotected is an element of weakness in the method.

253. Zinc-tanning process. The last two methods described (as well as some others employing similar chemicals) are open to the objection that since the wood is impregnated with an aqueous solution, it is liable to be washed out very rapidly if the wood is placed under water, and will even disappear, although more slowly, under the action of moisture and rain. Several processes have been proposed or patented to prevent this. By one of these processes the timber is successively subjected to the action of chemicals, each individually soluble in water, and hence readily impregnating the timber, but the chemicals when brought in contact form insoluble compounds which cannot be washed out of the wood-cells. After injecting the zinc-chloride, as before described, the solution is run off and the ties drained for 15 minutes. Then a 2% solution of tannic acid, made from $6\frac{2}{3}$ lbs. of 30% extract of tannin and 100 lbs. of water is run in and maintained at 100 lbs. pressure for one-half hour. Then a solution of glue made by dissolving 2.1 lbs. of glue containing 50% gelatine in 100 lbs. of water is run in and maintained at 100 lbs. pressure for one-half hour. The glue and tannin combine to form an insoluble leathery compound in the cells, which will prevent the zinc chloride from being washed out.

254. Zinc-creosote emulsion process. The chemical is an emulsion which will leave in the wood an equivalent of 0.4 lb. of dry, soluble zinc-chloride and from 1.25 to 1.5 lbs. of creosote per cubic foot. The zinc-chloride must not be stronger than 3.5%. The emulsion must be effectively mixed in a storage tank and heated to at least 140° F. before it enters the cylinder, where the pressure is raised to 100 lbs. per square inch and maintained there until the required amount of chemical has been absorbed by the wood.

255. Two-injection zinc-creosote process. The zinc-chloride and creosote are injected separately. The zinc-chloride must be as weak as possible (not more than 5%), and yet strong

enough so that the equivalent of 0.3 lb. can be injected per cubic foot. After impregnation, the remaining zinc-chloride is run out and the creosote is forced in and maintained at 100 lbs. pressure until the wood has absorbed about 3 lbs. of oil per cubic foot.

256. Cost of Treating. The cost of treating ties by the various methods has been estimated as follows*—assuming that the plant was of sufficient capacity to do the work economically; creosoting, 25 cents per tie; vulcanizing, 25 cents per tie; burnettizing (chloride of zinc), 8.25 cents per tie; kyanizing (steeping in corrosive sublimate), 14.6 cents per tie; Wellhouse process (chloride of zinc and tannin), 11.25 c. per tie. These estimates are only for the net cost at the works and do not include the cost of hauling the ties to and from the works, which may mean 5 to 10 c. per tie. Some of these processes have been installed on cars which are transported over the road and operated where most convenient. An estimate made in 1907 by Prof. Gellert Alleman on the cost of treating ties, each assumed to have a volume of 3 cubic feet, the cost "not including royalty on patents, profit, interest, or depreciation, all of which vary widely at the various plants," is as follows:

Zinc chloride	16 cents
" " and creosote	27 "
Creosote, 10 pounds to the cubic foot	55 "

The very great increase in these prices, especially for creosoting, is due to the enormous increase in late years in the consumption and in the price of creosote.

257. Economics of treated ties. The fact that treated ties are not universally adopted is due to the argument that the added life of the tie is not worth the extra cost. If ties can be bought for 25 c., and cost 25 c. for treatment, and the treatment only doubles their life, there is apparently but little gained except the work of placing the extra tie in the track, which is more or less offset by the interest on 25 c. for the life of the untreated tie, and the larger initial outlay makes a stronger impression on the mind than the computed ultimate economy. But when (utilizing some statistics from the Pittsburg, Ft. Wayne &

* Bull. No. 9, U. S. Dept. of Agric., Div. of Forestry. App. No. 1, by Henry Flad.

Chicago Railroad) it is found that white oak ties laid in rock ballast had a life of 10.17 years, and that hemlock ties treated with the zinc-tannin process and laid in the same kind of ballast lasted 10.71 years, then the economy is far more apparent. Unfortunately no figures were given for the cost of these ties nor for the cost of the treatment; but if we assume that the white oak ties cost 75 c. and the hemlock ties 35 c. plus 20 c. for treatment, there is not only a saving of 20 c. on each tie, but also the advantage of the slightly longer life of the treated tie. In the above case the total life of the two kinds of ties is so nearly the same that we may make an approximation of their relative worth by merely comparing the initial cost; but usually it is necessary to compare the value of two ties one of which may cost more than the other, but will last considerably longer. The mathematical comparison of the real value of two ties under such conditions may be developed as follows: The real cost of a tie, or any other similar item of constructive work, is measured by the cost of perpetually maintaining that item in proper condition in the structure. It will be here assumed that the annual cost of the trackwork, which is assignable to the tie, is the same for all kinds of ties, although the difference probably lies in favor of the more expensive and most durable ties. By assuming this expense as constant, the remaining expense may be considered as that due to the cost of the new ties whenever necessary, plus the cost of placing them in the track. We also may combine these two items in one, and consider that the cost of placing a tie in the track, which we will assume at the constant value of 20 c. per tie, regardless of the kind of tie, is merely an item of 20 c. in the total cost of the tie. We will assume that T_1 is the present cost of a tie, the cost including the preservative treatment if any, and the cost of placing in the track. The tie is assumed to last n years. At the end of n years another tie is placed in the track, and, for lack of more precise knowledge, we will assume that this cost T_2 equals T_1. The "present worth" of T_2 is the sum which, placed at compound interest, would equal T_2 at the end of n years, and is expressed by the quantity $\frac{T_2}{(1+r)^n}$, in which r equals the rate of interest. Similarly at the end of $2n$ years we must expend a sum T_3 to put in the third tie, and the present worth of the cost of that third tie is ex-

pressed by the fraction $\frac{T_3}{(1+r)^{2n}}$. We may similarly express the present worths of the cost of ties for that particular spot for an indefinite period. The sum of all these present worths is given by the sum of a converging series and equals (assuming that all the T's are equal) $\frac{T\times(1+r)^n}{(1+r)^n-1}$. But instead of laying aside a sum of money which will maintain a tie in that particular place in perpetuity, we may compute the annual sum which must be paid at the end of each year, which would be the equivalent. We will call that annual payment A, and then the present worths of all these items are as follows:

For the first payment .... $\frac{A}{(1+r)}$;

For the second payment .... $\frac{A}{(1+r)^2}$;

For the third payment .... $\frac{A}{(1+r)^3}$;

For the nth payment .... $\frac{A}{(1+r)^n}$.

After the next tie is put in place we have the present worths of the annual payments on the second tie, of which the first one would be

For the $(n+1)$ payment .... $\frac{A}{(1+r)^{(n+1)}}$.

Similarly after x ties have been put in place the last payment for the x tie would have a present worth $\frac{A}{(1+r)^{nx}}$. The sum of all these present worths is represented by the sum of a converging series and equals the very simple expression $\frac{A}{r}$. But since the sum of the present worths of these annual payments must equal the sum of the present worths of the payments made at intervals of n years, we may place these two summations equal to each other, and say that

$$A=\frac{r\times T\times(1+r)^n}{(1+r)^n-1}.$$

Values of A for various costs of a tie T on the basis that r equals 5% have been computed and placed in Table XVIII. To illustrate the use of this table, assume that we are comparing the relative values of two ties, both untreated, one of them a white oak tie which will cost, say 75 c., and will last twelve years, the other a yellow pine tie which will cost, say 35 c., and will last six years. Assuming a charge for each case of 20 c. for placing the tie in the track, we have as the annual charge against the white oak tie, which costs 95 c. in the track, 10.72 c. The pine tie, costing 55 c. in the track and lasting six years, will be charged with an annual cost of 10.48 c., which shows that the costs are practically equal. It is probably true that the track work for maintaining the white oak would be less than that for the pine tie, but since the initial cost of the pine tie is less than that of the oak tie, it would probably be preferred in this case, especially if money was difficult to obtain. It may be interesting to note that if a comparison is made from a similar table which is computed on the basis of compounding the money at 4% instead of 5%, the annual charges would be 10.13 and 10.49 c. for the oak and pine ties respectively, thus showing that when money is "easier" the higher priced tie has the greater advantage.

EXAMPLE 2. Considering again the comparison previously made of a white oak untreated tie which was assumed to cost 75 c., and a hemlock treated tie, which cost 35 c. for the tie and 20 c. for the treatment, the total costs of these ties laid in the track would therefore be 95 c. and 75 c. respectively. These ties had practically the same life (10.17 and 10.71 years), but in order to use the table, we will call it ten years for each tie. The annual charge against the oak tie would therefore be 12.30 c., while that against the hemlock tie would be 9.72 c. This gives an advantage in the use of the treated tie of 2.58 c. per year, which capitalized at 5% would have a capitalized value of 51.6 c.

The Atchison, Topeka and Santa Fé R. R. has compiled a record of treated pine ties removed in 1897, '98, '99, and 1900, showing that the *average* life of the ties removed had been about 11 years. On the Chicago, Rock Island and Pacific R. R., the average life of a very large number of treated hemlock and tamarack ties was found to be 10.57 years. Of one lot of 21,850 ties, 12% still remained in the track after 15 years' exposure.

It has been demonstrated that much depends on the minor

details of the process—whatever it may be. As an illustration, an examination of a batch of ties, treated by the zinc-creosote process, showed 84% in service after 13 years' exposure; another batch, treated by another contractor by the same process (nominally), showed 50% worthless after a service of six years.

METAL TIES.

258. Extent of use. In 1894 * there were nearly 35000 miles of "metal track" in various parts of the world. Of this total, there were 3645 miles of "longitudinals" (see § 264), found exclusively in Europe, nearly all of it being in Germany. There were over 12000 miles of "bowls and plates" (see § 263), found almost entirely in British India and in the Argentine Republic. The remainder, over 18000 miles, was laid with metal cross-ties of various designs. There were over 8000 miles of metal cross-ties in Germany alone, about 1500 miles in the rest of Europe, over 6000 miles in British India, nearly 1000 miles in the rest of Asia, and about 1500 miles more in various other parts of the world. Several railroads in this country have tried various designs of these ties, but their use has never passed the experimental stage. These 35000 miles represent about 9% of the total railroad mileage of the world—nearly 400000 miles. They represent about 17.6% of the total railroad mileage, exclusive of the United States and Canada, where they are not used at all, except experimentally.† In the four years from 1890 to 1894 the use of metal track increased from less than 25000 miles to nearly 35000 miles. This increase was practically equal to the total increase in railroad mileage during that time, exclusive of the increase in the United States and Canada. This indicates a large growth in the percentage of metal track to total mileage, and therefore an increased appreciation of the advantages to be derived from their use.

259. Durability. The durability of metal ties is still far from being a settled question, due largely to the fact that the best form for such ties is not yet determined, and that a large part of the apparent failures in metal ties have been evidently due to defective design. Those in favor of them estimate the life as from 30 to 50 years. The opponents place it at not more

* Bulletin No. 9, U. S. Dept. of Agriculture, Div. of Forestry.

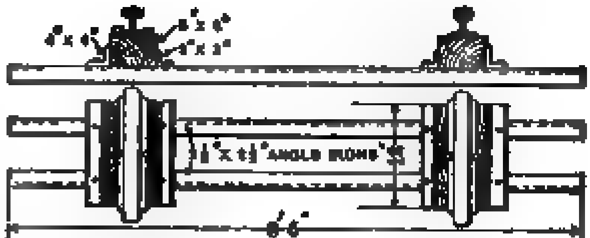
† See § 260 for a later development.

than 20 years, or perhaps as long as the best of wooden ties. Unlike the wooden tie, however, which deteriorates as much with time as with usage, the life of a metal tie is more largely a function of the traffic. The life of a well-designed metal tie has been estimated at 150000 to 200000 trains; for 20 trains per day, or say 6000 per year, this would mean from 25 to 33 years. 20 trains per day on a *single* track is a much larger number than will be found on the majority of railroads. Metal ties are found to be subject to rust, especially when in damp localities, such as tunnels; but on the other hand it is in such confined localities, where renewals are troublesome, that it is especially desirable to employ the best and longest-lived ties. Paint, tar, etc., have been tried as a protection against rust, but the efficacy of such protection is as yet uncertain, the conditions preventing any renewal of the protection—such as may be done by repainting a bridge, for example. Failures in metal cross-ties have been largely due to cracks which begin at a corner of one of the *square* holes which are generally *punched* through the tie, the holes being made for the bolts by which the rails are fastened to the tie. The holes are generally *punched* because it is cheaper. Reaming the holes after punching is thought to be a safeguard against this frequent cause of failure. Another method is to round the corners of the square punch with a radius of about $\frac{1}{8}''$. If a crack has already started, the spread of the crack may be prevented by drilling a small hole at the end of it.

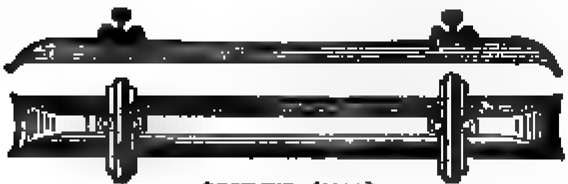
260. Form and dimensions of metal cross-ties. Since stability in the ballast is an essential quality for a tie, this must be accomplished either by turning down the end of the tie or by having some form of lug extending downward from one or more points of the tie. The ties are sometimes depressed in the center (see Plate VI, N. Y. C. & H. R. R. R. tie) to allow for a thick covering of ballast on top in order to increase its stability in the ballast. This form requires that the ties should be sufficiently well tamped to prevent a tendency to bend out straight, thus widening the gauge. Many designs of ties are objectionable because they cannot be placed in the track without disturbing adjacent ties. The failure of many metal cross-ties, otherwise of good design, may be ascribed to too light weight. Those weighing much less than 100 pounds have proved too light. From 100 to 130 pounds weight is being used satisfactorily on German railroads. The general outside dimensions are about

the same as for wooden ties, except as to thickness. The metal is generally from $\frac{1}{4}''$ to $\frac{3}{8}''$ thick. They are, of course, only made of wrought iron or steel, cast iron being used only for "bowls" or "plates" (see § 263). The details of construction for some of the most commonly used ties may be seen by a study of Plate VI.

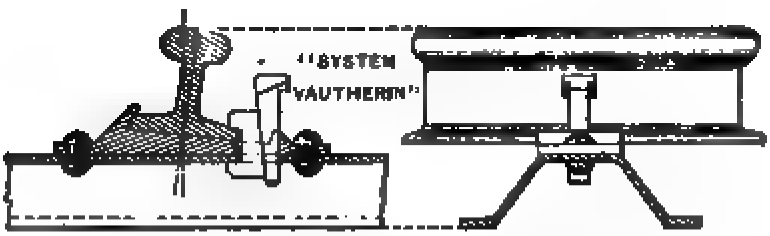
The Carnegie tie is perhaps the only tie whose use on steam railroads in this country has passed the experimental stage. The Bessemer and Lake Erie R. R. in 1910 had 188 miles of track laid with these ties, and other roads are making extensive experiments. One practical difficulty, which is not of course, insuperable, arises from the common practice of using the rails as parts of an electrical circuit for a block-signal system, which requires that the rails shall be insulated from each other. This requires that these metal ties shall be insulated from the rails. A method of insulation which is altogether satisfactory and inexpensive is yet to be determined. It is claimed that, on account of the better connection between the rail and the tie, there is less wear and more uniform wear to the rail. It is also claimed that there is greater lateral rigidity in the rails and ties (considered as a structure) and that this decreases the trackwork necessary to maintain alinement. These ties weigh 19.7 pounds per linear foot, or about 167 pounds for an 8 foot 6 inch tie. Even at the lowest possible price per pound the cost of the tie and its fastenings must be two or three times that of the best oak tie with spikes and even tie plates. It has been impossible to estimate the probable life of these ties. Until a reasonably close estimate of the life of steel ties can be determined, no proper comparison can be made of their economy relative to that of wooden ties. A study of Table XVIII will show that a tie which costs, say three times as much as a cheap tie, must last more than three times as long in order that the annual charge against the tie shall be as low as that of the cheaper tie. For example, let us assume that the cost of a metal tie, laid in the track, is \$2.55 and that it will last 20 years. From Table XVIII we may find that the annual charge against \$2.55 at 5% for 20 years $=(2\times8.02)+4.41=20.45$ cents. Compared with a tie costing 65 cents, plus 20 cents for track laying, we find that the cheaper tie will only cost 19.63 cents per year even if it only lasts 5 years. Of course the claimed advantage of better track and less cost for track maintenance, using steel ties, will tend to offset, so far as it is true, the disadvantage of

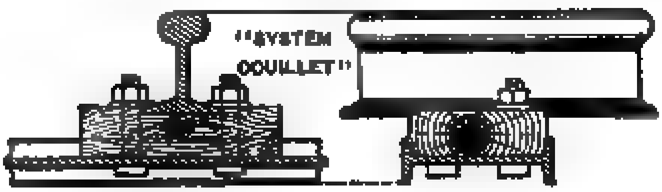


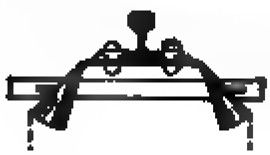
COOK-HICKS TIE. D. L. & W. R. R. (1890)

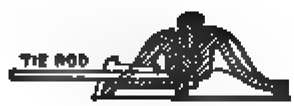


POST TIE. (1884)

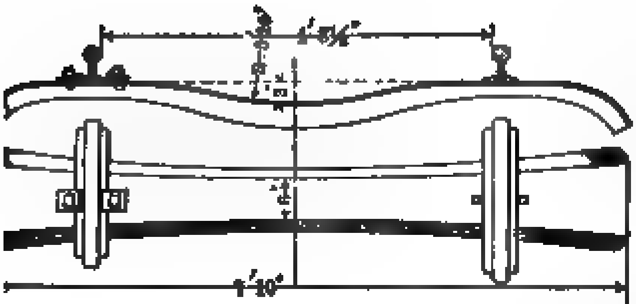




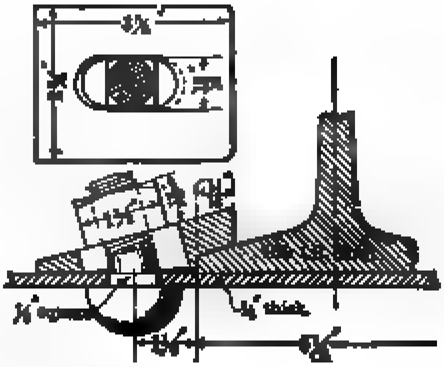


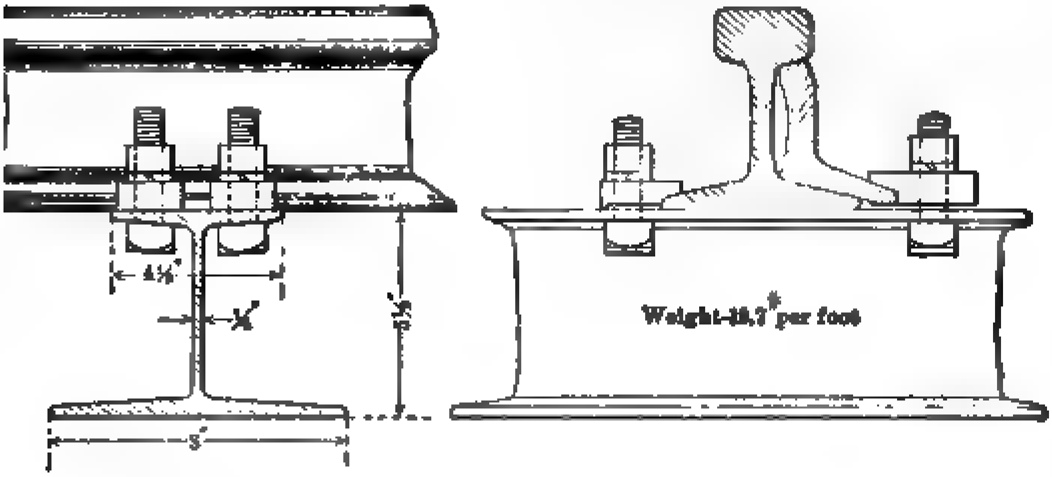


LIVESEY BOWL. (1884)

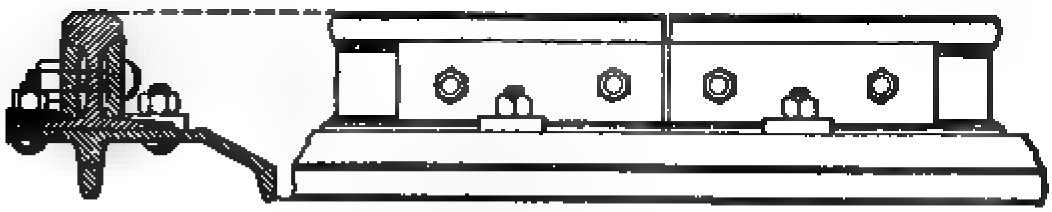


N.Y.C. & H.R.R.R. ([illegible])





CARNEGIE STEEL TIE (1906)



"SYSTEM KÖSTLIN u. BATTIG"—LONGITUDINAL.

PLATE VI.—SOME FORMS OF METAL TIES.

(*Between pp. 292 and 293.*)

the extra cost of the metal tie. Even if the extra work per tie amounts to only one-half hour for one man in a year, the cost of it, say 6 cents, will utterly change the relative economics of the two ties.

261. Fastenings, The devices for fastening the rails to the ties should be such that the gauge may be widened if desired on curves, also that the gauge can be made true regardless of slight inaccuracies in the manufacture of the ties, and also that shims may be placed under the rail if necessary during cold weather when the tie is frozen into the ballast and cannot be easily disturbed. Some methods of fastening require that the base of the rail be placed against a lug which is riveted to the tie or which forms a part of it. This has the advantage of reducing the number of pieces, but is apt to have one or more of the disadvantages named above. Metal keys or wooden wedges are sometimes used, but the majority of designs employ some form of bolted clamp. The form adopted for the experimental ties used by the N. Y. C. & H. R. R. R. (see Plate VI) is especially ingenious in the method used to vary the gauge or allow for inaccuracies of manufacture. Plate VI shows some of the methods of fastening adopted on the principal types of ties.

262 Cost. The cost of metal cross-ties in Germany averages about 1.6 c. per pound or about $1.60 for a 100-lb. tie. The ties manufactured for the N. Y. C. & H. R. R. R. in 1892 weighed about 100 lbs. and cost $2.50 per tie, but if they had been made in larger quantities and with the present price of steel the cost would possibly have been much lower. The item of freight from the place of manufacture to the place where used is no inconsiderable item of cost with some roads. Metal cross-ties have been used by some street railroads in this country. Those used on the Terre Haute Street Railway weigh 60 pounds and cost about 66 c. for the tie, or 74 c. per tie with the fastenings.

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263. Bowls or plates. As mentioned before, over 12000 miles of railway, chiefly in British India and in the Argentine Republic, are laid with this form of track. It consists essentially of large cast-iron inverted "bowls" laid at intervals under each rail and opposite each other, the opposite bowls being tied together with tie-rods. A suitable chair is riveted or bolted on to the top of each bowl so as to properly hold the rail. Being

made of cast iron, they are not so subject to corrosion as steel or wrought iron. They have the advantage that when old and worn out their scrap value is from 60% to 80% of their initial cost, while the scrap value of a steel or wrought-iron tie is practically nothing. Failure generally occurs from breakage, the failures from this cause in India being about 0.4% per annum. They weigh about 250 lbs. apiece and are therefore quite expensive in first cost and transportation charges. There are miles of them in India which have already lasted 25 years and are still in a serviceable condition. Some illustrations of this form of tie are shown in Plate VI.

264. Longitudinals.* This form, the use of which is confined almost exclusively to Germany, is being gradually replaced on many lines by metal cross-ties. The system generally consists of a compound rail of several parts, the upper bearing rail being very light and supported throughout its length by other rails, which are suitably tied together with tie-rods so as to maintain the proper gauge, and which have a sufficiently broad base to be properly supported in the ballast. One great objection to this method of construction is the difficulty of obtaining proper drainage especially on grades, the drainage having a tendency to follow along the lines of the rails. The construction is much more complicated on sharp curves and at frogs and switches. Another fundamentally different form of longitudinal is the Haarman compound "self-bearing" rail, having a base 12" wide and a height of 8", the alternate sections breaking joints so as to form a practically continuous rail.

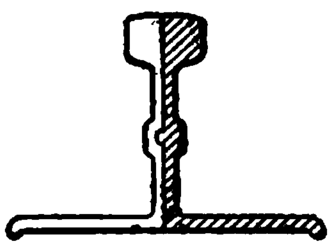

FIG. 114.

Some of the other forms of longitudinals are illustrated in Plate VI.

For a very complete discussion of the subject of metal ties, see the "Report on the Substitution of Metal for Wood in Railroad Ties" by E. E. Russell Tratman, it being Bulletin No. 4, Forestry Division of the U. S. Dept. of Agriculture.

* Although the discussion of longitudinals might be considered to belong more properly to the subject of RAILS, yet the essential idea of all designs must necessarily be the *support* of a rail-head on which the rolling stock may run, and therefore this form, unused in this country, will be briefly described here.

265. Reinforced Concrete Ties. The wide application of reinforced concrete to various structural purposes, combined with its freedom from decay, has led to its attempted adoption for ties. In the annual Proceedings of the American Railway Engineering Association for 1907 is a report on over a dozen different designs, the most of which were shown to be incapable of enduring traffic except on sidings. The ties are particularly subject to fracture if struck by a derailed car. A similar progress report, made in 1911, again indicated that a practicable concrete tie for general use has not yet been invented.

The annual report for 1915 again contained a review of all such ties then in service. In no case was there any considerable stretch of track laid with concrete ties—merely a few used experimentally in scattered places. The reports are full of instances of ties being fractured by a derailment after short service.

CHAPTER IX.

RAILS.

266. Early forms. The first rails ever laid were wooden stringers which were used on very short tram-roads around coal-mines. As the necessity for a more durable rail increased, owing chiefly to the invention of the locomotive as a motive power, there were invented successively the cast-iron "fish-belly" rail and various forms of wrought-iron strap rails which finally developed into the T rail used in this country and the double-headed rail, supported by chairs, used so extensively in England. The cast-iron rails were cast in lengths of about 3 feet and were supported in iron chairs which were sometimes set upon stone piers. A great deal of the first railroad track of this country was laid with longitudinal stringers of wood placed upon cross-ties, the inner edge of the stringers being protected by wrought-iron straps. The "bridge" rails were first rolled in this country in 1844. The "pear" section was an approach to the present form, but was very defective on account of the difficulty of designing a good form of joint. The "Stevens" section was designed in 1830 by Col. Robert L. Stevens, Chief Engineer of the Camden and Amboy Railroad; although quite defective in its proportions, according to the present knowledge of the requirements, it is essentially the present form. In 1836, Charles Vignoles invented essentially the same form in England; this form is therefore known throughout England and Europe as the Vignoles rail.

267. Present standard forms. The larger part of modern railroad track is laid with rails which are either "T" rails or the double-headed or "bull-headed" rails which are carried in chairs. The double-headed rail was designed with a symmetrical form with the idea that after one head had been worn out by traffic the rail could be reversed, and that its life would be practically doubled. Experience has shown that the wear of the

rail in the chairs is very great; so much so that when one head has been worn out by traffic the whole rail is generally useless.

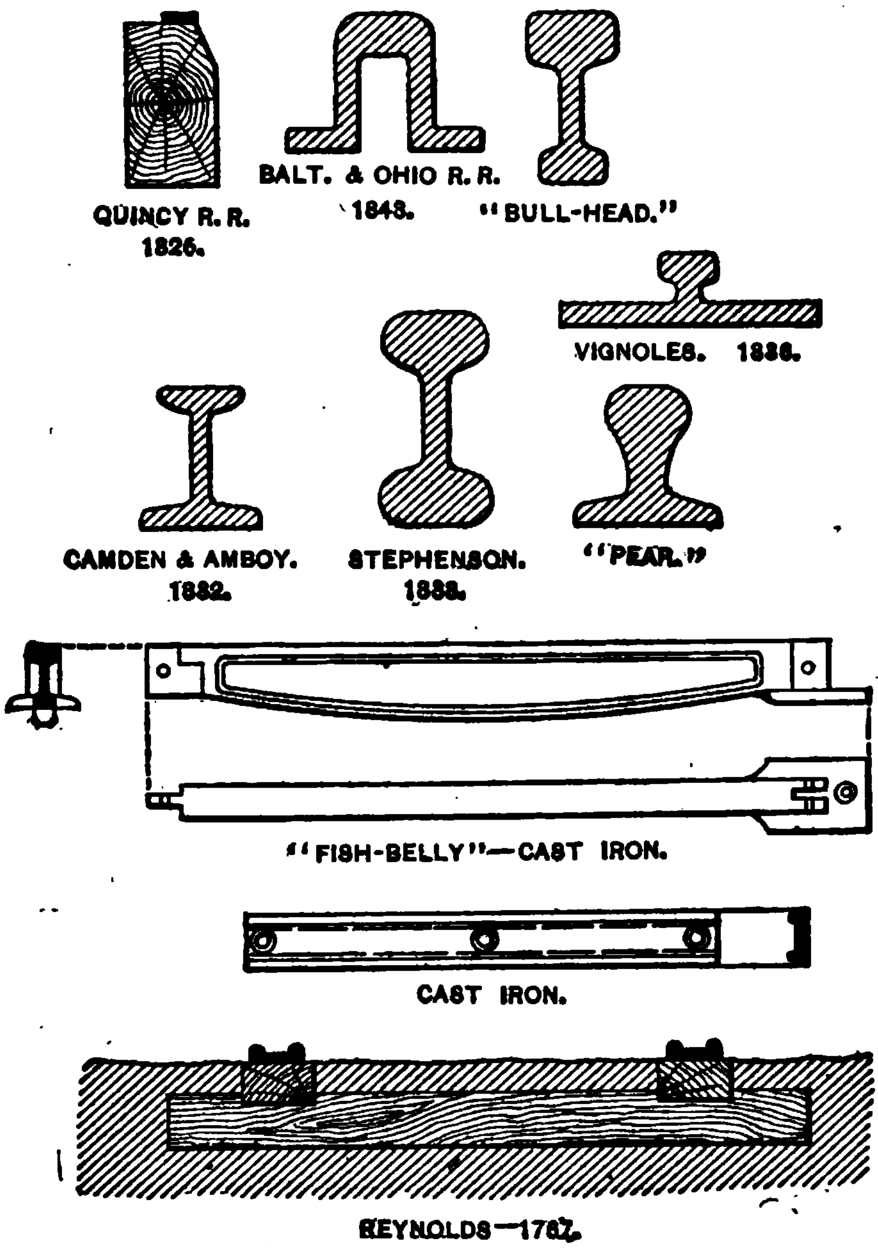

Fig. 115.—Early Forms of Rails.

If the rail is turned over, the worn places, caused by the chairs, make a rough track and the rail appears to be more brittle and subject to fracture, possibly due to the crystallization that may have occurred during the previous usage and to the reversal of stresses in the fibers. Whatever the explanation, experience has demonstrated the *fact*. The "bull-headed" rail has the lower head only large enough to properly hold the wooden keys with which the rail is secured to the chairs (see Fig. 116) and furnish the necessary strength. The use of these rails requires the use of two cast-iron chairs for each tie. It is claimed that such track is better for heavy and fast traffic, but it is more

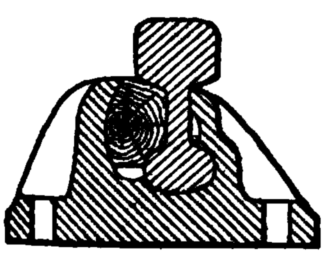

Fig. 116. — Bull-headed Rail and Chair.

expensive to build and maintain. It is the standard form of track in England and some parts of Europe.

Until after 1893 there was a very great multiplicity in the designs of "T" rails as used in this country, nearly every prominent railroad having its own special design, which perhaps differed from that of some other road by only a very minute and insignificant detail, but which nevertheless would require a complete new set of rolls for rolling. This had a very appreciable effect on the cost of rails. In 1893, the American Society of Civil Engineers, after a very exhaustive investigation of the

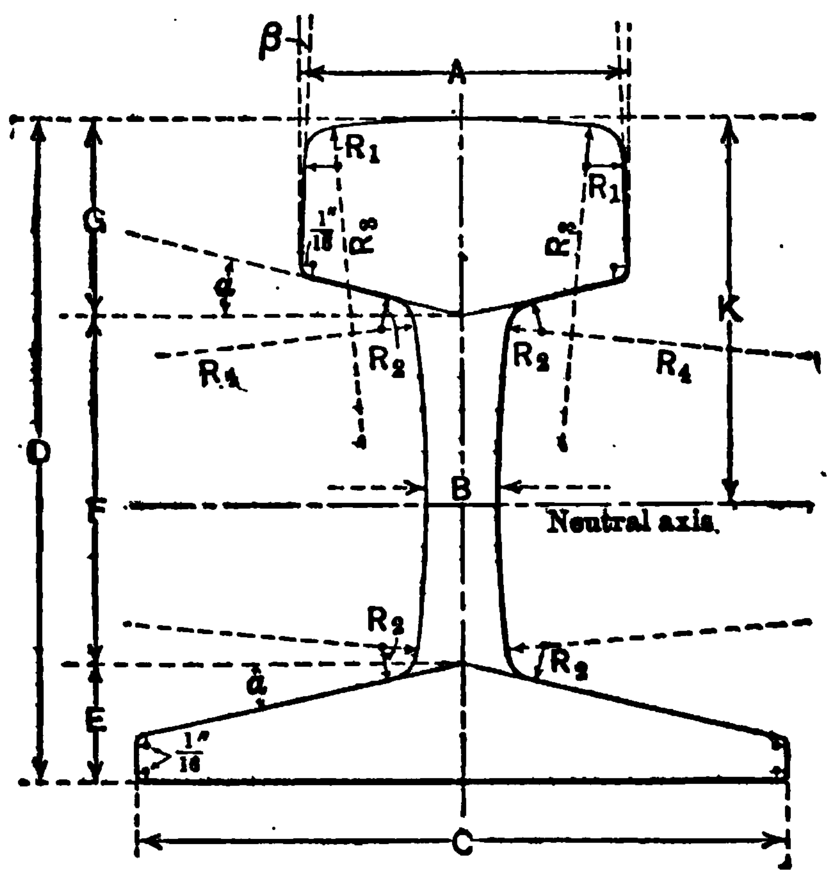

FIG. 117.—STANDARD RAIL SECTIONS.

subject, extending over several years, having obtained the opinions of the best experts of the country, adopted a series of sections which have been very extensively adopted by the railroads of this country.

In 1909 the American Railway Association and the American Railway Engineering Association, by combined action, developed a series of sections. Fig. 117 shows diagrammatically all of these sections and their variations with different weights and systems are shown by the tabular values for the lettered dimensions. It may be noted that the radii of the upper and lower corners of the flanges and of the lower corners of the head are constant ($\frac{1}{16}''$) for all weights of rail and for all systems.

TABLE XXIII.—ANGLES AND DIMENSIONS OF STANDARD DESIGNS FOR RAILS.

System.		Radii, inches.				Angles.		Weight of rail, lbs. per yard.	Dimensions, inches.							
		Upper corner of head. R_1	Fillet corners. R_2	Top of head. R_3	Side of web. R_4	Bottom of head and top of flange. α	Side of head. β		A	B	C	D	E	F	G	K
American Society of Civil Engineers		5/16	1/4	12	12	13°	Vert.	60 70 80 90 100	2 3/8 2 7/16 2 1/2 2 5/8 2 3/4	33/64 35/64 35/64 9/16 9/16	4 1/4 4 5/8 5 5 3/8 5 3/4	4 1/4 4 5/8 5 5 3/8 5 3/4	41/64 11/16 7/8 57/64 31/32	2 17/64 2 15/32 2 5/8 2 55/64 3 5/64	1 7/32 1 11/32 1 1/2 1 19/32 1 45/64	2.20 2.40 2.62 2.82 3.02
American Railway Association and American Railway Engineering Association	A	3/8	3/8	14	14	4:1 14° 02′	1:16 3° 35′	60 70 80 90 100	2 1/4 2 3/8 2 1/2 2 9/16 2 5/8	15/32 1/2 33/64 9/16 9/16	4 4 1/4 4 5/8 5 1/8 5 1/2	4 1/4 4 3/4 5 1/8 5 5/8 6	11/16 23/32 27/32 1 1 1/16	2 21/64 2 1/2 2 45/64 3 3/32 3 3/8	1 17/32 1 11/16 1 7/16 1 15/32 1 9/16	2.37 2.55 2.81 3.09 3.25
	B	3/8	5/16	12	12	13°	3°	60 70 80 90 100	2 1/8 2 3/8 2 7/16 2 9/16 2 21/32	33/64 35/64 35/64 9/16 9/16	3 11/16 4 3/32 4 7/16 4 49/64 5 3/64	4 3/16 4 35/64 4 13/16 5 9/64 5 41/64	7/8 63/64 1 1 1/32 1 5/64	2 1/16 2 17/64 2 15/32 2 5/8 2 53/64	1 1/4 1 21/64 1 13/32 1 33/64 1 41/64	2 15/64 2 29/64 2 87/128 2 115/128 3 7/64
	*	3/8	† 3/8 and 5/8	14	14	4:1 14° 02′	1:16	100 110 120	2 11/16 2 23/32 2 7/8	9/16 19/32 5/8	5 3/8 5 1/2 5 3/4	6 6 1/4 6 1/2	1 1/16 1 1/8 1 3/16	3 9/32 3 15/32 3 21/32	1 21/32 1 23/32 1 25/32	3 1/4 3 27/64 3 37/64

* Last three sections adopted by the A. R. E. A. in 1915. † Fillet radius under head, 3/8″; that above base, 5/8″.

The chief features of disagreement among railroad men relate to the radius of the upper corner of the head and the slope of the side of the head. The radius ($\frac{5}{16}''$) adopted by the A. S. C. E. for the upper corner (constant for all weights) is a little more than is advocated by those in favor of "sharp corners" who prefer a radius of $\frac{1}{4}''$. On the other hand it is much less than is advocated by those who consider that it should be nearly equal to (or even greater than) the larger radius universally adopted for the corner of the wheel-flange. The discussion turns on the relative rapidity of rail wear and the wear of the wheel-flanges as affected by the relation of the form of the wheel-tread to that of the rail. It is argued that sharp rail corners wear the wheel-flanges so as to produce sharp flanges, which are liable to cause derailment at switches and also to require that the tires of engine-drivers must be more frequently turned down to their true form. On the other hand it is generally believed that rail wear is much less rapid when the area of contact between the rail and wheel-flange is small, and that when the rail has worn down, as it invariably does, to nearly the same form as the wheel-flange, the rail wears away very quickly. The A. R. E. A. system uses $\frac{3}{8}''$ radius for all rail weights. The "B" sections were proposed to satisfy those that desired that the head should be narrower and deeper than as found in the "A" sections. The A. R. E. A. Manual (1915), suggests that if a section is found to be inadequate because of lack of depth of head, the next heavier section will be found more desirable and economical.

FIG. 118.—RELATION OF RAIL TO WHEEL-TREAD.

268. Weight for various kinds of traffic. The heaviest rails in regular use weigh 120 lbs. per yard, and even these are only used on some of the heaviest traffic sections of such roads as the N. Y. Central, the Pennsylvania, the N. Y., N. H. & H., and a few others. Probably the larger part of the mileage of the country is laid with 70- to 80-lb. rails—considering the fact that "the larger part of the mileage" consists of comparatively light-traffic roads and may exclude all the heavy trunk lines. Very light-traffic roads are sometimes laid with 56-lb. rails. Roads with fairly heavy traffic generally use 85- to 95-lb. rails, espe-

cially when grades are heavy and there is much and sharp curvature. The tendency on all roads is toward an increase in the weight, rendered necessary on account of the increase in the weight and capacity of rolling stock, and due also to the fact that the price of rails has been so reduced that it is both better and cheaper to obtain a more solid and durable track by increasing the weight of the rail rather than by attempting to support a weak rail by an excessive number of ties or by excessive track labor in tamping. It should be remembered that in buying rails the mere weight is, in one sense, of no importance. The important thing to consider is the STRENGTH and the STIFFNESS. If we assume that all weights of rails have *similar* cross-sections (which is nearly although not exactly true), then, since for beams of similar cross-sections the *strength* varies as the *cube* of the homologous dimensions and the *stiffness* as the *fourth power*, while the area (and therefore the weight per unit of length) only varies as the *square*, it follows that the stiffness varies as the square of the weight, and the strength as the $\frac{3}{2}$ power of the weight. Since for ordinary variations of weight the price per ton is the same, adding (say) 10% to the weight (and cost) adds 21% to the stiffness and over 15% to the strength. As another illustration, using an 80-lb. rail instead of a 75-lb. rail adds only $6\frac{2}{3}$% to the cost, but adds about 14% to the stiffness and nearly 11% to the strength. This shows why heavier rails are more economical and are being adopted even when they are not absolutely needed on account of heavier rolling stock. The stiffness, strength, and consequent durability are increased in a much greater ratio than the cost.

The relation between weight of rail and the weight on the drivers of the locomotives which are to run on it has been briefly expressed by the Baldwin Locomotive Works as "300 pounds of wheel per pound of rail per yard." This rule may be utilized by making a diagram as shown in Fig. 119. For example, if it is desired to use a type of locomotive with 170,000 lbs. on the drivers and also 75-lb. rails, four pairs of drivers will be needed and such a type of locomotive should be used. By using 95-lb. rails the same weight on the drivers could be placed on three axles. As another example, a Pacific-type locomotive, with 150,000 lbs. on its six drivers, should have a rail with a minimum weight of 83 lbs., or say an 85-lb. rail. Whatever elements are given, the corresponding proper value for the other element may be derived.

269. Effect of stiffness on traction. A very important but generally unconsidered feature of a stiff rail is its effect on tractive force. An extreme illustration of this principle is seen when a vehicle is drawn over a soft sandy road. The constant compression of the sand in front of the wheel has virtually the same effect on traction as drawing the wheel up a grade whose

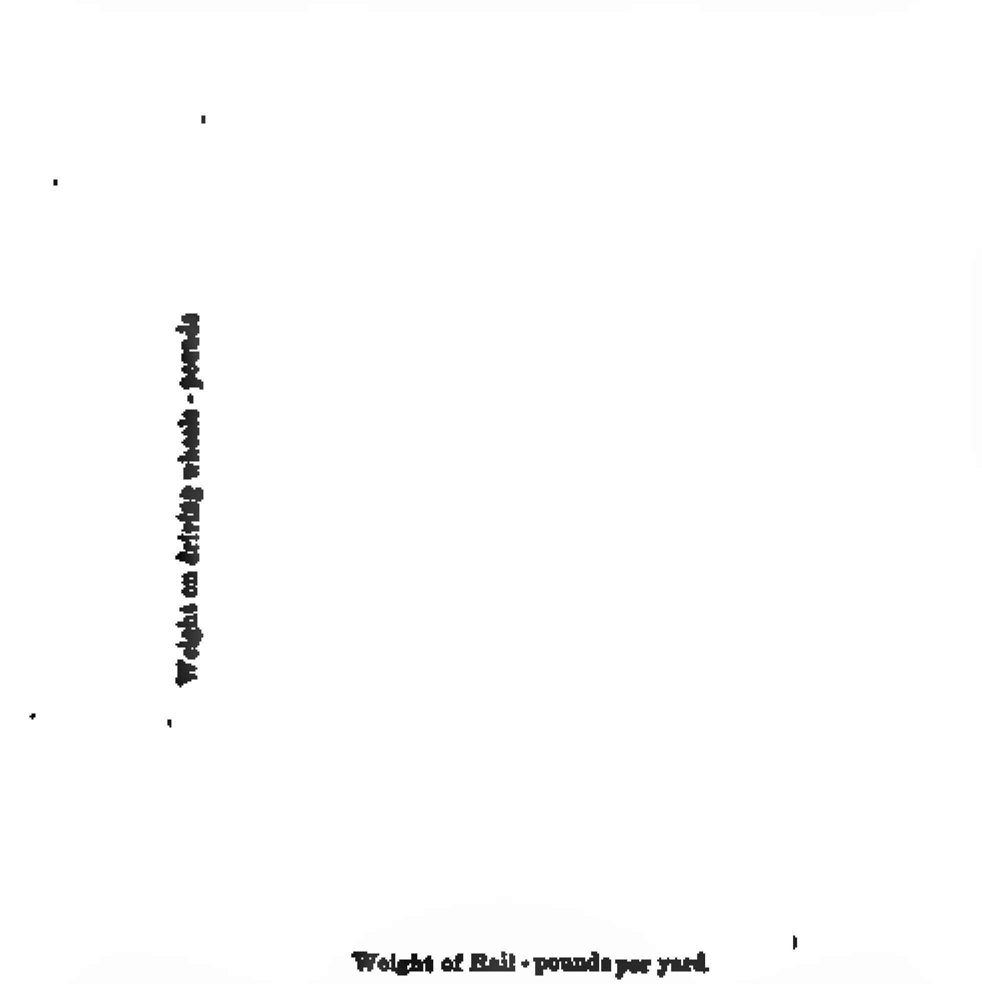

FIG. 119.—CURVES FOR FINDING THE NUMBER OF DRIVERS NEEDED FOR GIVEN WEIGHT ON DRIVING WHEELS AND WEIGHT OF RAILS.

steepness depends on the radius of the wheel and the depth of the rut. On the other hand, if a wheel, made of perfectly elastic material, is rolled over a surface which, while supported with absolute rigidity, is also perfectly elastic, there would be a forward component, caused by the expanding of the compressed metal just behind the center of contact, which would just balance the backward component. If the rail was supported throughout its length by an absolutely rigid support, the high elasticity of the wheel-tires and rails would reduce this form of

resistance to an insignificant quantity, but the ballast and even the ties are comparatively inelastic. When a weak rail yields, the ballast is more or less compressed or displaced, and even though the elasticity of the rail brings it back to nearly its former place, the work done in compressing an inelastic material is wholly lost. The effect of this on the fuel account is certainly very considerable and yet is frequently entirely overlooked. It is practically impossible to compute the saving in tractive power, and therefore in cost of fuel, resulting from a given increase in the weight and stiffness of the rail, since the yielding of the rail is so dependent on the spacing of the ties, the tamping, etc. But it is not difficult to perceive in a general way that such an economy is possible and that it should not be neglected in considering the value of stiffness in rails.

270. Length of rails. The recommended standard minimum length of rails is 33 feet. In recent years many roads have been trying 45-foot and even 60-foot rails. The argument in favor of longer rails is chiefly that of the reduction in track-joints, which are costly to construct and to maintain and are a fruitful source of accidents. Mr. Morrison of the Lehigh Valley R. R.* declares that, as a result of extensive experience with 45-foot rails on that road, he finds that they are much less expensive to handle, and that, being so long, they can be laid around sharp curves without being curved in a machine, as is necessary with the shorter rails. The great objection to longer rails lies in the difficulty in allowing for the expansion, which will require, in the coldest weather, an opening at the joint of nearly $\frac{3}{4}''$ for a 60-foot rail. The Pennsylvania R. R. and the Norfolk and Western R. R. each have a considerable mileage laid with 60-foot rails.

271. Expansion of rails. Steel expands at the rate of .0000065 of its length per degree Fahrenheit. The extreme range of temperature to which any rail will be subjected will be about 160°, or say from $-20°$ F. to $+140°$ F. With the above coefficient and a rail length of 60 feet the expansion would be 0.0624 foot, or about $\frac{3}{4}$ inch. But it is doubtful whether there would ever be such a range of motion even if there were such a range of temperature. Mr. A. Torrey, chief engineer of the Mich. Cent. R. R., experimented with a section over 500 feet long, which,

* Report, Roadmasters Association, 1895.

although not a single rail, was made "continuous" by rigid splicing, and he found that there was no appreciable additional contraction of the rail at any temperature below +20° F. The reason is not clear, but the *fact* is undeniable.

The heavy girder rails, used by the street railroads of the country, are bonded together with perfectly tight rigid joints which do not permit expansion. If the rails are laid at a temperature of 60° F. and the temperature sinks to 0°, the rails have a *tendency* to contract .00039 of their length. If this tendency is resisted by the friction of the pavement in which the rails are buried, it only results in a tension amounting to .00039 of the modulus of elasticity, or say 10920 pounds per square inch, assuming 28 000 000 as the modulus of elasticity. This stress is not dangerous and may be permitted. If the temperature rises to 120° F., a tendency to expansion and buckling will take place, which will be resisted as before by the pavement, and a compression of 10920 pounds per square inch will be induced, which will likewise be harmless. The range of temperature of rails which are buried in pavement is much less than when they are entirely above the ground and will probably never reach the above extremes. Rails supported on ties which are only held in place by ballast must be allowed to expand and contract almost freely, as the ballast cannot be depended on to resist the distortion induced by any considerable range of temperature, especially on curves.

272. Rules for allowing for temperature. Track regulations generally require that the track foremen shall use iron (*not* wooden) shims for placing between the ends of the rails while splicing them. The thickness of these shims should vary with the temperature. Some roads use such approximate rules as the following: "The proper thickness for coldest weather is $\frac{5}{16}$ of an inch; during spring and fall use $\frac{1}{8}$ of an inch, and in the very hottest weather $\frac{1}{16}$ of an inch should be allowed." This is on the basis of a 30-foot rail. When a more accurate adjustment than this is desired, it may be done by assuming some very high temperature (100° to 125° F.) as a maximum, when the joints should be *tight;* then compute in tabular form the spacing for each temperature, varying by 25°, allowing 0″.0643 (very nearly $\frac{1}{16}$″) for each 25° change. Such a tabular form would be about as follows (rail length 33 feet):

Temperature . .	Over 100°	100°–75°	75°–50°	50°–25°	25°–0°	Below 0°
Rail opening. . .	Close	$\frac{1}{16}''$	$\frac{1}{8}''$	$\frac{3}{16}''$	$\frac{1}{4}''$	$\frac{5}{16}''$

One practical difficulty in the way of great refinement in this work is the determination of the real temperature of the rail when it is laid. A rail lying in the hot sun has a very much higher temperature than the air. The temperature of the rail cannot be obtained even by exposing a thermometer directly to the sun, although such a result might be the best that is easily obtainable. On a cloudy or rainy day the rail has practically the same temperature as the air; therefore on such days there need be no such trouble.

273. **Chemical composition.** About 98 to 99.5% of the composition of steel rails is iron, but the value of the rail, as a rail, is almost wholly dependent upon the large number of other chemical elements which are, or may be, present in very small amounts. The manager of a steel-rail mill once declared that their aim was to produce rails having in them—

Carbon.	0.32 to 0.40%
Silicon.	0.04 to 0.06%
Phosphorus.	0.09 to 0.105%
Manganese.	1.00 to 1.50%

The analysis of 32 specimens of rails on the Chic., Mil. & St. Paul R. R. showed variations as follows:

Carbon.	0.211 to 0.52%
Silicon.	0.013 to 0.256%
Phosphorus.	0.055 to 0.181%
Manganese.	0.35 to 1.63%

These quantities have the same general relative proportions as the rail-mill standard given above, the differences lying mainly in the broadening of the limits. Increasing the percentage of carbon by even a few hundredths of one per cent makes the rail harder, but likewise more brittle. If a track is well ballasted and not subject to heaving by frost, a harder and more brittle rail may be used without excessive danger of breakage, and such a rail will wear much longer than a softer tougher

rail, although the softer tougher rail may be the better rail for a road having a less perfect roadbed.

A small but objectionable percentage of sulphur is sometimes found in rails, and very delicate analysis will often show the presence, in very minute quantities, of several other chemical elements. The use of a very small quantity of nickel or aluminum has often been suggested as a means of producing a more durable rail. The added cost and the uncertainty of the amount of advantage to be gained has hitherto prevented the practical use or manufacture of such rails.

274. Proposed standard specifications for steel rails. The following specifications for steel rails are those proposed by a committee of the American Railway Engineering Association in March, 1910:

PROCESS OF MANUFACTURE.

1. The entire process of manufacture shall be in accordance with the best current state of the art.

(*a*) Ingots shall be kept in a vertical position until ready to be rolled, or until the metal in the interior has had time to solidify.

(*b*) Bled ingots shall not be used.

CHEMICAL COMPOSITION.

2. The chemical composition of the steel from which the rails are rolled shall be within the following limits:

	Bessemer.		Open-hearth.	
	80 lbs. and under.	85 to 100 lbs. inclusive.	80 lbs. and under.	85 to 100 lbs. inclusive.
Carbon	0.40 to 0.50	0.45 to 0.55	0.53 to 0.66	0.63 to 0.76
Manganese	0.80 to 1.10	0.85 to 1.15	0.75 to 1.00	0.75 to 1.00
Silicon	0.10 to 0.20	0.10 to 0.20	0.10 to 0.20	0.10 to 0.20
Phosphorus not to exceed	0.10	0.10	0.04	0.04
Sulphur not to exceed	0.075	0.075	0.06	0.06

3. When lower phosphorus can be secured in Bessemer or open-hearth steel, the carbon shall be increased at the rate of 0.035 for each 0.01 reduction in phosphorus.

The percentages of carbon, manganese, and silicon in an entire order of rails shall average as high as the mean percentages between the upper and lower limits.

SHEARING.

4. There shall be sheared from the end of the bloom formed from the top of the ingot, sufficient discard to insure sound rails. All metal from the top of the ingot, whether cut from the bloom or the rail, is the top discard.

SHRINKAGE.

5. The number and passes and speed of train shall be so regulated that, on leaving the rolls at the final pass, the temperature of the rails will not exceed that which requires a shrinkage allowance at the hot saws, for a 33-ft. rail of 100 lbs. section of $6\frac{1}{2}$ ins., and $\frac{1}{8}$ in. less for each 10 lbs. decrease of section, these allowances to be decreased at the rate of $\frac{1}{100}$ in. for each second. of time elapsed between the rail leaving the finishing rolls and being sawed. The bars shall not be held for the purpose of reducing their temperature, nor shall any artificial means of cooling them be used between the leading and finishing passes, nor after they leave the finishing pass.

SECTION.

6. The section of rail shall conform as accurately as possible to the templet furnished by the railroad company. A variation in height of $\frac{1}{64}$ in. less or $\frac{1}{32}$ in. greater than the specified height, and $\frac{1}{16}$ in. in width of flange, will be permitted; but no variations shall be allowed in the dimensions affecting the fit of splice bars.

WEIGHT.

7. The weight of the rail shall be maintained as nearly as possible, after complying with the preceding paragraph, to that specified in the contract.

A variation of one-half of one per cent from the calculated weight of section, as applied to an entire order, will be allowed.

Rails will be accepted and paid for according to actual weight.

LENGTH.

8. The standard length of rail shall be 33 ft. Ten per cent of the entire order will be accepted in shorter lengths varying

as follows: 30 ft., 28 ft., and 26 ft. A variation of $\frac{1}{4}$ in. from the specified length will be allowed.

All No. 1 rails less than 33 ft. shall be painted green on both ends.

STRAIGHTENING.

9. Care shall be taken in hot-straightening rails, and it shall result in their being left in such condition that they will not vary throughout their entire length more than four (4) ins. from a straight line in any direction when delivered to the cold-straightening presses. Those which vary beyond that amount, or have short kinks, shall be classed as second quality rails and be so marked. The distance between supports of rails in the straightening press shall not be less than forty-two (42) ins.; supports to have flat surfaces and out of wind. Rails shall be straight in line and surface and smooth on head when finished, final straightening being done while cold. They shall be sawed square at ends, variations to be not more than $\frac{1}{32}$ in., and prior to shipment shall have the burr caused by the saw cutting removed and the ends made clean.

DRILLING.

10. Circular holes for joint bolts shall be drilled in accordance with specifications of the purchaser. They shall in every respect conform accurately to drawing and dimensions furnished and shall be free from burrs.

BRANDING.

11. The name of the maker, the weight of the rail, and the month and year of manufacture shall be rolled in raised letters and figures on the side of the web. The number of the heat and a letter indicating the portion of the ingot from which the rail was made shall be plainly stamped on the web of each rail, where it will not be covered by the splice bars. Rails to be lettered consecutively A, B, C, etc., the rail from the top of the ingot being A. In case of a top discard of twenty or more per cent the letter A will be omitted. Open-hearth rails to be branded "O. H."

DROP TESTS.

12. Drop tests shall be made on pieces of rail rolled from the top of the ingot, not less than four (4) ft. and not more than six (6) ft. long, from each heat of steel. These test pieces shall be

cut from the rail bar next to either end of the top rail, as selected by the inspector.

The temperature of the test piece shall be between forty (40) and one hundred (100) degrees Fahrenheit.

The test pieces shall be placed head upward on solid supports, five (5) ins. top radius, three (3) ft. between centers, and subjected to impact tests, the tup falling free from the following heights:

60- and 70-lb. rail	16 ft.
80-, 85-, and 90-lb. rail	18 ft.
100-lb. rail	20 ft.

The test pieces which do not break under the first drop shall be nicked and tested to destruction.

DEFLECTION.

13. It is proposed to prescribe, under this head, the requirements in regard to deflection, fixing maximum and minimum limits, as soon as proper deflection limits have been decided on.

(*a*) Two pieces shall be tested from each heat of steel. If either of these test pieces breaks, a third piece shall be tested. If two of the test pieces break without showing physical defect, all rails of the heat will be rejected absolutely. If two of the test pieces do not break, all rails of the heat will be accepted as No. 1 or No. 2 classification, according as the deflection is less or more, respectively, than the prescribed limit.*

(*b*) If, however, any test piece broken under test "A" shows physical defect, the top rail from each ingot of that heat shall be rejected.

(*c*) Additional tests shall then be made of test pieces selected by the inspector from the top end of any second rails of the same heat. If two of out three of these second test pieces break, the remainder of the rails of the heat will also be rejected. If two out of three of these second test pieces do not break, the remainder of the rails of the heat will be accepted, provided they conform to the other requirements of these specifications, as No. 1 or No. 2 classification, according as the deflection is less or more, respectively, than the prescribed limit.*

(*d*) If any test piece, test "A," does not break, but when nicked

* This clause to be added when the deflection limits are specified.

and tested to destruction shows interior defect, the top rails from each ingot of that heat shall be rejected.

DROP-TESTING MACHINE.

14. The drop-testing machine shall be the standard of the American Railway Engineering and Maintenance of Way Association, and have a tup of 2000 lbs. weight, the striking face of which shall have a radius of five (5) ins.

The anvil block shall be adequately supported and shall weigh 20 000 lbs.

The supports shall be a part of or firmly secured to the anvil.

NO. 1 RAILS.

15. No. 1 rails shall be free from injurious defects and flaws of all kinds.

NO. 2 RAILS.

16. Rails which, by reason of surface imperfections, are not accepted as No. 1 rails, will be classed as No. 2 rails, but rails which in the judgment of the inspector contain physical defects which impair their strength, shall be rejected.

No. 2 rails to the extent of five (5) per cent of the whole order will be received. All rails accepted as No. 2 rails must have the ends painted white, and shall have two prick punch marks on the side of the web near the heat number near the end of the rail, so placed as not to be covered by the splice bars.

Rails improperly drilled or straightened, or from which the burrs have not been properly removed, shall be rejected, but may be accepted after being properly finished.

All classes of rails must be kept separate from each other and shipped in separate cars.

All rails must be loaded in the presence of the inspector.

INSPECTION.

17. (*a*) Inspectors representing the purchaser shall have free entry to the works of the manufacturer at all times while the contract is being executed, and shall have all reasonable facilities afforded them by the manufacturer to satisfy them that the rails have been made in accordance with the terms of the specifications.

(*b*) For Bessemer steel the manufacturer shall, before the rails

are shipped, furnish the inspector daily with carbon determinations for each heat, and two complete chemical analyses every twenty-four hours representing the average of the other elements contained in the steel, for each day and night turn. These analyses shall be made on drillings taken from small test ingots. The drillings for analyses shall be taken from the ladle test ingot at a distance of ¼ in. beneath the surface.

For open-hearth steel, the makers shall furnish the inspectors with the complete chemical analysis for each melt.

(c) On request of the inspector, the manufacturer shall furnish a portion of the test ingot for check analysis.

(d) All tests and inspections shall be made at the place of manufacture, prior to shipment, and shall be so conducted as not to unnecessarily interfere with the operation of the mill.

(e) Rails to be accepted must meet all of the requirements of the specifications.

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275. Rail wear on tangents. When the wheel loads on a rail are abnormally heavy, and particularly when the rail has but little carbon and is unusually soft, the concentrated pressure on the rail is frequently greater than the elastic limit, and the metal "flows" so that the head, although greatly abraded, will spread somewhat outside of its original lines, as shown in Fig. 120. The rail wear that occurs on tangents is almost exclusively on top.

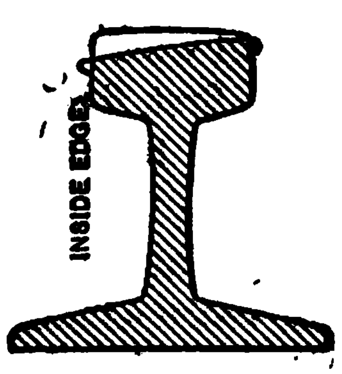

FIG. 120.

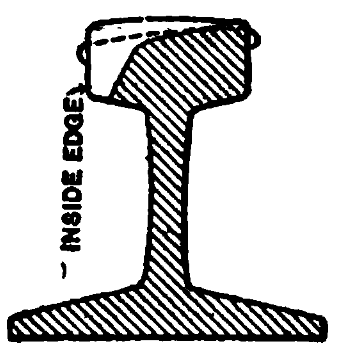

FIG. 121.

276. Rail wear on curves. On curves the maximum rail wear occurs on the inner side of the head of the outer rail, giving a worn form somewhat as shown in Fig. 121. The dotted line shows the nature and progress of the rail wear on the inner rail of a curve. Since the pressure on the outer rail is somewhat lateral rather than vertical, the "flow" does not take place to the same extent, if at all, on the outside, and whatever flow would take place on the inside is immediately worn off by the wheel-

flange. Unlike the wear on tangents, the wear on curves is at a greater rate as the rail becomes more worn.

The inside rail on curves wears chiefly on top, the same as on a tangent, except that the wear is much greater owing to the longitudinal slipping of the wheels on the rail, and the lateral slipping that must occur when a rigid four-wheeled truck is guided around a curve. The outside rail is subjected to a greater or less proportion of the longitudinal slipping, likewise to the lateral slipping, and, worst of all, to the grinding action of the flange of the wheel, which grinds off the side of the head.

277. Experimental determination of rail wear. Several years ago a series of tests for rail wear were made on the Northern Pacific R. R. by taking up, weighing, and replacing, each year, the several groups of rails under test. Some of these rails were on tangents, the others on curves of various curvature. Some of the rails of each group were made of Bessemer steel, the others of open-hearth steel. No tests were made to determine the loss of weight through mere oxidation. All of the rails were in service for five years and some lasted for six years or more, but the loss in weight during the sixth year was nearly always equal to, and in some cases twice as much as, the loss during the preceding five years. Some of the rails lost over 10% of their weight, or about one-fourth the weight of the head, before being removed. Although the tests were too few to establish any positive laws, some tendencies which may be observed will give at least an approximate idea of the laws of rail wear.

1. The average loss of weight during the first five years on 20 rails on tangents was 0.412 lb. per yard per 10,000,000 tons of traffic.

2. Ten of these same rails were kept in place at least one year longer and during the sixth year lost almost twice as much metal as during the previous five years; in other words, about two-thirds of the entire loss occurred during the sixth year.

3. The average loss of weight during the first five years from 20 rails on a tangent was 0.463 lb. per yard per 10,000 trains. The relation between mere tonnage and number of trains could not be even indicated by so few tests. There is reason to believe that engine drivers are more responsible for rail wear than mere car-wheel tonnage. This practically means that one effect of grade is to increase rail wear, since more (or heavier) engines are needed to haul a given car tonnage.

4. The wear of the outer rail of curves is, of course, far greater than that of the inner rail, but the figures obtained did not seem to follow any rational law, the ratio of outer to inner rail wear varying from 144 to 244%, with an average of 182%.

5. The average rail wear on curves, averaging inner and outer rails, per yard, per degree of curve, per 10,000,000 tons traffic, varied from 0.145 lb. for a 4° .04′ curve down to 0.102 lb. per degree for a 10° 13′ curve. Based on the four curves tested, the results seemed to point to the law that rail wear on curves does *not* increase as fast as the degree of the curve.

6. Although the tests were too few to establish any law, the increase of the mean rail wear on curves with increase in degree of curve was very regular and indicated that the average rail wear on a curve of about 6° 40′ is about twice as great as that on a tangent.

7. The wear on open-hearth rails was almost invariably less than that on Bessemer rails, under identical conditions.

278. Cost of rails. In 1873 the cost of steel rails was about $120 per ton, and the cost of iron rails about $70 per ton. Although the steel rails were at once recognized as superior to iron rails on account of more uniform wear, they were an expensive luxury. The manufacture of steel rails by the Bessemer process created a revolution in prices, and they steadily dropped in price until, many years ago, steel rails were manufactured and sold for $22 per ton. For several years since then the price was very uniform at $28 per ton at the mill. But now (1916) the advantages of open-hearth steel are better appreciated and a large proportion of rails are being rolled from open-hearth steel, which commands about $2 per ton more. At present (1916) the current prices at Pittsburgh mills run at about $33 per ton for Bessemer and $35 for open-hearth.

At such prices there is no longer any demand for iron rails, since the cost of manufacturing them is substantially the same as that of steel rails, while their durability is unquestionably inferior to that of steel rails. Rail quotations are generally on the basis of "long tons" of 2240 lbs.

The freight charge for transporting rails from the mill to the place where used is usually so large that it adds a very appreciable amount to the cost per ton. As an approximation, the freight may be estimated as 0.6 cent per ton-mile, or $3.00 per ton for a haul of 500 miles.

CHAPTER X.

RAIL-FASTENINGS.

RAIL-JOINTS

279. Theoretical requirements for a perfect joint. A perfect rail-joint is one that has the *same strength* and *stiffness*—no more and no less—as the rails which it joins, and which will not interfere with the regular and uniform spacing of ties. It should also be reasonably cheap both in first cost and in cost of maintenance. Since the action of heavy loads on an elastic rail is to cause a wave of translation in front of each wheel, any change in the stiffness or elasticity of the rail structure will cause more or less of a shock, which must be taken up and resisted by the joint. The greater the change in stiffness the greater the shock, and the greater the destructive action of the shock. The perfect rail-joint must keep both rail-ends truly in line both laterally and vertically, so that the flange or tread of the wheel need not jump or change its direction of motion suddenly in passing from one rail to the other. A consideration of all the above requirements will show that only a perfect welding of rail-ends would produce a joint of uniform strength and stiffness which would give a uniform elastic wave ahead of each wheel. As welding is impracticable for ordinary railroad work (see § 271), some other contrivance is necessary which will approach this ideal as closely as may be.

280. Efficiency of the ordinary angle-bar. Throughout the middle portion of a rail the rail acts as a continuous girder. If we consider for simplicity that the ties are unyielding, the deflection of such a continuous girder between the ties will be but one-fourth of the deflection that would be found if the rail were cut half-way between the ties and an equal concentrated load were divided equally between the two unconnected ends. The maximum stress for the continuous girder would be but one-half of that in the cantilevers. Joining these ends with rail-joints will give the ordinary "suspended" joint. In order to main-

tain uniform strength and stiffness the angle-bars must supply the deficiency. These theoretical relations are modified to an unknown extent by the unknown and variable yielding of the ties. From some experiments made by the Association of Engineers of Maintenance of Way of the P. R. R.* the following deductions were made:

1. The capacity of a "suspended" joint is greater than that of a "supported" joint—whether supported on one or three ties. (See § 282.)

2. That (with the particular patterns tested) the angle-bars alone can carry only 53 to 56% of a concentrated load placed on a joint.

3. That the capacity of the whole joint (angle-bars and rail) is only 52.4% of the strength of the unbroken rail.

4. That the ineffectiveness of the angle-bar is due chiefly to a deficiency in compressive resistance.

Although it has been universally recognized that the angle-bar is not a perfect form of joint, its simplicity, cheapness, and reliability have caused its almost universal adoption. Within a very few years other forms (to be described later) have been adopted on trial sections and have been more and more extended, until their present use is very large. These designs all agree in using metal below the base of the rail, as is shown in the several designs on Plate VII, but the general type shown in Fig. 119 is still (1916) in most common use.

281. Effect of rail gap at joints. It has been found that the jar at a joint is due almost entirely to the *deflection* of the joint and scarcely at all to the small gap required for expansion. This gap causes a drop equal to the versed sine of the arc having a chord equal to the gap and a radius equal to the radius of the wheel. Taking the extreme case (for a 30-foot rail) of a $\frac{3}{8}''$ gap and a 33" freight-car wheel, the drop is about $\frac{1}{1000}''$. In order to test how much the jarring at a joint is due to a gap between the rails, the experiment was tried of cutting shallow notches in the top of an otherwise solid rail and running a locomotive and an inspection car over them. The resulting jarring was practically imperceptible and not comparable to the jar produced at joints. Notwithstanding this fact, many plans have

* Roadmasters Association of America—Reports for 1897.

been tried for avoiding this gap. The most of these plans consist essentially of some form of compound rail, the sections breaking joints. (Of course the design of the compound rail has also several other objects in view.) In Fig. 122 are shown a

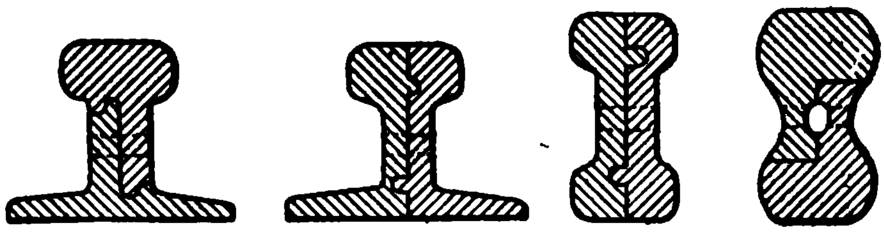

FIG. 122.—COMPOUND RAIL SECTIONS.

few of the very many designs which have been proposed. These designs have invariably been abandoned after trial. Another plan, which has been extensively tried on the Lehigh Valley R. R., is the use of mitered joints. The advantages gained by their use are as yet doubtful, while the added expense is unquestionable. The "Roadmasters Association of America" in 1895 adopted a resolution recommending mitered joints for double track, but their use has been abandoned.

282. "Supported," "suspended," and "bridge" joints. In a supported joint the ends of the rails are on a tie. If the angle-plates are short, the joint is entirely supported on one tie; if very long, it may be possible to place three ties under one angle-bar and thus the joint is virtually supported on three ties rather than one. In a suspended joint the ends of the rails are midway between two ties and the joint is supported by the two. There have always been advocates of both methods, but suspended joints are more generally used than supported joints. The opponents of three-tie joints claim that either the middle tie will be too strongly tamped, thus making it a supported joint, or that, if the middle tie is weakest, the joint becomes a very long (and therefore weak) suspended joint between the outer joint-ties, or that possibly one of the outer joint-ties gives way, thus breaking the angle-plate at the joint. Another objection which is urged is that unless the bars are very long (say 44 inches, as used on the Mich. Cent. R. R.) the ties are too close for proper tamping. The best answer to these objections is the successful use of these joints on several heavy-traffic roads

"Bridge"-joints are similar to suspended joints in that the joint is supported on two ties, but there is the important difference that the bridge joint supports the rail from *underneath* and

there is no transverse stress in the rail, whereas the suspended joint requires the combined transverse strength of both angle-bars and rail. A serious objection to bridge-joints lies in the fact of their considerable thickness between the rail base and the tie. When joints are placed "staggered" (as is now the invariable standard practice), rather than "opposite," the ties supporting a bridge-joint must either be notched down, thus weakening the tie and promoting decay at the cut, or else the tie must be laid on a slope and the joint and the opposite rail do not get a fair bearing.

283. Failures of rail-joints. It has been observed on double-track roads that the maximum rail wear occurs a few inches beyond the rail gap at the joint in the direction of the traffic. On single-track roads the maximum rail wear is found a few inches *each* side of the joint rather than at the extreme ends of the rail, thus showing that the rail end deflects down under the wheel until (with fast trains especially) the wheel actually jumps the space and strikes the rail a few inches beyond the joint, the impact producing excessive wear. This action, which is called the "drop," is apt to cause the first tie beyond the joint to become depressed, and unless this tie is carefully watched and maintained at its proper level, the stresses in the angle-bar may actually become reversed and the bar may break at the top. The angle-bars of a suspended joint are normally in compression at the top. The mere reversal of the stresses would cause the bars

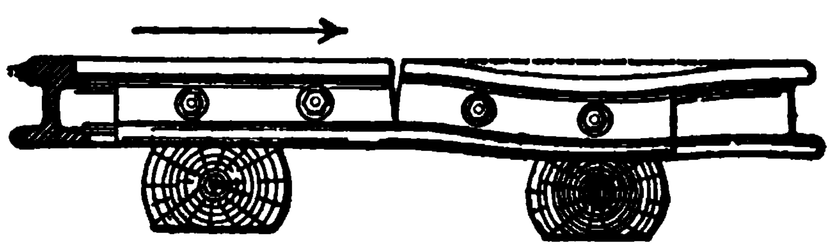

FIG. 123.—EFFECT OF "WHEEL DROP" (EXAGGERATED).

to give way with a less stress than if the stress were always the same in kind. A supported joint, and especially a three-tie joint (see § 282), is apt to be broken in the same manner.

284. Standard angle-bars. An angle-bar must be so made as to closely fit the rails. The great multiplicity in the designs of rails (referred to in Chapter IX) results in a corresponding variety in the detailed dimensions of the angle-bars. The absolutely essential features required for a fit are (1) the angles of the upper and lower surfaces of the bar where they fit against

the rail, and (2) the height of the bar. The bolt-holes in the bar and rail must also correspond. The holes in the angle-plates are elongated or made oval, so that the track-bolts, which are made of corresponding shape immediately under the head, will not be turned by jarring or vibration. The holes in the rails are made of larger diameter (by about $\frac{3}{16}''$) than the bolts, so as to allow the rail to expand with temperature.

In Table XXIV and in Fig. 124 are shown the angles and dimensions for angle-plates to fit the standard rail sections shown in §267. Note that the dimension a for the splice-bar corresponds with dimension F for the rail and that R_4 and the angle α are the same for both for each type of rail. These dimensions were copied from the 1916 Handbook of the Carnegie Steel Co. Although they correspond perfectly with the rail standards of the A. R. E. A., that association has not yet adopted any such definite standard dimensions for a rail-joint.

FIG. 124.—STANDARD ANGLE BAR.

The standard drilling for bolt-holes in splice-bar, as adopted by the A. R. E. A. in 1914, is as follows:

For 6-bolt splices, 5 spaces of 5½ inches.
For 4-bolt splices, 3 spaces of 5½ inches.

No definite recommendation was made by the Association as to the total length of angle bars, but the committee recommended that, on the basis of the above spacing of holes, 24 inches is a satisfactory length for a 4-bolt splice and 32 inches for a 6-bolt splice, in both cases using suspended joints. On this basis, the spacing from the center of the last hole to the end of the bar would be 3¾ inches for the 4-bolt splice and 2¼ inches for the 6-bolt splice.

In Plate VII are shown some of the many designs which have been competing for favor and which have been more or less extensively tried out for both steam and electric railroad work. While many thousands in the aggregate have been placed on various roads, no one design has succeeded in displacing the

TABLE XXIV.—ANGLES AND DIMENSIONS OF STANDARD DESIGNS FOR SPLICE-BARS.

System.		Weight of rail.	R_4-inches.	Angle or slope ratio.			Dimensions, inches.													
				α	δ	γ	a	b	c	d	e	f	g	g'	h	i	j	k	l
A. S. C. E.		60	12	13°	20°	—	2 17/64	1 19/64	43/64		25/64	1 3/16	7/16	7/16	89/128	43/64	21/128	2 5/16	3/8
		70					2 15/32	1 27/64	47/64	11/16	27/64	1 7/32	7/16	7/16	51/64	23/32	11/64	2 1/2	7/16
		80					2 5/8	1 17/32	25/32	3/4	29/64	1 1/4	7/16	7/16	7/8	3/4	3/16	2 3/4	7/16
		90					2 55/64	1 5/8	13/16	13/16	15/32	1 5/16	1/2	1/2	119/128	51/64	13/64	2 15/16	1/2
		100					3 5/64	1 23/32	27/32	7/8	15/32	1 3/8	1/2	1/2	1 5/128	27/32	9/32	3 1/8	1/2
Am. Rwy. Eng. Assoc. and Am. Rwy. Assoc.	A	60	14	14° 02′	23°	14° 02′	2 29/64	1 21/64	45/64	5/8	25/64	1 1/8	1 7/128	65/128	51/128	3/4	45/128	2 7/16	5/8
		70		4:1		4:1	2 1/2	1 27/64	51/64	5/8	13/32	1 3/16	1 7/64	33/64	25/64	27/32	23/64	2 9/16	11/16
		80					2 23/32	1 17/32	7/8	21/32	57/128	1 1/4	1 13/64	39/64	23/64	29/32	25/64	2 3/4	3/4
		90					3 5/32	1 21/32	15/16	23/32	15/32	1 9/32	1 11/32	19/32	9/16	15/16	7/16	3	13/16
		100					3 3/8	1 23/32	31/32	3/4	15/32	1 3/8	1 7/32	3/4	21/32	1	15/32	3 3/16	7/8
	B	60	14	14° 02′	17°	14° 02′	2 1/16	1 3/8	11/16	11/16	51/128	1 1/16	55/64	3/4	11/64	3/4	7/32	2 9/32	23/32
		70		4:1		4:1	2 17/64	1 9/16	13/16	3/4	53/128	1 3/16	55/64	3/4	35/128	51/64	35/128	2 69/128	3/4
		80					2 15/32	1 19/32	27/32	3/4	59/128	1 7/32	1 1/128	27/32	29/128	7/8	17/64	2 31/32	13/16
		90					2 5/8	1 23/32	29/32	13/16	15/32	1 9/32	1/32	29/32	9/32	29/32	17/64	2 105/128	27/64
		100					2 55/64	1 23/32	29/32	13/16	15/32	1 21/64	1 1/32	29/32	51/128	61/64	45/128	3 1/128	7/8

angle-bar. There are necessarily as many variations in the details of the angle-bars as there are variations in the sizes of rails, beside other slight variations, but all cross-sections are similar to that shown in Fig. 124. This general design probably represents the majority of all the splice-plates in the country.

285. Specifications for steel splice-bars. Formerly these were made of either Bessemer or open-hearth steel. Now (1916), the specifications of the A. E. R. A. require open-hearth steel exclusively. Two grades are used. The special requirements of the "high-carbon steel joint bars" are that the phosphorus shall not exceed 0.04%; that the tensile strength of a ½-inch test specimen shall be at least 85,000 lbs. per square inch, the elongation at least 16% in 2 inches and that it shall bend 90° without fracture on the outside around an arc, the diameter of which is three times the thickness of the test piece. Also, they must be punched, slotted and shaped at a temperature of not less than 800° C. or 1470° F. The other grade is "heat-treated, oil-quenched steel joint bars." These must have a tensile strength of at least 100,000 pounds per square inch, a yield point of at least 70,000, an elongation in 2 inches of not less than (1,500,000 ÷ tensile strength) per cent, which must not be less than 12, and also that it shall bend 90° without fracture on the outside around an arc, the diameter of which is 1½ times the thickness of the test piece. The joint bars shall be heated and quenched in an oil bath from a temperature of about 810° C. (1490° F.) and shall be kept in the oil bath until cold enough to be handled. As before, they must be punched, slotted and shaped at a temperature of not less than 800° C. or 1470° F. There are the usual specifications about accuracy of workmanship, marks rolled in the steel, inspection, etc.

TIE-PLATES.

286. Advantages. (*a*) As already indicated in § 242, the life of a soft-wood tie is very much reduced by "rail-cutting" and "spike-killing," such ties frequently requiring renewal long before any serious decay has set in. It has been practically demonstrated that the "rail-cutting" is not due to the mere pressure of the rail on the tie, even with a maximum load on the rail, but is due to the impact resulting from vibration and

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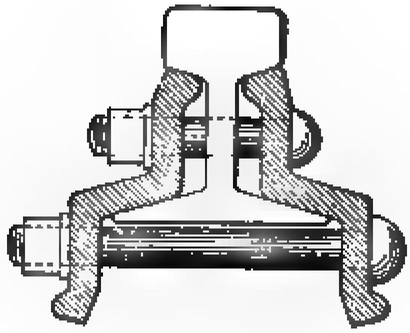
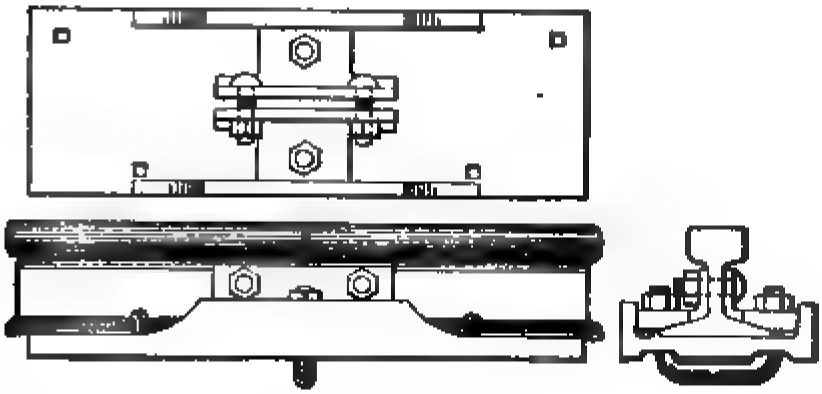
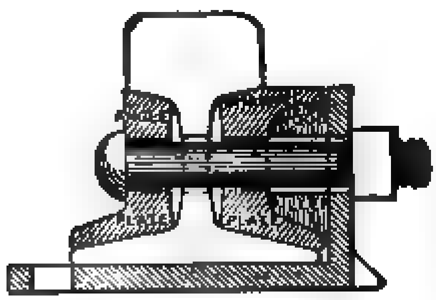


CLOUD JOINT.



FISHER BRIDGE JOINT.



WOLHAUPTER JOINT

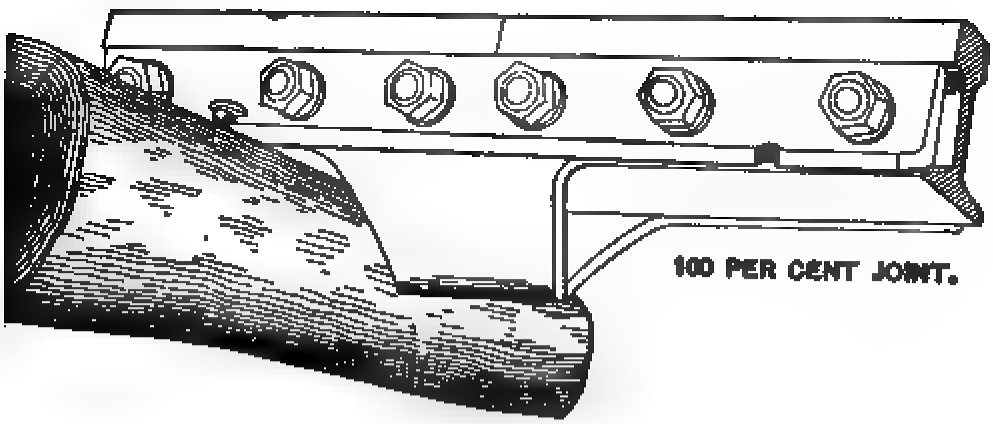
WEBER RAIL JOINT.

PLATE VII.—SOME FORMS OF RAIL JOINTS.

(Between pp. 320 and 321.)

CONTINUOUS RAIL JOINT.

ATLAS SUSPENDED RAIL JOINT.



100 PER CENT JOINT.

BONZANO RAIL JOINT.

to the longitudinal working of the rail. It has been proved that this rail-cutting is practically prevented by the use of tie-plates. (*b*) On curves there is a tendency to overturn the outer rail due to the lateral pressure on the side of the head. This produces a concentrated pressure of the outer edge of the base on the tie which produces rail-cutting and also draws the inner spikes. Formerly the only method of guarding against this was by the use of "rail-braces," one pattern of which is shown in Fig. 125. But shoulder tie-plates serve the purpose even better and rail-braces are now only used for guard rails and stock rails at switches. (*c*) Driving spikes through holes in the plate enables the spikes on *each* side of the rail to mutually support each other, no matter in which (lateral) direction the rail may tend to move, and this probably accounts in large measure for the added stability obtained by the use of tie-plates. (*d*) The wear in spikes, called "necking," caused by the vertical vibration of the rail against them, is very greatly reduced. (*e*) The cost is very small compared with the value of the added life of the tie, the large reduction in the work of track maintenance, and the smoother running on the better track which is obtained. It has been estimated that by the use of tie-plates the life of hard-wood ties is increased from one to three years and the life of soft-wood ties is increased from three to six years. From the very nature of the case, the value of tie-plates is greater when they are used to protect soft ties.

FIG. 125.—ATLAS BRACE K.

287. Elements of the design. The Am. Rwy. Eng. Assoc. has stated these principles in its Manual, as follows:

1. "Plates shall not be less than 6 inches in width, and as much wider as consistent with the class of ties to be used." The use of a wide tie presumes heavy traffic and heavy wheel loads and, therefore, the area of the plate should be increased by widening the plate.

2. "The length of the plates [parallel with the length of the tie] shall not be less than the safe-bearing area of the ties divided

by the width of the plate, and, when made for screw spikes, shall be so shaped as to provide proper support for the screw spikes." 335 lbs. per square inch is declared to be, by test, the minimum safe-bearing load. Tie-plates sometimes sink quickly and deeply into the tie, thus proving that the area is inadequate for the wheel loads and traffic on them.

3. "The thickness of the plate shall be properly proportioned to the length." Tie-plates have been used as thin as $\frac{3}{16}$ inch, but it is now being realized that the real function of the plate is to be a *bearing* plate which shall distribute the load, rather than a mere surface plate which shall protect the tie from abrasion. The Track Committee of the A. R. E. A. recommended that the plates should be at least $\frac{5}{8}$ inch thick under either edge of the rail. Although the Association refused to concur, the discussion developed the fact that the thin plates formerly used have been found to be too thin and that thicker plates are more satisfactory.

4. "Plates shall have a shoulder at least $\frac{1}{2}$ of an inch high. The distance from the edge of rail base to the end of the tie-plate on the outer side must be uniform, and in excess of the projection inside of the rail base."

5. "Where treated ties are used or where plates are for screw spikes, a flat-bottom plate is preferable. Where ribs of any kind are used on base of plate, these shall be few in number and not to exceed $\frac{1}{4}$ inch in depth." This specification is in direct contrast to the older designs which had been corrugations and even "claws" which were forced deeply into the tie, in order to anchor the plate immovably to the tie. But experience has proved that these corrugations hasten deterioration. In spite of this, the type using claws (see Fig. 126) is still the standard on some roads.

6. "Punching must correspond to the slotting in the splice-bars and, where advisable, may be so arranged that the plates may be used for joints. Spike holes may be punched for varying widths of rail base where the slotting will permit such punching without the holes interfering with each other and when the plate is of such design that the additional holes will not impair the strength of the plate."

Tie-plates are variously made of steel, wrought iron and malleable iron. Tie-plates are peculiarly subject to rust, especially as an effect of brine drippings from refrigerator cars. The comparative immunity from rust of malleable iron explains its

use for this purpose. The specifications for steel and wrought iron are similar to other physical tests for such a metal when toughness rather than high ultimate strength is desired. The malleable iron tie-plates have lugs cast on them for testing purposes. When this lug is broken off, it must not break easily, as cast iron, but must show toughness. The fracture must show a narrow band of white metal on the surface, the center portion

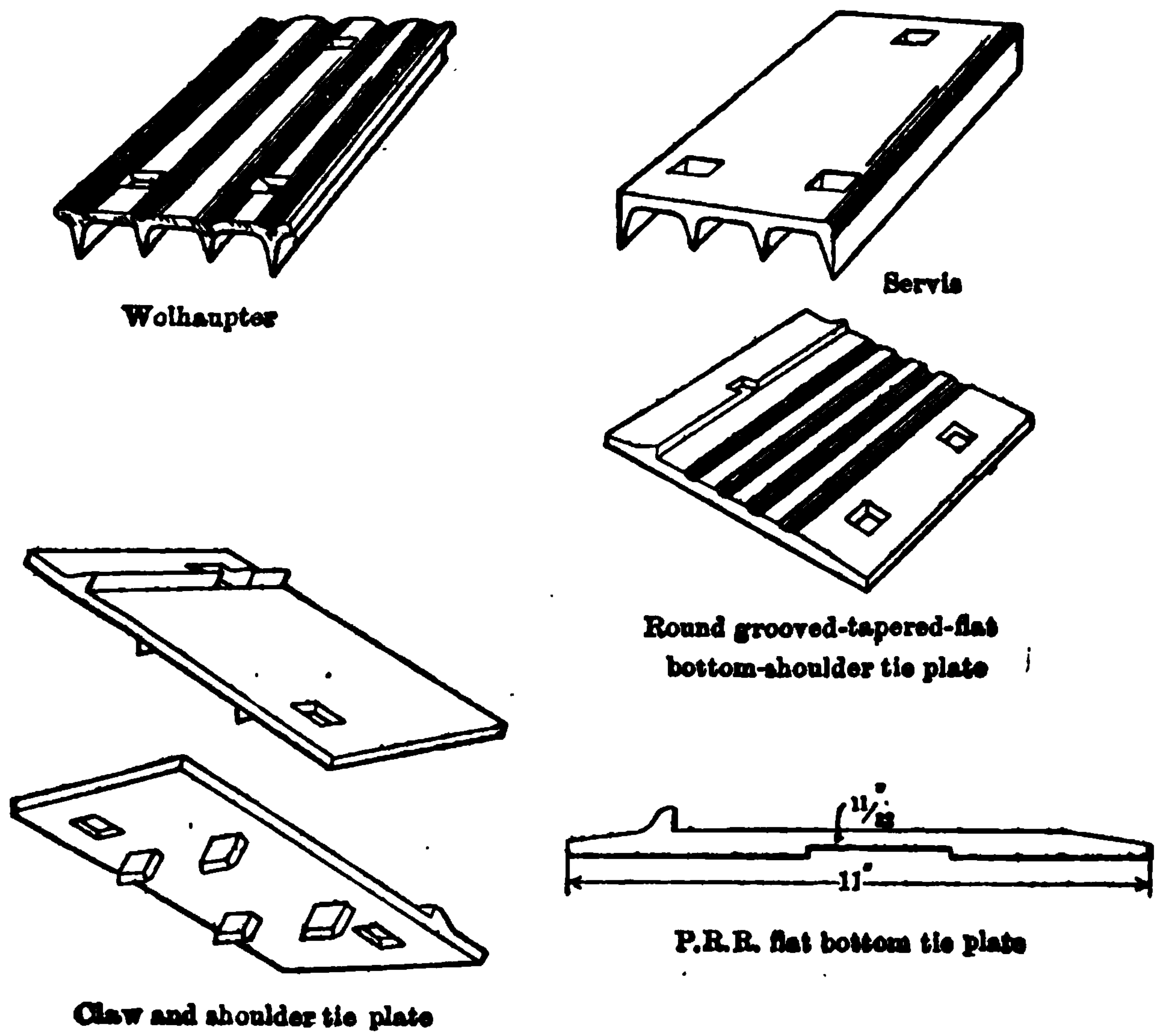


FIG. 126.—VARIOUS FORMS OF TIE-PLATES.

being dark and fiberless. The plates must, when tested, bend sufficiently to prove thorough annealing.

The holes in a tie-plate should be about $\frac{1}{16}''$ larger than the size of the intended spike. The length of the plate, perpendicular to the rail, should be such that there is a shoulder of $1\frac{3}{4}$ to $2\frac{1}{4}$ in. on each side of the rail base; a little more on the outside than on the inside. For very heavy traffic the thickness should be $\frac{1}{2}''$ to $\frac{3}{4}''$; for lighter traffic, they may be as thin as $\frac{3}{8}''$. Flat-bottom plates should be at least $\frac{1}{2}''$ thick; corrugated plates, being somewhat stiffer, may be thinner for the same service. The tie-plates under the joint ties must be somewhat longer than the

intermediates, in order to allow for the extra length from out to out of the angle-plates.

288. Method of setting. A very important detail in the process of setting the tie-plates on the ties is that the plates should be rigidly attached to the ties in their intended position during the process of setting. If tie-plates with flat bottoms are used, the surface of the tie must be adzed, so that it is not only plane but level, so that there will be no danger that the plate will rock on the tie. When using tie-plates which are corrugated on the under surface, it is necessary to force them into the tie until the under side of the plate is flush with the surface of the tie. This requires a pressure of several thousand pounds. Sometimes trackmen have depended on the easy process of waiting for passing trains to force the corrugations into the tie until the plate is in its intended position. Until the plates are finally set the spikes cannot be driven home, and this apparently cheap and easy process generally results in loose spikes and rails. The best method for new work is to drive the plates into the tie before setting the tie in position. A tie-plate gauge holds both tie-plates in their proper relative position, and both plates may be driven by the use of heavy beetles. When it is necessary to place the plate under the rail and drive it in, it is somewhat difficult to drive it by striking the plate with a swage on each side of the rail alternately. When it is struck on one side, the other side flies up unless held down by a wedge driven between the plate and the rail on the other side of the rail. A straddler, which straddles the rail somewhat like an inverted U, is very useful for this purpose, since it makes it possible to strike the head of the straddler and force down both sides of the plate at once. The Southern Pacific Railroad Company has rigged up a small pile-driver on a hand-car, which is used in connection with a straddler to drive the tie-plates into position. Some western railroads have even adopted the process of rigging up a flat car with a machine which will press the tie-plates into place in the ties before the ties are placed in the track.

SPIKES.

289. Requirements. The rails must be held to the ties by a fastening which will not only give sufficient resistance, but which

will retain its capacity for resistance. It must also be cheap and easily applied. The ordinary track-spike fulfills the last requirements, but has comparatively small resisting power, compared with screws or bolts. Worse than all, the tendency to

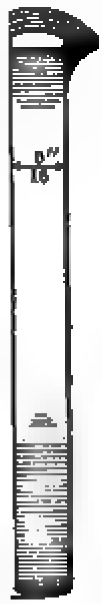


FIG. 127. FIG. 128.

vertical vibration in the rail produces a series of upward pulls on the spike that soon loosens it. When motion has once begun the capacity for resistance is greatly reduced, and but little more vibration is required to pull the spike out so much that redriving is necessary. Driving the spike to place again in the same hole is of small value except as a very temporary expedient, as its holding power is then very small. Redriving the spikes in new holes very soon "spike-kills" the tie. Many plans have been devised to increase the holding power of spikes, such as making them jagged, twisting the spike, swelling the spike at about the center of its length, etc. But it has been easily demonstrated that the fibers of the wood are generally so crushed and torn by driving such spikes that their holding power is less than that of the plain spike, and the durability is greatly diminished.

The ordinary spike (see Fig. 127) is made with a square cross-section which is uniform through the middle of its length, the lower $1\frac{1}{4}$ in. tapering down to a chisel edge, the upper part swelling out to the head. The Goldie spike (see Fig. 128) aims to improve this form by reducing to a minimum the destruction of the

fibers. To this end, the sides are made smooth, the edges are clean-cut, and the point, instead of being chisel-shaped, is ground down to a pyramidal form. Such fiber-cutting as occurs is thus accomplished without much crushing, and the fibers are thus pressed away from the spike and slightly downward. Any tendency to draw the spike will, therefore, cause the fibers to press still harder on the spike and thus increase the resistance.

A series of tests made by a committee of the A. R. E. A. and reported to the 1914 Convention, established some very valuable conclusions with respect to the use of the ordinary cut spike. Spikes with sharp pyramidal points and with various degrees of bluntness, and also the ordinary chisel-pointed spike, were driven into ties and other timbers and were withdrawn by a testing machine. Then the timbers were cut so as to expose the holes to their full length, so that the crushing of the fibers by the spike driving could be observed. A series of photographs illustrated this feature. In some cases the spikes were driven into ⅜-in. bored holes, some of which were 2½ ins. deep, but the most of them were 4 ins. deep. In other cases, the spikes were driven without previous boring. The following conclusions were unmistakable.

1. The spike with a pyramidal point about 1 in. long (virtually the "Goldie" design Fig. 128), has greater holding power, not only when it first begins to yield, but also afterward while the spike is being drawn out.

2. The long-pointed spikes crushed the fiber far less than any other type.

3. The chisel-pointed spike, virtually as shown in Fig. 127, and which is the type now in most common use, has the least holding power and is more destructive in crushing the fibers.

4. Spikes driven into ⅜-in. bored holes have greater holding power than when driven without boring, and the crushing of the fiber is much less. This indicates the very real economy in boring holes where the life of the tie is an economical consideration.

290. Driving. The holding power of a spike depends largely on how it is driven. If the blows are eccentric and irregular in direction, the hole will be somewhat enlarged and the holding power largely decreased. The spikes on each side of the rail in any one tie should not be directly opposite, but should be staggered. Placing them directly opposite will tend to split the tie, or at least decrease the holding power of the spikes. The direction of staggering should be reversed in the two pairs

of spikes in any one tie (see Fig. 129). This will tend to prevent any twisting of the tie in the ballast, which would otherwise loosen the rail from the tie.

291. **Screw spikes.** The D., L. & W. R. R. began the general use of screw spikes for all new work and for extensive track renewals in 1910. In five years they used over 12,000,000 screw spikes. The design is shown in Fig. 130. From a report made

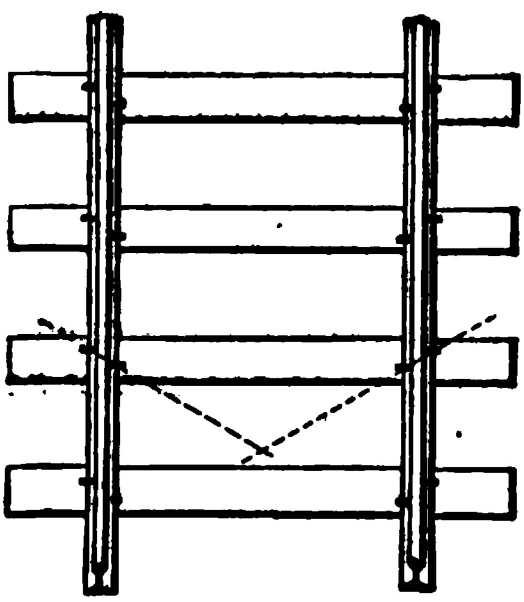


FIG. 129.—SPIKE-DRIVING.

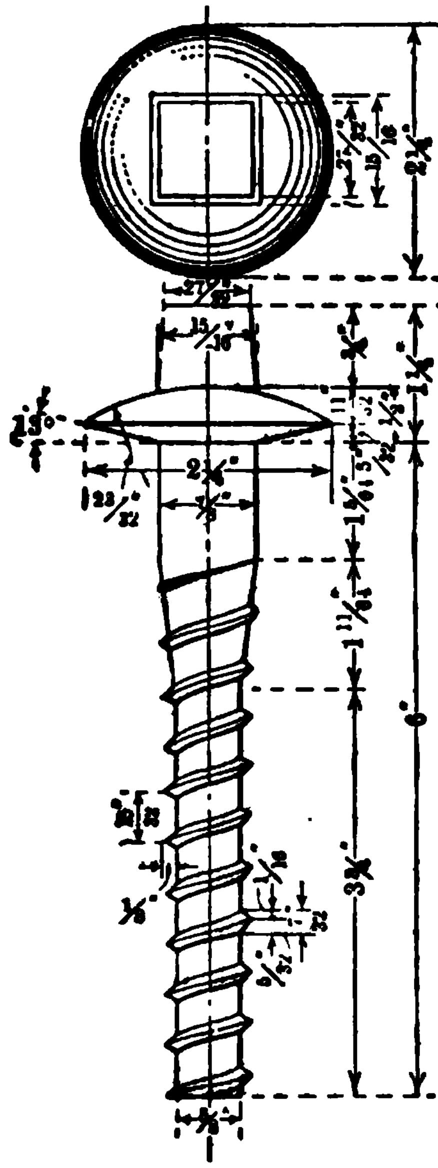


FIG. 130.—SCREW SPIKE, D. L. & W. R. R.

by Mr. G. J. Ray, Chief Engineer, to the A. R. E. A., the following facts and conclusions are deduced:

1. The use of screw spikes, in conjunction with suitable tie-plates, is almost a necessity in order to fully utilize the durability of a treated tie. A treated tie is seldom removed on account of decay in the body of the tie. Its destruction is generally due to "spike-killing," rail cutting, or to the decay which comes immediately after mechanical injury to the wood under the rail. Screw spikes and tie-plates largely prevent this mechanical injury.

2. "As a rule, with woods which it will pay to treat, the poorer the quality of the timber the more elaborate and expensive the fastening must be if the mechanical life of the tie is made to approach the life of the treated timber."

3. "Tie-plates should be used on all ties where screw spikes are used."

4. "Four holes should be provided for screw spikes, so that two extra holes will be available if needed."

5. "The size of screw spikes and the design of the thread should be carefully considered before a screw spike is adopted. Thereafter no changes should be made; otherwise the new screw spikes cannot be used in old holes without damaging the wood fiber."

6. "The screw-spike head should have tapering sides to prevent turning in the wrench socket after the size of the head has been diminished by rust."

7. "When screw spikes are fully seated, no further strain should be put on them, as this will tend to destroy the threads in the wood or injure the spikes."

8. "All ties should be bored at the treating plant before treatment. This can be done while the ties are being adzed, and not only insures that the holes are bored sufficiently deep, but provides for good treatment of all wood adjacent to the spike holes."

9. "Where the ties are bored before treatment, the track must be to proper gauge before the ties can be placed."

10. "The holes for screw spikes should be of proper dimensions for the class of wood used, with due regard to the size of screw spike used."

11. "A limited number of holes can be bored with one bit, after which its size will diminish so as to make it unfit for a hole of a given size." [The paper nowhere makes any statement as to the size of the bored hole in comparison with the diameter of the screw. The bored hole should have *about* the same diameter as the diameter of the screw at the base of the screw thread, but the hardness of the wood requires some variation, since, if the hole is too small, it will be impossible to turn the screw. The exact diameter must be determined for each kind of wood and must be strictly maintained.]

12. "Holes should be bored somewhat deeper than the length of the screw spike. There is no serious objection to boring the holes clear through the ties."

13. "Not only is the lateral and vertical resistance of a screw spike greater than that of a cut spike when both are first applied, but the lateral and vertical resistance of a loose screw spike is considerably greater than the lateral and vertical resistance of a loose cut spike."

14. "When the threads in the tie are entirely destroyed, a screw lining (any one of several different varieties) may be used with good results."

15. "All ties should be bored and adzed before treatment. This insures good gauge, a perfect bearing for the tie-plates and good treatment under the rail seat and around the screw-spike holes."

16. "In placing screw spikes, they should be driven by hammer only sufficient to make the threads take hold. If rigid instructions are not carried out, laborers will continually overdrive spikes and thus destroy the wood fibers near the top of the holes."

17. "The best results with the screw spikes can be expected in new construction, and where the number of screw spikes in tie renewals predominate over cut spikes."

18. "The use of screw spikes for the past five years has not made it necessary to increase the number of sectionmen per mile of track."

19. "Whether or not it will pay to use screw spikes will depend upon the cost of ties, their probable life and the amount of traffic."

292. "Wooden spikes." Among the regulations for track-laying given in § 246, mention was made of wooden "spikes," or plugs, which are used to fill up the holes when spikes are withdrawn. The value of the policy of filling up these holes is unquestionable, since the expense is insignificant compared with the loss due to the quick and certain decay of the tie if these holes are allowed to fill with water and remain so. But the method of making these plugs is variable. On some roads they are "hand-made" by the trackmen out of otherwise useless scraps of lumber, the work being done at odd moments. This policy, while apparently cheap, is not necessarily so, for the hand-made plugs are irregular in size and therefore more or less inefficient. It is also quite probable that if the trackmen are required to make their own plugs, they would spend time on these very cheap articles which could be more profitably employed otherwise. Since the holes made by the spikes are larger at the top than they are near the bottom, the plugs should *not* be of uniform cross-section but should be slightly wedge-shaped. The "Goldie tie-plug" (see Fig. 131) has been designed to fill these requirements. Being machine-made, they are uniform in size; they are of a shape which will best fit the hole; they can be furnished of any desired wood, and at a cost which makes it a wasteful economy to attempt to cut them by hand.

FIG. 131.

TRACK-BOLTS.

293. Essential requirements. The track-bolts must have sufficient strength and must be screwed up tight enough to hold the angle-plates against the rail with sufficient force to develop the full transverse strength of the angle-bars. On the other hand the bolts should not be screwed so tight that slipping may not take place when the rail expands or contracts with temperature. It would be impossible to screw the bolts tight enough to prevent slipping during the contraction due to a considerable fall of temperature on a straight track, but when the track is curved, or when expansion takes place, it is conceivable that the resistance of the ties in the ballast to lateral motion may be less than the resistance at the joint. A test to determine this resistance was made by Mr. A. Torrey, chief engineer of the Mich. Cent. R. R., using 80-lb. rails and ordinary angle-bars, the bolts being screwed up as usual. If required a force of about 31000 to 35000 lbs. to start the joint, which would be equivalent to the stress induced by a change of temperature of about 22°. But if the central angle of any given curve is small, a comparatively small lateral component will be sufficient to resist a compression of even 35000 lbs. in the rails. Therefore there will ordinarily be no trouble about having the joints screwed too tight. The vibration caused by the passage of a train reduces the resistance to slipping. This vibration also facilitates an objectionable feature, viz., loosening of the nuts of the track-bolts. The bolt is readily prevented from turning by giving it a form which is *not* circular immediately under the head and making corresponding holes in the angle-plate. Square holes would answer the purpose, except that the square corners in the holes in the angle-plates would increase the danger of fracture of the plates. Therefore the holes (and also the bolts, under the head) are made of an oval form, or perhaps a square form with rounded corners, avoiding angles in the outline.

"As a rule, as large track-bolts should be used as the rail and splice-bars will permit." [From 1915 Manual, A. R. E. A.] There is always some danger that a trackman may stretch a bolt beyond its elastic limit. A pull of 100 lbs. on a 33-inch track wrench will induce a stress of about 45000 lbs. per square inch in a $\frac{7}{8}$-inch track bolt. The same work on a 1-inch bolt would produce a stress of about 35000 lbs. per square inch. In order to

obtain the necessary toughness, bolts must be made of low-carbon steel or of nickel-steel, untreated or heat-treated. When made of carbon steel, specifications require an elastic limit of at least 35,000 lbs. per square inch but at the same time an elongation of 25% in 2 inches and a reduction of area of at least 50%. A harder steel would have a higher elastic limit, but would not be sufficienlty ductile. Higher elastic limits, with sufficient ductility, may be obtained by using untreated nickel or other alloy steel (at least 45,000 lbs. per square inch), or heat-treated nickel or other alloy steel (at least 75,000 lbs. per square inch). The elastic limit shall not be less than 50% of the ultimate. Added strength can only be obtained by using larger bolts or a more expensive metal.

294. Design of track-bolts. In Fig. 132 is shown a common design of track-bolt. In its general form this represents the bolt used on nearly all roads, being used not only with the common angle-plates, but also with many of the improved designs of rail-joints. The variations are chiefly a general increase in size to correspond with the increased weight of rails, besides variations in detail dimensions which are frequently unimportant. The diameter is usually ¾″ to ⅞″; 1″ bolts are used for 100-lb. rails. As to length, the bolt should not extend more than ½″ outside of the nut when it is screwed up. If it extends farther than this it is liable to be broken off by a possible derailment at that point. The lengths used vary from 3½″, which may be used with 60-lb. rails, to 5″, which is required with 100-lb. rails. The length required depends somewhat on the type of nut-lock used.

Fig. 132.—Track-bolt.

NUT-LOCKS.

295. Design of nut-locks. The designs for nut-locks may be divided into three classes: (a) those depending entirely on an

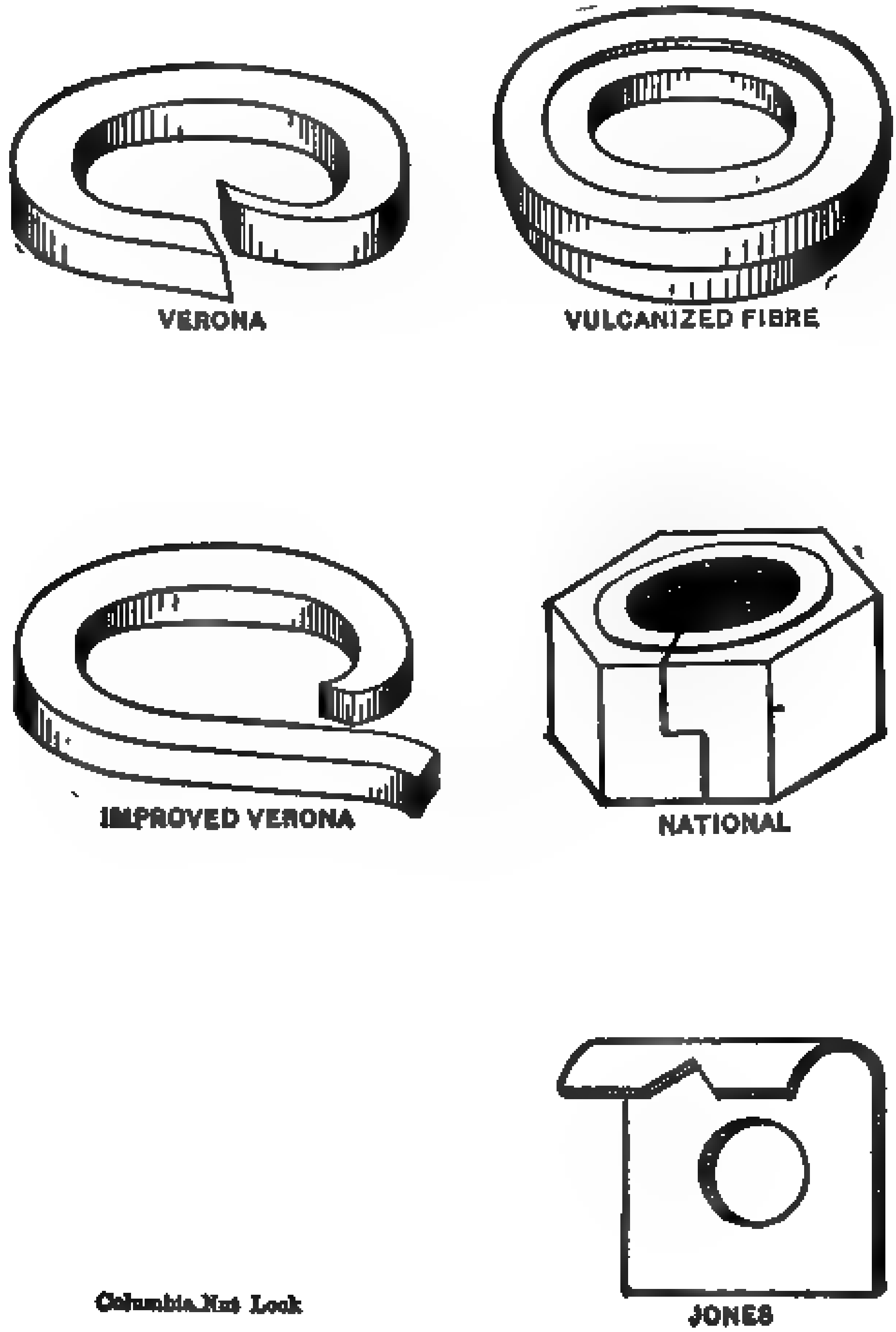


FIG. 133.—TYPES OF NUT-LOCKS.

elastic washer which absorbs the vibration which might otherwise induce turning; (*b*) those which jam the threads of the bolt and nut so that, when screwed up, the frictional resistance is too great to be overcome by vibration; (*c*) the "positive" nut-locks—those which mechanically hold the nut from turning. Some of the designs combine these principles to some extent. The "vulcanized fiber" nut-lock is an example of the first class. It consists essentially of a rubber washer which is protected by an iron ring. When first placed this lock is effective, but the rubber soon hardens and loses its elasticity and it is then ineffective and worthless. Another illustration of class (*a*) is the use of wooden blocks, generally 1″ to 2″ oak, which extend the entire length of the angle-bar, a single piece forming the washer for the four or six bolts of a joint. This form is cheap, but the wood soon shrinks, loses its elasticity, or decays so that it soon becomes worthless, and it requires constant adjustment to keep it in even tolerable condition. The "Verona" nut-lock is another illustration of class (*a*) which also combines some of the positive elements of class (*c*). It is made of tempered steel and, as shown in Fig. 133, is warped and has sharp edges or points. The warped form furnishes the element of elastic pressure when the nut is screwed up. The steel being harder than the iron of the angle-bar or of the nut, it bites into them, owing to the great pressure that must exist when the washer is squeezed nearly flat, and thus prevents any *backward* movement, although forward movement (or tightening the bolt) is not interfered with. The "National" nut-lock is a type of the second class (*b*), in which, like the "Harvey" nut-lock, the nut and lock are combined in one piece. With six-bolt angle-bars and 30-foot rails, this means a saving of 2112 pieces on each mile of single track. The "National" nuts are open on one side. The hole is drilled and the thread is cut slightly smaller than the bolt, so that when the nut is screwed up it is forced slightly open and therefore presses on the threads of the bolt with such force that vibration cannot jar it loose. Unlike the "National" nut, the "Harvey" nut is solid, but the form of the thread is progressively varied so that the thread pinches the thread of the bolt and the frictional resistance to turning is too great to be affected by vibration.

The "Columbia" nut-lock is a two-piece nut, both parts of which must turn simultaneously. As shown in the figure, one

section wedges into the other. The greater the tension in the bolt, the greater the wedging action and the greater the friction to prevent turning.

The "Jones" nut-lock, belonging to class (*c*), is a type of a nut-lock that does not depend on elasticity or jamming of screw-threads. It is made of a thin flexible plate, the square part of which is so large that it will not turn after being placed on the bolt. After the nut is screwed up, the thin plate is bent over so that the re-entrant angle of the plate engages the corner of the nut and thus mechanically prevents any turning. The metal is supposed to be sufficiently tough to endure without fracture as many bendings of the plate as will ever be desired. Nut-locks of class (*c*) are not in common use.

The above types have been discussed in order to show the development of the various devices. With but few exceptions, the standard nut-lock is a steel spring ring of the same general class as the Verona. The A. R. E. A. have prepared specifications for such nut-locks which include the following:

"After the finished nut-lock has been subjected for one hour to pressure sufficient to compress it flat and has been released, its reaction shall be not less than two-thirds its height or thickness of section, provided thickness is less than width of section. If the section is square, the reaction must be not less than one-half its thickness. If height or thickness of section is more than width, the reaction shall be not less than the width of the section. The internal diameters naturally affect the percentage of reaction, and the above specifications apply to nut-locks of internal diameters from $\frac{13}{16}$ in. to 1 $\frac{5}{16}$ ins. Owing to the difficulty of establishing a common rate of percentage that shall be uniformly applicable to any internal diameter of any nut-lock of any section it has been sought to cover the matter as above. Amount and durability of reactionary power under constant pressure is the true test of any spiral spring nut-lock. The percentage of reaction increases proportionately with the increased internal diameter of any given section."

"With one end of the finished nut-lock secured in a vise, and the opposite end twisted to 45 degrees, there must be no sign of fracture. When further twisted until broken, the fracture must show a good quality of steel."

CHAPTER XI.

SWITCHES AND CROSSINGS.

SWITCH CONSTRUCTION.

296. Essential elements of a switch. Flanges of some sort are a necessity to prevent car-wheels from running off from the rails on which they may be moving. But the flanges, although a necessity, are also a source of complication in that they require some special mechanism which will, when desired, guide the wheels out from the controlling influence of the main-line rails. This must either be done by raising the wheels high enough so that the flanges may pass *over* the rails, or by breaking the continuity of the rails in such a way that channels or "flange spaces" are formed *through* the rails. An ordinary stub-switch breaks the continuity of the main-line rails in three places, two of them at the switch-block and one at the frog. The Wharton switch avoids two of these breaks by so placing inclined planes that the wheels, rolling on their flanges, will surmount these inclines until they are a little higher than the rails. Then the wheels on the side toward which the switch runs are guided over and across the main rail on that side This rise being accomplished in a short distance, it becomes impracticable to operate these switches except at slow speeds, as any sudden change in the path of the center of gravity of a car causes very destructive jars both to the switch and to the rolling stock. The other general method makes a break in one main rail (or both) at the switch-block. In both methods the wheels are led to one side by means of the "lead rails," and finally one line of wheels passes *through* the main rail on that side by means of a "frog." There are some designs by which even this break in the main rail is avoided, the wheels being led *over* the main rail by means of a short *movable* rail which is on occasion placed across the main rail, but such designs have not come into general use.

297. Frogs. Frogs are provided with two channel-ways or "flange spaces" through which the flanges of the wheels move.

Each channel cuts out a parallelogram from the tread area. Since the wheel-tread is always wider than the rail, the wing rails will support the wheel not only across the space cut out by the channel, but also until the tread has passed the point of the frog and can obtain a broad area of contact on the tongue of the frog. This is the theoretical idea, but it is very imperfectly

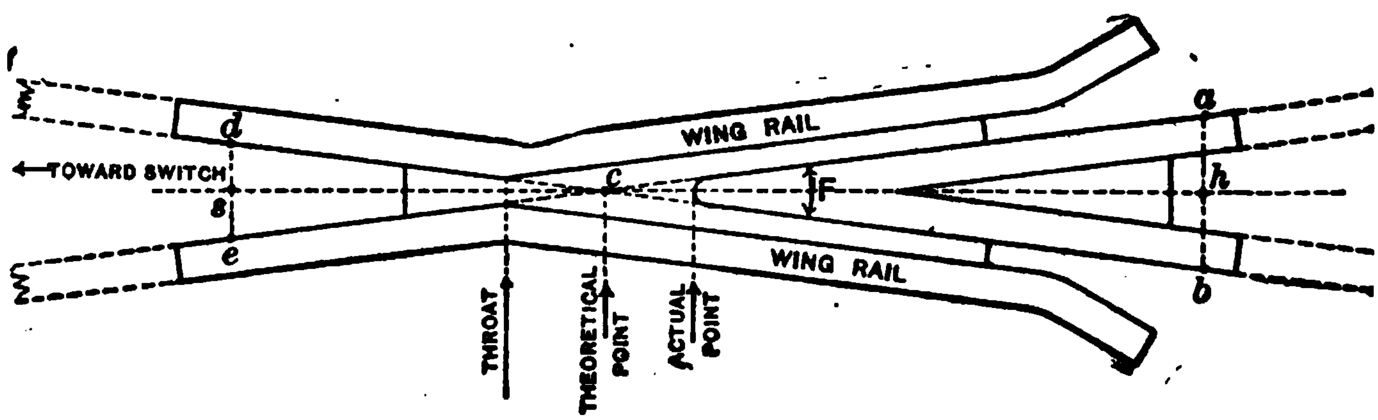


FIG. 134.—DIAGRAMMATIC DESIGN OF FROG.

realized. The wing rails are sometimes subjected to excessive wear owing to "hollow treads" on the wheels—owing also to the frog being so flexible that the point "ducks" when the wheel approaches it. On the other hand the sharp point of the frog will sometimes cause destructive wear on the tread of the wheel. Therefore the tongue of the frog is not carried out to the sharp theoretical point, but is purposely somewhat blunted. But the break which these channels make in the continuity of the tread area becomes extremely objectionable at high speeds, being mutually destructive to the rolling stock and to the frog. The jarring has been materially reduced by the device of "spring frogs"—to be described later. Frogs were originally made of cast iron—then of cast iron with wearing parts of cast steel, which were fitted into suitable notches in the cast iron. This form proved extremely heavy and devoid of that elasticity of track which is necessary for the safety of rolling stock and track at high speeds. The present standard practice is to build the frog up of pieces of rails which are cut or bent as required. There are always four pieces for single-pointed frogs. For heavy work they are assembled by bolting them together, the flangeways being provided by the use of fillers made of cast iron, cast steel or rolled steel. For still heavier work the above combination is riveted to a base plate. For light or street railway work, the rails are riveted to a base plate without using

BOLTED FROG.

BOLTED FROG RIVETED TO BASE PLATE.

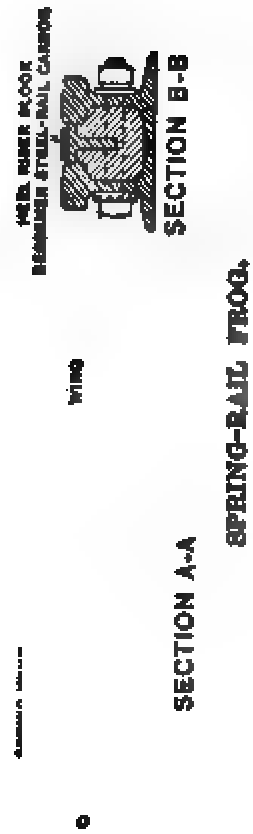


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PLATE VIII.—SOME TYPES OF FROGS.
(As made by Ramapo Iron Works.)

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fillers. For details, study Plate VIII. The operation of a spring-rail frog is evident from the figure. Since a siding is usually operated at slow speed, while the main track may be operated at fast speed, a spring-rail frog will be so set that the tread is continuous for the main track and broken for the siding. This also means that the spring-rail will only be moved by trains moving at a (presumably) slow speed on to the siding. For the fast trains on the main line such a frog is substantially a "fixed" frog and has a tread which is practically continuous.

298. To find the frog number. The frog number (n) equals the ratio of the distance of any point on the tongue of the frog from the theoretical point of the frog divided by the width of the tongue at that point, i.e. $=hc \div ab$ (Fig. 134). This value may be directly measured by applying any convenient unit of measure (even a knife, a short pencil, etc.) to some point of the tongue where the width just equals the unit of measure, and then noting how many times the unit of measure is contained in the distance from that place to the theoretical point. But since c, the theoretical point, is not so readily determinable with exactitude, it being the imaginary intersection of the gauge lines, it may be more accurate to measure de, ab, and hs; then n, the frog number, $=hs \div (ab+de)$. If the frog angle be called F, then

$$n = hc \div ab = hs \div (ab+de) = \tfrac{1}{2} \cot \tfrac{1}{2}F;$$

i.e., $$\cot \tfrac{1}{2}F = 2n.$$

299. Stub switches. The use of these, although once nearly universal, has been practically abandoned as turnouts from *main track* except for the poorest and cheapest roads. In some States their use on main track is prohibited by law. They have the sole merit of cheapness with adaptability to the circumstances of very light traffic operated at slow speed when a considerable element of danger may be tolerated for the sake of economy. The rails from A to B (see Fig. 135*) are not fastened

* The student should at once appreciate that in Fig. 135, as well as in nearly all the remaining figures in this chapter, it becomes necessary to use excessively large frog angles, short radii, and a very wide gauge in order to illustrate the desired principles with figures which are sufficiently small for the page. In fact, the proportions used in the figures are such that serious mechanical difficulties would be encountered if they were used. These difficulties are here ignored because they can be neglected in the proportions used in practice.

to the ties; they are fastened to each other by tie-rods which keep them at the proper gauge; at and back of *B* they are securely spiked to the ties, and at *A* they are kept in place by

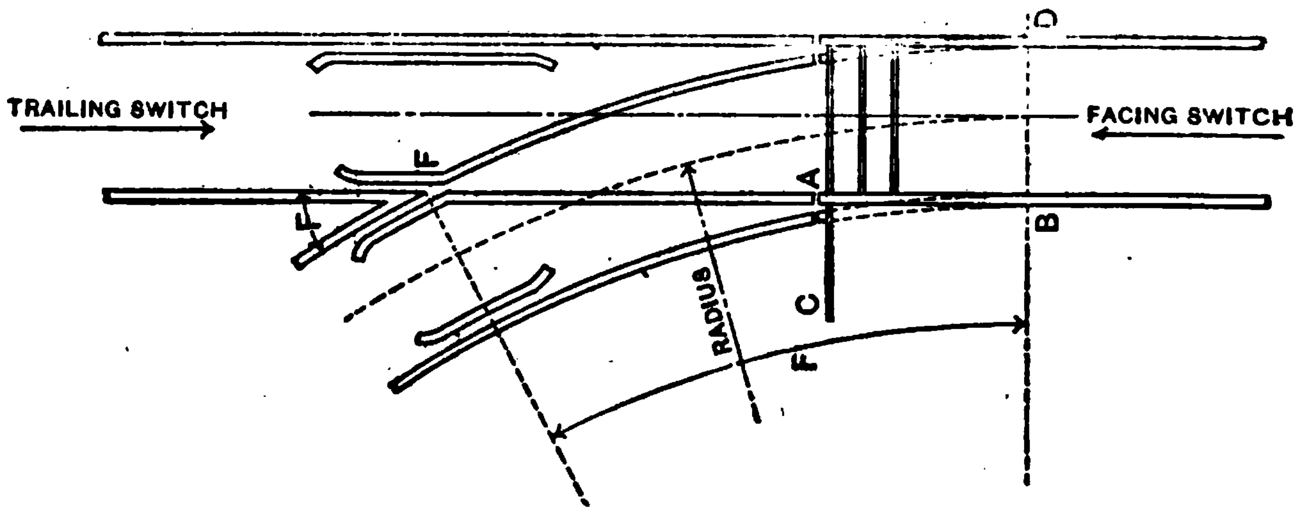

FIG. 135.—STUB SWITCH.

the connecting bar (*C*) fastened to the switch-stand. One great objection to the switch is that, in its usual form, when operated as a trailing switch, a derailment is inevitable if the switch is misplaced. The very least damage resulting from such a derailment must include the bending or breaking of the tie-rods of the switch-rail. Several devices have been invented to obviate this objection, some of which succeed very well mechanically, although their added cost precludes any economy in the total cost of the switch. Another objection to the switch is the looseness of construction which makes the switches objectionable at high speeds. The gap of the rails at the head-block is always considerable, and is sometimes as much as two inches. A driving-

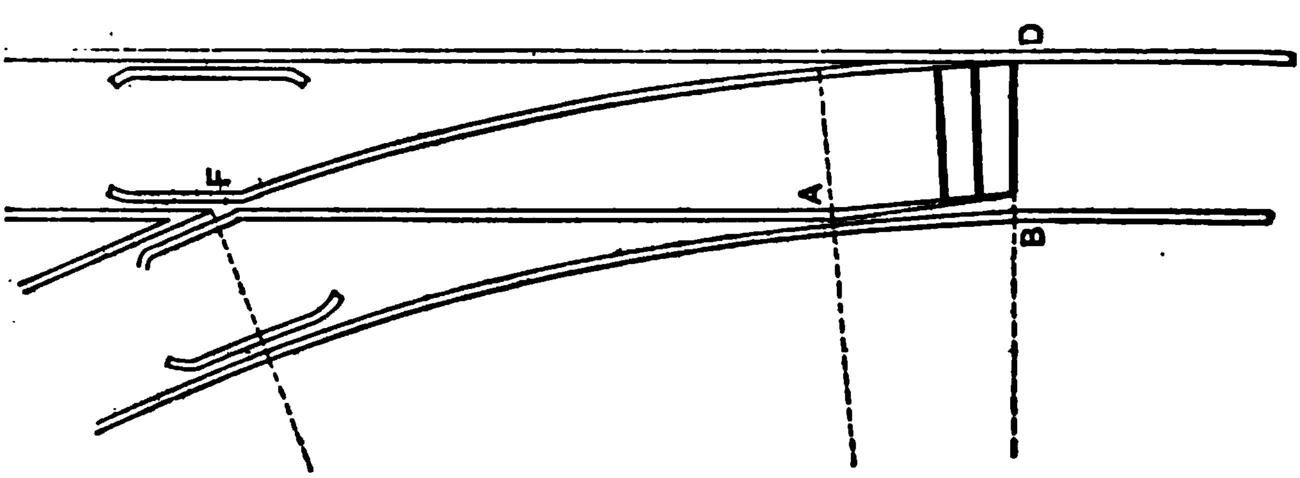

FIG. 136.—POINT SWITCH.

wheel with a load of 20000 to 30000 pounds, jumping this gap with any considerable velocity will do immense damage to the

farther rail end, besides producing such a stress in the construction that a breakage is rendered quite likely, and such a breakage might have very serious consequences.

300. Point switches. The essential principle of a point switch is illustrated in Fig. 136. As is shown, one main rail and also one of the switch-rails is unbroken and immovable. The other main rail (from A to F) and the corresponding portion of the other lead rail are substantially the same as in a stub switch. A portion of the main rail (AB) and an equal length of the opposite lead rail (usually 16.5 to 22 feet long) are fastened together by tie-rods. The end at A is jointed as usual and the other end is pointed, both sides being trimmed down so that the feather edge at B includes the web of the rail. In order to retain in it as much strength as possible, the point-rail is raised so that it rests on the base of the stock-rail, one side of the base of the point-rail being nearly cut away. As may be seen in Fig. 137, although the influence of the point of the rail in moving the wheel-flange away from the stock-rail is really zero at that point, yet the rail has all the strength of the web, more than one-half that of the base, and is also reinforced. The planing runs back in *straight* lines, until at about six or seven feet back from the point the full width of the head is obtained. The full width of the base will only be obtained at about 13 feet from the point. The A. R. E. A. standard switch rail is always cut on the basis that the distance between gauge lines at the heel of the switch (the distance MN in Fig. 143) is $6\frac{1}{4}$ inches and that the "point" is $\frac{1}{4}$ inch wide. Then, using four standard lengths, 11, $16\frac{1}{2}$, 22 and 33 feet, the angles vary from 2° 36′ 19″ to 0° 52′ 05″, as shown in Table III.

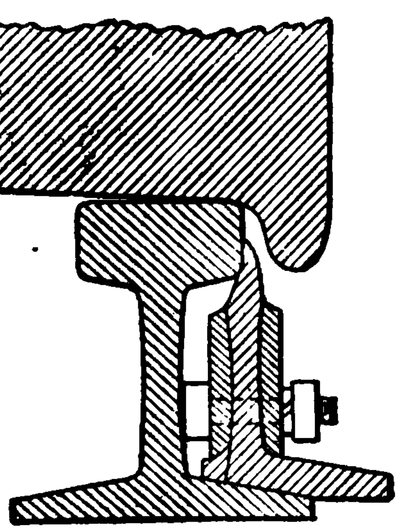

FIG. 137.

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301. Switch-stands. The simplest and cheapest form is the "ground lever," which has no target. The radius of the circle described by the connecting-rod pin is precisely one-half the throw. From the nature of the motion the device is practically

self-locking in either position, padlocks being only used to prevent malicious tampering.

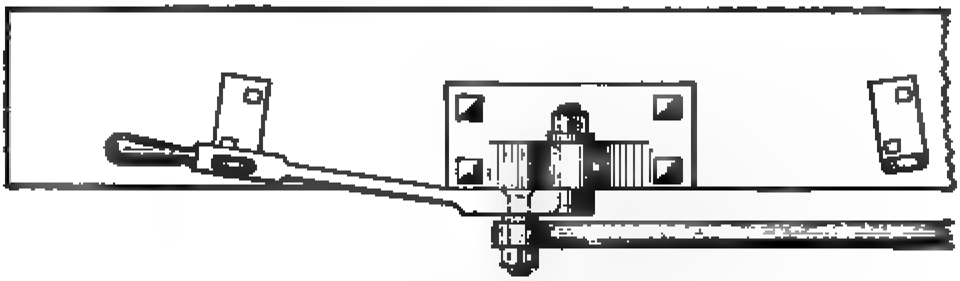


FIG. 138.—GROUND LEVER FOR THROWING A SWITCH.

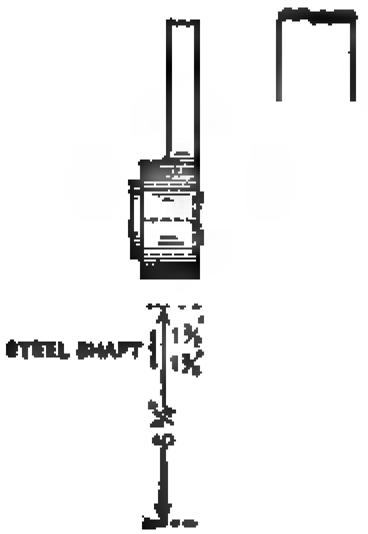


FIG. 139.—RAMAPO PATENT SWITCH STAND. NON-AUTOMATIC.

In Fig. 139 is shown a design in which the arc of the throwing lever is parallel to the track, an important feature in quick switching work.

302. Tie-rods. These are fastened to the webs of the rails by means of lugs which are bolted on, there being usually a hinge-

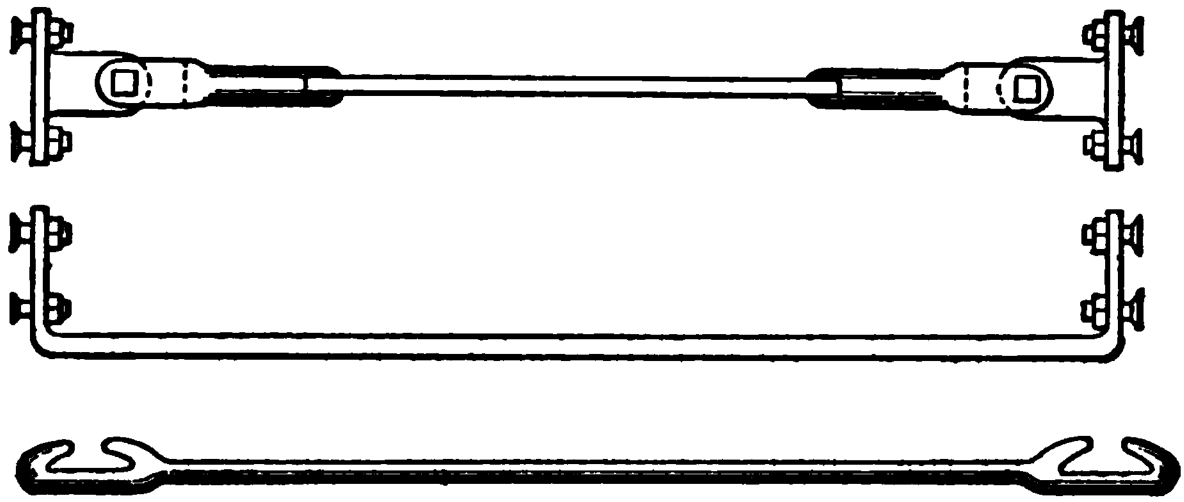


FIG. 140.—FORMS OF TIE-RODS.

joint between the rod and the lug. Two such tie-rods (three for a 30-foot switch) are generally necessary. The first rod is sometimes made without hinges, which gives additional stiffness to the comparatively weak rail-points. The old-fashioned tie-rod, having jaws fitting the base of the rail, was almost universally used in the days of stub switches. One great inconvenience in their use lies in the fact that they must be slipped on, one by one, over the *free* ends of the switch-rails.

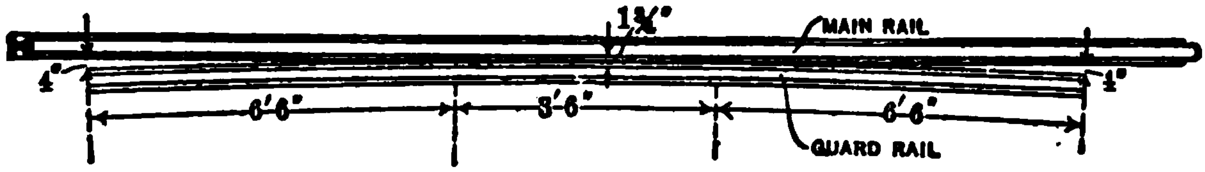


FIG. 141.—STANDARD GUARD-RAIL.

303. Guard-rails. As shown in Figs. 135 and 136, guard-rails are used on both the main and switch tracks opposite the frog-point. Their function is not only to prevent the possibility of the wheel-flanges passing on the wrong side of the frog-point, but also to save the side of the frog-tongue from excessive wear. The flange-way space between the heads of the guard-rail and wheel-rail should equal $1\frac{3}{4}$ inches. Since this is less than the space between the heads of ordinary (say 80-pound) rails when

placed base to base, to say nothing of the $\frac{3}{4}''$ required for spikes, it becomes necessary to cut the flange of the guard-rail. The length of the rail should be 16 feet 6 inches, the middle portion being straight for a length of 3 feet 6 inches, and the ends, each being 6 feet 6 inches long, curved out so that the side of the rail head at each end is 4 inches from the main rail head, when the flange-way at the center is $1\frac{3}{4}$ inches. See Fig. 141.

MATHEMATICAL DESIGN OF SWITCHES.

In all of the following demonstrations regarding switches, turnouts, and crossovers, the lines are assumed to represent the *gauge-lines*—i.e., the lines of the *inside* of the head of the rails.

304. **Design with circular lead-rails.** The simplest method is to consider that the lead-rails curve out from the main track-

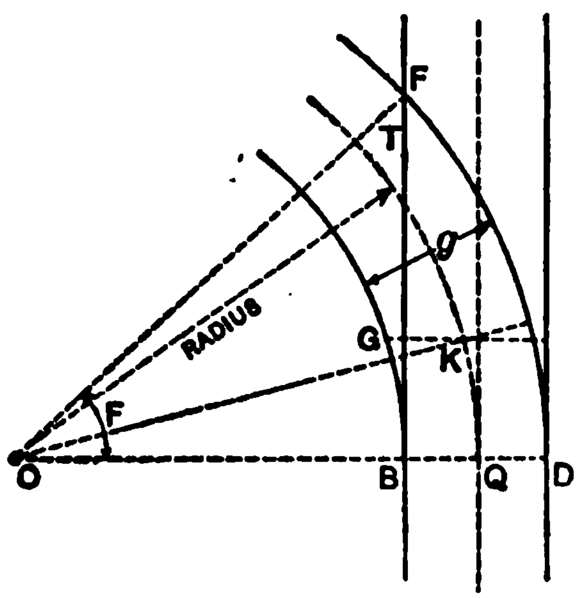


Fig. 142.

rails by arcs of circles which are tangent to the main rails and which extend to the frog-point F. The simple curve from D to F is of such radius that $(r+\frac{1}{2}g)$ vers $F=g$, in which $F=$the frog angle $g=$gauge, $L=$the "lead" (BF), and $r=$the radius of the *center* of the switch-rails.

$$\therefore\ r+\tfrac{1}{2}g=\frac{g}{\text{vers } F}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

Also, $\qquad BF+BD=\cot\tfrac{1}{2}F;\ BD=g;\ BF=L.$

$$\therefore\ L=g\cot F. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

Also, $$L = (r + \tfrac{1}{2}g)\sin F; \quad \ldots \ldots \quad (71)$$

$$QT = 2r \sin \tfrac{1}{2}F. \quad \ldots \ldots \quad (72)$$

These formulæ involve the angle F. As shown in Table III, the angles (F) are always odd quantities, and their trigonometric functions are somewhat troublesome to obtain closely with ordinary tables. The formulæ may be simplified by substituting the frog-number n, from the relation that $n = \frac{1}{2} \cot \frac{1}{2}F$. Since

$$r - \tfrac{1}{2}g = L \cot F \quad \text{and} \quad r + \tfrac{1}{2}g = L \operatorname{cosec} F,$$

then
$$\begin{aligned} r &= \tfrac{1}{2}L\,(\cot F + \operatorname{cosec} F) \\ &= \tfrac{1}{2}g \cot \tfrac{1}{2}F(\cot F + \operatorname{cosec} F) \\ &= \tfrac{1}{2}g \cot^2 \tfrac{1}{2}F, \text{ since } (\cot a + \operatorname{cosec} a) = \cot \tfrac{1}{2}a \\ &= 2gn^2. \quad \ldots \ldots \quad (73) \end{aligned}$$

Also, $$L = 2gn, \quad \ldots \ldots \quad (74)$$

from which $$r = n \times L. \quad \ldots \ldots \quad (75)$$

These extremely simple relations may obviate altogether the necessity for tables, since they involve only the frog-number and the gauge. On account of the great simplicity of these rules, they are frequently used as they are, regardless of the fact that the curve is never a uniform simple curve from switch-block to frog. In the first place there is a considerable length of the gauge-line within the frog, which is straight unless it is purposely curved to the proper curve while being manufactured, which is seldom if ever done—except for the very large-angled frogs used for street-railway work, etc. It is also doubtful whether the switch-rails (BA, Fig. 135) are bent to the computed curve when the rails are set for the switch. The switch-rails of point switches are *straight*, thus introducing a stretch of straight track which is about one-fifth of the total length of the lead-rails. The effect of these modifications on the length and radius of the lead-rails will be developed and discussed in the following sections.

The throw (t) of a stub switch depends on the weight of the rail, or rather on the width of its base. The throw must be at

least $\frac{3}{4}''$ more than that width. The head-block should therefore be placed at such a distance from the heel of the switch (B) that the versed sine of the arc equals the throw. These points *must* be opposite on the two rails, but the points on the two rails where these relations are exactly true will not be opposite. Therefore, instead of considering either of the two radii $(r+\frac{1}{2}g)$ and $(r-\frac{1}{2}g)$, the mean radius r is used. Then (see Fig. 142)

$$\text{vers } KOQ = t \div r,$$

and the length of the switch-rails is

$$QK = r \sin KOQ. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$

Stub-switches are generally used with large frog angles. For small frog angles (large frog-numbers) the values of QK are so great that the length of rail left unspiked is too great for a safe track. If this were obviated by spiking down a portion of the lead the theoretical accuracy of the switch would be lost.

The use of stub switches may now be considered obsolete. But the above demonstration has been retained in this edition for its educational value as an introduction to the more complicated method which is now the standard.

305. Standard design, using straight frog-rails and straight point-rails. It becomes necessary in this case to find a curve which shall be tangent to both the point-rail and the frog-rail. The curve therefore begins at M, its tangent making an angle of α (varying from 0° 52′ to 2° 36′) with the main rail, and runs to H. $FJ = W =$ the length of the "wing-rail" from the theoretical point of the frog (F) to the toe, J or J'. $FK = K =$ the length from the theoretical point to the heel of the frog. $MN = H =$ the "heel distance," or the distance of the gauge line of the switch-rail at the heel from the gauge line of the main track rail.

The central angle of the curve equals $(F-a)$. The angle of the chord HM with the main rails is therefore

$$\tfrac{1}{2}(F-a)+a=\tfrac{1}{2}(F+a);$$

$$JM = \frac{g - W \sin F - H}{\sin \frac{1}{2}(F+a)};$$

$$r+\tfrac{1}{2}g=\frac{JM}{2\sin\frac{1}{2}(F-a)}$$

$$=\frac{g-W\sin F-H}{2\sin\frac{1}{2}(F+a)\sin\frac{1}{2}(F-a)}$$

$$=\frac{g-W\sin F-H}{\cos a-\cos F}; \quad \ldots \ldots \quad (77)$$

$$DN=s\cos\alpha, \quad \ldots \ldots \quad (78)$$

in which S = length of switch-rail.

$$BF=L=JM\cos\tfrac{1}{2}(F+a)+W\cos F+S\cos\alpha$$

$$=(g-W\sin F-H)\cos\tfrac{1}{2}(F+a)+W\cos F+S\cos\alpha. \quad (79)$$

It may be more simple, if $(r+\frac{1}{2}g)$ has already been computed, to write

$$L=2(r+\tfrac{1}{2}g)\sin\tfrac{1}{2}(F-a)\cos\tfrac{1}{2}(F+a)+W\cos F+S\cos\alpha$$

$$=(r+\tfrac{1}{2}g)(\sin F-\sin a)+W\cos F+S\cos\alpha. \quad \ldots \quad (80)$$

The above equations for L give the distance from the actual (blunt) point of the switch-rail to the *theoretical* point of the frog. The lead (L') given in Table III is the distance from the actual point of the switch-rail to the actual (blunt) point of the frog. The difference (L'—L) is the "frog bluntness," which in each case equals the width of the frog point ($\frac{1}{2}$ inch = .04166 foot) multiplied by the frog number. The values of the frog bluntness for the various frogs is given in the second column of Part B, Table III.

The value of $MN=H$ has been standardized by the A. R. E. A. as $6\frac{1}{4}$ inches for all lengths of switch-rail and for all values of α. The point of the switch-rail (at D) is invariably $\frac{1}{4}$-inch thick. When it is necessary to calculate MN for other standards of construction, it may be computed (calling S = length of switch-rail) to be

$$MN=S\sin\alpha+(\text{thickness of point of switch rail}).$$

The length of the wing rail of the frog ($W=FJ$) is given for each frog in the third column of Table III, Part B. The several values of F and α are also given in Table III. g is the gauge $=$ 4 feet $8\frac{1}{2}$ inches $=$ 4.7083 feet.

The solution of Eq. 77–80 for various frog angles will give a series of "theoretical leads," as given in Table III, Part B. The table also gives the "closure values," or the lengths of the arc MJ and of the straight rail $M'J'$. But these closure lengths are invariably such odd quantities that rails must be cut and

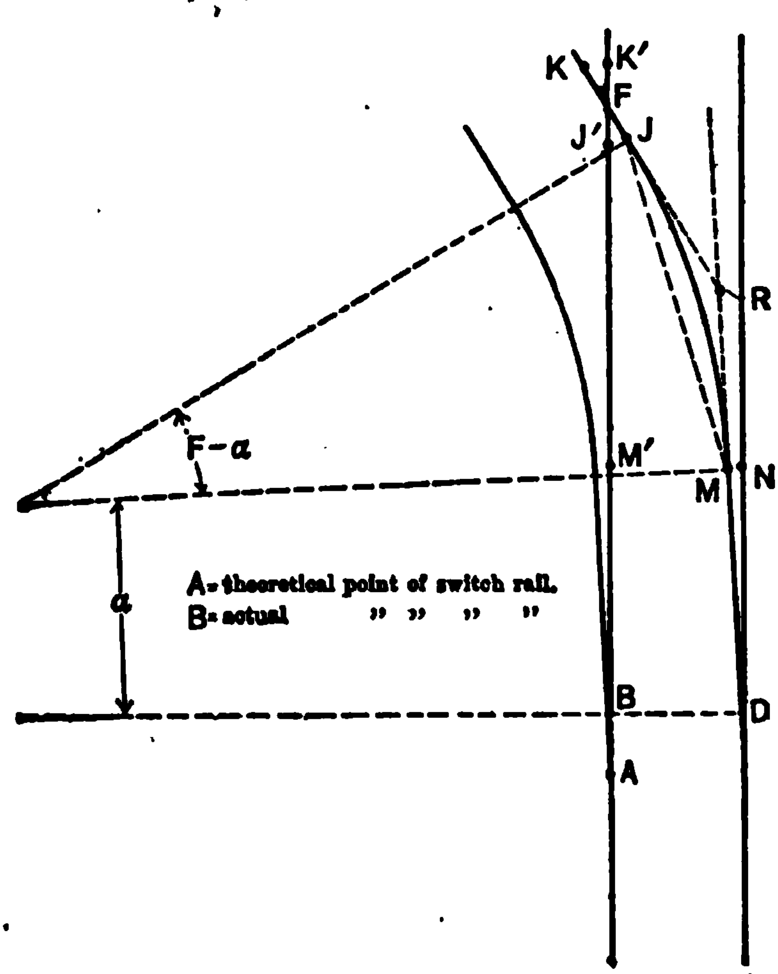


FIG. 143.

more or less rail must be wasted. By shortening the radius of the connecting curve very slightly and inserting a very short length of tangent either between the curve and switch-rail at M, or between the curve and wing-rail at J, all of which will change very slightly the length of lead, the closure lengths can be made such that the rail cutting and wastage is minimized, and yet the combinations of curves and tangents are mathematically perfect. The detailed method of computing these combinations is tedious and will not be elaborated here, but a series of results developed by the A. R. E. A. is given under the heading of "practical leads" in Table III, Part C.

The above computations and tabular values assume that the two switch points (at B and D) are directly opposite. This would always mean that the straight rail (BF) is somewhat shorter than the curved rail from D to F. In the maximum case the difference is less than 5 inches. Therefore, assuming that rails are obtainable at even-foot lengths down to 27 feet, or 24 feet for a No. 4 frog switch, the system of practical leads never requires more than one rail cutting. But even this is sometimes avoided by using for the straight-rail closure the same number and lengths of uncut rails as are specified for the closure of the curved part. The chief effect of this is that the point of the switch-rail will be located a few inches below its normal position at B and that the gauge at the switch-point will be slightly widened when the switch is open. This effect is possibly an advantage rather than a disadvantage.

306. Design for a turnout from the OUTER side of a curved track. Fig. 144 is a diagram of what the construction would be

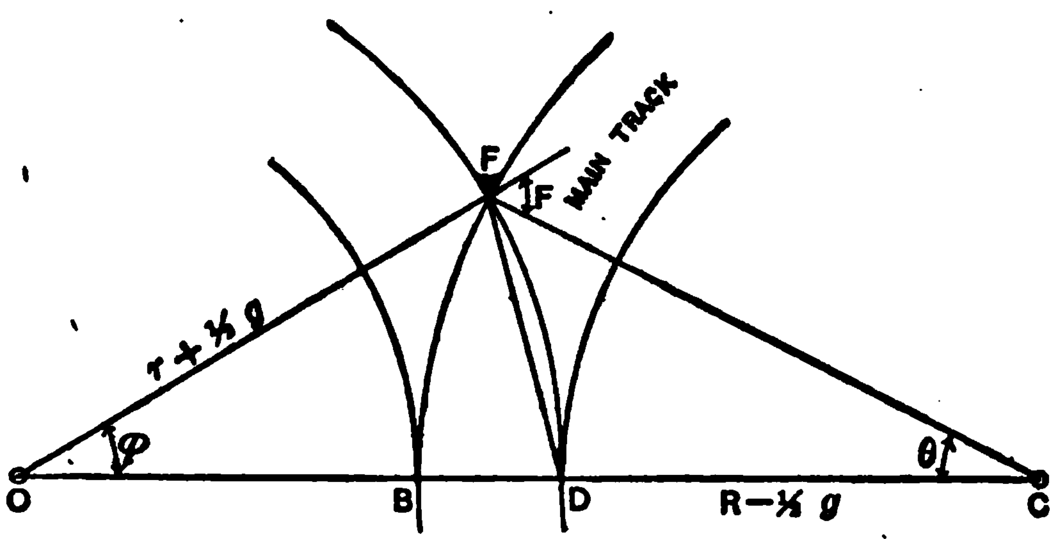


FIG. 144.

if the switch-rails were circular throughout. Before the invention of point switches and when stub switches were in universal use, the lead-rails were considered to be circular, both for straight and for curved main track. If Eqs. 70 and 75 and the corresponding Eqs. 77 to 80 are solved for any given frog, it is found that the lead, when using straight switch-rails and straight frog-rails, is considerably less than when using circular lead-rails throughout; also the curvature is considerably sharper. But stub-rail switches are obsolete and the mathematical solutions used for them cannot be utilized, even approximately, for point switches. If such a diagram as Fig. 144 is worked out in detail, as has been done in previous editions, it is found that

(*a*) the lead (*BF*) is almost identical with that computed from Eq. 70 or 74, when the main line is straight.

(*b*) the degree of curve (*d*) of the circular switch-rails would be *very nearly* equal to the degree of curve (*d'*) of the circular switch-rails for a straight track minus the degree of curve (*D*) of the main track; or, $d=d'-D$.

These statements are more exactly true when the degree of curvature of the main track is small. Even for a 10° curve on the main track the errors are not large. It has been found to be a needless refinement to compute the precise mathematical properties of the switch-rails from a curved main track, any more than as given by the two principles stated above: Therefore

(*a*) the length of the lead is assumed to be the same as that for a straight track, using the same frog, and

(*b*) the degree of curve of the switch-rails is found as stated above—in principle (*b*). As the curvature of the main track sharpens, the curvature of the switch-rails becomes less until they become straight. For still sharper main track, the center of curvature is on the same side. This is illustrated in Fig. 145, if we consider the sharper curved track to be the main track and the easier curve the switch. The above rule is still applicable, the algebraic sign of the result showing the location of the center.

307. Design for a turnout from the INNER side of a curved track. As in the previous section, Fig. 145 illustrates the dia-

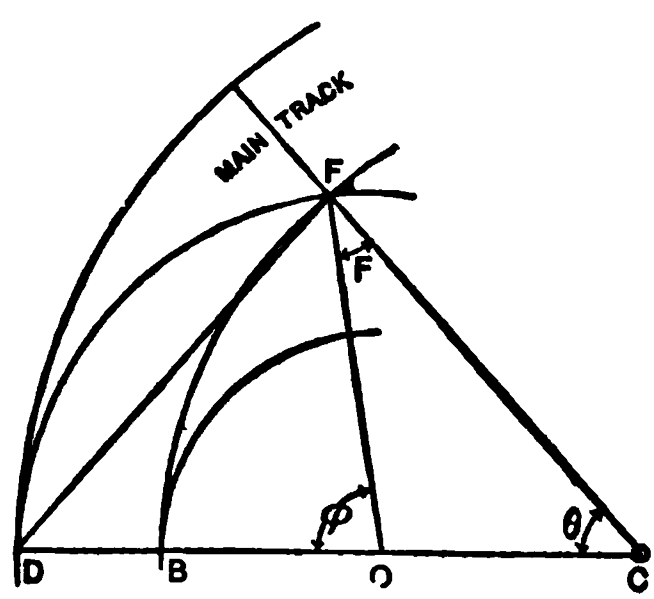


FIG. 145.

gram for circular lead rails. It may be shown that the degree of the turnout (*d*) is *nearly* the *sum* of the degree of the main

track (D) and the degree (d') of a turnout from a straight track when the frog angle is the same. The discrepancy in this case is somewhat greater than in the other, especially when the curvature of the main track is sharp. If the frog angle is also large, the curvature of the turnout is excessively sharp. If the frog angle is very small, the liability to derailment is great. Turnouts to the inside of a curved track should therefore be avoided, unless the curvature of the main track is small.

308. Connecting curve from a straight track. The "connecting curve" is the track lying between the frog and the side

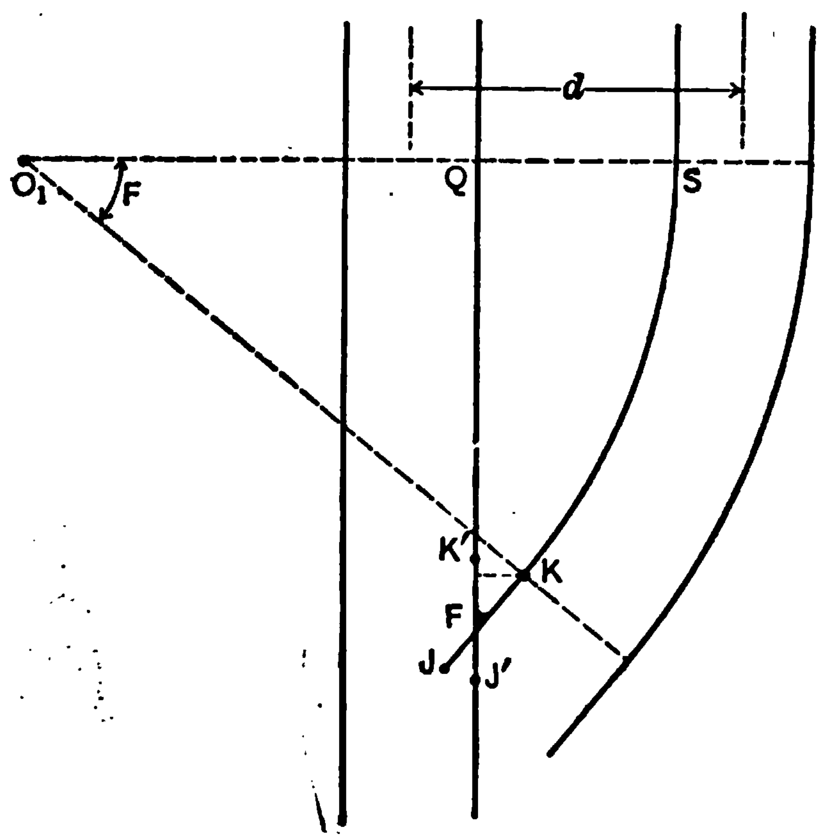


FIG. 146.

track where it becomes parallel to the main track (FS in Fig. 146 or 147). Call d the distance between track centers. The angle $KO_1S = F$ (see Fig. 146). Call r' the radius of the connecting curve. Then

$$(r' - \tfrac{1}{2}g) = \frac{d - g \ - K \sin f}{\text{vers } F}; \quad . \quad . \quad . \quad . \quad . \quad . \qquad (81)$$

$$FQ = (r' - \tfrac{1}{2}g) \sin F + K \cos f \quad . \quad . \quad . \qquad (82)$$

In these equations (and in several that follow) K is the distance from the theoretical point of the frog to the heel. The length, for each standard frog, is found in Table III, Part B.

309. Connecting curve from a curved track to the OUTSIDE. When the main track is curved, the required quantities are the radius of the connecting curve from K to S, Fig. 147, and its length or central angle.

The accuracy of all these computations on switches and frogs in curved main track is vitiated by the fact that the frog-rails are straight. The design might be mathematically more perfect if the main track curve were transformed into two curves on either side of the frog which had centers separated as far as the

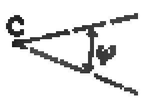


FIG. 147.

length of the frog, but this would introduce a very great and needless complication and is never done. The more simple solution is to consider that the frog-rail is a chord of the original curve, which (*a*) narrows the track gauge by an amount equal to the middle ordinate of that chord and which (*b*) is not tangent to the curve at either end. For all ordinary curvature neither of these theoretical defects is vitally objectionable or even appreciable. In Fig. 147 KC is practically perpendicular to one frog-rail and KO_1 is exactly perpendicular to the other frog-rail. Therefore, the angle CKO_1 equals the frog angle F. While the following calculations are amply precise for practical purposes, the discrepancy from strict mathematical accuracy should be noted and properly valued.

In the triangle CSK

$$CS+CK : CS-CK :: \tan \tfrac{1}{2}(CKS+CSK) : \tan \tfrac{1}{2}(CKS-CSK);$$

but $\frac{1}{2}(CKS+CSK)=90-\frac{1}{2}\psi$; and, since the triangle O_1SK is isosceles, $\frac{1}{2}(CKS-CSK)=\frac{1}{2}F$;

$$\therefore\ 2R+d+K\sin F : d-g-K\sin F :: \cot\tfrac{1}{2}\psi : \tan\tfrac{1}{2}F$$
$$:: \cot\tfrac{1}{2}F : \tan\tfrac{1}{2}\psi;$$

$$\therefore\ \tan\tfrac{1}{2}\psi=\frac{2n(d-g-K\sin F)}{2R+d+K\sin F}. \quad . \quad . \quad . \quad . \quad (83)$$

From the triangle CO_1K we may derive

$$r-\tfrac{1}{2}g : R+\tfrac{1}{2}g+K\sin F :: \sin\psi : \sin(F+\psi);$$

$$r-\tfrac{1}{2}g=(R+\tfrac{1}{2}g+K\sin F)\frac{\sin\psi}{\sin(F+\psi)}. \quad . \quad . \quad . \quad . \quad (84)$$

Also

$$KS=2(r-\tfrac{1}{2}g)\sin\tfrac{1}{2}(F+\psi). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (85)$$

310. **Connecting curve from a curved track to the** INSIDE. As above, it may readily be deduced from the triangle CKS (see Fig. 148) that

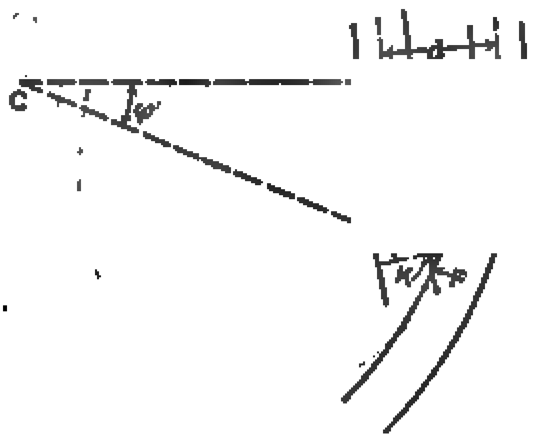


FIG. 148.

$$CK+CS : CK-CS :: \tan\tfrac{1}{2}(CSK+CKS) : \tan\tfrac{1}{2}(CSK-CKS);$$
$$(2R-d-K\sin F) : (d-g-K\sin F :: \cot\tfrac{1}{2}\psi : \tan\tfrac{1}{2}F;$$

$$\tan\tfrac{1}{2}\psi=\frac{2n(d-g-K\sin F)}{2R-d-K\sin F}. \quad . \quad . \quad . \quad . \quad (86)$$

From triangle CO_1K,

$$O_1K : CK :: \sin\psi : \sin(F-\psi);$$
$$(r-\tfrac{1}{2}g) : (R-\tfrac{1}{2}g-K\sin F) :: \sin\psi : \sin(F-\psi);$$

$$(r-\tfrac{1}{2}g)=(R-\tfrac{1}{2}g-K\sin F)\frac{\sin\psi}{\sin(F-\psi)}. \quad . \quad . \quad (87)$$

Also

$$KS = 2(r - \tfrac{1}{2}g) \sin \tfrac{1}{2}(F - \psi). \quad . \quad . \quad . \quad . \quad . \quad (88)$$

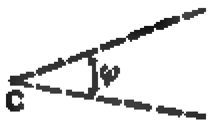


FIG. 149.

Two other cases are possible. (*a*) r may increase until it becomes infinite (see Fig. 149), then $F = \psi$. In such a case we may write, by substituting in Eq. 86,

$$2R - d - K \sin F = 4n^2(d - g - K \sin F). \quad . \quad . \quad . \quad (89)$$

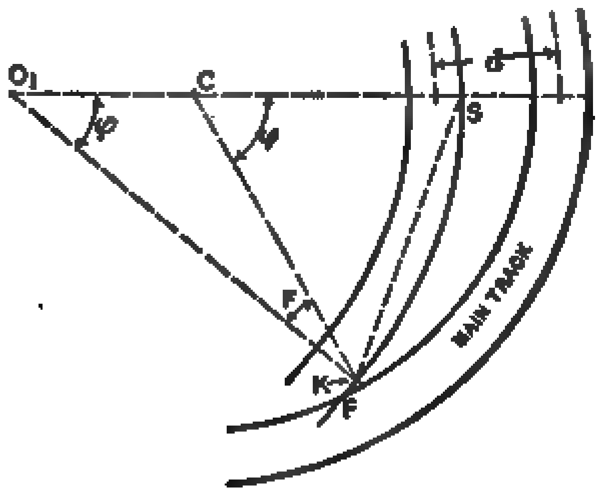


FIG. 150.

This equation shows the value of R which renders this case possible. (*b*) ψ may be greater than F. As before (see Fig. 150).

$$(2R - d - K \sin F) : (d - g - K \sin F) :: \cot \tfrac{1}{2}\psi : \tan \tfrac{1}{2}F;$$

$$\tan \tfrac{1}{2}\psi = \frac{2n(d - g - K \sin F)}{2R - d - K \sin F}$$

the same as Eq. 86, but

$$(r + \tfrac{1}{2}g = (R - \tfrac{1}{2}g - K \sin f)\frac{\sin \psi}{\sin (\psi - F)}. \quad . \quad . \quad . \quad (90)$$

Problem. To find the dimensions of a connecting curve running to the INSIDE of a curved main track; number 9 frog, 4° 30′ curve, $d=13'$, $g=4'\,8\frac{1}{2}''$.

Solution.

[Eq. 86] $d=13.000$	$K=10'\,0''$ $K\sin F=1.108$	$\log 2n=1.2552\bar{7}$
5.816	$g=4.708$	
7.184	5.816	
		$\log 7.184=0.8563\bar{6}$
$R=1273.6$	$2R-d-K\sin F=2533.1$	
$2R=2547.2$	$\log=3.40365$	
$(d+K\sin F)=14.108$	co-log $=6.59635$	
		co-log $=6.59635$
		$\log\tan\frac{1}{2}\psi=8.70799$
		$\frac{1}{2}\psi=2°\,55'\,20''$
		$\psi=5°\,50'\,40''$
		$F=6°\,21'\,35''$
		$F-\psi=0°\,30'.55'$

Since $F>\psi$, we must use Eq. 87, rather than Eq. 90.

$\frac{1}{2}g=2.354$	$R-\frac{1}{2}g-K\sin F=1270.1$	$\log=3.10384$
$K\sin F=1.108$	$(F-\psi)=1855''$; $\log=3.2683\bar{4}$	$\log\sin\psi=9.00787$
sum $=3.462$	4.68557	
	$7.9539\bar{1}$	
	co-log $=2.04608$	co-log $=2.0460\bar{8}$
	$r-\frac{1}{2}g=14381.2$	$4.1577\bar{9}$
	$r=14383.5$	
	$d=0°\,24'$	

[Eq. 88].	2	0.30103
$\frac{1}{2}(F-\psi)=927.5''$; $\log=2.9673\bar{1}$		
$4.6855\bar{7}$	$r-\frac{1}{2}g$	$4.1577\bar{9}$
$\sin\frac{1}{2}(F-\psi)=7.65289$		7.65289
	$KS=129.33$	$2.1117\bar{1}$

311. Crossover between two parallel straight tracks. (See Fig. 151.) The turnouts are as usual. The cross-over track may be straight, or it may be a reversed curve. The reversed curve shortens the total length of track required, but is somewhat objectionable. The first method requires that both frogs must be equal. The second method permits unequal frogs, although equal frogs are preferable. The length of straight crossover track is F_1T.

$$F_1T \sin F_1 + g \cos F_1 = d - g;$$

$$F_1T = \frac{d-g}{\sin F_1} - g \cot F_1. \quad . \quad . \quad (91)$$

The total distance along the track may be derived as follows:

$$DZ = D_1F_1 + D_2F_2 + F_2Y$$
$$= DF_1 + D_2F_2 + XY - XF_2;$$

$$XY = (d-g) \cot F_1;$$

$$XF_2 = g \div \sin F_2;$$

$$\therefore\ D_1Z = 2D_1F_1 + (d-g) \cot F_1 - \frac{g}{\sin F_2}. \quad (92)$$

FIG. 151.

312. Crossover between two parallel curved tracks. Using a straight connecting curve. This solution has limitations. If one frog (F_1) is chosen, F_2 must be determined, being a function of F_1. If F_1 is less than some limit, depending on the width (d) between the parallel tracks, this solution becomes impossible. In Fig. 152 assume F_1 as known. Then $K_1N = g \sec F_1$. In the triangle NOK_2 we have

FIG. 152.

$$\sin NK_2O : \sin K_2NO :: NO : K_2O;$$
$$\sin K_2NO = \cos F_1; \quad NK_2O = 90° + F_2;$$
$$\therefore \sin NK_2O = \cos F_2.$$

$$NO = R + \tfrac{1}{2}d - \tfrac{1}{2}g - K_1 \sin F_1 - g \sec F_1; \quad K_2O = R - \tfrac{1}{2}d + \tfrac{1}{2}g + K_2 \sin F_2;$$

$$\therefore \cos F_2 = \cos F_1 \frac{R + \frac{1}{2}d - \frac{1}{2}g - K_1 \sin F_1 - g \sec F_1}{R - \frac{1}{2}d + \frac{1}{2}g + K_2 \sin F_2}. \quad . \quad (93)$$

The solution of this equation involves the frog angle F_2, which is the angle sought, but there is little error in considering in this solution that $K_2 \sin F_2$ is numerically equal to $K_1 \sin F_1$ and solving accordingly. If the computed value of F_2 is very different from F_1, it would be more precise to recompute Eq. 93 by substituting for $K_2 \sin F_2$ the more exact quantities obtainable from the first trial solution. The relative position of the frogs F_1 and F_2 may be determined as follows:

$$NO_2K = 180° - (90° - F_1) - (90° + F_2) = F_1 - F_2.$$

Then $\quad GF_1 = 2(R + \tfrac{1}{2}d - \tfrac{1}{2}g) \sin \tfrac{1}{2}(F_1 - F_2) + K_1 \cos F_1. \quad . \quad (94)$

There is a theoretical, but practically inappreciable, inaccuracy in Eq. 94, since the chord GF_1 is really the sum of two chords of which one is the chord from the point G to the point where ON produced intersects the gauge line. After locating G, the point radially opposite, on the outer gauge line of the inner track, may be located, from which the frog-point F_2 is located at a distance of $K_2 \cos F_2$. Note that these frog-points referred to are the *theoretical* points. Due allowance must be made during location for the "frog bluntness."

In general, the value of F_2 computed from Eq. 93 is *not* the angle of any standard number-frog, and a strict compliance with theory would require that the frog should be made to order. This is needlessly expensive and the nearest size frog may generally be used without appreciable error.

Example. A crossover between parallel tracks on a 6° curve, the track spacing d being 13 feet. F_1 assumed a No. 9 frog.

[Eq. 93]

$R = 955.37$	$\tfrac{1}{2}g = 2.35$	$K_1 = 10$ ft.
$\tfrac{1}{2}d = \;\;6.5$	$K_1 \sin F_1 = 1.11$	$\sin F_1 = .11077$
961.87	$g \sec F_1 = 4.74$	
$-\;\;8.20$	8.20	

$953.67 \ldots\ldots\ldots\ldots\ldots \log = 2.97940$

$R = 955.37$
$\tfrac{1}{2}g = \;\;2.35$
$K_2 \sin F_2 = \;\;1.11$ (assumed = to $K_1 \sin F_1$)
958.83
$-\tfrac{1}{2}d = -6.5$
$952.33 \ldots\ldots\ldots\ldots\ldots \log = 2.97879$

0.00061

$\log \cos F_1 \quad . \; . \; 9.99732$

$F_2 = 5° \; 35' \; 30''$ $\qquad \log \cos 5° \; 35' \; 30'' \;\; 9.99793$

This angle is within 8 minutes of the angle of a No. 10 frog, which could be used without appreciable error. The point K_2 would be shifted laterally .023 foot, or about ¼ inch, but there would be no visible irregularity in alinement.

$$NOK_2 = F_1 - F_2 = 6°\ 21'\ 35'' - 5°\ 35'\ 30'' = 0°\ 46'.$$

[Eq. 94] $R + \frac{1}{2}d = 961.87$

$-\frac{1}{2}g = -2.35$ 2 . . $\log = 0.30103$

959.52 . . . . $\log = 2.98205$

$\sin \frac{1}{2}NOK_2 = \sin 0°\ 23' = 7.82545$

12.84 $\log = 1.10853$

$K_1 \cos F_1 = 9.94$

$GF_1 = 22.78$

It is instructive to note that if the same crossover problem is worked out for a straight track, as in § 311, using No. 9 frogs on both tracks, the distance between frog points, measured parallel with the track, is nearly the same as in the above problem, especially when the distance 12.84, measured on the outer track, is reduced by bringing it in to the center line. This is analogous to the statement, previously made, that the lead of a switch on a curved track is nearly the same as that for a straight track.

It is theoretically possible to find two standard frog angles which may be so located that the connecting curve consists of straight lines and circular curves, which connect tangentially, making perfect alinement, but such methods are very complicated and the above method is sufficiently exact for practical purposes.

313. **Practical rules for switch-laying.** A consideration of the previous sections will show that the formulæ are comparatively simple when the lead-rails are assumed as circular; that they become complicated, even for turnouts from a straight main track, when the effect of straight frog and point rails is allowed for, and that they become hopelessly complicated when allowing for this effect on turnouts from a curved main track. It is also shown (§ 306) that the length of the lead is practically the same whether the main track is straight or is curved with such curves as are commonly used, and that the degree of curve of the lead-rails from a curved main track may be found with close approximation by mere addition or subtraction. From this it may be assumed that if the length of lead (L) and the

radius of the lead-rails (r) are computed from Eq. 77 and 80 for various frog angles, the same leads may be used for curved main track; also, that the degree of curve of the lead-rails may be found by addition or subtraction, as indicated in § 306, and that the approximations involved will not be of practical detriment. In accordance with this plan Table III has been computed from Eq. 77, 78 and 80. The *leads* there given may be used for all main tracks, straight or curved. The table gives the degree of curve of the lead-rails for *straight* main track; for a turnout to the *inside*, *add* the degree of curve of the main track; for a turnout to the *outside*, *subtract* it.

But there are complications resulting from practical and economical switch construction. A committee of the A. R. E. A., in 1910, adopted certain standards in details, which, when applied to Eqs. 77 to 80 give the values for switch dimensions as quoted in the second section of Table III. They adopted four lengths of switch-rails. In each case the " point " is always ¼″ thick. The gauge line at the other end is always to be placed 6¼″ from the gauge line of the main rail, and the planing is so done that when in this position the switch-rail lies against the main rail. Therefore the angle α is always an angle whose sine equals 6 inches (or 0.5 foot) divided by the length of the switch-rail in feet. In Fig. 153, the point D is not on the gauge line of the main rail but at a point ¼″ away from it, and the point M 6¼″ away from it. The straight rail BF consists of a point-rail at one end, the "closure rails," and one of the wing rails of the frog at the other end. The closure rails will in general consist of one rail cut to a computed length and one or more rails from 24 to 33 feet long, the lengths being in even feet. The curved rail DF will also consist of a point-rail, a frog wing-rail, and one or more lengths of closure rail, but the closure rails in this case are slightly longer than those for the straight rail. Since it is always practically easier to measure to the "actual point" of a frog (see Fig. 134),

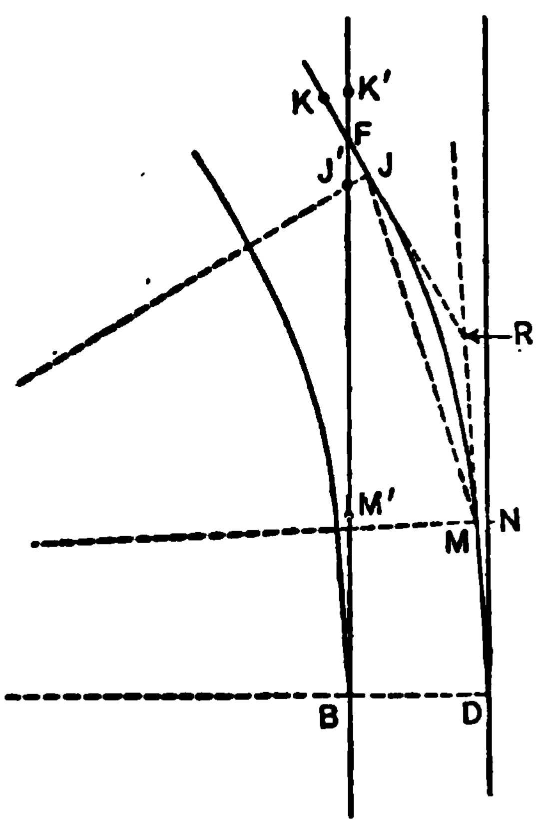


FIG. 153.

rather than to the theoretical point, Table III gives the distance L', which is the distance $L, = BF$, plus the "frog bluntness," which is found by multiplying ½″ (=0.0417 foot) by the frog number.

The curvature for a curved switch-rail (for a straight track) is most readily determined by measuring off a series of ordinates whose origin is at the switch-point D, Fig. 153, the points being the center and the quarter points of the actual curve. These ordinates, as computed on the basis of "practical leads," by the A. R. E. A. committee, are quoted below. It should be remembered that the system of practical leads usually involves a very short tangent adjacent to either M or J, and that the line MJ for "practical leads" is not entirely an arc.

TABLE XXV.—RECTANGULAR COORDINATES TO THE QUARTER AND CENTER POINTS ON THE GAUGE SIDE OF CURVED RAIL, REFERRED TO POINT OF SWITCH-RAIL AS ORIGIN.

Frog No.	Measured along main rail.			Measured perpendicular to main rail.		
	X	X_1	X_2	Y	Y_1	Y_2
4	17.74	23.44	29.75	0.97	1.67	2.79
5	17.78	24.54	31.27	0.95	1.61	2.62
6	19.07	27.13	35.15	1.01	1.74	2.72
7	26.72	36.93	47.11	0.97	1.71	2.74
8	28.37	39.91	51.45	1.02	1.78	2.91
9	28.75	40.98	53.19	1.02	1.76	2.75
9½	30.31	43.35	56.37	1.06	1.82	2.83
10	30.28	44.05	57.81	1.06	1.84	2.85
11	40.74	56.47	72.19	1.08	1.84	2.87
12	43.99	60.65	77.28	1.15	1.90	2.91
15	55.49	77.98	100.45	1.01	1.78	2.84
16	58.16	81.76	105.35	1.04	1.82	2.87
18	58.73	84.46	110.10	1.04	1.82	2.86
20	61.84	90.21	118.59	1.08	1.88	2.93
24	67.82	100.21	132.59	1.27	1.97	3.00

If the position of the switch-block is definitely determined, then the rails must be cut accordingly; but when some freedom is allowable (which never need exceed 16.5 feet and may require but a few inches), one rail-cutting may be avoided. Mark on the rails at B, F, and D; measure off the length DN and locate the point M at the distance H from N. If the frog must be placed during the brief period between the running times of

trains, it will be easier to joint up to the heel of the frog (the point K', Fig. 153), a piece of rail, the farther end of which will just reach the next joint and also joint up to the toe of the frog the straight closure rail and the point-rail. Then, when all is ready, the rails are loosened from the ties back to B, the joint beyond the frog is removed and the whole rail back to B is swung outward. The new combination is shoved into place and spiked, even the point-rail being temporarily spiked to hold it in place as a main track rail, until the other switch-rail and the tie rods can be placed. When the frog is thus in place, the point J becomes located. The curved closure rails, as called for in Table III, should prove to be just long enough, when properly curved, to fill in the gap between M and J. Using the proper pairs of values for X and Y as given above, the three values of X may be measured on the main track rail from the point D, and the corresponding offsets will give points on the curved switch-rail. The old main track rail which was bent outward from B may be utilized as the other switch-rail and set to gauge from the rail just located.

Example.—Given a main track on a 4° curve—a turnout to the outside, using a No. 9 frog; gauge 4′ 8½″; $W=6'.00$; $H=6\frac{1}{4}''$; $S=16'\ 6''$ and $a=1°\ 44'\ 11''$ Then for a *straight* track r would equal 605.18 [$d=9°\ 28'\ 42''$]. For this curved track d will be nearly $9°\ 29'-4°=5°\ 29'$, or r will be 1045.3. L' for a *straight* track would be 72.28, and is here considered to be the same. The closure rails have a total arc length of 49.59, and will here be taken the same. Note that the curved and straight closure rails each have odd lengths which are made by one cut of a 33-foot rail. This avoids all rail waste and also one rail-cutting and the boring of holes.

314. **Slips.** Track movements in crowded yards are facilitated by using "slips" (see Fig. 154), which may be "single" or "double." The crossing of two rails is done either by operating two movable rails or by using fixed "frogs," but a comparison of the continuity of the running rails, using ordinary frogs (see Fig. 134) and these frogs, will show their radical difference. These slips can be used for frog angles from No. 6 to No. 15. The levers are so connected that the several operations necessary to set the rails for any desired train movement are accomplished by one motion.

CROSSINGS.

tracks. When two
ıch other, four frogs
les of two of them
› the angles of the
ssings are sometimes
de, they should be

SECTION ON C-D

SECTION ON A-B

FIG. 155.—CROSSING.

very strongly constructed, and the angles should preferably be 90° or as near that as possible. The frogs will not in general be "stock" frogs of an even number, especially if the angles are large, but must be made to order with the required angles as measured. In Fig 155 are shown the details of such a crossing. Note the fillers, bolts, and guard-rails.

316. One straight and one curved track. Structurally the crossing is about the same as above, but the frog angles are all unequal. In Fig. 156, R is known, and the angle M, made by

the center lines of the tracks at their point of intersection, is also known. $M = NCM. \quad NC = R \cos M.$

$$\left.\begin{array}{l}(R-\tfrac{1}{2}g)\cos F_1 = NC+\tfrac{1}{2}g; \quad \therefore \cos F_1 = \dfrac{R\cos M+\tfrac{1}{2}g}{R-\tfrac{1}{2}g}.\\ \text{Similarly} \quad \cos F_2 = \dfrac{R\cos M+\tfrac{1}{2}g}{R+\tfrac{1}{2}g}, \cos F_3 = \dfrac{R\cos M-\tfrac{1}{2}g}{R+\tfrac{1}{2}g},\\ \qquad\qquad \cos F_4 = \dfrac{R\cos M-\tfrac{1}{2}g}{R-\tfrac{1}{2}g}.\end{array}\right\} \quad (95)$$

$$\left.\begin{array}{l}F_3F_4 = (R+\tfrac{1}{2}g)\sin F_3 - (R-\tfrac{1}{2}g)\sin F_4;\\ HF_4 = (R-\tfrac{1}{2}g)(\sin F_4 - \sin F_1).\end{array}\right\} \quad . \quad . \quad (96)$$

Fig. 156.

317. Two curved tracks. The four frogs are unequal, and the angle of each must be computed. The radii R_1 and R_2 are known; also the angle M. r_1, r_2, r_3 and r_4 are therefore known by adding or subtracting $\frac{1}{2}g$, but the lines are so indicated for brevity. Call the angle $MC_1C_2 = C_1$, the angle $MC_2C_1 = C_2$, and the line $C_1C_2 = c$. Then

$$\tfrac{1}{2}(C_1+C_2) = 90° - \tfrac{1}{2}M$$

and

$$\tan \tfrac{1}{2}(C_1-C_2) = \cot \tfrac{1}{2}M \frac{R_2-R_1}{R_2+R_1} \quad . \quad . \quad . \quad . \quad (97)$$

C_1 and C_2 then become known and

$$c = C_1C_2 = R_2 \frac{\sin M}{\sin C_1} \quad . \quad . \quad . \quad . \quad . \quad (98)$$

In the triangle $F_1C_1C_2$, call $\frac{1}{2}(c+r_1+r_4)=s_1$; $s_2=\frac{1}{2}(c+r_2+r_4)$;

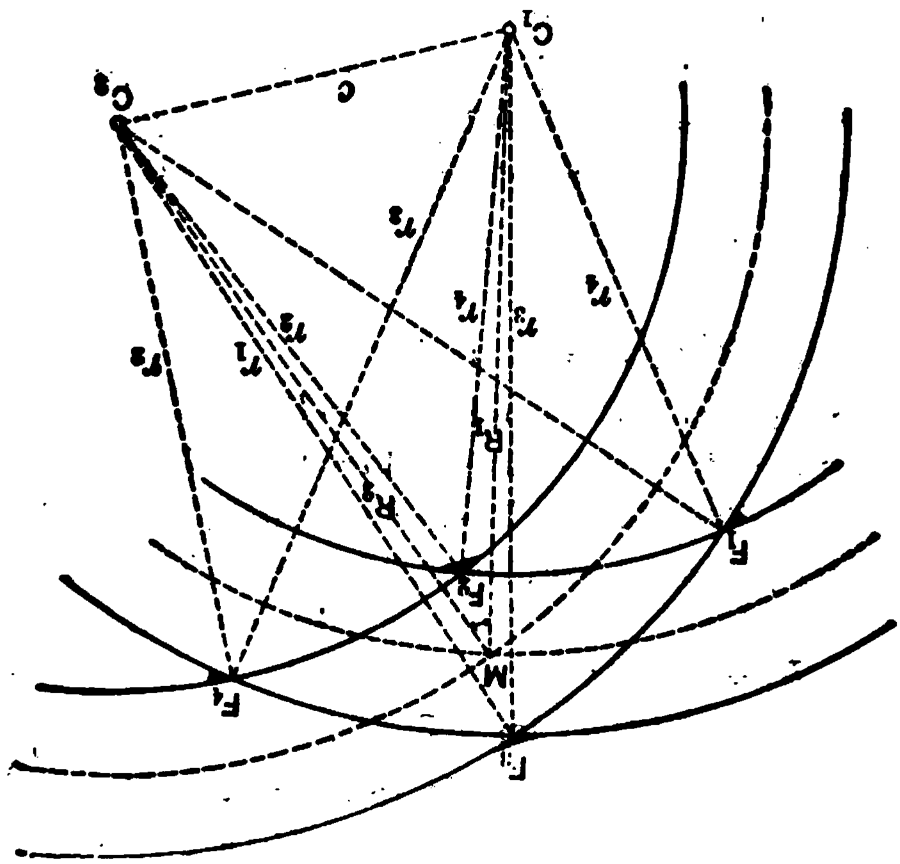


FIG. 157.

$s_3=\frac{1}{2}(c+r_1+r_3)$; and $s_4=\frac{1}{2}(c+r_2+r_3)$. Then, by formula 29, Table XIV,

$$\left.\begin{aligned} \operatorname{vers} F_1 &= \frac{2(s_1-r_1)(s_1-r_4)}{r_1r_4}. \\ \text{Similarly}\qquad \operatorname{vers} F_2 &= \frac{2(s_2-r_2)(s_2-r_4)}{r_2r_4}, \\ \operatorname{vers} F_3 &= \frac{2(s_3-r_1)(s_3-r_3)}{r_1r_3}, \\ \operatorname{vers} F_4 &= \frac{2(s_4-r_2)(s_4-r_3)}{r_2r_3}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (99)$$

$$\sin C_1C_2F_4 = \sin F_4 \frac{r_3}{c};$$

$$\sin C_1C_2F_2 = \sin F_2 \frac{r_4}{c};$$

$$\therefore \quad F_2C_2F_4 = C_1C_2F_4 - C_1C_2F_2, \quad . \quad . \quad . \quad . \quad . \quad (100)$$

$$\sin F_1C_1C_2 = \sin F_1 \frac{r_1}{c};$$

$$\sin F_2C_1C_2 = \sin F_2 \frac{r_2}{c},$$

$$\therefore \quad F_1C_1F_2 = F_1C_1C_2 - F_2C_1C_2; \quad . \quad . \quad . \quad . \quad . \quad (101)$$

from which the chords F_1F_2 and F_2F_4 are readily computed.

F_1F_2 and F_3F_4 are nearly equal. When the tracks are straight and the gauges equal, the quadrilateral is equilateral.

Problem. Required the frog angles and dimensions for a crossing of two curves ($D_1=4°$; $D_2=3°$) when the angle of their tangents at the point of intersection $=62° \; 28'$ (the angle M in Fig. 157).

Solution

$$R_1=1432.7; \quad R_2=1910.1;$$
$$r_1=R_2+\tfrac{1}{2}g=1910.1+2.35=1912.45;$$
$$r_2=R_2-\tfrac{1}{2}g=1910.1-2.35=1907.75;$$
$$r_3=R_1+\tfrac{1}{2}g=1432.7+2.35=1435.05;$$
$$r_4=R_1-\tfrac{1}{2}g=1432.7-2.35=1430.35.$$

Eq. 97. $\log \cot \tfrac{1}{2}M=0.21723$

$R_2-R_1=477.4$; $\log =2.67888$

$R_2+R_1=3342.8$; $\log=3.52411$; co-log $=6.47589$

$\tfrac{1}{2}(C_1-C_2)=13° \; 15' \; 07''$; $\tan 13° \; 15' \; 07''=9.37200$

$\tfrac{1}{2}(C_1+C_2)=58° \; 46'$ $\quad [\tfrac{1}{2}(C_1+C_2)=90°-\tfrac{1}{2}M]$

$$C_1=72° \; 01' \; 07''$$
$$C_2=45° \; 30' \; 53''$$

Eq. 98. $\log R_2=3.28105$

$\log \sin M=9.94779$

$\log \sin C_1=9.97825$; co-log $=0.02175$

$c=C_1C_2=1780.7$; $\quad \log C_1C_2=3.25059$

Eq. 99.

$c=1780.7$	$c=1780.7$	$c=1780.7$	$c=1780.7$
$r_1=1912.45$	$r_2=1907.75$	$r_1=1912.45$	$r_2=1907.75$
$r_4=1430.35$	$r_4=1430.35$	$r_3=1435.05$	$r_3=1435.05$
$2 \mid 5123.50$	$2 \mid 5118.80$	$2 \mid 5128.20$	$2 \mid 5123.50$
$s_1=2561.75$	$s_2=2559.40$	$s_3=2564.10$	$s_4=2561.75$
$s_1-r_1=649.30$	$s_2-r_2=651.65$	$s_3-r_1=651.65$	$s_4-r_2=654.00$
$s_1-r_4=1131.40$	$-r_4=1129.05$	$s_3-r_3=1129.05$	$s_4-r_3=1126.70$

$\log 2=0.30103$

(s_1-r_1); $\log 649.30=2.81244$

(s_1-r_4); $\log 1131.40=3.05361$

$r_1=1912.45$; $\log=3.28159$; co-log $=6.71841$

$r_4=1430.35$; $\log=3.15544$; co-log $=6.84456$

$F_1=62° \; 25' \; 31''$; $\quad \log \text{vers} \; 62° \; 25' \; 31''=9.73006$

$\log 2=0.30103$

(s_2-r_2); $\log 651.65=2.81401$

(s_2-r_4); $\log 1129.05=3.05271$

$r_2=1907.75$; $\log=3.28052$; co-log $=6.71948$

$r_4=1430.35$; $\log=3.15544$; co-log $=6.84456$

$F_2=62° \; 33' \; 55''$; $\quad \log \text{vers} \; 62° \; 33' \; 55''=9.73180$

	$\log 2 = 0.30103$
	$(s_3 - r_1)$; $\log 651.65 = 2.81401$
	$(s_3 - r_3)$; $\log 1129.05 = 3.05271$
$r_1 = 1912.45$; $\log = 3.28159$;	co-log $= 6.71841$
$r_3 = 1435.05$; $\log = 3.15686$;	co-log $= 6.84313$
$F_3 = 62°\ 21'\ 57''$;	log vers $62°\ 21'\ 57'' = 9.72930$
	$\log 2 = 0.30103$
	$(s_4 - r_2)$; $\log 654.00 = 2.81558$
	$(s_4 - r_3)$; $\log 1126.70 = 3.05181$
$r_2 = 1907.75$; $\log = 3.28052$;	co-log $= 6.71948$
$r_3 = 1435.05$; $\log = 3.15686$;	co-log $= 6.84313$
$F_4 = 62°\ 30'\ 14''$;	log vers $62°\ 30'\ 14'' = 9.73103$

As a check, the *mean* of the frog angles $= 62°\ 27'\ 54''$, which is within $6''$ of the value of M.

Eq. 100.	$\log \sin F_4 = 9.94794$
	$\log r_3 = 3.15686$
$\log c = 3.25059$;	co-log $c = 6.74940$
$C_1C_2F_4 = 45°\ 37'\ 51''$;	$\sin C_1C_2F_4 = 9.85421$
	$\log \sin F_2 = 9.94818$
	$\log r_4 = 3.15544$
	co-log $c = 6.74940$
$C_1C_2F_2 = 45°\ 28'\ 17''$;	$\sin C_1C_2F_2 = 9.85303$
$F_2C_2F_4 = 45°\ 37'\ 51'' - 45°\ 28'\ 17'' = 0°\ 09'\ 34''$.	
	$\log 2 = 0.30103$
	$\log r_2 = 3.28052$
$\frac{1}{2}(0°\ 09'\ 34'') = 0°\ 04'\ 47''$;	$\log \sin = \begin{cases} 4.68557 \\ 2.45788 \end{cases}$
$F_2F_4 = 5.309$;	$\log F_2F_4 = 0.72500$
Eq. 101.	$\sin F_1 = 9.94763$
	$\log r_1 = 3.28159$
	co-log $c = 6.74940$
$F_1C_1C_2 = 72°\ 10'\ 22''$;	$\sin F_1C_1C_2 = 9.97863$
	$\sin F_2 = 9.94818$
	$\log r_2 = 3.28052$
	co-log $c = 6.74940$
$F_3C_1C_2 = 71°\ 57'\ 38''$;	$\sin F_3C_1C_2 = 9.97811$
$F_1C_1F_3 = 72°\ 10'\ 22'' - 71°\ 57'\ 38'' = 0°\ 12'\ 44''$.	
	$\log 2 = 0.30103$
	$\log r_4 = 3.15544$
$\frac{1}{2}(0°\ 12'\ 44'') = 0°\ 06'\ 22''$;	$\log \sin = \begin{cases} 4.68557 \\ 2.58206 \end{cases}$
$F_1F_3 = 5.298$;	$\log F_1F_3 = 0.72411$

As a check, F_2F_4 and F_1F_3 are very nearly equal, as they should be.

The foregoing problems on switches, connecting curves and crossings cover only a few of the most common of the problems encountered by the engineer. For the solution of a far wider range of problems, the engineer is referred to "Track Formulæ and Tables," by S. S. Roberts. [Wiley & Sons.]

CHAPTER XII.

MISCELLANEOUS STRUCTURES AND BUILDINGS.

WATER-STATIONS AND WATER-SUPPLY.

318. Location. The water-tank on the tender of a locomotive has a capacity of from 3000 to 10000 gallons—sometimes less, rarely very much more. The consumption of water is very variable, and will correspond very closely with the work done by the engine. On a long down grade it is very small; on a ruling grade, going up, using full stroke, an engine with 28-in. cylinders, 30-in. stroke, 180 lbs. boiler pressure, will use 4.59 lbs. of steam, or water, per stroke or 18.36 pounds per revolution. With 63-in. drivers, the circumference is 16.5 feet and there will be 320 revolutions per mile. The engine will use 5875 lbs. or 700 gallons of water per mile. This engine has a tank capacity of 9000 gallons, which would permit running about 12 miles at full stroke. But it is very rare that a locomotive must work for such long distances at full stroke. After starting and attaining full normal speed, the valves may be set to cut off at one-fourth stroke, or even at one-fifth or one-sixth for high speed running. With ordinary grades, such an engine might average 200 gallons per mile, in both directions. A quoted numerical case is that of a 106-ton engine using 7,500,000 gallons during an annual mileage of 45000 miles. This means an average of 167 gallons per mile. Observations were taken in 1910, on the N. Y. Central R.R., where the grades are moderate, showing that the heavy passenger trains of eight to twelve cars consumed 80 to 100 gallons of water per mile and that freight trains of about fifty loaded cars consumed from 110 to 130 gallons per mile. These figures are far less than those given above, but the grades on the N. Y. Central are very light.

Freight engines, running at lower speeds and longer cut-off, require more frequent water-tanks than passenger engines. Even before a road is built, the water-tank requirements and the minimum spacing may be computed on the basis of the steam consumption (see § 454), of the locomotives with which it is expected to handle the estimated traffic of the road. Usually tanks will be located at intervals of 10 to 20 miles.

In the early history of some of the Pacific railroads it was necessary to attach one or more tank-cars to each train in order to maintain the supply for the engine over stretches of 100 miles and over where there was no water. Since then water-stations have been obtained at great expense by boring artesian wells. The individual locations depend largely on the facility with which a sufficient supply of suitable water may be obtained. Streams intersecting the railroad are sometimes utilized, but if such a stream passes through a limestone region the water is apt to be too hard for use in the boilers. More frequently wells are dug or bored. When the local supply at some determined point is unsuitable, and yet it is necessary to locate a water-station there, it may be found justifiable to pipe the water several miles. The construction of municipal water-works at suitable places along the line has led to the frequent utilization of such supplies. In such cases the railroad is frequently the largest single consumer and obtains the most favorable rates. When possible, water-stations are located at regular stopping points and at division termini.

319. Required qualities of water. Chemically pure water is unknown except as a laboratory product. The water supplied by wells, springs, etc., is always more or less charged with calcium and magnesium carbonates and sulphates, as well as other impurities. The evaporation of water in a boiler precipitates these impurities to the lower surface of the boiler, where they sometimes become incrusted and are difficult to remove. The protection of the iron or steel of a boiler from the fierce heat of the fire depends on the presence of water on the other side of the surface, which will absorb the heat and prevent the metal from assuming an excessively high temperature. If the water side of the metal becomes covered or incrusted with a deposit of chemicals, the conduction of heat to the water is much less free, the metal will become more heated and its deterioration or destruction will be much more rapid. An especially common effect is the production of leaks around the joints between tubes and tube-sheets and the joints in the boiler-plates. Such injury can only be prevented by the application of one (or more) of three general methods—(*a*) the mechanical cleaning of the boilers, (*b*) the chemical purification of the water before its introduction into the boiler, and (*c*) the use of some "boiler compound" which is introduced directly into the boiler and which

causes precipitation of the harmful ingredients as non-incrusting solids which can be readily blown out.

320. **Mechanical cleaning,** as a sole dependence is impracticable except in the comparatively rare localities where the water is so "soft" that no incrusting deposits will be made and such precipitation as does take place is of such a character that it is removable by blowing out the boiler. There are many railroads, especially the smaller ones, which do not give any chemical treatment to any of their engine water-supply, and yet which are not fortunate enough to obtain even approximately soft water. The only method by which such roads can prevent a great waste of heat and the rapid deterioration of boiler tubes and sheets is by frequent mechanical cleaning.

321. **Chemical purification** before the water enters the boiler has the advantage of removing the troublesome ingredients, leaving nothing further to be done except the occasional removal, by blowing out, of the suspended matter or harmless matter precipitated by boiling. Sodium carbonate is the most common reagent. It is commercially sold as "soda crystals, sal soda, washing soda, Scotch soda, concentrated crystal soda, sesquicarbonate of soda, crystal carbonate of soda, black ash, soda ash and pure alkali." Although often chemically impure, it can now readily be obtained with a purity of 97 to 99%. The chemicals which are most common as incrustants are calcium and magnesium carbonates and sulphates. The effect of sodium carbonate on calcium sulphate is to produce soluble sodium sulphate—which is non-incrustant—and calcium carbonate, which precipitates into a sludge at the bottom of the water softener tank. The action on magnesium sulphate is similar. When this is done in a purifying tank, the purified water is drawn off from the top of the tank and supplied pure to the engines. The precipitants are drawn off from the settling-basin at the bottom of the tank. This purification, which makes no pretense of being chemically perfect, may be accomplished for a few cents per 1000 gallons. There are manufacturers which make a specialty of machinery, working more or less automatically, which introduces into the raw water a measured amount of chemical which, by analysis, has been calculated to be necessary with that particular quality of water. In spite of the automatic features, such machinery needs constant attention, and the water, both raw and treated, needs frequent analysis to

insure efficiency, since the character of the raw water may change.

Sodium hydrate, or "caustic soda," has the same general chemical effect as sodium carbonate, and acts more quickly and powerfully, but its caustic nature makes it somewhat objectionable to handle. Common lime, barium hydrate, and many other chemicals are also more or less used.

In the following tabular form is given the quantities of reagents required per unit of scaling or corroding substance held in solution, the table being copied from the 1915 Manual of the Amer. Rwy. Eng. Assoc. "Where the commercial product is not chemically pure, the proportion of reagents should be increased to correspond with an equivalent quantity of pure reagent. Given the analysis of a water, the pounds of incrusting or corrosive matter held in solution per 1000 gallons can be obtained by dividing the grains per gallon of each substance by seven, or the parts per 100,000 by twelve. In order to obtain the full amount of lime necessary, the amount of free carbonic acid contained in the water should be determined, as well as the solids contained in solution, since this free acid must be eliminated in

TABLE XXVI. QUANTITY OF PURE REAGENTS REQUIRED TO REMOVE ONE POUND OF INCRUSTING OR CORROSIVE MATTER FROM THE WATER.

Incrusting or corrosive substance held in solution.	Amount of reagent (pure).	Foaming matter increased
Sulphuric acid	0.57-lb. lime plus 1.08 lbs. soda ash	1.45 lbs.
Free carbonic acid	1.27 lbs. lime	None
Calcium carbonate	0.56-lb. lime	None
Calcium sulphate	0.78-lb. soda ash	1.04 lbs.
Calcium chloride	0.96-lb. soda ash	1.05 "
Calcium nitrate	0.65-lb. soda ash	1.04 "
Magnesium carbonate	1.33 lbs. lime	None
Magnesium sulphate	0.47-lb. lime plus 0.88 lb. soda ash	1.18 lbs.
Magnesium chloride	0.59-lb. lime plus 1.11 lbs. soda ash	1.22 "
Magnesium nitrate	0.38-lb. lime plus 0.72-lb. soda ash	1.15 "
Calcium carbonate	3.15 lbs. barium hydrate	None
Magnesium carbonate	3.76 lbs. barium hydrate	None
Magnesium sulphate	2.62 lbs. barium hydrate	None
Calcium sulphate *	2.32 lbs. barium sulphate	None

* In precipitating the calcium sulphate, there would also be precipitated 0.74 lb. of calcium carbonate or 0.31 lb. of magnesium carbonate, the 2.32 lbs. of barium hydrate performing the work of 0.41 lb. of lime and 0.78 lb. of soda ash, or for reacting on either magnesium or calcium sulphate, 1 lb. of barium hydrate performs the work of 0.18 lb. of lime plus 0.34 lb. of soda ash, and the lime treatment can be correspondingly reduced.

order to obtain efficient treatment of water and reduce scaling matter to the minimum.

322. Foaming and priming. This phenomenon is the foaming or frothing of the water for a considerable height above its normal level in the boiler. The rapid flow of steam into the steam pipe in the dome mechanically carries some of this froth into the steam pipe and causes water to accumulate in the steam pipe and also in the cylinders, with considerable resulting loss in efficiency. Foaming in treated water is largely due to the presence of sodium salts as a result of treatment for incrusting sulphates, and this constitutes one of the objections to the use of soda in treating water. The presence of suspended matter in the water aggravates and even causes foaming. The constant withdrawal of the water from the boiler leaves these suspended solids in the boiler and they keep accumulating until the concentrations reach a critical point, which is about 100 grains per gallon. Beyond this point foaming will be experienced unless the water is changed, which is done by a systematic blowing-off and an occasional complete blowing-down and washing. But blowing-off involves the wastage of water which has been heated to boiler temperature and which has, perhaps, been chemically treated. Even the raw water costs something, perhaps several cents per 1000 gallons. The blowing-off required to keep the concentration below the proper limit may be so excessive that some anti-foaming agent may be necessary. The required effect is physical rather than chemical, the object being to reduce the surface tension, which is done chiefly by the use of oils, petroleum and castor oil being used. Tannic acids are also used for such a purpose.

323. Boiler compounds. Chemical treatment at special plants along the road is unquestionably the most efficient method, but it is costly. The use of boiler compounds, often patented, obviates the erection of any plant, but, since the water at each water-supply station has its own characteristics and it is impracticable to vary the chemicals used at each supply-station according to the character of the water, the treatment is very imperfect. Minute instructions to enginemen to introduce definite amounts of chemical at each water-station have proved unsatisfactory and impractical. Sometimes the chemical is mixed with enough water to partially suspend it and then it is thrown into the tender tank, this method having the advantage that a considerable part of the precipitation takes place promptly and the sludge

never enters the boiler. Sometimes a siphon attached to the feed-pipe outside of the injector, or, perhaps, a special injector, leads from a reservoir in which the chemical, suspended in water, has been placed. Sometimes a stick or "brick" of the chemical is placed directly in the boiler, through a hand-hole, during one of its periodical cleanings. In spite of the inefficiency of the method, 70% of replies to a circular inquiry reported the use of some kind of boiler compound. The chemicals used, some of which are patented compounds, are in general the same as those used in the outside chemical plants. Sodium carbonate is the most common constituent.

324. Tanks. Whatever the source, the water must be led or pumped into tanks which are supported on frames so that the bottoms of the tanks are about 12 feet above the rails. Wooden tanks having a diameter of 24 feet, 16 feet high, and with a capacity of over 50000 gallons, are frequently employed. Iron or steel tanks are also used.

FIG. 158.—WATER-TANK.

In Table XXVII is shown the capacity of cylindrical water-tanks in United States standard gallons of 231 cubic inches. From this table the dimensions of a tank of any desired capacity may readily be found. Two or more tanks are sometimes used rather than construct one of excessive size. The smaller sizes shown in the table are of course too small for ordinary use, but that part of the table was filled out for its possible convenience otherwise. On single-track roads where all engines use one track the tank may be placed 8′ 5″ from the track center; this gives sufficient clearance and yet permits the use of a single swinging pipe which will reach from the bottom of the tank to the tender manhole. In Fig. 158 is illustrated one form of wooden tank. They are preferably manufactured by those who make a special business of it and who by the use

TABLE XXVII—CAPACITY OF CYLINDRICAL WATER-TANKS IN UNITED STATES STANDARD GALLONS OF 231 CUBIC INCHES.

Height in feet.	Diameter of tank in feet.							
	10	12	14	16	18	20	22	24
6	3525	5076	6909	9024	11421	14101	17062	20305
7	4113	5922	8061	10528	13325	16451	19905	23689
8	4700	6768	9212	12032	15229	18801	22749	27073
9	5288	7614	10364	13536	17132	21151	25592	30457
10	5875	8460	11515	15041	19036	23501	28436	33841
11	6463	9306	12667	16545	20939	25851	31280	37225
12	7050	10152	13819	18049	22843	28201	34123	40609
13	7638	10998	14970	19553	24746	30551	36967	43994
14	8225	11844	16122	21057	26650	32901	39810	47378
15	8813	12690	17273	22561	28554	35251	42654	50762
16	9400	13536	18425	24065	30457	37601	45498	54146
17	9988	14383	19576	25569	32361	39951	48341	57530
18	10575	15229	20728	27073	34264	42301	51185	60914
19	11163	16075	21879	28577	36168	44652	54028	64298
20	11750	16921	23031	30081	38071	47002	56872	67682
21	12338	17767	24182	31585	39975	49352	59716	71067
22	12925	18613	25334	33089	41879	51702	62559	74451
23	13513	19459	26485	34593	43782	54052	65403	77835
24	14101	20305	27637	36097	45686	56402	68246	81219
25	14688	21151	28789	37601	47589	58752	71090	84603

of special machinery can insure tight joints. When it is inconvenient to place the tank near the track, or when there is a double track, a "stand-pipe" becomes necessary. See § 327. One of the most difficult and troublesome problems is to prevent freezing, particularly in the valves and pipes. Not only are the pipes carefully covered but fires must be maintained during cold weather. When the pumping is accomplished by means of a steam-pump, supplied from a steam-boiler in the pump-house under the tank, coils of steam-pipe may be employed to heat the water or to heat the pipes. Partial protection may be obtained by means of a double roof and double bottom, the spaces being filled with sawdust or some other non-conductor of heat.

325. Pumping. (*a*) **Steam-pumps.** When coal is very cheap or "when 100 lbs. of coal in the pumphouse is cheaper than one gallon of fuel oil in the storage tank," and especially when steam can be procured from the railroad repair-shop plant, direct-acting steam pumps may be preferable and more economical, but they always require skilled attendance. (*b*) **Gasoline-engines.** These have been so highly developed in recent years that they are very efficient and are nearly "fool-proof," so that they may be oper-

TABLE XXVIII.—COST OF FUEL FOR VARIOUS TYPES OF PUMPS AND ENGINES.

Type.		Fuel.		B. H. P. hour.		Eff. H. P.	
Pump.	Engine.	Kind.	Price.	Fuel used.	Cost.	No.	Cost 10 hrs.
Reciprocating . . .	Steam (slide valve)	Bit. coal	$2.00 per ton	14 lbs.	$0.0126	40	$3.15
" . . .	Internal combustion	Gasoline	0.16 " gal.	1/8 gal.	0.0200	50	4.00
" . . .	" "	Ill. gas	0.75 M cu. ft.	12 cu. ft.	0.0090	50	1.90
" . . .	" "	Nat. "	0.25 " " "	8 cu. ft.	0.0020	50	0.40
" . . .	" "	Fuel oil	0.06 per gal.	1/8 gal.	0.0075	50	1.50
" . . .	Electric motor	Electric	0.03 K. W. hr.	.746 K.W.	0.0224	50	4.48
Centrifugal.	" "	"	0.03 " " "	.746 "	0.0224	50	4.48
"	Internal combustion	Gasoline	0.16 per gal.	1/8 gal.	0.0200	50	4.00
"	" "	Fuel oil	0.06 " "	1/8 "	0.0075	50	1.50

NOTE.—The last column "Eff. H. P., Cost 10 hrs." covers the work required to elevate 400 gal. per minute 100 ft., this being equivalent to a delivery of 240,000 gal. per day of 10 hours and is an average requirement condition of a railroad water-station.

ated by unskilled labor, although skilled attention is periodically necessary. But the rising cost of gasoline has directed attention to other fuels. (*c*) Oil-engines. Crude petroleum, when refined, will give off approximately the following: Ether, 2%; gasoline, 6%; naphtha and benzine, 8%; kerosene, 44%; 39° power distillate, 10%; gas oil, 10%; lubricating oils and petrolatum, 15%, and "slops" 5%. The "fuel oil," as supplied for oil engines, is a mixture of the slops with enough of some other constituent, usually the "power distillate," which is at the time the cheapest, to make the gravity of the mixture about 29°. The fuel oil costs approximately 40% as much as gasoline. Gasoline engines have been converted into fuel oil engines by attaching a mixing chamber in which the oil is heated by the exhaust of the engine. (*d*) Gas-engines, using natural gas. Where natural gas is available at 25 cents per 1000 cu.ft. or less, it is an economical fuel. (*e*) **Electric power.** Where this is obtainable at a low rate, it may be

a cheaper source of power than steam, gasolene or fuel oil. The electric motor either operates a centrifugal pump, or a slow-speed motor is direct-connected to a triplex reciprocating pump.

A Committee of the Amer. Rwy. Eng. Assoc. reported in 1915 the preceding (see p. 374) tabular costs of pumping 240,000 gallons per day of 10 hours. By comparing the data with that of any given locality a fair idea of relative costs and of the proper choice for that particular station may be made.

326. Track tanks. These are chiefly required as one of the means of avoiding delays during fast-train service. A trough, made of steel plate, is placed between the rails on a stretch of *perfectly level* track. A scoop on the end of a pipe is lowered from under the tender into the tank while the train is in motion. The rapid motion scoops up the water, which then flows into the tender tank. They should preferably be located on tangents, although the Penn. R. R. has track tanks at Atglen on a 2° curve where the track has 4 inches superelevation. Since the inside width of the tank (19″) is almost exactly $\frac{1}{3}$ of the gauge, the water is about $1\frac{1}{3}$ inches deeper on the side toward the inner rail, but this much lack of symmetry does not seem to have interfered with successful operation. The length of the tanks varies from 1200 to 2500 feet; the net inside width is usually 19 inches. The scoops are usually 12 to 13 inches wide, which gives allowance for swaying. The tanks are made of sheet steel $\frac{3}{16}''$ to $\frac{1}{4}''$ thick. The usual cross-section is that of a wide and shallow U, 19″ wide, 6″ to $7\frac{1}{2}''$ deep, reinforced on the sides with angles. The ties are usually dapped, especially for the deeper tanks, so that the upper edges will not be higher than the rail. At each end there is a double inclined plane on which the scoops may slide without catching if the scoop should be lowered too soon or if it is not raised before the far end of the tank is reached. Experiments have shown that, at a speed as low as 20 m.p.h., more water is wasted by slopping over the sides than the amount collected by the scoop. At a speed of 45 to 50 m.p.h. the amount wasted becomes minimum and the amount scooped up becomes maximum. At higher speeds the amount scooped up decreases and the wastage increases. The best results show a wastage of at least one-eighth of the total. These same tests showed that at 45 to 50 m.p.h. the 13″ scoop in a 19″ tank will scoop up about 625 gallons per inch of immersion per 1000 feet of tank, or say 2500 gallons per 1000 feet for a 4-inch immersion.

The amount scooped up is practically proportional to the depth of immersion when that depth is over 2¾ inches. **Heating.** The water must be heated in winter to prevent freezing. There are two general methods: (*a*) Live steam is forced into the tank through nozzles about 40 feet apart; (*b*) a "circulatory system" by which steam is forced into a water main which feeds the tank in such a way that the water is in constant circulation through the main, into the tank and then back again into the main to be reheated. For the climatic conditions of the N. Y. Central R. R. a steam capacity of 100 H. P. is considered essentail to heat 7000 sq. ft. of tank surface, which means about 4400 lineal feet of 19-inch tank, or two good-length tanks on a double track. On account of the great amount of water splashed over the track and its scouring action on any ordinary ballast, a large item in the cost of an installation is the reconstruction of the track. The certainty of quick freezing in winter, at least in high latitudes, demands that a drainage system, to carry away the spilled water, shall be effective and thorough. Scouring is prevented by a pavement of cobbles, 6-inch quarry spalls, or large flat stones, laid over the ballast. A layer of large stones under the ballast facilitates drainage to numerous cross drains and to longitudinal drains laid between the tracks. For further details the student is referred to a monograph by Geo. W. Vaughan, Eng. Main. of Way, N. Y. Central R. R., in Vol. XIV, Proc. Am. Rwy. Eng. Assoc.

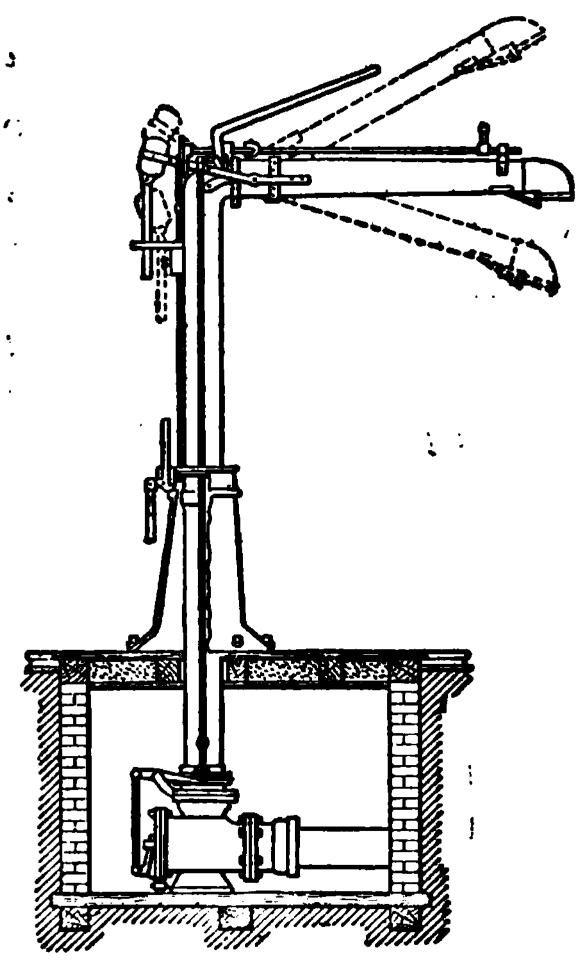


FIG. 159.—STAND-PIPE.

327. Stand-pipes. These are usually manufactured by those who make a specialty of such track accessories, and who can ordinarily be trusted to furnish a correctly designed article. In Fig. 159 is shown a form manufactured by the Sheffield Car Co. Attention is called to the position of the valve and to the device for holding the arm parallel to the track when not in use so that

it will not be struck by a passing train. When a stand-pipe is located between parallel tracks, the strict requirements of clearance demand that the tracks shall be bowed outward slightly. If the tracks were originally straight, they may be shoved over by the trackmen, the shifting gradually running out at about 100 feet each side of the stand-pipe. If the tracks were originally curved, a slight change in radius will suffice to give the necessary extra distance between the tracks.

BUILDINGS.

328. Station platforms. These are most commonly made of planks at minor stations. Concrete is used in better-class work, also paving brick. An estimate of the cost of a platform of paving brick, laid at Topeka, Kan., was $4.89 per 100 square feet when laid flat and $7.24 per 100 square feet when laid on edge. The curbing cost 36 cents per linear foot. Cinders, curbed by timbers or stone, bound by iron rods, make a cheap and fairly durable platform, but in wet weather the cinders will be tracked into the stations and cars. Three inches of crushed stone on a cinder foundation is considered to be still better, after it is once thoroughly packed, than a cinder surface.

Elevation.—The elevation of the platform with respect to the rail has long been a fruitful source of discussion. Some roads make the platforms on a level with the top of the rail, others 3 inches above, others still higher. As a matter of convenience to the passengers, the majority find it easier to enter the car from a high platform, but experience proves that accidents are more numerous with the higher platforms, unless steps are discarded altogether and the cars are entered from level platforms, as is done on elevated roads. As a railroad must generally pay damages to the stumbling passenger, they prefer to build the lower platform. Convenience requires that the rise from the platform to the lowest step should not be greater than the rise of the car steps. This rise is variable, but with the figures usually employed the application of the rule will make the platform 5 ins. to 15 ins. above the rail.

Position with respect to tracks.—Low platforms are generally built to the ends of the ties, or, if at the level of the top of the rail, are built to the rail head. Car steps usually extend 4 ft. 6 ins. from the track center and are 14 ins. to 24 ins. above

the rail. The platform must have plenty of clearance, and when the platform is high its edge is generally required to be 5 ft. 6 ins. from the track center.

329. Minor stations. The Amer. Rwy. Eng. Assoc. recommend one general waiting room (without reference to separate waiting room for colored people), for a passenger station of medium size for the following reasons: (See 1915 Manual, p. 187).

(1) It permits the general waiting room to be properly proportioned.

(2) It permits proper development of a retiring room for women, with private entrance to the lavatory.

FIG. 160.—DIVISION OF FLOOR AREA RECOMMENDED FOR PASSENGER STATIONS WITH ONE GENERAL WAITING ROOM.

(3) It readily admits of the other rooms being properly proportioned.

(4) It permits ease of access from the agent's office to the trains, to the baggage room and to the waiting room.

(5) It permits the ticket office to be of proper size and location for general office purposes.

(6) It admits of the station being contracted in size without detriment to facilities.

(7) It offers economy in heating.

In the Southern States a separate waiting room for colored people is provided and is sometimes even required by law. The older design, combining a residence for the agent with the station, is now obsolete for new construction, although many such still exist. "Combination stations" (for both passenger and freight business) were formerly quite popular for very small stations and

are still considered desirable when all responsible freight and passenger business must be handled by one man. But it is desirable to separate them whenever the volume of business will justify the employment of two responsible men.

In Gillette's Handbook of Cost Data (1910 ed.), is given in detail the cost of several station buildings. Such figures can be utilized when unit prices are given or can be derived. For example, in one case the building was 24×60 ft., exclusive of platforms; there was no masonry foundation nor plastering. The summary was as follows:

Materials.	Total.	Per cent.	Per sq. ft. of floor.
30,057 ft. B. M. at $13.23 (aver.)	$296.97	33.2	21 ft. B. M.
20 M shingles at $1.10	22.00	2.4	
Millwork	55.75	6.1	3.9 cents
Hardware	37.50	4.1	2.6 "
23 gal. paint at 70 cents	16.10	1.8	1.1 "
1100 brick, at $8.00 per M	8.80	1.0	
Total materials	$437.12	48.6	30.4 "
Labor:			
176.2 days' labor, building at $2.32	$406.38	45.3	28.2 cents
2 days' labor, put up ladders, at $2.50	5.00	0.6	
14 days' labor, painting at $1.75	24.50	2.8	1.7 cents
4 days' labor, building chimney, at $4.00	16.00	1.8	
8 days' labor, filling cinders, at $1.20	8.50	0.9	
Total labor	$460.38	51.4	31.9 "
Total, materials and labor	$897.50	100.0	
Freight, 55 tons, 200 miles ½c. ton-m.	55.00		
Tools (excessive in this case)	38.50		
Grand total	$990.00		68.8 cents

The cost of lumber was very low and even the unit cost of labor (carpenters, $2.50; masons, $4.00; average of all, $2.32), were lower than must frequently be paid. But the figures can be utilized by noting the percentages of the various items to the total and applying local unit costs for material and labor. The total cost per square foot ($0.688), is abnormally low, partly because of no masonry foundation nor cellar, which would add 40 to 50 cents per square foot. Note also that no expenses were included for lighting, plumbing, or heating—except a chimney.

FREIGHT HOUSES.

330. Two types. The freight house, or freight room, at a station where the business is small, is merely a small ordinary building or a room attached to the station building. As the business

becomes larger, efficient operation requires that two types of buildings must be designed—the **inbound** and the **outbound** freight house. These types agree in requiring certain details in common, but there are also differences.

331. Fire-risk. A small freight house in the country usually has a minimum of actual fire-risk and of valuable freight stored at any one time. This may justify an inexpensive type of frame building which is in no sense fireproof. On the other hand, a building in the heart of a city, closely surrounded by other buildings and stored with a large amount of valuable freight, justifies an expensive type of fireproof construction. The term "fireproof" is only relative. Certain devices and added expenditures will reduce more and more the probability of destructive fires. Certain principles of construction which reduce fire-risk are as follows: (*a*) Use of noncombustible materials for floor, side walls and roof; (*b*) avoidance of space under wooden main floor, between foundations, where combustible rubbish may accumulate; (*c*) fire-walls dividing large houses so that there is not more than 5000 square feet of floor between fire-walls; fire-walls to be never more than 200 feet apart; (*d*) minimum number of doors through a fire-wall; no door larger than 80 square feet; all doors fireproof and automatically self-closing; (*e*) fireproofing protection of walls and roof for at least five feet each side of a fire-wall; (*f*) provision for fire stand-pipes and hose racks not more than 150 feet apart; the stand-pipe should run up about 8 feet above floor where there should be 50 feet of 2-inch linen hose in a hose rack; the valve should be in a pit (*always* accessible), and so far below floor level that there is little or no danger of freezing, since freight houses are ordinarily *not* heated.

332. Dimensions. A freight house usually has a track on one side and a vehicle driveway on the other, the floor being utilized for the more or less temporary storage of freight, which in this case is always in "less than carload" (L. C. L.) lots, carload shipments being transferred directly between cars and vehicles. Since small shipments can usually be loaded into cars (outbound shipments) with less delay than the delivery of freight to vehicles (inbound shipments), the required space for outbound shipments can be less than that for inbound. Experience has shown that for outbound freight only, a **width** of 30 feet is desirable; for both outbound and inbound, the width may be 30 to 40 feet;

for inbound only it should be 40 to 60 feet. Too great a width needlessly increases the amount of hand-trucking. The length is indefinite and should correspond to the amount of business to be handled. Freight houses are usually single-storied, except where galleries or partial second stories are built to accommodate offices, file and stationery rooms, toilet and locker rooms, the room for "over, short and damaged" freight and the cooperage room for repairing broken packages.

333. **Platforms.** The platform on the track side should preferably be 8 to 10 feet wide, which will avoid the necessity of spotting cars with their doors directly in front of freight-house doors. The platform should be not more than 4 feet above the top of the rail. Even this would be too high to permit opening the doors of refrigerator cars, which swing outward. An occasional refrigerator car could be handled, even with a high platform, by opening the doors before placing the car. The M. C. B. standard, for regular use of refrigerator cars, is "not more than 3 ft. 8 ins. The P. R. R. standard is 3 ft. 5 ins. The minimum distance from track center to edge of platform is 5 ft. 9 ins. The P. R. R. standard is 6 ft. 1½ ins. If there is a platform on the driveway side, it should be 3 to 4 feet above the driveway level. At an outbound house, where the freight is delivered from the vehicle into the freight house, the height should be not more than 3 feet. Platforms should slope away from the house with a grade of about 1 in. to 8 ft. for drainage.

334. **Floors.** The designed floor loading should be 250 lbs. per square foot. In § 347 are described several types of floors suitable for engine houses, many of which are also suitable for freight houses. In selecting a type, it should be remembered that hand-trucking is apt to be concentrated along certain rather narrow paths and that this wears out the floor surface, requiring premature renewals along these paths, unless these paths are overlaid with iron or steel plates. When a solid type of floor is used (supported on sub-soil), the flooring should be independent of the side walls, which avoids trouble due to floor settlement. For inbound freight houses the floor should slope about 1 inch in 8 feet from the track side toward the driveway side, the slope continuing to the outer edge of the driveway platform, since this is in the direction of traffic and aids it, but the track platform must slope the other way for drainage. For outbound freight houses, the slope is exactly reversed.

335. Doors. Ordinary swinging doors are unsuitable. Lifting doors, counterbalanced, which sometimes fold as they lift, are used. Rolling metal shutters are, perhaps, most satisfactory, but are expensive. Sliding doors require that a guarded space be made so that stored freight does not interfere with the sliding. They also limit the possible total door width to less than half the side of the house. All lifting types permit opening up the whole side of the house (if desired), except the space occupied by the posts. Continuous doors are particularly necessary when there is no platform between the house and the track. Doors should be at least 8 feet high. On the track side this is sufficient, since the car door cannot be higher. On the driveway side a greater height might be desirable.

336. Roofs projecting over platforms. These are desirable as a protection when loading or unloading during storms. That over the driveway platform should be at least 10 feet above the platform or 14 feet above the driveway. When not forbidden by State laws, the roof may be extended beyond the edge of the track platform, but it should be, at least, 17 feet above the rail and 18 inches from the track center, thus leaving a walking space on top of the car.

337. Lighting. Daylight lighting should be obtained by windows through the side-walls above the doors, or by vertical sashes in a monitor roof, which will also provide for ventilation. Skylights, especially when nearly flat, are expensive both for construction and for maintenance. Artificial lighting should be obtained from electricity, with wires run according to the strictest specifications of the National Board of Underwriters. Platforms should be illuminated. A series of push plugs should be placed along the platform wall face, from which extension cords with bulbs may be run to light car interiors.

338. Scales. Outbound houses need scales, with capacity of 8000 lbs., to weigh outgoing freight. "From 50 to 80 feet apart is good practice."

339. Ramps. These are slopes from the driveway level to the car level which facilitate the loading or unloading of agricultural implements and all heavy vehicles running on their own wheels. They are usually built at the end of an extension of the platform, with as low a grade as the circumstances will permit.

"Buildings and Structures of American Railroads," by Walter

G. Berg, although now (1916) somewhat old, contains many plans, showing considerable detail, of station and other buildings. "Railroad Structures and Estimates" by J. W. Orrock, also shows some plans.

340. Section houses. These are houses built along the right-of-way by the railroad company as residences for the trackmen. The liability of a wreck or washout at any time and at any part of the road, as well as the convenience of these houses for ordinary track labor, makes it all but essential that the trackmen should live on the right-of-way of the road, so that they may be easily called on for emergency service at any time of day or night. This is especially true when the road passes through a thinly settled section, where it would be difficult if not impossible to obtain suitable boarding places. It is in no sense an extravagance for a railroad to build such houses. Even from the direct financial standpoint the expense is compensated by the corresponding reduction in wages, which are thus paid partly in free house rent. And the value of having men on hand for emergencies will often repay the cost in a single night. Where the country is thickly settled the need for such houses is not so great, and railroads will utilize or perhaps build any sort of suitable house, but on Southern or Western roads, where the need for such houses is greater, standard plans have been studied with great care, so as to obtain a maximum of durability, usefulness, comfort, and economy of construction. (See Berg's Buildings, etc., noted above.) On Northwestern roads, protection against cold and rain or snow is the chief characteristic; on Southern roads good ventilation and durability must be chiefly considered. Such houses may be divided into two general classes—(*a*) those which are intended for trackmen only and which may be built with great simplicity, the only essential requirements being a living room and a dormitory, and (*b*) those which are intended for families, the houses being then distinguished as "dwelling-houses for employees."

ENGINE HOUSES.*

341. Form. When not more than three or four engines are to be housed at once and when no turntable is to be provided,

* Condensed and abbreviated from Committee Report, Am. Ry. Eng. Assoc., 1915.

the rectangular form is preferable. All large engine houses are "circular," with a turntable at the center of the circle, except some very large houses, which are really repair shops, where it seems advisable to install a transfer table.

342. Doors. The clear opening should be not less than 13 feet wide by 16 feet high. The doors should fold outward and should have such a design that a pilot door may be inserted.

343. Length. The length of stall along the center line of the track should be 15 feet greater than the overall length of the longest locomotive, which will provide a walkway behind the tender, a trucking space in front of the pilot and a sufficient distance in which to stop the engine so that the side rods will be in any desired position.

344. Materials of construction. Wood was formerly very commonly used, but it is too inflammable. The walls should be made of brick, stone, or plain concrete—not reinforced, at least "for that portion of the wall directly in line of track where engine is liable to run into it." The roof is the difficult problem, since wood is inflammable and iron or steel, even for framing, is very rapidly corroded by coal gas from the engines. Reinforced concrete is the only thoroughly satisfactory material but "when the roof is of reinforced concrete, the columns and roof beams should be of the same material," i. e., it is useless to support a reinforced concrete slab on steel beams.

345. Engine pits. These "should be not less than 60 feet in length, with convex floor, with drainage toward the turntable. The walls and floors may be of concrete. Proper provision should be made for the support of the jacking timbers." The engine should stand with its tender toward the turntable.

346. Smokejacks. Locomotives leave an engine house under their own steam, which requires starting their fires considerably beforehand, and the smoke must be removed. The precise position of the locomotive on the track is variable, since it must be adjusted to the place where the side rods are in a proper position for repairs. A smokejack is essentially a funnel whose base is at the minimum height above the track which will give the smokestack a proper clearance. The base should be 42 inches wide and long enough for the adjustment as stated above, which means at least 10 feet. The sides should slope upward gradually to a flue whose area should be not less than 7 square feet. There should be a drip trough around the base of the jack.

The material should be "non-combustible," but the choice is troublesome. Sheet iron, even when heavily painted, corrodes rapidly. Wood, covered with "fireproof paint," has been tried. Cast iron has been tried but is exceedingly heavy as well as expensive. Asbestos is being used on several important roads. Patented designs, of which there are several, are used on the majority of roads.

347. **Floors.** (*a*) **Stone screenings.** Subsoil should be good; all soft spots cleaned out and filled with good material; subsoil rolled. Foundation of cinders or gravel, 6 ins. thick. Top coat, 2 inches of stone screenings, perhaps mixed with a little clay or crude oil, the surface being thoroughly rolled. Special foundations for machinery necessary. Surface is not good for heavy wheeling. (*b*) **Planks.** Subsoil same as above; 6 ins. cinders or gravel, with 4″×6″ creosoted sleepers, spaced about 3 feet, embedded in upper surface of cinders; then 3-inch plank. Again, special foundations for machinery and at jacking-up places are necessary. (*c*) **Creosoted wood-block.** The wood blocks, 4 ins. deep, fiber vertical, should be laid on a 1-inch cushion coat of sand which is supported by a 6-inch layer of concrete. A 6-inch layer of cinders, as specified above, is also recommended as a bed for the concrete, but this may depend on the character of the subsoil. The joints should be filled with asphaltic mastic, and an expansion joint 1 inch wide should be provided every 50 feet. (*d*) **Wood floor on concrete.** Sleepers, spaced about 3 feet, trapezoidal, 4-inch top, 6-inch bottom, 4 inches deep, embedded in a 6-inch layer of concrete, so that the sleepers project ½ inch above concrete. Then layer of 2-inch plank, covered with 1⅛-inch maple flooring. (*e*) **Brick.** Same as (*c*) except that bricks are used in place of wood block. (*f*) **Concrete.** Same foundation as above; 6-inch course of concrete overlaid with 1-inch surface coat (1:2) laid on before base has taken initial set. (*g*) **Asphalt.** Same as (*f*) except that surface coat is 1½ inches of rock mastic. Expert workmen are needed for satisfactorily mixing and laying the asphalt, but the floor is ideal.

348. **Drop pits** are necessary, where pairs of truck, driving and trailer wheels may be dropped from their journals and removed from the engine for repairs or renewals.

349. **Heating.** The primary object of heating is to thaw out the engines so that they may be returned to service as quickly

as possible, rather than to heat the building, whose general temperature should be kept at 50° to 60°. Therefore heat should be concentrated at the pits. Hot air should be forced through permanent ducts, preferably laid under the floor. The outlets should have dampers, which may be closed when men are working in the pits. Fresh air should be drawn from outdoors and no recirculation permitted. The air should be heated by passing over coils containing exhaust steam, supplemented by live steam, if necessary. The air passes out of the building through annular openings around the smokejacks, and also through openings between the wall plates and the roof rafters. These openings should extend entirely around the building.

350. **Window lighting.** Skylights are undesirable because of preponderant disadvantages. The windows in the outer walls should be as large, wide and as high as safe construction will permit, the sill not more than 4 feet from the floor. Windows should be placed over the locomotive doors. Windows set into locomotive doors cause heavy maintenance charges on the doors.

351. **Electric lighting.** Numerous lights should be provided to avoid shadows. Plugged outlets for incandescent lights in alternate spaces between pits should be provided.

352. **Piping.** Pipes for air, steam and water supply should be provided, and where desired, piping for a washout and refilling system should be installed. Where this system is installed, the blow-off lines should be led to a central reservoir; where it is not used, the blow-off lines should be led outside the house. The steam outlet should be located near the front end of the boiler. The blow-off pipe, the air, the washout and refilling water and the cold water connections should be near the front end of the firebox. Connections need only be provided in alternate spaces between stalls.

353. **Tools.** There should ordinarily be facilities provided for hand tools and for the location of a few machine tools, preferably electrically driven.

354. **Hoists.** Hoists with differential blocks are generally used for handling heavy repair parts, and suitable provision should be made for supporting them.

355. **Turntables.** The turntable should be long enough to balance the engine when the tender is empty. The deck form is preferable to the through form. Power should be provided at turntables having great service. Electric power is best and least

expensive when it is available. Compressed air, supplied either by a pumping plant or by the locomotive itself, is sometimes used. The turntable pit should be thoroughly drained and preferably paved. The circle wall should be of concrete or brick, with proper supports and fastenings for rails on the coping. The circle rail should preferably bear directly on concrete base. The use of wood ties and tie-plates supported by masonry is desirable for the circle rail under some conditions. Easy access to the parts of a turntable for the oiling of bearings, painting and inspection should be provided in the design of the turntable pit, unless ample provision is made in the turntable itself.

LOCOMOTIVE COALING STATIONS.

356. **Hand shoveling.** For roads of the smallest traffic, particularly at terminals where locomotives lie overnight, hand shoveling direct from coal cars or from platforms provided with a jib crane and one-ton buckets, is the most economical.

357. **Locomotive crane.** A locomotive crane, equipped with buckets, provides an efficient method of transferring coal from the coal car to a tender, particularly when the crane can be profitably employed at other times.

358. **Coaling trestle.** This method requires a trestle with an approach not exceeding 5%, so that coal may fall from bottom-dumping cars into a pocket and then be discharged through chutes into the tender on a track on either side of the trestle. This method is satisfactory when two coaling tracks are sufficient and when there is available space for the approach track.

359. **Coal conveyors.** When more than two coaling tracks are essential, a conveyor system may be preferable. The coal is brought to the plant in bottom-dumping gondola cars, which dump the coal on to a conveyor which conveys it up and drops it into the bin, from which it may fall either into the tender or into an elevated conveyor car which runs it across a system of parallel tracks and dumps it into a tender, spotted there for the purpose. Incidentally, such a plant usually has also an **ash conveyor** onto which ashes are dumped from the engine. This conveyor carries the ashes to a place where the conveyor buckets dump them into a waiting gondola car, which when full is hauled away.

360. Oil houses * should be fireproof and should be separated from other buildings. Above ground there should be a masonry building, 20′×40′, or perhaps less, with one fireproof door and one or more windows, having wire glass. This room contains a row of pumps, one for each kind of oil; also a series of inlet pipes in the floor leading to tanks in the basement. The floor should be 4 feet above the track rail outside and there should be a

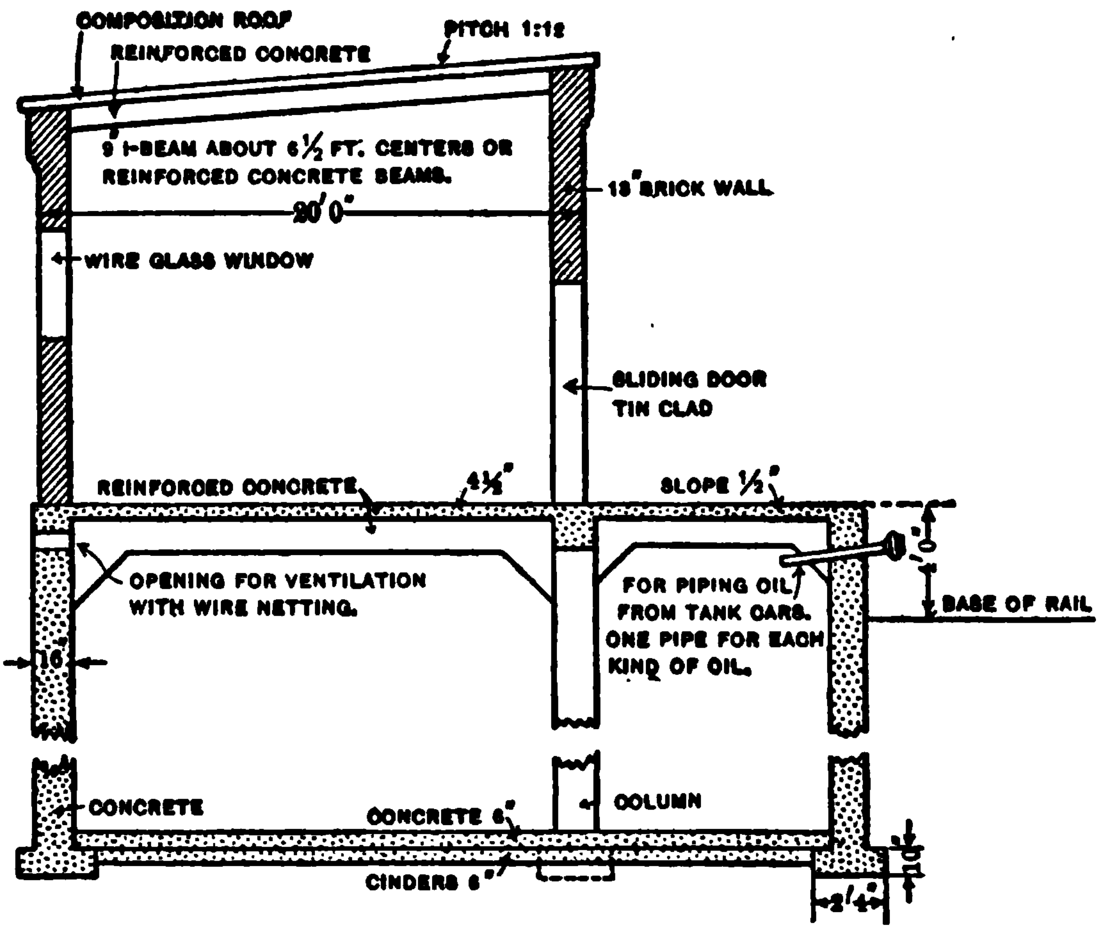


FIG. 161.—CROSS-SECTION OF TYPICAL OIL-HOUSE.

platform between the house and the track. The storage space for oil is entirely in the basement and includes the area under the floor and also the area under the platform. The height depends on the required storage space for tanks. A series of pipes, one for each kind of oil, pass through the outer vertical face of the platform, for the convenient emptying of tank cars into the storage tanks. The inlet pipes through the floor are only for small quantities of oil drawn from barrels.

The delivery system from the storage tanks to the faucets should be such that the oil can be delivered quickly and measured automatically. The delivery should also be such that there will

* Condensed from the Manual of the Am. Rwy. Eng. Assoc., 1915 Ed.

be a minimum of dripping at the faucet and that the dripping may drain back to the storage tanks. Openings for ventilation should be provided above the level of the top of the tanks. Lighting, when required, should be by electricity and heating by steam. For fire protection purposes a live-steam line should be run to the oil storage space, controlled by a valve outside the house.

361. Section tool houses. For small-traffic roads these should be 10′×14′, the short dimension parallel with the track, with double swinging doors, swinging out on the end nearest the track. For roads of larger traffic the dimension parallel with the track should be 18 to 20 feet and the other dimension 12 to 14 feet. There should be a sliding door, 8 feet in clear, at extreme end, on track side, to permit the storing of hand car. A sliding wooden shutter (instead of glass) may serve as a window for fair weather. It should *not* be made so convenient and comfortable that it will become a lounging place for trackmen in stormy or wintry weather. The building should be of wooden frame construction, resting on wooden posts, or on masonry piers if the location can be considered permanent. Drop siding on the sides and some kind of prepared roofing will usually be most economical.

362. Sand houses. Sand is a necessity in the operation of locomotives. Ordinarily it is obtained in a more or less moist and caked condition. It must be made thoroughly dry, so that it will flow readily through a pipe having sufficient slope. The plant consists essentially of a "wet storage bin," about 12′×16′, which adjoins a "drying room" of about the same size. This room contains a screen, which is usually necessary to screen out the coarser particles; also a furnace to dry the sand, and a coal bin. For small traffic roads it may be sufficient to store the dry sand in a bin or even in buckets which are lifted by hand to the engine. For heavier traffic it may be justifiable to raise the sand to a bin or hopper whose lowest point is at least 22 feet above the rail, from which the sand may flow through a jointed pipe, somewhat similar to a water-supply pipe, directly into the sand box on the engine. Of course the bottom of the hopper must have sufficient slope so that the sand will always flow over it. The sand is hoisted to the hopper, either by some mechanical conveyor system, or is forced through a pipe by compressed air. The building should be located about 8 feet from the nearest track center.

363. Ash pits. A locomotive must dump the ashes from its ash pan at frequent intervals. The operation is usually timed to be done at terminal or divisional points, just before taking on water, coal, etc. These several plants are, therefore, grouped together in the yard. When there are no facilities for removing ashes by a conveyor at the same time that coal is being loaded on to the tender (see §§ 356–359), the ashes are dumped into a pit. The poorest roads dump them on the track under the engine, but this burns the ties, is dangerous, and is uneconomical, since they must be immediately removed. The simplest form of ash pit is made by dropping the ties about a foot, and then laying the rails on a pair of stringers about 12″×12″. The stringers and ties must be covered with sheet iron to protect them from hot ashes. The capacity of such a pit is so small that the ashes must be removed quite frequently, which must usually be done by hand shoveling over the side of a gondola car on an adjacent track. The next development is a deeper pit, with concrete walls. Even then, the rails must be fastened to longitudinal wooden stringers, protected with sheet iron, or to cast-iron chairs which are embedded in the concrete. The ashes may be shoveled out by hand after the locomotive has passed, or they may be dropped from the ash pan into buckets or small cars, which run on a narrow track at the bottom of the pit, and which may be lifted out by a jib crane. Another development is to widen the pit, running one rail on one wall and the other rail on a series of cast-iron columns. The pit has much greater capacity and the ashes may be hoisted out at any time, even if the locomotive is still on the ash track. Great economy in the disposal of ashes is obtained when it is practicable to construct a depressed track, with its track center about 14 feet away from the ash track and 9 feet or more lower. The ashes may then be dropped onto a platform about 3 feet below the ash track, the platform extending to the top of a vertical retaining wall whose face is 5 ft. 6 ins. from the center of the depressed track, and from there the ashes are easily shoveled over the side of a gondola car placed on the lower track. No lifting of the ashes by hand is necessary. As in the previous plan, one rail of the ash track is supported by a wall, while the rail toward the depressed track is supported on cast-iron columns. The platform space is thus 10 to 11 feet wide.

Ashes should be quenched promptly after being deposited,

so as to reduce their heating effect even on metal and masonry. This requires a hose and a water supply. The pits should be graded so as to drain to a sump, which should have an overflow sufficiently above the bottom so that periodical cleaning out will suffice to keep the drain pipe from getting clogged with detritus from the ashes.

SNOW STRUCTURES.

364. Snow-fences. Snow structures are of two distinct kinds—fences and sheds. A snow-fence implies drifting snow—snow carried by wind—and aims to cause all drifting snow to be deposited away from the track. Some designs actually succeed in making the wind an agent for clearing snow from the track where it has naturally fallen. A snow-fence is placed at right angles to the prevailing direction of the wind and 50 to 100 feet away from the tracks. When the road line is at right angles to the prevailing wind, the right-of-way fence may be built as a snow-fence—high and with tight boarding. Hedges have sometimes been planted to serve this purpose. When the prevailing wind is oblique, the snow fences must be built in sections where they will serve the best purpose. The fences act as wind breakers, suddenly lowering the velocity of the wind and causing the snow carried by the wind to be deposited along the fence. Portable fences are frequently used, which are placed (by permission of the adjoining property owners) outside of the right-of-way. If a drift forms to the height of the portable fence the fence may be replaced on the top of the drift, where it may act as before, forming a still higher drift. When the prevailing wind runs along the track line, snow-fences built in short sections on the sides will cause snow to deposit around them while it scours its way along the track line, actually clearing it. Such a method is in successful operation at some places on the White Mountain and Concord divisions of the Boston & Maine Railroad. Snow-fences, in connection with a moderate amount of shoveling and plowing, suffice to keep the tracks clear on railroads not troubled with avalanches. In such cases snow-sheds are the only alternative.

365. Snow-sheds. These are structures which will actually keep the tracks clear from snow regardless of its depth outside. Fortunately they are only necessary in the comparatively rare situations where the snowfall is excessive and where the snow

is liable to slide down steep mountain slopes in avalanches. These avalanches frequently bring down with them rocks, trees, and earth, which would otherwise choke up the road-bed and render it in a moment utterly impassable for weeks to come. The sheds are usually built of 12″×12″ timber framed in about the same manner as trestle timbering; the "bents" are sometimes placed as close as 5 feet, and even this has proved insufficient to withstand the force of avalanches. The sheds are there-

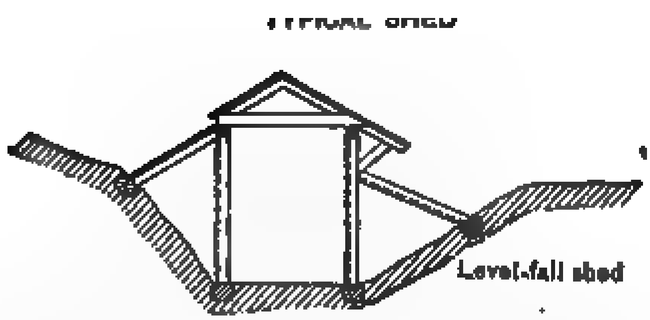


FIG. 102.—SNOW-SHEDS—CANADIAN PACIFIC RAILROAD.

fore so designed that the avalanche will be *deflected* over them instead of spending its force against them. Although these sheds are only used in especially exposed places, yet their length is frequently very great and they are liable to destruction by fire. To confine such a fire to a limited section, "fire-breaks" are made—i.e., the shed is discontinued for a length of perhaps 100 feet. Then, to protect that section of track, a V-shaped deflector will be placed on the uphill side which will deflect all descending material so that it passes over the sheds. Solid crib

work is largely used for these structures. Fortunately suitable timber for such construction is usually plentiful and cheap where these structures are necessary. Sufficient ventilation is obtained by longitudinal openings along one side immediately under the roof. "Summer" tracks are usually built outside the sheds to avoid the discomfort of passing through these semi-tunnels in pleasant weather. The fundamental elements in the design of such structures is shown in Fig. 162, which illustrates some of the sheds used on the Canadian Pacific Railroad.

FENCES.

366. Wire fences. The following is condensed from the conclusions adopted by the Amer. Rwy. Eng. Assoc. and incorporated in their 1915 Manual. The recommended standard right-of-way fence is a wire fence, supported on wood or concrete posts. The wiring is to consist of five to nine longitudinal strands, with vertical stay wires spaced 12 to 24 inches apart. The longitudinal and vertical wires are to be locked or fastened with a mechanical lock which will prevent slipping either longitudinally or vertically, or the wires shall be electrically welded. The wire shall be galvanized so as to stand the following test: "The galvanizing shall consist of an even coating of zinc, which shall withstand one-minute immersion tests in a solution of commercial sulphate of copper crystals and water, the specific gravity of which shall be 1.185 and whose temperature shall be from 60° to 70° F. Immediately after each immersion the sample shall be washed in water and wiped dry. If the zinc is removed, or a copper-colored deposit formed at the end of the fourth immersion, the lot of material from which the sample is taken shall be rejected. The fence shall be so fabricated as not to remove the galvanizing or impair the tensile strength of the wire." Electrically welded fencing should be galvanized after it has been fabricated.

367. Types. Class A fence has 9 horizontal smooth wires whose spacing, starting at the ground, is 5, 4, 4½, 5, 5½, 6, 7, 8 and 9 inches. To make it "hog-tight" the bottom space (5") is reduced to 3 inches and a barbed wire is inserted midway in the 3-inch space. The top and bottom smooth wires are No. 7 gauge wire and the 7 intermediate wires are No. 9. The vertical stay wires, spaced 12 inches, shall be No. 9 gauge.

Class B fence has 7 horizontal wires, and 2 vertical wires spaced 18 inches—all wires No. 9 gauge. The spacing, starting at the ground, is 7, 6½, 7, 7½, 8, 8½ and 9 inches.

Class C fence has 5 horizontal wires, and 2 vertical wires spaced 24 inches—all wires No. 9 gauge. The spacing, starting at the ground, is 9, 7½, 8, 8½ and 9 inches.

Class D fence has 5 horizontal wires and no vertical stay wires, the wires being No. 9 gauge. The spacing, starting at the ground, is 10, 10, 10, 12 and 12 inches.

368. **Posts.** End, corner, anchor and gate posts shall be at least 8 feet long and set 3 feet 4 inches in the ground, even if blasting must be resorted to. Intermediate posts shall be at least 7 feet long and set 2 feet 4 inches in the ground. Where rock is encountered at intermediate post holes, the intermediate posts, if of wood and not more than two in succession, may be set on sills, 6″×6″×4′ 0″, braced on both sides by braces 2″×6″×3′ 0″. End, corner, anchor and gate posts, when of wood, shall be 8 inches in diameter at the small end; when of concrete, shall be 6 inches square at the top, 8 inches square at the base and shall be reinforced with four ⅜-inch square twisted rods. Intermediate wood posts shall be at least 4 inches in diameter at the small end; intermediate concrete posts shall be 4 inches thick at the top, 5½ inches at the bottom and reinforced with three (or four, depending on design) ¼-inch square twisted rods.

369. **Braces.** End, corner, anchor and gate posts shall be braced by 4″×4″ sawed lumber, or round posts at least 4 inches in diameter, or by concrete struts, 4″×4″, reinforced with four ¼-inch twisted rods. The strut braces shall extend from a point about 12″ below the top of the braced post to a point about 12″ from the ground line at the adjacent intermediate post. In addition, a tie, made of a double strand of No. 9 galvanized soft wire, looped around the end, corner, anchor or gate post near the ground line, and around the next intermediate or line post about 12 inches from the top, shall be put on and twisted until the top of the next intermediate or line post is drawn back about 2 inches.

370. **Concrete posts.** These are recommended. They may be made of one part of cement to four parts of pit gravel; or one part cement, two parts sand and four parts of stone of low absorption or screened gravel, the aggregate in any case being not less than ¼″ nor more than ½″. The molds should be oiled

or soaped and should be vibrated while concrete is poured to make the concrete more compact. The concrete should have a "quaking" consistency. The pouring should not be done out of doors in freezing weather. The concrete should not be exposed to sun, should be sprinkled every day for 8 or 10 days and should have 90 days for curing. They should be packed in sawdust or straw for shipment. Posts are usually made tapering and the cross-section is variously a square, a rectangle, or an isosceles triangle, the corners being chamfered. The reinforcement should be placed not more than ½″ from the surface and should be wired by bands spaced about 12″. The fencing is sometimes fastened to the posts merely by wires tied tightly about the post or may be fastened to metal lugs which are embedded in the soft concrete during molding.

371. **Construction details.** Wood posts shall be **anchored** by gaining and spiking two cleats, 2″×6″×2′ 0″, on the side of the post below the ground line. **Staples** shall be 1 inch long for hard wood, and 1½ inch for soft wood, made of No. 9 galvanized steel wire. They shall be driven diagonally with the grain of the wood, the top wires double-stapled. Staples, No. 9 wire, 1 inch long, weigh 108 to the pound; 1½ inch long, 72 to the pound.

Wire. No. 7 wire is 0.177 inch in diameter, weighs 439 pounds to the mile, or 12.05 feet to the pound. No. 9 wire is 0.148 inch in diameter, weighs 306 pounds to the mile or 17.24 feet to the pound. Smooth wire is preferable to barbed. A heavy smooth wire or a plank should be used at the top of a barbed-wire fence. Wires shall be placed on the side of the post away from the track. **Splicing** shall be done as follows: "The ends of the wires shall be carried 3 inches past the splicing tools and wrapped around both wires backward from the tool for at least five turns, and after the tool is removed, the space occupied by it shall be closed by pulling the ends together." After erection, wood posts should be sawed off, on a one-fourth pitch, the high side being next to the wire and 2 inches above it.

Gates should be hinged to swing away from the track; should be at least 12 feet wide and 4 feet 6 inches above the ground; should swing shut by gravity, and the free end should overlap the post so that it cannot be swung open toward the track. All-metal construction is preferable.

SIGNS.

372. Highway signs. The crossing sign recommended by the Amer. Rwy. Eng. Assoc. is essentially as follows: Two wooden blades, 12 inches wide, 8 feet long, with mitered ends, are placed diagonally, with an angle of 50° between the blades, on an 8″×8″×16′ 0″ wooden post sunk 4 feet in the ground. The lower 9 feet is painted black, the upper 7 feet white. The blades are painted white with black letters and a ½-inch black border around the blades. The border and lettering is on both sides. The lettering is Egyptian style 9 inches high with the exception of the connecting terms, as "for the" in the recommended sign, which should be 4 inches high. The recommended wording is "RAILROAD CROSSING" on one blade and "LOOK OUT FOR THE LOCOMOTIVE" on the other blade. The width of band of the letters is 1¼ inches. If two railroads parallel each other within 400 feet, another blade marked "TWO CROSSINGS" should be added. The laws in some states prescribe what the lettering shall be.

373. Trespass signs. The specifications for these signs are applicable to many other public warnings which must be displayed. A cast-iron plate, ¼ inch thick, stiffened on the back by ⅜-inch diagonal cast ribs and having thé letters and border cast on the front by raising the surface about ⅛ inch, is set on an iron post 10 feet long, which is embedded 2 feet in a block of concrete, which serves as foundation. The letters should be about 2 inches high. A socket is cast on the rear side of the plate of such dimensions that it will set on the pipe and be fastened with a ½-inch set screw. The posts may be made of 2½-inch wrought iron pipe or of good second-hand boiler tubes, which should be filled with cement grout. The face of the letters and the borders should be painted black while the background is painted white. The tablet will usually be about 30 inches wide by 18 inches high with rounded corners, although the dimensions will vary in accordance with the lettering to be placed on it. The following trespass signs frequently need to be displayed:

RAILROAD PROPERTY TRESPASSING FORBIDDEN UNDER PENALTY OF LAW	DANGER DO NOT TRESPASS ON THE RAILROAD

DANGER DO NOT TRESPASS ON THIS BRIDGE

374. **Marker posts.** **Mile posts** are most economically made, considering their durability, of skeletonized cast iron. The post is made up of two slabs of cast iron ½ inch thick, 8 feet long, the width tapering from 10 inches to 12 inches, the two slabs being formed in one piece and connected at intervals by ½-inch webs and a top and bottom plate. They should be set 3 feet 6 inches in the ground and have a 4-inch slab of concrete or a heavy, flat stone as a base. The mile post numbers should be cast in raised letters on the face, the letters being 4½ inches high. The two faces should be at right angles with each other and should each stand at an angle of 45° with the track. They should be set at least 8 feet from the gauge line of the nearest rail and 11 feet away, where it is practicable. The numbers should be so set that, on approach, the distance to the terminus or division point beyond will be indicated.

The separating line between divisions is indicated to track men by an iron sign, called a **division post**, which is structurally the same as that of the mile posts. The two divisions are indicated by raised lettering on the faces of the posts. Of course there must be a variation in the lettering or numbering and a special post must be cast for each location of division post or mile post.

Whistle signs are made similarly except that there is but one slab, suitably reinforced with ribs, and which faces in the desired direction. The letter W 7½ ins. high is cast in raised letters near the top. The **ring sign** is made similarly by using the letter R. The separating line between sections is indicated to the trackmen by a cast-iron sign, called a **section post**, which is made similarly to the Trespass Signs, except that the tablet is much smaller. Such a sign will have two consecutive numbers, for example, 24–25, to indicate that the sign is at the separating line between section 24 and section 25.

375. **Bridge warning.** When possible the headroom beneath overhead bridges is made at least 22 ft., which will make it safe for a trainman to stand on the top of a freight car which is

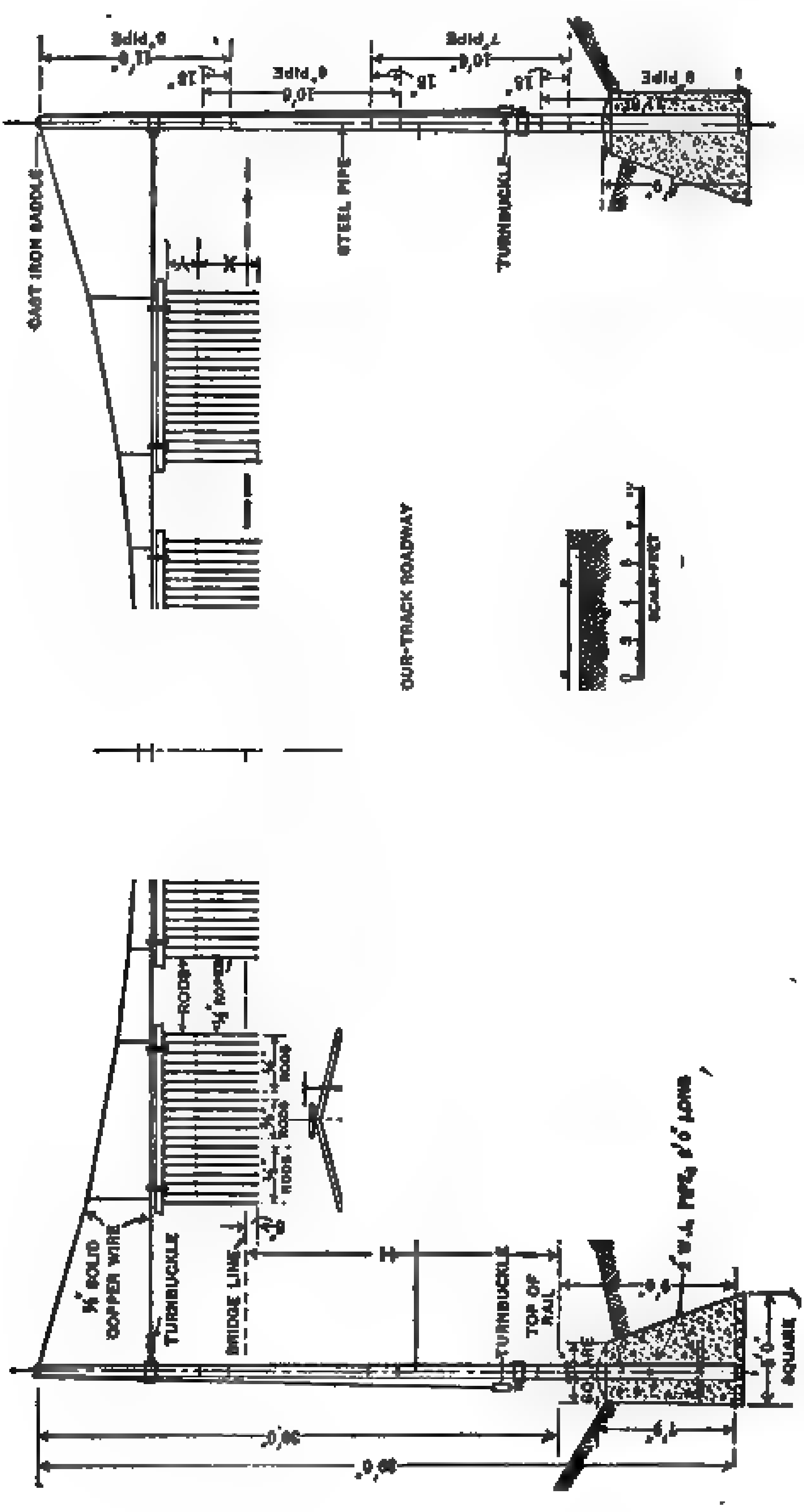


FIG. 163.—BRIDGE WARNING. Penn. R. R. Standard (1911).

passing under the bridge, but it is not always possible to have that amount of headroom. Under such circumstances, a warning for trainmen is necessary. These are made by suspending "ticklers," which are a series of ropes spaced 6 ins. apart which are suspended over the track at a sufficient distance from the bridge or tunnel so that the trainman shall have sufficient warning if he is struck by the dangling ropes. For a single track road the tickler may be suspended from a horizontal arm fastened to a pole planted at least 10 ft. from the track center, the arm being braced by a tie from the top of the pole and also by a short strut underneath. When several tracks are to be spanned, two poles will be used and a catenary cable, between the tops of the poles, supports a horizontal cable by means of a pair of suspenders over each track. The standard on the Pennsylvania Railroad has 19 ticklers 6 ins. apart over each track. The bottoms of the several ropes are 6 ins. below the bottom line of the bridge, the ropes having a length varying from 3 ft. to 5 ft. 3 ins. The ropes are fastened to $\frac{1}{4}$ in. or $\frac{3}{8}$ in. iron rods which swing on ring-bolts which are run through a wooden arm or hanger. The distance from the warning to the bridge or tunnel should be about 100 to 200 ft., depending somewhat on the grade, since that affects the time of the average freight train in passing the interval.

CHAPTER XIII.

YARDS AND TERMINALS.

376. Value of proper design. A large part of the total cost of handling traffic, particularly freight, is that incurred at terminals and stations. In illustration of this, consider the relative total cost of handling a car-load of coal and a car-load (of equal weight) of mixed merchandise. The coal will be loaded in bulk on the cars at the mines, where land is comparatively cheap, and the cars grouped into a train without regard to order, since they are (usually) uniform in structure, loading, and contents. When the terminal or local station is reached they are run on tracks occupying property which is usually much cheaper than the site of the terminal tracks and freight-houses; they are unloaded by gravity into pockets or machine conveyors and the empty cars are rapidly hauled by the train-load out of the way. On the other hand, the merchandise is loaded by hand on the car from a freight-house occupying a central and valuable location, the car is hauled out into a yard occupying valuable ground, is drilled over the yard tracks for a considerable aggregate mileage before starting for its destination, where the same process is repeated in inverse order. In either case the terminal expenses are evidently a large percentage of the total cost and, once loaded, it makes but little difference just how far the car is hauled to the other terminal. But the very evident increase in terminal charges for general merchandise over those for coal (large as they are) gives a better idea of the magnitude of terminal charges.

Many yards are the result of growth, adding a few tracks at a time, without much evidence of any original plan. In such cases the yard is apt to be very inefficient, requiring a much larger aggregate of drilling to accomplish desired results, requiring much more *time* and hence blocking traffic and finally adding greatly to the cost of terminal service, although the fact of its being a needless addition to cost may be unsuspected or not fully appreciated. An unwillingness or inability to spend money for

the necessary changes, and the difficulty of making the changes while the yard is being used, only prolong the bad state of affairs and an inefficient makeshift is frequently adopted. Assume that an improvement in the design of the yard will permit a saving of the use of one switching engine, or for example, that the work may be accomplished with three switching engines instead of four. Assuming a daily cost of $25, we have in 313 working days an annual saving of $7825, which, capitalized at 5%, gives $156,500, enough to reconstruct any ordinary yard.*

377. Divisions of the subject. The subject naturally divides itself into three heads—(*a*) Yards for receiving, classifying, and distributing freight cars, called more briefly freight yards; (*b*) yards and conveniences for the care of engines, such as ash tracks, turn-tables, coal-chutes, sand-houses, water-tanks, or water stand-pipes, etc., and (*c*) passenger terminals.

FREIGHT YARDS.

378. General principles. It should be recognized at the start that at many places an ideally perfect yard is impossible, or at least impracticable, generally because ground of the required shape or area is practically unobtainable. But there are some general principles which may and should be followed in every yard and other ideals which should be approached as nearly as possible. Nevertheless every yard is an independent problem. Before taking up the design of freight yards, it is first necessary to consider the general object of such yards and the general principles by which the object is accomplished. These may be briefly stated as follows:

1. A yard is a device, a machine, by which incoming cars are sorted and classified—some sent to warehouses for unloading, some sent to connecting railroads, some made up for local distribution along the road, some sent for repairs, and, in short a device by which all cars are sent *through* and *out* of the yard as quickly as possible.

2. Except when a road's business is decreasing, or when its equipment is greater than its needs and its cars must be stored, efficiency of management is indicated by the rapidity with which the passage of cars *through* the yard is accomplished.

3. When a yard is the terminal of a "division," the freight

* Estimate of Mr. H. G. Hetzler, C., B. & Q. Ry., now Pres. Chi. & West. Ind. Rwy.

trains will be pulled into a "receiving track" and the engine and caboose detached. The caboose will be run on to a "caboose track," which should be conveniently near, and the engine is run off to the engine yard. If the train is a "through" train and no change is to be made in its make-up, it will only need to wait for another engine and perhaps another caboose. If the cars are to be distributed, they will be drawn off by a switching engine to the "classification yard."

4. The design of a yard is best studied by first picking out the ladder tracks and the through tracks which lead from one division of the yard to another. These are tracks which must always be kept open for the passage of trains, in contradistinction to the tracks on which cars may be left standing, even though it is only for a few moments, while drilling is being done. Such a set of tracks, which may be called the skeleton of the yard, is shown by heavy lines in Fig. 164. Each line indicates a pair of rails. The tracks of the storage yards are shown by the lighter lines.

5. There is a distinct advantage in having all storage tracks double-ended—except "team tracks." Team tracks are those which have spaces for the accommodation of teams, so that loading or unloading may be done directly between the cars and teams. To avoid the necessity of teams passing over the tracks, these are best placed on the outskirts of the yard and consist of short stub-sidings arranged in pairs. But storage tracks should have an outlet at each end so as to reduce the amount of drilling necessary to reach a car which may be at the extreme end of a long string of cars. This is done usually by means of two "ladder" tracks, parallel to each other, which thus make the storage tracks between them of equal length.

6. The equality of length of these storage tracks is a point insisted on by many, but on the other hand, trains are not always of uniform length even on any one division. Loaded trains and trains of empties will vary greatly in length, and the various styles and weights of freight engines employed necessitate other variations in the weights and lengths of trains hauled. With storage tracks of somewhat variable length a larger percentage of track length may be utilized, there will be less hauling over a useless length of track, and (assuming that the plot of ground available for yard purposes has equally favorable conditions for yard design) more business may be handled in a yard of given area.

Fig. 164.—Plan of the New Shops and Yards of the Colorado & Southern Railway at Denver.

7. Although not absolutely necessary, there is an advantage in having all frog numbers and switch dimensions uniform. No. 8 frogs are recommended. Sharper-angled frogs make easier riding, less resistance and less chance of derailment, but on the other hand require longer leads and more space. No. 7 and even No. 6 frogs are sometimes used on account of economy of space, but they have the disadvantages of greater tractive resistance, greater wear and tear on track and rolling stock, and greater danger of derailment.

8. The spacing of " body tracks " (the parallel storage tracks which are headed by ladder tracks), should be 13 to 14 ft. and when they are parallel to a main track, or important running track, the first body track should be at least 15 ft. from the main track.

9. When practicable, caboose tracks should be so located that cabooses can be placed on and removed from them in the order of their arrival, and should be so graded that cabooses can be dropped by gravity on to the rear of trains made up for departure.

10. " Bad-order " tracks are those onto which damaged cars may be conveniently placed and from which they may be easily run to double-ended " repair tracks," which should have a capacity of about 15 cars each, and laid out in pairs which are spaced 16 and 24 ft. alternately.

11. Car capacity should be rated at 42 ft. of track per car.

379. **Hump yards.** A great economy in the movement of cars in a classification yard is obtained by the use of humps. A hump is a grade summit in a receiving track which has such an elevation that cars will run by gravity from it to any desired point in the classification yard. If a yard is practically level, an engine must push or " kick " every separate " cut " of the train on to its particular track, which involves not only a great waste of time but also a very large switch-engine mileage. By pushing the cars over the hump and successively cutting off, or uncoupling, one or more cars which are to be run down a ladder track to any one body track, the cars quickly acquire a desired velocity on a short stretch of perhaps 4% grade. This grade reduces to about 1% along the ladder track and through the switches, which allows for the added resistance through them, and then the grade is dropped to about 0.5% or less. The 4% grade, for about 50 ft., followed by a vertical curve about 150 ft. long,

at the end of which the grade is reduced to 1%, develops the required velocity in the car, the 1% grade maintains it and the momentum thus acquired is sufficient to move the cars to the farthest point of the body tracks. A brakeman, or "rider," accompanies each car, or group of cars. To avoid the great waste of time required for these riders to walk back to the hump, it has been found economical in some large yards to have a track for the exclusive use of a car, especially fitted for easy jumping on or off, operated, perhaps, by a switching engine, or possibly by gasoline, which picks up the riders and carries them back to the hump. The aggregate time saved justifies the expenditure. Since empty cars have a greater tractive resistance per ton than loaded cars, they require a steeper grade to maintain the same velocity, and, therefore, when tracks are set aside for the use of empty cars, the grade leading to such empty tracks should be increased if possible. To operate such a hump efficiently, the yard clerk makes up a triple (or quadruple) list for each freight train arriving at the yard for distribution. One of these lists is given to the man cutting off the cars at the top of the hump, and one to the towerman, if the switches are operated from the tower, or one to each switch tender if the switches are hand-operated. Each list contains in the first column the consecutive number of the cut, in the second column the number of the track on which that cut of cars is to be placed, and in the third column the number of cars cut. Cut No. 1 is the first car (or cars) to go over the hump. The grade from the receiving track to the hump should be such that one engine can push the maximum train over the hump. Since track resistance is greater in winter than in summer, the summit of the hump may be raised in winter sufficiently to develop the required added gravity force, and lowered again when the added height is not needed. The length of track required to be raised is not very great, while the saving in not being obliged to lift every train the required extra height, during the many months each year when the extra height is not needed, usually justifies the two changes each year.

380. Relation of yard to main tracks. Safety requires that there should be no connection between the yard tracks and the main tracks except at each end of the yard, where the switches should be amply protected by signals. Sometimes the main tracks run through the yard, making practically two yards—one for the traffic in either direction—but this either requires a double

layout of tracks and houses (such as ash tracks, coal-chutes, sand-houses, etc.), or a very objectionable amount of crossing of the main-line tracks. The preferable method is to have the main-line tracks entirely on the outside of the yard. A method which is in one respect still better is to spread the main tracks so that they run on each side of the yard. In this case there is never any necessity to cross one main track to pass from the yard to the other main track; a train may pass from the yard to either main track and still leave the other main track free and open. The ideal arrangement is that by which some of the tracks cross *over* or *under* all opposing tracks. By this means all connections between the yard and the main tracks may be by "trailing" switches; that is, trains will run on to the main track in the direction of motion on that main track. Of course all this applies only to *double* main track.

An important element of yard design is to have a few tracks immediately adjoining the main tracks and separate from the yard proper on which outgoing trains may await their orders to take the main track. When the orders come, they may start at once without any delay, without interfering with any yard operations, and they are not occupying tracks which may form part of the system needed for switching.

381. Minor freight yards. The term here refers to the sub-stations, only found in the largest cities, to which cars will be sent to save in the amount of necessary team hauling and also to relieve a congestion of such loading and unloading at the main freight terminal. The cars are brought to these yards sometimes on floats (as is done so extensively at various points around New York Harbor), or they are run down on a long siding running perhaps through the city streets. But the essential feature of these yards is the maximum utilization of every square foot of yard space, which is always very valuable and which is frequently of such an inconvenient shape that a great ingenuity is required to obtain good results. There is generally a temptation to use excessively sharp curves. When the radii are greater than 175 feet no especial trouble is encountered. Curves with radius as short as 50 feet have been used in some yards. On such curves the long cars now generally used make a sharper angle with each other than that for which the couplers were designed and special coupler-bars become necessary. The two general methods of construction are (*a*) a series of parallel team tracks (as pre-

Fig. 165.—Minor Freight Yard.

viously described and as illustrated further in Fig. 165), and (*b*) the "loop system," as is illustrated in Fig. 166.

382. Transfer cranes. These are almost an essential feature for yards doing a large business. The transportation of built-up girders, castings for excessively heavy machinery, etc., which weigh five to thirty tons and even more, creates a necessity for machinery which will easily transfer the loads from the car to the truck and *vice versa*. An ordinary "gin-pole" will serve the purpose for loads which do not much exceed five tons. A fixed framework, covering a span long enough for a car track and a team space, with a trolley traveling along the upper chord, is the next design in the order of cost and convenience. Increasing the span so that it covers two car tracks and two team spaces will very materially increase the capacity. Making the frame movable so that it travels on tracks which are parallel to the car tracks, giving the frame a longitudinal motion equal to two or three car lengths, and finally operating the raising and traveling mechanism by power, the facility for rapidly disposing of heavy articles of freight is greatly increased. Of course only a very small proportion of freight requires such handling, and the business of a yard must be large or perhaps of a special character to justify and pay for the installation of such a mechanism. Figs. 165 and 166 each indicate a transfer crane, evidently of the fixed type.

383. Track scales. The location of these should be on one of the receiving tracks near the entrance to the yard, but not on the main track nor on any track where drilling must be done. It is usually best to have a "dead track" over the scales—i.e., a track which has one rail on the solid side wall of the scale pit and the other supported at short intervals by posts which come up through the scale platform and yet do not touch it. These rails and the regular scale rails switch into one track by means of point rails a few feet beyond each end of the scales. The switches should be normally set so that all trains will use the dead track, unless the scales are to be operated. It has been found possible in a gravity yard to weigh a train with very little loss of time by running each car slowly and separately by gravity over the scales and weighing them as they pass over.

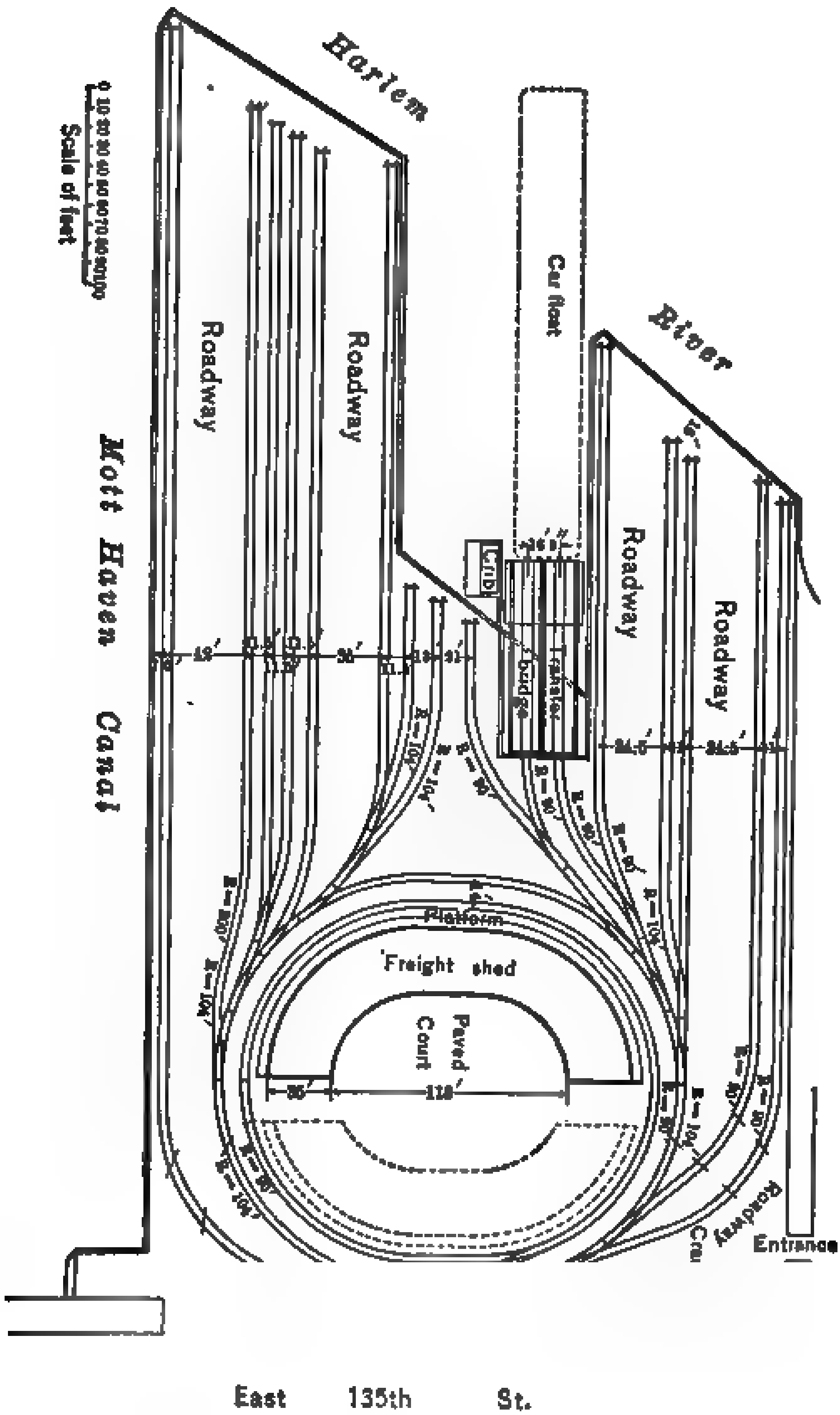


FIG. 166.—MINOR FREIGHT YARD ON A HARBOR FRONT.

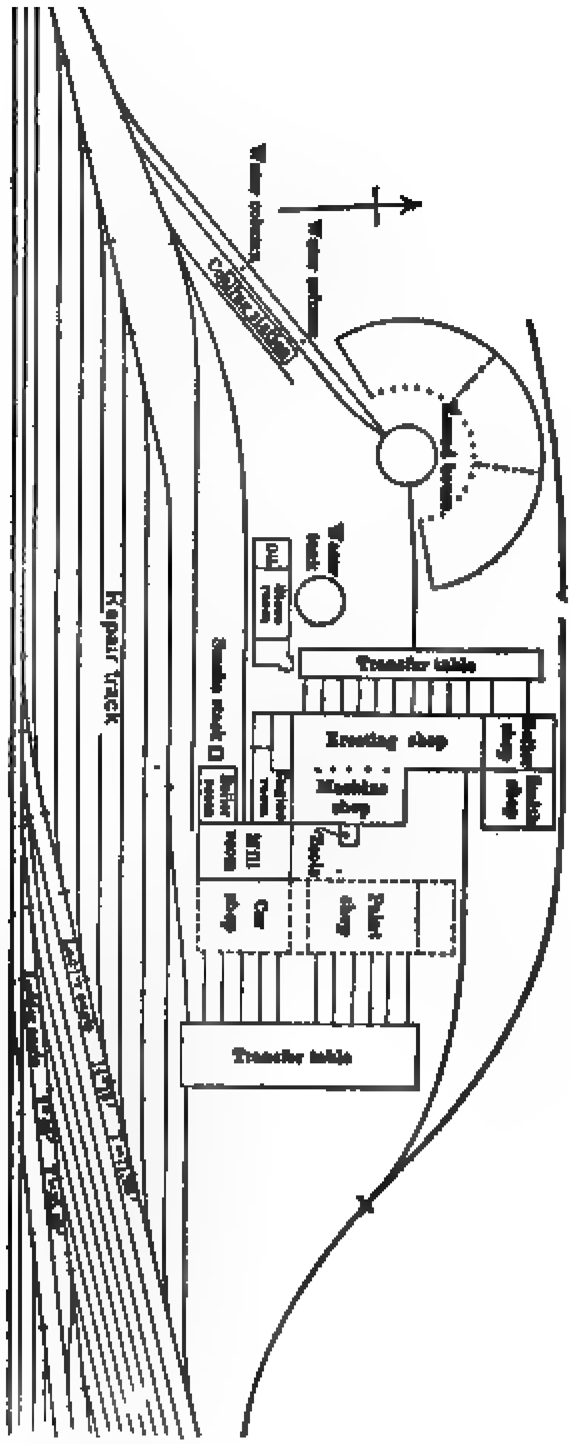


FIG. 167.—ENGINE YARD AND SHOPS, URBANA, ILL.

SOUTH BOSTON TERMINAL.

(To face p. 411.) (Published through courtesy of Union Switch and Signal Co.)

ENGINE YARDS.

384. General principles. Engine yards must contain all the tracks, buildings, structures, and facilities which are necessary for the maintenance, care, and storage of locomotives and for providing them with all needed supplies. The supplies are fuel, water, sand, oil, waste, tallow, etc. Ash-pits are generally necessary for the prompt and economical disposition of ashes; engine-houses are necessary for the storage of engines and as a place where minor repairs can be quickly made. A turntable is another all but essential requirement. The arrangement of all these facilities in an engine yard should properly depend on the form of the yard. In general they should be grouped together and should be as near as possible to the place where through engines drop the trains just brought in and where they couple on to assembled outgoing trains, so that all unnecessary running light may be avoided. Switching engines should be able to dump ashes, take their supplies and pass around waiting road engines. In Figs. 164 and 167 are shown two designs which should be studied with reference to the relative arrangement of the yard facilities.

PASSENGER TERMINALS.

(Passenger terminals are one of the logical subdivisions of this chapter, but their construction does not concern one engineer in a thousand. The local conditions attending their construction are so varied that each case is a special problem in itself—a problem which demands in many respects the services of the architect rather than the engineer. The student who wishes to pursue this subject is referred to an admirable chapter in "Buildings and Structures of American Railroads," by Walter G. Berg, Chief Engineer of the Lehigh Valley Railroad.)

CHAPTER XIV.

BLOCK SIGNALING.

GENERAL PRINCIPLES.

385. **Two fundamental systems.** The growth of systems of block signaling has been enormous within the last few years—both in the amount of it and in the development of greater perfection of detail. The development has been along two general lines: (*a*) the *manual*, in which every change of signal is the result of some definite action on the part of some signalman, but in which every action is so controlled or limited or subject to the inspection of others that a mistake is nearly, if not quite, impossible; (*b*) the *automatic*, in which the signals are operated by mechanism, which cannot set a wrong signal as long as the mechanism is maintained in proper order. The fundamental principles of the two systems will be briefly outlined, after which the chief details of the most common systems will be pointed out.

386. **Manual systems.** Small traffic roads are usually operated on the basis of the "train-order system." A "train dispatcher" controls the movement of every train on his division and telegraphs orders to men (who are frequently station agents) at various points along the line, who transmit these orders to the trainmen as the trains reach these points. A train-order signal station, whether at a regular traffic station or in a special cabin, has "train-order signals" which, when in the stop position, inform the engineman and conductor that they are to receive orders at the telegraph office; the clear position informs them that there are no orders for them. When more than one train is allowed on a single track between two consecutive train-order stations, the engineman and conductor of each train has strict orders with reference to the other train, for example, that the trains are to pass at some siding where there is no telegraphic station. A very strict code of rules has been developed which, when literally followed, ensures safety of operation, but these rules cannot eliminate the human element, or the liability of personal negligence or error. When such a system is applied to a double-track road, or even to a single-track road, with train-order signal

stations located so frequently that only one train will be allowed between two consecutive offices at once, it virtually becomes a block system even though it is not called such. When such a system is adhered to rigidly, it is called an *absolute block* system. But when operating on this system, a delay of one train will necessarily delay every other train that follows closely after. A portion, if not all, of the delay to subsequent trains may be avoided, although at some loss of safety, by a system of permissive blocking. By this system an operator may give to a succeeding train a "clearance card" which permits it to pass into the next block, but at a reduced speed and with the train under such control that it may be stopped on very short notice, especially near curves. One element of the danger of this system is the *discretionary* power with which it invests the signalmen, a discretion which may be wrongfully exercised. A modification (which is a fruitful source of collisions on single-track roads) is to order two trains to enter a block approaching each other, and with instructions to pass each other at a passing siding at which there is no telegraph-station. When the instructions are properly made out and literally obeyed, there is no trouble, but every thousandth or ten thousandth time there is a mistake in the orders, or a misunderstanding or disobedience, and a collision is the result. The telegraph line, a code of rules, a corps of operators, and signals under the immediate control of the operators, are all that is absolutely needed for the simple manual system.

387. Development of the manual system. One great difficulty with the simple system just described is that each operator is practically independent of others except as he may receive general or specific orders from a train-dispatcher at the division headquarters. Such difficulties are somewhat overcome by a very rigid system of rules requiring the signalmen at each station to keep the adjacent signalmen or the train-dispatcher informed of the movements of all trains past their own stations. When these rules (which are too extensive for quotation here) are strictly observed, there is but little danger of accident, and a neglect by any one to observe any rule will generally be apparent to at least one other man. Nevertheless the safety of trains depends on *each* signalman doing his duty, and a little carelessness or forgetfulness on the part of any one man may cause an accident. The signaling between stations *may* be done by

ordinary telegraphic messages or by telephone, but is frequently done by electric bells, according to a code of signals, since these may be readily learned by men who would have more difficulty in learning the Morse code.

In order to have the signalmen mutually control each other, the "controlled manual" system has been devised. The first successful system of this kind which was brought into extensive use is the "Sykes" system, of which a brief description is as follows: Each signal is worked by a lever; the lever is locked by a latch, operated by an electro-magnet, which, with other necessary apparatus, is inclosed in a box. When a signal is set at danger, the latch falls and locks the lever, which cannot be again set free until the electro-magnet raises the latch. The magnet is energized only by a current, the circuit of which is closed by a "plunger" at the *next* station ahead; just above the plunger is an "indicator," also operated by the current, which displays the words *clear* or *blocked*. (There are variations on this detail.) When a train arrives at a block station (A), the signalman should have previously signaled to the station *ahead* (B) for permission to free the signal. The man ahead (B) pushes in the "plunger" on his instrument (assuming that the previous train has already passed him), which electrically opens the lock on the lever at the previous station (A). The signal at A can then be set at "safety." As soon as the train has passed A the signal at A must be set at "danger." A further development is a device by which the mere passage of the train over the track for a few feet beyond the signal will automatically throw the signal to "danger." After the signal once goes to danger, it is automatically locked and cannot be released except by the man in advance (B), who will not do so until the train has passed him. The "indicator" on B's instrument shows "blocked" when A's signal goes to danger after the train has passed A, and B's plunger is then locked, so that he cannot release A's signal while a train is in the block. As soon as the train has passed A, B should prepare to get his signals ready by signaling ahead to C, so that if the block between B and C is not obstructed, B may have his signals at "safety" so that the train may pass B without pausing. The student should note the great advance in safety made by the Sykes system; a signal cannot be set free except by the combined action of two men, one the man who actually operates the signal and

the other the man at the station ahead, who frees the signal electrically and who by his action certifies that the block immediately ahead of the train is clear.

A still further development makes the system still more "automatic" (as described later), and causes the signal to fall to danger or to be kept locked at danger, if even a single pair of wheels comes on the rails of a block, or if a switch leading from a main track is opened.

388. Permissive blocking. "Absolute" blocking renders accidents due to collisions almost impossible unless an engineer runs by an adverse signal. The signal mechanism is usually so designed that, if it gets out of order, it will inevitably fall to "danger," i.e., as described later, the signal-board is counterbalanced by a weight which is much heavier. If the wire breaks, the counterweight will fall and the board will assume the horizontal position, which always indicates "danger."* But it sometimes happens that when a train arrives at a signal-station, the signalman is unable to set the signal at safety. This may be because the previous train has broken down somewhere in the next block, or because a switch has been left open, or a rail has become broken, or there is a defect of some kind in the electrical connections. In such cases, in order to avoid an indefinite blocking of the whole traffic of the road, the signalman may give the engineer a "caution-card" or a "clearance card," which authorizes him to proceed slowly and with his train under complete control into the block and through it if possible. If he arrives at the next station without meeting any obstruction it merely indicates a defective condition of the mechanism, which will, of course, be promptly remedied. Usually the next section will be found clear, and the train may proceed as usual. On roads where the "controlled manual" system has received its highest development, the rules for permissive blocking are so rigid that there is but little danger in the practice, unless there is an absolute disobedience of orders.

389. Automatic systems. By the very nature of the case, such systems can only be used to indicate to the engineers of trains something with reference to the passage of previous

* This was written on the basis of the older system, in which the semaphore swings through the *lower* right-hand quadrant. The most recent practice swings the semaphore through the *upper* right-hand quadrant. A break in the wire holding the semaphore vertical will cause it to fall to horizontal position without the aid of a counterweight.

trains. The complicated shifting of switches and signals which is required in the operation of yards and terminals can only be accomplished by "manual" methods, and the only automatic features of these methods consist in the mechanical checks (electric and otherwise), which will prevent wrong combinations of signals. But for long stretches of the road, where it is only required to separate trains by at least one block length, an automatic system is generally considered to be more reliable. As expressed forcibly by a railroad manager, "an automatic system does not go to sleep, get drunk, become insane, or tell lies when there is any trouble." The same cannot always be said of the employés of the manual system.

The basic idea of all such systems is that when a train passes a signal-station (A), the signal automatically assumes the "danger" position. This may be accomplished electrically, pneumatically, or even by a direct mechanism. When the train reaches the end of the block at B and passes into the next one, the signal at B will be set at danger and the signal at A will be set at safety. The lengths of the blocks are usually so great that the only practicable method of controlling from B a mechanism at A is by electricity, although the actual motive power at A may be pneumatic or mechanical. At one time the current from A to B was run only through wires. This method has the very positive advantage of reliability, definite resistance to the current, and small probability of short-circuiting or other derangement. But now all such systems use the rails for a track circuit and this makes it possible to detect the presence of a single pair of wheels on the track anywhere in the block, or an open switch, or a broken rail. Any such circumstances, as well as a defect in the mechanism, will break or short-circuit the current and will cause the signal to be set at danger. To prevent an indefinite blocking of traffic owing to a signal persistently indicating danger, most roads employing such a system have a rule substantially as follows: When a train finds a signal at danger, after waiting one minute (or more, depending on the rules), it may proceed slowly, expecting to find an obstruction of some sort; if it reaches the next block without finding any obstruction and finds the next signal clear, it may proceed as usual, but must promptly report the case to the superintendent. Further details regarding these methods will be given later. See § 394.

390. "Distant" signals. The close running of trains that is required on heavy-traffic roads, especially where several branches combine to enter a common terminal, necessitates the use of very short blocks. A heavy train running at high speed can hardly make a "service" stop in less than 2000 feet, while the curves of a road (or other obstructions) frequently make it difficult to locate a signal so that it can be seen more than a few hundred feet away. It would therefore be impracticable to maintain the speed now used with heavy trains if the engineer had no foreknowledge of the condition in which he will find a signal until he arrives within a short distance of it. To overcome this difficulty the "distant" signal was devised. This is placed about 1800 or 2000 feet from the "home" signal, and is interlocked with it so that it gives the *same* signal. The distant signal is frequently placed on the same pole as the home signal of the previous block. When the engineer finds the distant signal "clear," it indicates that the succeeding home signal is also clear, and that he may proceed at full speed and not expect to be stopped at the next signal; for the distant signal cannot be cleared until the succeeding home signal is cleared, which cannot be done until the block succeeding that is clear. A clear distant signal therefore indicates a clear track for two succeeding blocks. When the engineer finds the distant signal blocked, he need not stop (providing the home signal is clear). It simply indicates that he must be prepared to stop at the next home signal and must reduce speed if necessary. It may happen that by the time he reaches the succeeding home signal it has already been cleared, and he may proceed without stopping. This device facilitates the rapid running of trains, with no loss of safety, and yet with but a moderate addition to the signaling plant.

391. "Advance" signals. It sometimes becomes necessary to locate a signal a few hundred feet short of a regular passenger-station. A train might be halted at such a signal because it was not cleared from the signal-station ahead—perhaps a mile or two ahead. For convenience, an "advance" signal may be erected immediately beyond the passenger-station. The train will then be permitted to enter the block as far as the advance signal and may deliver its passengers at the station. The advance signal is interlocked with the home signal back of it, and cannot be cleared until the home signal is cleared and

the entire block ahead is clear. In one sense it adds another block, but the signal is entirely controlled from the signal station back of it.

MECHANICAL DETAILS.

308. **Signals.** The primitive signal is a mere cloth flag. A better signal is obtained when the flag is suspended in a suitable place from a fixed horizontal support, the flag weighted at the bottom, and so arranged that it may be drawn up and out of sight by a cord which is run back to the operator's office. The next step is the substitution of painted wood or sheet metal for the cloth flag, and from this it is but a step to the standard semaphore on a pole, as is illustrated in Fig. 168. The simple flag, operated for convenience with a cord, is the signal employed on thousands of miles of road, where they perhaps make no claim to a block-signal system, and where the trains are run on the "train-order system."

Semaphore boards. These are about 5 feet long, 8 inches wide at one end, and tapered to about 6 inches wide at the hinge end. The boards are fastened to a casting which has a ring to hold a red glass which may be swung over the face of a lantern, so as to indicate a red signal. "Distant" signal-boards usually have their ends notched or pointed; the "home" signal-boards are square ended. The boards are always to the *right* of the hinge when a train is approaching them. The "home" signals are generally painted red and the "distant" signals green, although these colors are not invariable. The backs of the boards are painted white. Therefore any signal-board which appears on the *left* side of its hinge will also appear *white*, and is a signal for traffic in the opposite direction, and is therefore of no concern to an engineman.

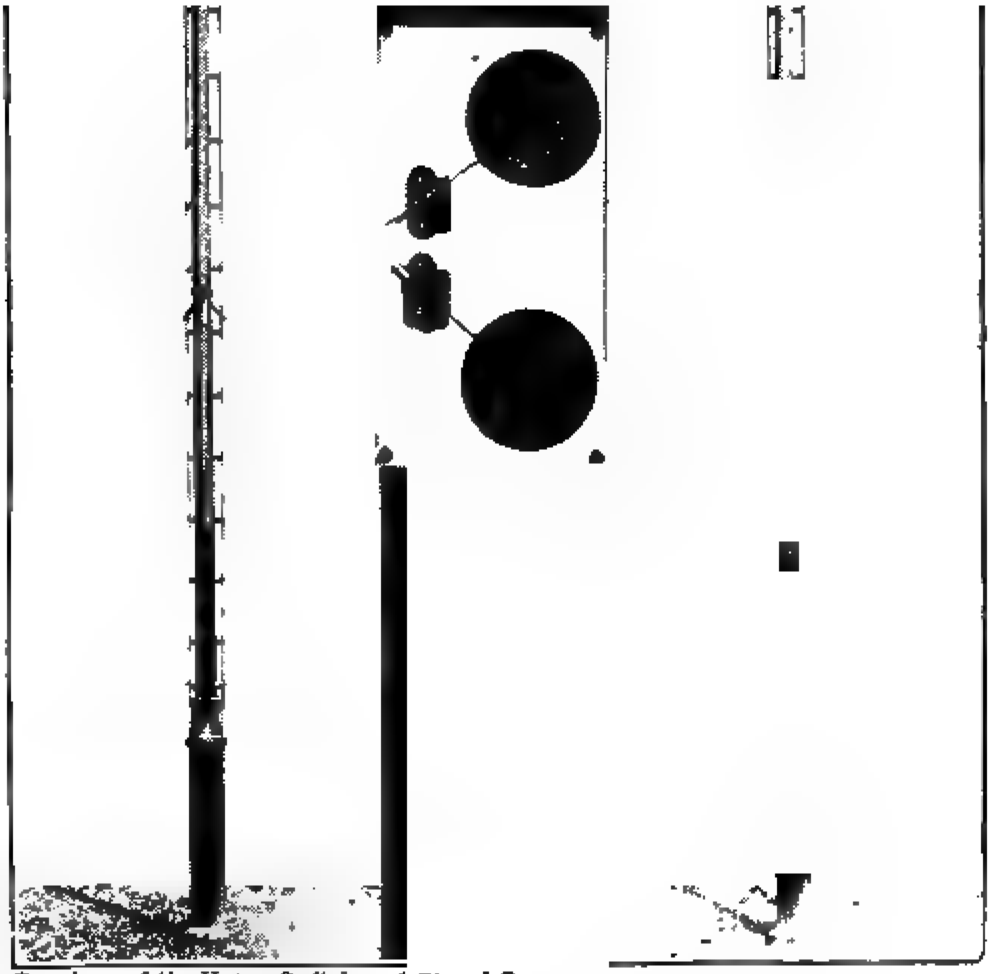
Poles and bridges. When the signals are set on poles, they are always placed on the right-hand side of the track. When there are several tracks, four or more, a bridge is frequently built and then each signal is over its own track. The signals for two tracks, operated in the same direction, may be placed on one pole by having a cross-piece which supports two "masts," see Fig. 168. In that figure the signals on the left-hand mast control the second track at the left of the signal; those on the right-hand mast control the track just to the left of the signal,

(To face page 418.)

FIG. 168.—SEMAPHORES.

Fig. 163.—Signal Bridge.

(*To face page* 418.)



Courtesy of the Union Switch and Signal Co.

FIG. 170.—"BANJO" SIGNALS.

A train movement, from the switch track at the right of the signal on to the main track, is controlled by the "dwarf" signal at the right of the switch track. The signals controlling the two tracks at the extreme left are not shown. The building at the left of the track in the extreme background is apparently the signal tower controlling this signal.

In Fig. 169 is shown a "bridge" and the two signals (home and distant), for each track. The two pairs of signals on the two right-hand poles are extended to the right and show that the movement of trains on those tracks is away from the observer. The darkness of the blades in the picture shows that they are painted dark, probably orange or red. The other blades show light (because painted white), and extend to the left but would appear to the right to an engineman on either left-hand track coming toward the observer. Incidentally the picture shows, over the two right-hand tracks, the ropes of a "tickler" (see § 375), to protect brakemen on the tops of cars which will enter the tunnel shown in the background.

"Banjo" signals. This name is given to a form of signal, illustrated in Fig. 170, in which the indication is taken from the *color* of a round disk inclosed with glass. The great argument in their favor is that they may be worked by an electric current of low voltage, which is therefore easily controlled; that the mechanism is entirely inside of a case, is therefore very light, and is not exposed to the weather. The argument urged against them is that it is a signal of *color* rather than *form* or *position*, and that in foggy weather the signal cannot be seen so easily; also that unsuspected color-blindness on the part of the engineman may lead to an accident. Notwithstanding these objections, this form of signal is used on thousands of miles of line in this country.

393. Wires and pipes. Signals are usually operated by levers in a signal-cabin, the levers being very similar to the reversing-lever of a locomotive. The distance from the levers to the signals is, of course, very variable, but it is sometimes 2000 feet. The connecting-link for the most distant signals is usually No. 9 wire; for nearer signals and for all switches operated from the cabin it may be 1-inch pipe. When not too long, one pipe will serve for both motions, forward and back. When wires are used, it is sometimes so designed (in the cheaper systems) that one wire serves for one motion, gravity being de-

pended on for the other, but now all good systems require two wires for each signal.

Compensators. Variations of temperature of a material with as high a coefficient as iron will cause very appreciable difference of length in a distance of several hundred feet, and a dangerous lack of adjustment is the result. To illustrate: A fall of 60° F. will change the length of 1000 feet of wire by

$$1000 \times 60 \times .0000065 = 0.39 \text{ foot} = 4.68 \text{ inches.}$$

A much less change than this will necessitate a readjustment of length, unless automatic compensators are used. A compensator for pipes is very readily made on the principle illustrated in Fig. 171. The problem is to preserve the distance between a and d constant regardless of the temperature. Place the compensator half-way between a and d, or so that $ab = cd$. A fall of temperature contracts ab to ab'. Moving b to b' will cause c to move to c', in which $bb' = cc'$. But cd has also shortened to $c'd$; therefore d remains fixed in position.

The regulations of the Am. Rwy. Eng. Assoc. require that "A compensator shall be provided for each pipe line over fifty (50) feet in length and under eight hundred (800) feet, with crank-arms eleven by thirteen (11×13) inch centers. From eight hundred (800) to twelve hundred (1200) feet in length, crank-arms shall be eleven by sixteen (11×16) inch centers. Pipe lines over twelve hundred (1200) feet in length shall be provided with an additional compensator.

"Compensators shall have one sixty (60) degree and one one hundred and twenty (120) degree angle-cranks and connecting link, mounted in cast iron base, having top of center pins supported. The distance between center of pin-holes shall be twenty-two (22) inches."

The compensator should be placed in the middle of the length when only one is used. When two are used they should be placed at the quarter points. Note that in operating through a compensator the *direction* of motion changes; i.e., if a moves to the right, d moves to the left, or if there is compression in ab there is tension in cd, and *vice versa*. Therefore this form of compensator can only be used with pipes which will withstand compression. It has seemed impracticable to design an equally satisfactory compensator for wires, although there are several designs on the market.

The change of length of these bars is so great that allowance must be made for the temperature at the time of installation. On the basis of 50° as the mean temperature, the pipes are so adjusted that the distance between the points b and c of Fig. 171 is made greater or less than 22 inches, according to the temperature of installation. For example, if the temperature were 80° and the length of the piping were 900 feet, the length of the pipes should be adjusted so that bc is *less* than 22 inches by an amount equal to $900 \times (80° - 50°) \times .0000065 = 0.1755$ feet =

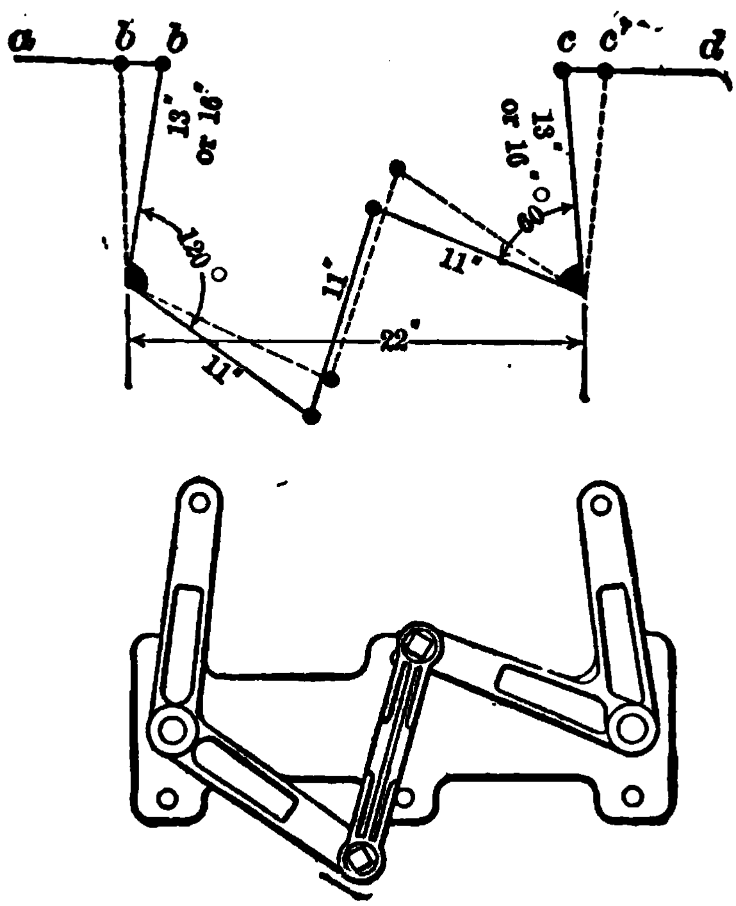


FIG. 171.—STANDARD PIPE COMPENSATOR.

2.106 inches. The length should therefore be 19.9 inches instead of 22 inches. If the mean temperature was very different (say in Florida) some higher temperature should be taken as normal, so that the extreme range above and below the normal shall be approximately the same.

Guides around curves and angles. When wires are required to pass around curves of large angle, pulleys are used, and a length of chain is substituted for the wire. For pipes, when the curve is easy the pipes are slightly bent and are guided through pulleys. When the angle is sharper, "angles" are used. The operation of these details is self-evident from an inspection of Fig. 172.

394. **Track circuit for automatic signaling.** The fundamental principle of the track circuit method of indicating a track obstruction or breakage, using direct current, is as follows: A current of low potential is run from a battery at one end of a section through one line of rails to the other end of the section, then through a relay, and then back to the battery through the other line of rails. To avoid the excessive resistance which would occur at rail joints which may become badly rusted, a wire

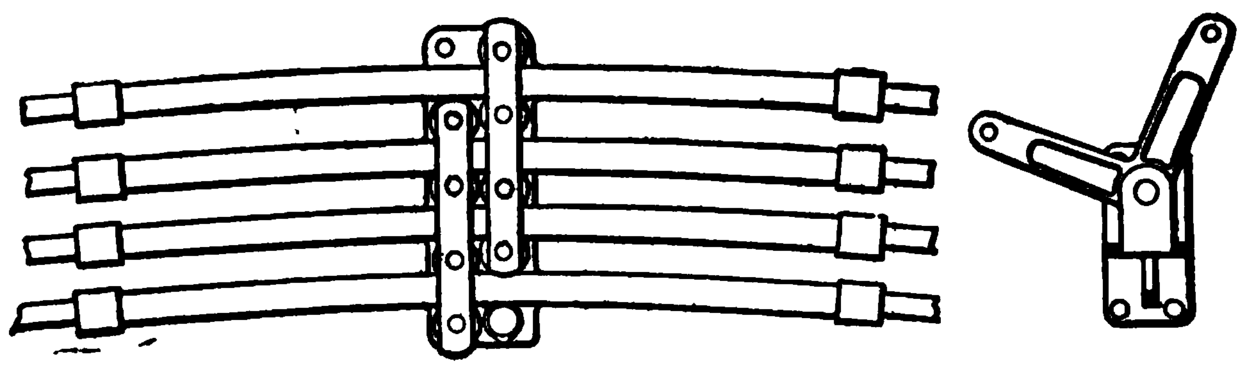


FIG. 172.—DEFLECTING-RODS AND ANGLE.

suitably attached to the rails is run around each joint. In order to insulate the rails of one section from the rails at either end and yet maintain the rails structurally continuous, the ends of the rails at these dividing points are separated by an insulator and the joint pieces are either made of wood or have some insulating material placed between the rails and the ordinary metal joint. The bolts must also be insulated. When the relay is energized by a current, it closes a local circuit at the signal-station, which will set the signal there at "safety." The resistance of the relay is such that it requires nearly the whole current to work it and to keep the local circuit closed. Therefore, when there is any considerable loss of current from one rail to the other, the relay will not be sufficiently energized, the local circuit will be broken, and the signal will automatically fall to danger. This diversion of current from one rail to the other before the current reaches the relay may be caused in several ways: the presence of a pair of wheels on the rails anywhere in the section will do it; also the breakage of a rail; also the opening of a switch anywhere in the section; also the presence of a pair of wheels on a siding between the "fouling point" and the switch. (The "fouling point" of a siding is that point where the rails first commence to approach the main track.) In Fig. 173 is shown all of the above details as well as some others.

At A, B, and the "fouling point" are shown the insulated joints. The batteries and signals are arranged for train motion to the *right*. When a train has passed the points near A, where the wires leave the rails for the relay, the current from the "track battery" at B will pass through the wheels and axles, and although no electrical connection is broken, so much current will be shunted through the wheels and axles that the weak current still passing through the relay is not strong enough to energize it against its spring and the "signal-magnet" circuit is broken, and the signal A goes to "danger." At the turnout the rails between the fouling point and the switch are so connected (and insulated) that a pair of wheels on these rails will produce the same effect as a pair of the main track. This is to guard against the effect of a car standing too near the switch, even though it is not on the main track. When the train passes B, if there is no other interruption of the current, the track battery at B again energizes the relay at A, the signal-magnet circuit at A is closed, and the signal is drawn to "safety."

About 1903 the application of *alternating current* to signaling circuits was invented. This not only permits the substitution of a. c. circuit for track batteries, but also makes it possible to utilise the track circuit method to indicate obstructions or rail breakages even when the track is the return circuit for an electrified road. But an explanation of this development would be too long for this text-book. It is

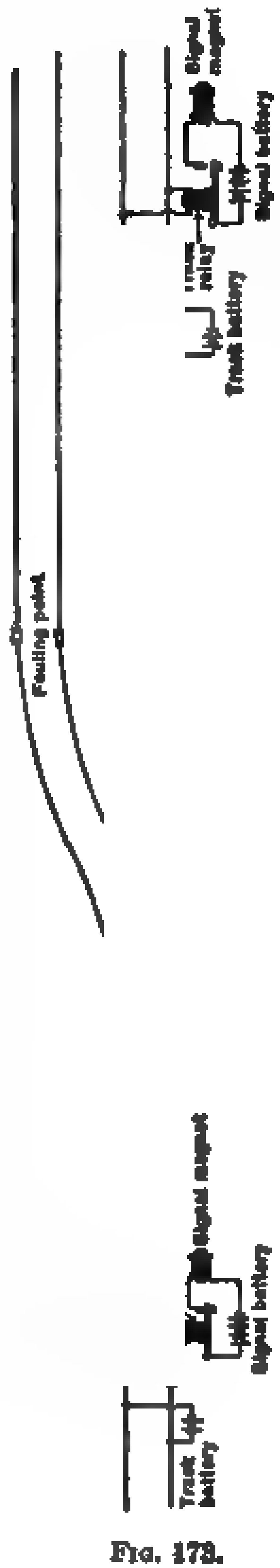


FIG. 172.

given in a 548-page book called "Alternating Current Signaling," published by the Union Switch & Signal Co., Swissvale, Pa.

This chapter also omits all references to "interlocking plants," which are essential features of the operation of large terminal yards. Even an elementary treatment of the present development of signaling and interlocking would require a large text-book, and, therefore, nothing more than the above brief outline will be here given.

CHAPTER XV.

ROLLING-STOCK.

(It is perhaps needless to say that the following chapter is in no sense a course in the design of locomotives and cars. Its chief idea is to give the student the elements of the construction of those vehicles which are to use the track which he may design—to point out the mutual actions and reactions of vehicle against track and to show the effect on track wear of variations in the design of rolling-stock. The most of the matter given has a direct practical bearing on track-work, and it is considered that all of it is so closely related to his work that the civil engineer may study it with profit.)

WHEELS AND RAILS.

395. **Effect of rigidly attaching wheels to their axles.** The wheels of railroad rolling-stock are invariably secured rigidly to the axles, which therefore revolve with the wheels. The chief reason for this is to avoid excessive wear between the axles and the wheels.

Any axle must always be somewhat loose in its journals. A sidewise force P (see Fig. 174) acting against the circumference of the wheel will produce a much greater pressure on the axle at S and S', and if the wheel moves on the axle, the wear at S and S' will be excessive. But when the axle is fitted to the wheel with a "forced fit" and does not revolve, the mere *pressure* produced at S is harmless. When two wheels are fitted tight to an axle, as in Fig. 175, and the axle revolves in the journals aa, a sidewise pressure of the rail against the wheel flange will only produce a slight and harmless increase of the journal pressure Q, although at Q there is sliding contact. Twist-

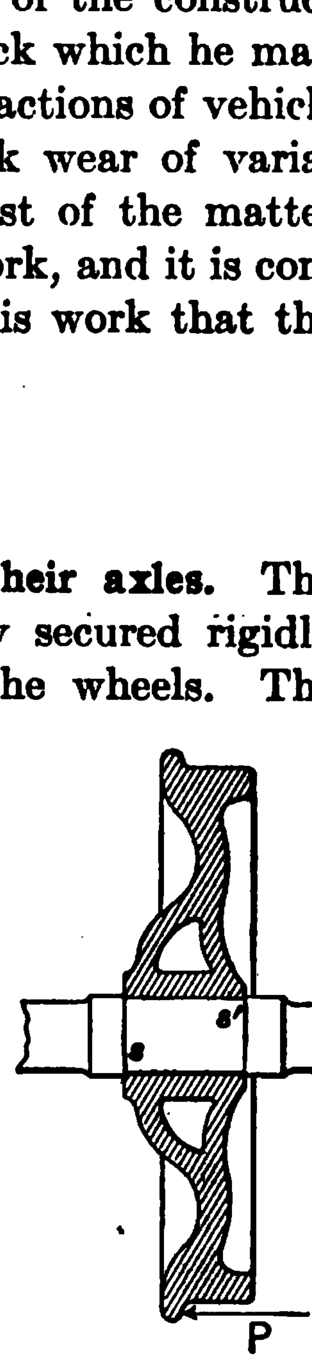


FIG. 174.

ing action in the journals is thus practically avoided, since a small pressure at the journal-boxes at each end of the axle suffices to keep the axle truly in line.

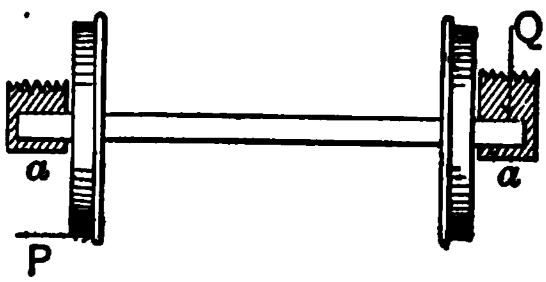


FIG. 175.

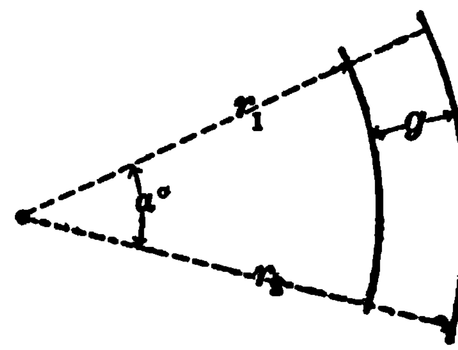


FIG. 176.

On the other hand, when the wheels are rigidly attached to their axles, both wheels must turn together, and when rounding curves, the inner rail being shorter than the outer rail, one wheel must slip by an amount equal to that difference of length. The amount of this slip is readily computable:

$$\text{Longitudinal slip} = \frac{2\pi a°}{360°}(r_2 - r_1) = \frac{2\pi g}{360°}a° = Ca°, \quad . \qquad (102)$$

in which C is a constant for any one gauge, and $g =$ the track gauge $= (r_2 - r_1)$. For standard gauge (4.708) the slip is .08218 foot per degree of *central angle*. This shows that the longitudinal slipping around any curve of any given central angle will be *independent of the degree of the curve*. The constant (.08218) here given is really somewhat too small, since the true gauge that should be considered is the distance between the lines of tread on the rails. This distance is a somewhat indeterminate and variable quantity, and probably averages 4.90 feet, which would increase the constant to .086. The slipping may occur by the inner wheel slipping ahead or the outer wheel slipping back, or by both wheels slipping. The total slipping will be constant in any case. The slipping not only consumes power, but wears both the wheels and the rail. But even these disadvantages are not sufficient to offset the advantages resulting from rigid wheels and axles.

396. Effect of parallel axles. Trucks are made with two or three parallel axles (except as noted later), in order that the axles shall mutually guide each other and be kept approximately

perpendicular to the rails. If the curvature is very sharp and the wheel-base comparatively long (as is notably the case on street railways at street corners), the front and rear wheels

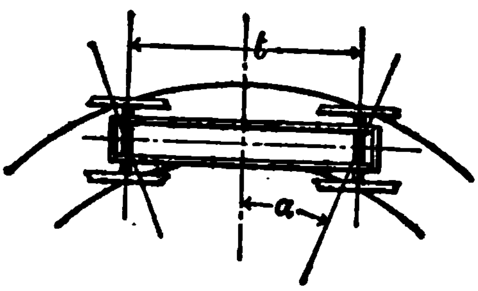


FIG. 177.

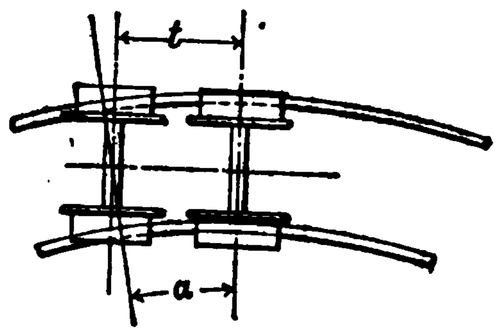


FIG. 178.

will stand at the same angle (a) with the track, as shown in Fig. 177. But it has been noticed that for ordinary degrees of curvature, the rear wheels stand radial to the curve (see Fig. 178), and for steam railroad work this is the normal case. When the two parallel axles are on a curve (as shown), the wheels tend to run in a straight line. In order that they shall run on a curve they must slip laterally. The principle is illustrated in an exaggerated form in Fig. 179. The wheel *tends* to roll from a toward b. Therefore in passing along the track from a to c it must actually slip laterally an amount bc which equals ac sin a.

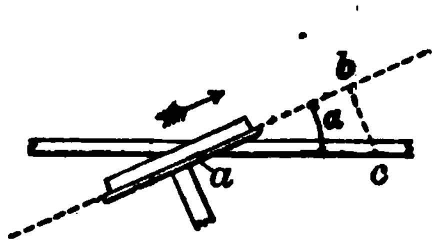


FIG. 179.

Let t = length of the wheel-base (Figs. 177 and 178); r = radius of curve; then for the first case (Fig. 177), $\sin a = t \div 2r$; for the second and usual case (Fig. 178), $\sin a = t \div r$; for $t = 5$ feet and r = radius of a 1° curve, $a = 0° \, 03'$ for the second case. a varies (practically) as the degree of curve. The lateral slipping *per unit* of distance traveled therefore equals sin a. As an illustration, given a 5-foot wheel-base on a 5° curve, $a = 0° \, 15'$, $\sin a = .00436$, and for each 100 feet traveled along the curve the lateral slip of the front wheels would be 0.436 foot. There would be no lateral slipping of the rear wheels, assuming that the rear axle maintained itself radial.

From the above it might be inferred that the flanges of the forward wheels will have much greater wear than those of the rear wheels. Since cars are drawn in both directions about equally, no difference in flange wear due to this cause will occur, but locomotives (except switching-engines) run forward almost

exclusively, and the excess wear of the front wheels of the pilot- and tender-trucks is plainly observable.

For a given curve the angle a (and the accompanying resistance) is evidently greater the greater the distance between the axles. On the other hand, if the two axles are very close together, there will be a tendency for the truck to twist and the wheels to become jammed, especially if there is considerable play in the gauge. The flange friction would be greater and would perhaps exceed the saving in lateral slipping. A general rule is that the axles should never be closer together than the gauge.

Although the slipping per unit of length along the curve varies directly as the degree of curvature, the length of curve necessary to pass between two tangents is inversely as the degree of curve, and the total slipping between the two tangents is independent of the degree of curve. Therefore when a train passes between two tangents, the total slipping of the wheels on the rails, longitudinal and lateral, is a quantity which depends only on the central angle and is independent of the radius or degree of curve.

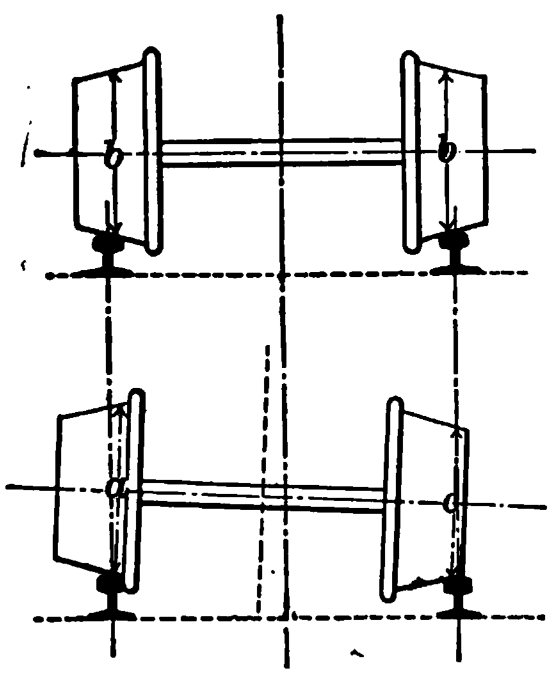


Fig. 180.

397. Effect of coning wheels. The wheels are always set on the axle so that there is some "play" or chance for lateral motion between the wheel-flanges and the rail. The treads of the wheel are also "coned." This coning and play of gauge are shown in an exaggerated form in Fig. 180. When the wheels are on a tangent, although there will be occasional oscillations from side to side, the normal position will be the symmetrical position in which the circles of tread bb are equal. When centrifugal force throws the wheel-flange against the rail, the circle of tread a is larger than b, and much larger than c; therefore the wheels will tend to roll in a circle whose radius equals the slant height of a cone whose elements would pass through the unequal circles a and c. If this radius equaled the radius of the track, and if the axle were free to assume a radial position, the wheels would roll freely on the rails without any

slipping or flange pressure. Under such ideal conditions, coning would be a valuable device, but it is impracticable to have all axles radial, and the radius of curvature of the track is an extremely variable quantity. It has been demonstrated that with parallel axles the influence of coning diminishes as the distance between the axle increases, and that the effect is practically inappreciable when the axles are spaced as they are on locomotives and car-trucks. The coning actually used is very slight (see Chapter XV, § 420) and has a different object. It is so slight that even if the axles were radial it would only prevent the slipping on a very light curve—say a 1° curve.

398. **Effect of flanging locomotive driving-wheels.** If all the wheels of all locomotives were flanged it would be practically impossible to run some of the longer types around sharp curves. The track-gauge is always widened on curves, and especially on sharp curves, but the widening would need to be excessive to permit a consolidation locomotive to pass around an 8° or 10° curve if all the drivers were flanged. The action of the wheels on a curve is illustrated in Figs. 181, 182, and 184. All small truck-wheels are flanged. The rear drivers are always flanged and four-driver engines usually have all the drivers flanged. Consolidation engines have only the front and rear drivers flanged. Mogul and ten-wheel engines have one pair of drivers blank. On Mogul engines it is always the middle pair. On ten-wheel engines, when used on a road having sharp curves, it is preferable to flange the front and rear driving-wheels and use a "swing bolster" (see § 399); when the curvature is easy, the middle and rear drivers may be flanged and the truck made with a rigid center. The blank drivers have the same total width as the other drivers and of course a much wider tread, which enables these drivers to remain on the rail, even though the curvature is so sharp that the tread overhangs the rail considerably.

399. **Action of a locomotive pilot-truck.** The purpose of the pilot-truck is to guide the front end of a locomotive around a curve and to relieve the otherwise excessive flange pressure that would be exerted against the driver-flanges. There are two classes of pilot-trucks—(*a*) those having fixed centers and (*b*) those having shifting centers. This second class is again subdivided into two classes, which are radically different in their action—(b_1) four-wheeled trucks having two parallel axles

and (b_2) two-wheeled trucks which are guided by a "radius-bar." The action of the four-wheeled fixed-centered truck (a) is shown in Fig. 181. Since the center of the truck is forced

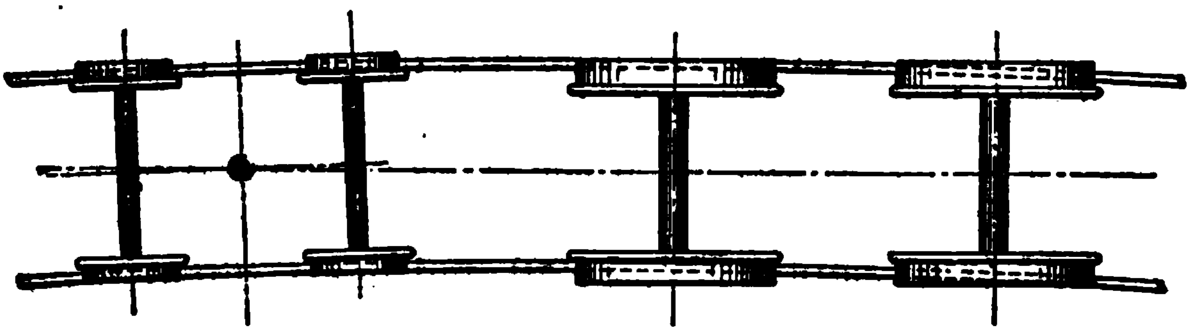


Fig. 181.—Fixed Center Pilot-truck.

to be in the center of the track, the front drivers are drawn away from the outer rail. The rear outer driver tends to roll *away* from the outer rail rather than toward it, and so the effect

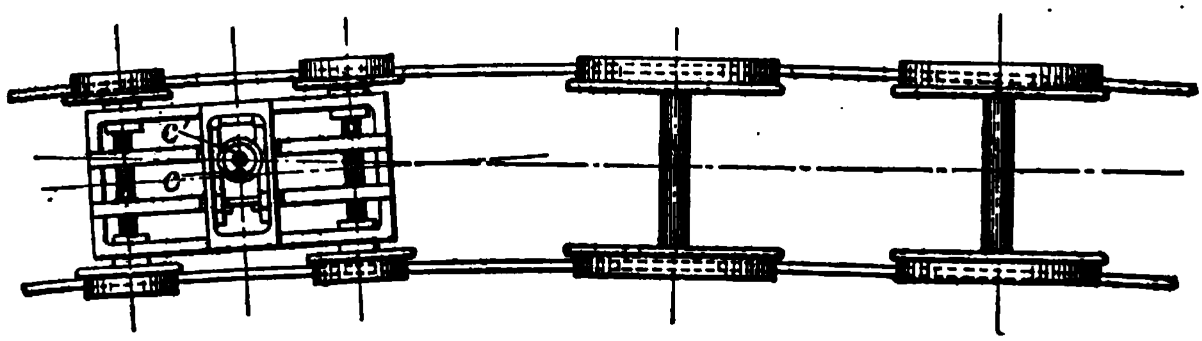


Fig. 182.—Four-wheeled Truck—Shifting Center.

of the truck is to relieve the driver-flanges of any excessive pressure due to curvature. The only exception to this is the case where the curvature is sharp. Then the front inner driver may be pressed against the *inner* rail, as indicated in Fig. 181. This limits the use of this type of wheel-base on the sharper curves.

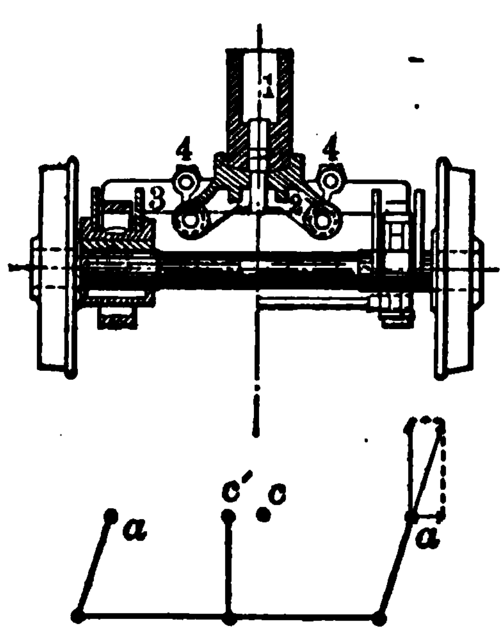


Fig. 183.—Action of Shifting Center.

The next type—(b_1) four-wheeled trucks with shifting centers—is much more flexible on sharp curvature; it likewise draws the front drivers away from the outer rail. The relative position of the wheels is shown in Fig. 182, in which c' represents the position of center-pin and c the displaced truck center. The structure and action of the truck is shown in Fig. 183. The "center-pin" (1) is supported on the "truck-bolster" (2), which is hung by the "links" (4) from the "cross-ties" (3). The links are therefore

in tension and when the wheels are forced to one side by the rails the *links* are inclined and the front of the engine is drawn inward by a force equal to the weight on the bolster times the tangent of the angle of inclination of the links. This assumes that all links are vertical when the truck is in the center. Frequently the opposite links are normally inclined to each other, which somewhat complicates the above simple relation of the forces, although the general principle remains identical.

The two-wheeled pilot-truck with shifting center is illustrated in Fig. 184. The figure shows the facility with which

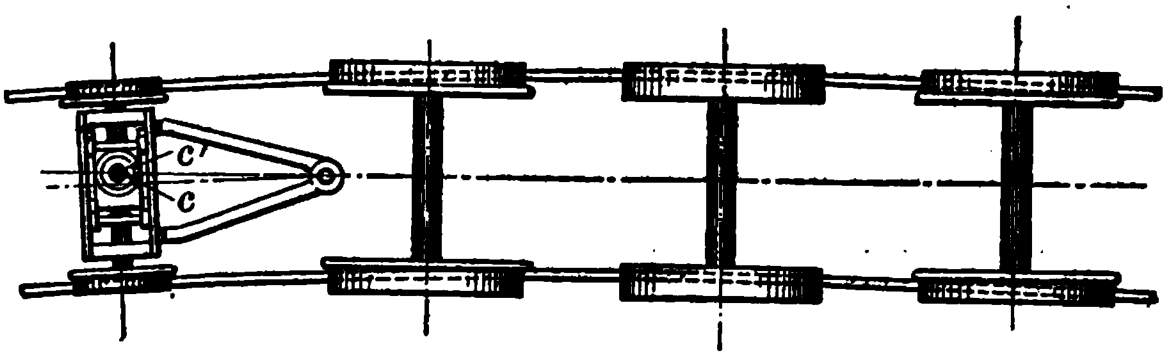


Fig. 184.—Two-wheeled Truck—Shifting Center.

an engine with long wheel-base may be made to pass around a comparatively sharp curve by omitting the flanges from the middle drivers and using this form of pilot-truck. As in the previous case, the eccentricity of the center of the truck relative to the center-pin induces a centripetal force which draws the front of the engine inward. But the swing-truck is not the only source of such a force. If the

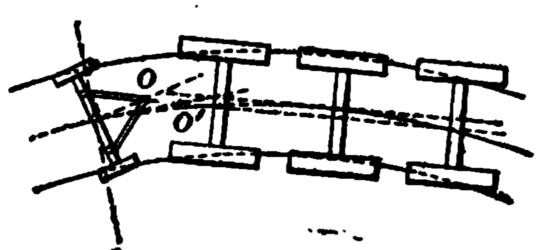


Fig. 185.—Action of Two-wheeled Truck.

"radius-bar pin" were placed at O' (see Fig. 185), the truck-axle would be radial. But the radius-bar is always made somewhat shorter than this, and the pin is placed at O, a considerable distance ahead of O', thus creating a tendency for the truck to run toward the inner rail and draw the front of the locomotive in that direction. This tendency will be objectionably great if the radius-bar is made too short, as has been practically demonstrated in cases when the radius-bar has been subsequently lengthened with a resulting improvement in the running of the engine. This type of pilot truck is used on both Mogul and Consolidation locomotives and explains why these long engines can so easily operate on sharp curves.

400. Types of locomotive wheel-bases. The variations in locomotive service have developed all conceivable types as to total weight, ratio of total weight to weight on drivers, types of running gear, relation of steaming capacity to tractive power, etc. The method of classification on the basis of the running gear is very simple. The number of wheels on both rails of the pilot truck, if any, is placed as the first of three numbers. If there is no pilot truck, the character 0 is used. This is followed by the number of drivers and then by the number of trailing wheels, if any. For example, a Pacific type engine has four wheels on the pilot truck, six driving wheels, and two trailing wheels under the rear of the boiler. The wheel-base is symbolized as 4–6–2. The most common types of locomotives, with their popular names and wheel base symbols, are

American	4–4–0	Consolidation	2–8–0
Columbia	2–4–2	Mikado	2–8–2
Atlantic	4–4–2	Mastodon	4–8–0
Mogul	2–6–0	Santa Fe	2–10–2
Prairie	2–6–2		
Ten-wheel	4–6–0	Mallet	A–B–B–A
Pacific	4–6–2	A = truckwheels, usually	2 or 0
Six-wheel switcher	0–6–0	B = drivers, varying from	4 to 10

The "**Mallet**" type of locomotive is one which combines sufficient flexibility to operate on ordinary railroad curves, wheel loads on the drivers which are not excessive, a very great increase in the total tractive power and yet operated by one engineman. In one respect it is like coupling two or three locomotives together, but the saving consists in reducing the number of enginemen and firemen which would be needed to run the two or three locomotives. Excluding freak variations, they are usually "four-cylinder compounds," one pair of cylinders discharging into the other pair and then exhausting. This type has from five to ten driving axles and has a length of engine wheel-base up to about 60 ft., but this wheel-base is flexible, so that it will bend on a curved track. Sometimes the boiler is made flexible by having a set of accordion-shaped steel rings forming a joint in the boiler shell. The boiler itself is on one side of this flexible joint and the feed-water heater, the reheater, and perhaps the superheater are on the other side of the joint. In this case each half of the flexible boiler is carried on a frame supported by one of the sets of driving wheels, the two frames being connected by a suitable joint. The boiler shell is made rigid; one end is rigidly attached to the frame carrying the high-pressure cylinders and

the other end is supported on a bearing on the truck frame which carries the low-pressure cylinders and the drivers operated by them. The low-pressure truck frame swings around a pivot in the fixed frame. This flexibility has been made so great that these locomotives are operated successfully on 20° curves. The Baldwin Locomotive Works have developed this type still further by building a locomotive for the Erie R. R. which has three wheel frames, mutually flexible with each other, the third frame being under the tender. Each wheel frame has eight driving wheels. The total load carried by the twenty-four drivers is 761,600 lbs. or an average of 31,733 lbs. per driver. There are six cylinders of equal size. The two cylinders on the center frame use high-pressure steam and exhaust into the other four cylinders. The total weight of locomotive and tender is 853,050 lbs. On a test trip it pulled a train with a total length of 8547 ft. or 1.6 miles, the total weight of the train being 18,338 tons. The maximum draw-bar pull, registered by the dynamometer car, was 130,000 lbs. The adhesion between the drivers and the rails must have been considerably more. Such engines are chiefly used for hauling long trains of slow-speed freight. Their boilers cannot produce steam fast enough to develop their enormous tractive power at high speeds and the power falls off rapidly with increase in speed. They are frequently equipped with automatic stokers for burning coal, or with oil-burning outfits, since the great amount of power developed can only be produced by the consumption of a corresponding amount of fuel, and a fireman would be physically incapable of shoveling coal as rapidly as the production of such an amount of power would demand.

LOCOMOTIVES.

GENERAL STRUCTURE.

401. Frame. The frame or skeleton of a locomotive consists chiefly of a collection of forged wrought-iron bars, as shown in Figs. 186 and 187. These bars are connected at the

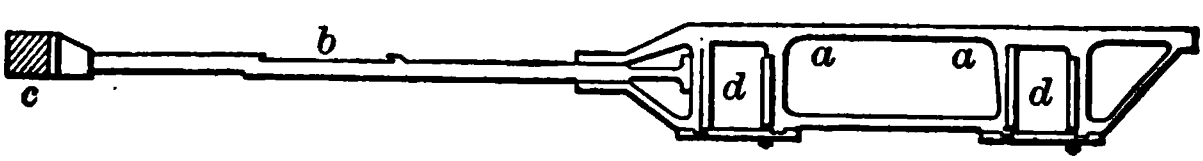


FIG. 186.—ENGINE-FRAME.

front end by the "bumper" (*c*), which is usually made of wood.

A little further back they are rigidly connected at *bb* by the cylinders and boiler-saddle. The boilers rest on the frames at *aaaa* by means of "pads," which are bolted to the fire-box, but which permit a free expansion of the boiler along the frame. This expansion is sometimes as much as $\frac{5}{16}''$. On a "consolidation" engine (frame shown in Fig. 187) it is frequently

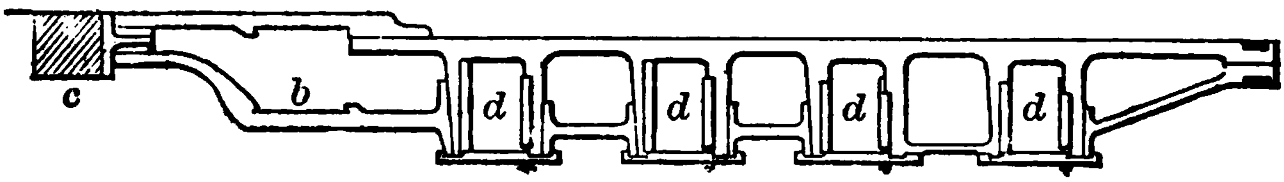


FIG. 187.—ENGINE-FRAME—CONSOLIDATION TYPE.

necessary to use vertical swing-levers about 12″ long instead of "pads." The swinging of the levers permit all necessary expansion. At the back the frames are rigidly connected by the iron "foot-plate." The driving-axles pass through the "jaws" *dddd*, which hold the axle-boxes. The frame-bars have a width (in plan) of 3″ to 4″. The depth (at *a*) is about the same. Fig. 186 shows a frame for an "American" type of locomotive; Fig. 187 shows a frame for a "Consolidation" type (see § 400).

402. Boiler. A boiler is a mechanism for transferring the latent heat of fuel to water, so that the water is transformed from cold water into high-pressure steam, which by its expansion will perform work. The efficiency of the boiler depends largely on its ability to do its work rapidly and to reduce to a minimum the waste of heat through radiation. The boiler contains a fire-box (see Fig. 188), in which the fuel is burned. The gases of consumption pass from the fire-box through the numerous boiler-tubes into the "smoke-box" *S* and out through the smoke-stack. The fire-box consists of an inner and outer shell separated by a layer of water 3″ to 5″ thick. The exposure of water-surface to the influence of the fire is thus very complete. The efficiency of this transferal of heat is somewhat indicated by the fact that, although the temperature of the gases in the fire-box is probably from 3000° to 4000° F., the temperature in the smoke-box is generally reduced to 500° to 600° F. If the steam pressure is 180 lbs., the temperature of the water is about 380° F., and, considering that heat will not pass from the gas to the water unless the gas is hotter than the water, the water evidently absorbs a large part of the theoretical maximum. Nevertheless gases at a temperature of

600° F. pass out of the smoke-stack and such heat is utterly wasted.

The tubes vary from 1¾″ to 2″, inside diameter, with a thickness of about 0″.10 to 0″.12. The aggregate cross-sectional

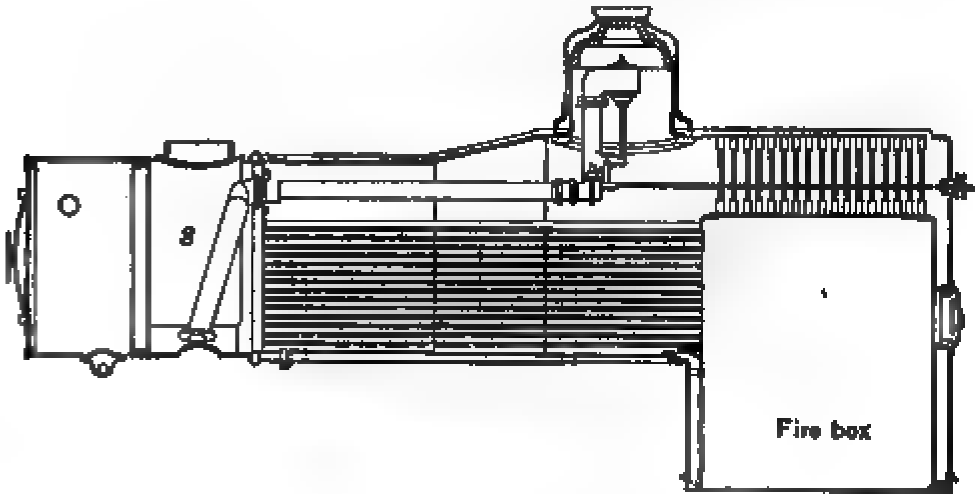


FIG. 188.—LOCOMOTIVE-BOILER.

area of the tubes should be about one-eighth of the grate area. The number will vary from 140 to 375. The length varies from 11′ to 21′, but the length is virtually determined by the type and length of engine.

403. **Fire-box.** The fire-box is surrounded by water on the four sides and the top, but since the water is subjected to the

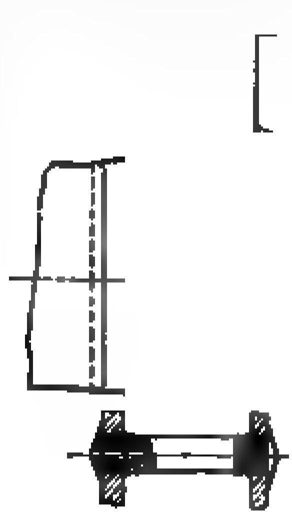


FIG. 189.

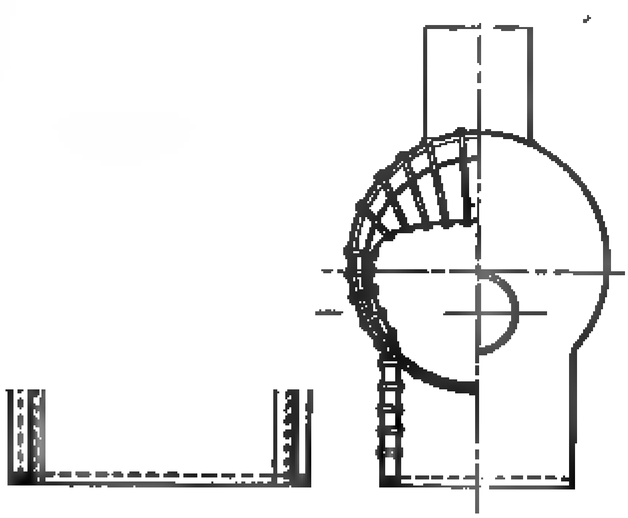


FIG. 190.

boiler pressure, the plates, which are $\frac{5}{16}$″ to ⅜″ thick, must be stayed to prevent the fire-box from collapsing This is easily accomplished over the larger part of the fire-box surface by

having the outside boiler-plates parallel to the fire-box plates and separated from them by a space of 3″ to 5″. The plates

FIG. 191.—BOILER SHOWING CROWN-BARS AND WATER-TABLE. Half-section through AB.

are then mutually held by "stay-bolts." See Fig. 189. These are about $\frac{7}{8}$″ in diameter and spaced 4″ to $4\frac{1}{2}$″. The $\frac{3}{16}$″ hole, drilled $1\frac{1}{4}$″ deep, indicated in the figure, will allow the escape

of steam if the bolt breaks just behind the plate, and thus calls attention to the break. The stay-bolts are turned down to a diameter equal to that at the root of the screw-threads. This method of supporting the fire-box sheets is used for the two sides, the entire rear, and for the front of the fire-box up to the boiler-barrel. The "furnace tube-sheet"—the upper part of the front of the fire-box—is stayed by the tubes. But the top of the fire-box is troublesome. It must always be covered with water so that it will not be "burned" by the intense heat. It must therefore be nearly, if not quite, flat. There are three general methods of accomplishing this.

FIG. 192.—"BELPAIRE" FIRE-BOX.

Half-section through *AB*. Half-section through *CD*.

(a) **Radial stays.** This construction is indicated in Fig. 190. Incidentally there is also shown the diagonal braces for resisting the pressure on the back end of the boiler above the fire-box. It may be seen that the stays are not perpendicular to either the crown-sheet or the boiler-plate. This is objectionable and is obviated by the other methods.

(b) **Crown-bars.** These bars are in pairs, rest on the side furnace-plates, and are further supported by stays. See Fig. 191.

(c) **Belpaire fire-box.** The boiler above the fire-box is rectangular, with rounded corners. The stays therefore are perpendicular to the plates. See Fig. 192.

Fire-brick arches. These are used, as shown in Fig. 193, to force all the gases to circulate through the upper part of the fire-box. Perfect combustion requires that all the carbon shall be turned into carbon dioxide, and this is facilitated by the forced circulation.

Water-tables. The same object is attained by using a water-table instead of a brick arch—as shown in Fig. 191. But it has the further advantages of giving additional heating-surface and avoiding the continual expense of maintaining the bricks. One feature of the design is the use of a number of steam-jets which force air into the fire-box and assist the combustion.

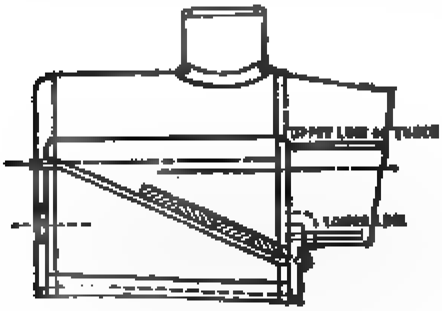


FIG. 193.—FIRE-BRICK ARCH. FIG. 194.—WOOTTEN FIRE-BOX.

404. Area of grate. The older types of engines, as represented by the "American," "Mogul" or "Consolidation" type, always had the fire-box set between the drivers, which practically meant that the maximum effective inside width of the fire-box was limited to about 3 ft. 5 ins. for standard-gauge locomotives. The maximum distance over which a fireman can properly control a fire is perhaps 10 to 11 ft., but such extreme lengths are objectionable. The grate area was thus quite definitely limited. The Wootten fire-box, illustrated in Fig. 194, obtained a fire-box eight feet wide by raising it above the level of the drivers, as shown, but this required that the drivers should be objectionably small in diameter, except for low-speed engines, or that the fire-box would be set objectionably high. The last difficulty has been solved by engines of the "Columbia," "Atlantic," "Pacific," "Mikado," and "Santa Fe" types, all of which have a pair of trailing wheels, 36 to 45 ins. in diameter, set back of the driving wheels and under the fire-box, which may thus be widened to 7 or 8 ft., the entire fire-box being placed back of the driving wheels.

405. Superheaters. Inside of a boiler the steam has a temperature corresponding to its pressure. For example, if the pressure is 180 lbs., the temperature is about 379° F. When the steam of a locomotive is superheated, the steam is conducted from the throttle to the cylinders through pipes which are pur-

posely placed in the path of the flue gases on their way to the smokestack. A simple form of superheater is a series of tubes and drums located in the smokebox. Here the temperature is perhaps 600° F., which is sufficient to heat the steam from 30° to 50° above the boiler temperature and to produce substantial economies. In another more effective but more costly type a considerable number of the ordinary 2¼-inch boiler tubes are replaced by 5½-inch tubes, inside of each of which is a pipe loop extending from the smokebox headers to within a short distance of the fire-box, where the temperature approaches the fire-box temperature, which is perhaps 2000° F. The live steam passes through these loops and is so heated that, even after it reaches the cylinder, it has a superheat of 150° to 200° over the boiler temperature, but since its pressure is substantially the boiler pressure, the *quantity* (or weight) of steam required to fill the cylinder at that temperature and pressure is much less than the quantity of steam at the same pressure but lower temperature. Superheating also has the advantage of making the steam more dry and of preventing condensation in the cylinders until the steam has lost in temperature at least the amount of its superheat. Superheating is chiefly advantageous for use with passenger engines, when they must work at high power for long, continuous runs. An economy of 15 to 25% in coal consumption (and even 30% in some tests), can ordinarily be obtained by the use of superheaters, but the economy is somewhat offset by the additional cost for installation and for subsequent repairs and maintenance.

406. Reheaters. A reheater is substantially the same as a superheater in its general principle of construction. When steam has been exhausted from a high-pressure cylinder, the temperature and pressure are both considerably lower than their boiler values. If the steam is to be again used, an economy is obtained and the steam is dried by passing it through a reheater. They are generally used on Mallet engines to reheat the steam in its passage from the high-pressure to the low-pressure cylinders.

407. Coal consumption. No form of steam-boiler (except a boiler for a steam fire-engine) requires as rapid production of steam, considering the size of the boiler and fire-box, as a locomotive. The combustion of coal per square foot of grate per hour for stationary boilers averages about 15 to 25 lbs. and seldom exceeds that amount. An ordinary maximum for a

locomotive is 125 lbs. of coal per square foot of grate-area per hour, and in some recent practice 220 lbs. have been used. Of course such excessive amounts are wasteful of coal, because a considerable percentage of the coal will be blown out of the smoke-stack unconsumed, the draft necessary for such rapid consumption being very great. The only justification of such rapid and wasteful coal consumption is the necessity for rapid production of steam. The best quality of coal is capable of evaporating about 14 lbs. of water per pound of coal, i.e., change it from water at 212° to steam at 212°; the heat required to change water at ordinary temperatures to steam at ordinary working pressure is (roughly) about 20% more. From 6 to 9 lbs. of water per pound of coal is the average performance of ordinary locomotives, the efficiency being less with the higher rates of combustion. Some careful tests of locomotive coal consumption gave the following figures: when the consumption of coal was 50 lbs. per square foot of grate-area per hour, the rate of evaporation was 8 lbs. of water per pound of coal. When the rate of coal consumption was raised to 180, the evaporation dropped to 5 lbs. of water per pound of coal. It has been demonstrated that the efficiency of the boiler is largely increased by an increased length of boiler-tubes. The actual consumption of coal per mile is of course an exceedingly variable quantity, depending on the size and type of the engine and also on the work it is doing—whether climbing a heavy grade with its maximum train-load or running easily over a level or down grade. A test of a 50-ton engine, running without any train at about 20 to 25 miles per hour, showed an average consumption of 21 lbs. of coal per mile. Statistics of the Pennsylvania Rail road show a large increase (as might be expected, considering the growth in size of engines and weight of trains) in the average number of pounds of coal burned per *train*-mile—some of the figures being 55 lbs. in 1863, 72 lbs. in 1872, and nearly 84 lbs. in 1883. Figures are published showing an average consumption of about 10 lbs. of coal per passenger-car mile, and 4 to 5 lbs. per freight-car mile. But these figures are always obtained by dividing the total consumption per train-mile by the number of cars, the coal due to the weight of the engine being thrown in. Wellington developed a rule, based on the actual performance of a very large number of passenger-trains, that the number of pounds of coal per mile $=21.1+6.74$ times

the number of passenger-cars The amount of coal assigned to the engine agrees remarkably with the test noted above For freight-trains the amount assigned to the engine should be much greater (since the engine is much heavier), and that assigned to the individual cars much less, although the great increase in freight-car weights in recent years has caused an increase in the coal required per car.

There is a physical limit to the amount of coal which can be shovelled into a firebox by a fireman. Tests have shown that the average fireman can handle about 4000 lbs. of coal per hour and keep up such work almost indefinitely. For a short time he can shovel coal at the rate of 80 or 90 lbs. per minute, and this may be necessary to keep up steam while the train is going over some hump, but it must be followed by some relief which will make the average about the same. **Automatic stokers** have been devised for locomotives which can feed as much as 6000 lbs. of coal per hour when the grate area is less than 70 square feet and up to 8000 lbs. per hour when the grate area is 70 square feet or over. These are necessary on some of the most powerful locomotives in order to produce steam fast enough to develop their maximum capacity.

408. Oil-burning locomotives. In 1912 over one-sixth of all the locomotives west of the Mississippi River used oil as fuel. Some of the advantages in using oil are as follows: (1) the British thermal units in one pound of oil vary from about 19,000 to 21,000; those in a pound of coal vary from perhaps 14,000 for the very best down to 5000 for the poorer grades of lignite found in the western parts of the United States, and this means a great reduction in the cost of carrying and storing fuel, measured in heat units; (2) the cost of handling fuel is reduced and that of disposing of ashes is eliminated; (3) engine repairs are reduced in many respects, although it is said that the increased cost of fire-box repairs, due to the intense heat of the oil flame, offsets any reduction in other items; (4) the fires can be more easily controlled and waste of heat reduced during stoppages or when drifting down grade; (5) wayside fires due to sparks are altogether eliminated; (6) there is a practical limitation (see § 407), to the amount of coal that one fireman can feed to a fire; but there is no such limitation when using oil; (7) there is an equality in cost of heat units when a 42-gallon barrel of oil, weighing 7.3 lbs. per gallon, costs 60 cents and a ton (2000 lbs.) of coal, having

two-thirds as many heat units per pound, costs $2.61, or 4.35 times as much. The other items of difference almost invariably favor the oil and might make it more desirable even when the ratio of cost seemed to favor the coal. The extensive use of oil west of the Mississippi River is due to the fact that in many localities a very suitable quality of crude oil is plentiful and cheap while coal is expensive and of low calorific power.

409. Heating-surface. The rapid production of steam requires that the hot gases shall have a large heating-surface to which they can impart their heat. From 50 to 75 square feet of heating-surface is usually designed for each square foot of grate-area. A more recently used rule is that there should be from 60 to 70 square feet of tube heating-surface per square foot of grate-area for bituminous coal. 40 or 50 to 1 is more desirable for anthracite coal. Almost the whole surface of the fire-box has water behind it, and hence constitutes heating-surface. Although this surface forms but a small part of the total (nominally), it is really the most effective portion, since the difference of temperature of the gases of combustion and the water is here a maximum, and the flow of heat is therefore the most rapid. The heating-surface of the tubes varies from 85 to 93% of the total, or about 7 to 15 times the heating-surface in the fire-box. By dividing the total weight of a well-designed engine (exclusive of tender) by the number of square feet of heating-surface (fire-box and tubes), we get a quotient which varies from 60 to 80 or over. For example, a light engine, weighing only 96,450 lbs. had a total heating surface of 1449 square feet, or about 67 lbs. per square foot. On the other hand, a Mikado engine, weighing 297,500 lbs., had 4359 square feet of heating surface, or 68 lbs. per square foot.

410. Loss of efficiency in steam pressure. The effective work done by the piston is never equal to the theoretical energy contained in the steam withdrawn from the boiler. This is due chiefly to the following causes:

(*a*) The steam is "**wire-drawn,**" i.e., the pressure in the cylinder is seldom more than 85 to 90% of the boiler pressure. This is due largely to the fact that the steam-ports are so small that the steam cannot get into the cylinder fast enough to exert its full pressure. Partially closing the throttle, so that the steam will be used less rapidly, also wire-draws the steam.

(*b*) **Entrained water.** Steam is always drawn from a dome

placed over the boiler so that the steam shall be as far above the water-surface as possible, and shall be as dry as possible. In spite of this the steam is not perfectly dry and carries with it water at a temperature of, say, 361°, and pressure of 140 lbs. per square inch. When the pressure falls during the expansion and exhaust, this hot water turns into steam and absorbs the necessary heat from the hot cylinder-walls. This heat is then carried out by the exhaust and wasted.

(*c*) The **back pressure** of the exhaust-steam, which depends on the form of the exhaust-passages, etc. This amounts to from 2 to 20% of the power developed.

(*d*) **Clearance-spaces.** When cutting off at full stroke this waste is considerable (7 to 9%), but when the steam is used expansively the steam in these clearance-spaces expands and so its power is not wholly lost.

(*e*) **Radiation.** In spite of all possible care in jacketing the cylinders, some heat is lost by radiation.

(*f*) **Radiation into the exhaust-steam.** This is somewhat analogous to (*b*). Steam enters the cylinder at a temperature of, say, 361°; the walls of the cylinder are much cooler, say 250°; some heat is used in raising the temperature of the cylinder-walls; some steam is vaporized in so doing; when the exhaust is opened the temperature and pressure fall; the heat temporarily absorbed by the cylinder-walls is reabsorbed by the exhaust-steam, re-evaporating the vapor previously formed, and thus a certain portion of heat-energy goes through the cylinder without doing any useful work. With an early cut-off the loss due to this cause is very great.

The sum of all these losses is exceedingly variable. They are usually less at lower speeds. The loss in *initial pressure* (the difference between boiler pressure and the cylinder pressure at the beginning of the stroke) is frequently over 20%, but this is not all a net loss With an early cut-off the average cylinder pressure for the whole stroke is but a small part of the boiler pressure, yet the horse-power developed may be as great as, or greater than that developed at a lower speed, later cut-off, and higher average pressure.

411. Tractive power The work done by the two cylinders during a complete revolution of the drivers evidently = area of pistons × average steam pressure × stroke × 2 × 2. The resistance overcome evidently = tractive force at circumference of

drivers times distance traveled by drivers (which is the circumference of the drivers) Therefore

$$\text{Tractive force} = \left\{ \frac{\begin{array}{c}\text{area pistons} \times \text{average steam pressure} \\ \times \text{stroke} \times 2 \times 2.\end{array}}{\text{circumference of drivers}} \right. .$$

Dividing numerator and denominator by π (3.1415), we have

$$\text{Tractive force} = \left\{ \frac{\begin{array}{c}(\text{diam piston})^2 \times \text{average steam} \\ \text{pressure} \times \text{stroke}\end{array}}{\text{diameter of driver}} \right. , \quad . \quad (103)$$

which is the usual rule Although the rule is generally stated in this form, there are several deductions In the first place the net effective area of the piston is less than the nominal on account of the area of the piston-rod. The ratio of the areas of the piston-rod and piston varies, but the effect of this reduction is usually from 1.3 to 1.7%. No allowance has been made for friction—of the piston, piston-rod, cross-head, and the various bearings. This would make a still further reduction of several per cent. Nevertheless the above simple rule is used, because, as will be shown, no great accuracy can be utilized.

The maximum draw bar pull is limited by the adhesion between the driving wheels and the rails. This is usually about one-fourth of the weight. The use of sand may increase it to one-third. But this ratio is important only when starting or at very low speeds. The adhesion is always ample for the much lower cylinder power which can be developed at higher speeds. This is considered more fully in Chapter XVIII.

RUNNING GEAR.

412. Equalizing-levers. The ideal condition of track, from the standpoint of smooth running of the rolling stock, is that the rails should always lie in a plane surface. While this condition is theoretically possible on tangents, it is unobtainable on curves, and especially on the approaches to curves when the outer rail is being raised. Even on tangents it is impossible to *maintain* a perfect surface, no matter how perfectly the track may have been laid. In consequence of this, the points

of contact of the wheels of a locomotive, or even of a four-wheeled truck, will not ordinarily lie in one plane. The rougher and more defective the track, the worse the condition in this respect. Since the frame of a locomotive is practically rigid, and the frame rests on the driver-axles through the medium of springs at each axle-bearing, the compression of the springs (and hence the pressure of the drivers on the rail) will be variable if the bearing-points of the drivers are not in one plane. This variable pressure affects the tractive power and severely strains the frame. Applying the principle that a tripod will stand on an uneven surface, a mechanism is employed which

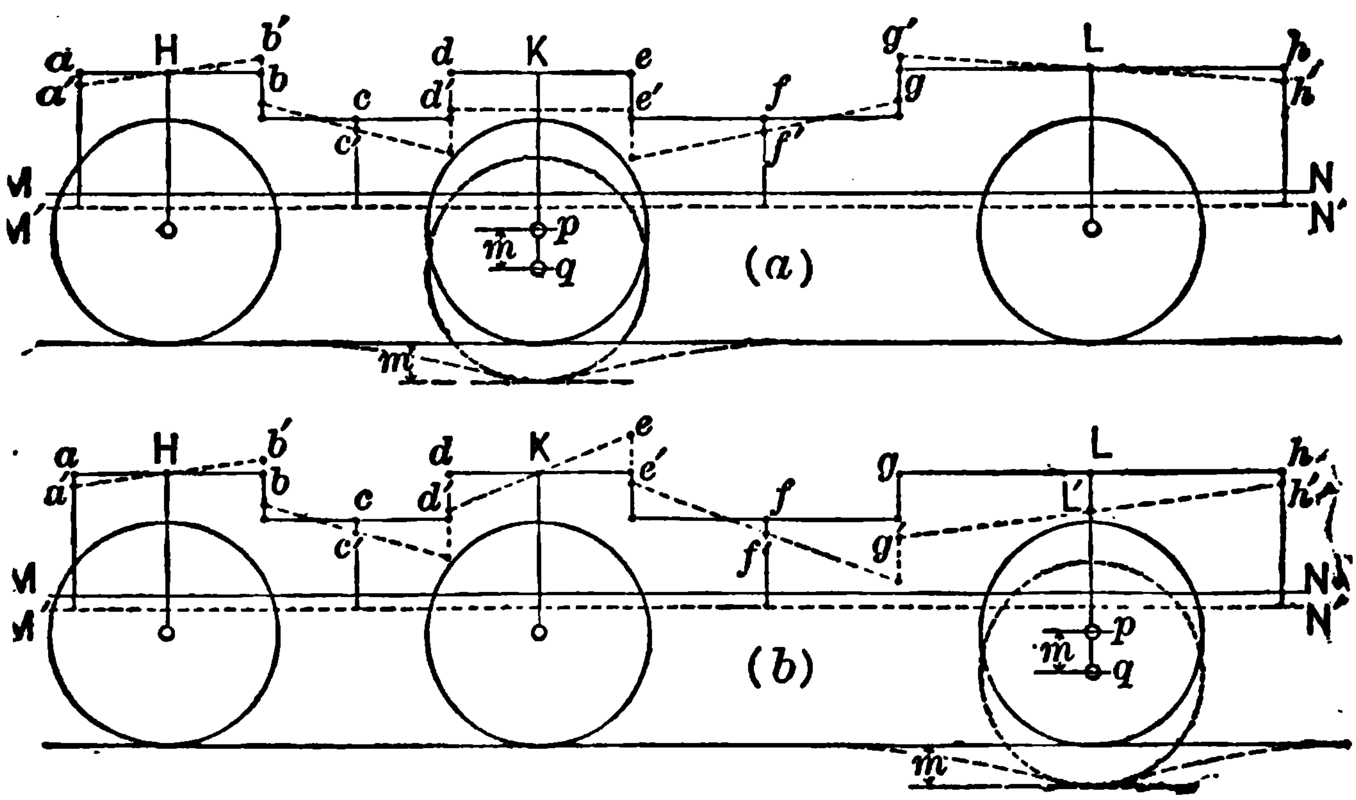


FIG. 195.—ACTION OF EQUALIZING-LEVERS.

virtually supports the locomotive on three points, of which one is usually the center-bearing of the forward truck. On each side the pressure is so distributed among the drivers that even if a driver rises or falls with reference to the others, the load carried by each driver is unaltered, and that side of the engine rises or falls by one *n*th of the rise or fall of the single driver, where *n* represents the number of wheels. The principle involved is shown in an exaggerated form in Fig. 195. In the diagram, *MN* represents the normal position of the frame when the wheels are on line. The frame is supported by the hangers at *a*, *c*, *f*, and *h*. *ab*, *de*, and *gh* are horizontal levers vibrating about the points *H*, *K*, and *L*, which are supported by the axles. While it is *possible* with such a system of levers to make

MN assume a position not parallel with its natural position, yet, by an extension of the principle that a beam balance loaded with equal weights will always be horizontal, the effect of raising or lowering a wheel will be to move MN parallel to itself. It only remains to determine *how much* is the motion of MN relative to the rise or drop of the wheel.

The dotted lines represent the positions of the wheels and levers when one wheel drops into a depression. The wheel center drops from p to q, a distance m. L drops to L', a distance m (see Fig. 195, b); M drops to M', an unknown distance x; therefore $aa'=x$; $bb'=x$; $cc'=x$; $dd'=3x=ee'$; $ff'=x$; $\therefore$ $gg'=5x$; $hh'=x$; $LL'=\frac{1}{2}(gg'+hh')=\frac{1}{2}(6x)=m$; $\therefore$ $x=\frac{1}{3}m$; i.e., MN drops, parallel to itself, $1/n$ as much as the wheel drops, where n is the number of wheels. The resultant effect caused by the simultaneous motion of two wheels with reference to the third is evidently the algebraic sum of the effects of each wheel taken separately.

The practical benefits of this device are therefore as follows:

(a) When any driver reaches a rough place in the track, a high place or a low place, the stress in all the various hangers and levers is unchanged.

(b) The motion of the frame (represented by the bar MN in Fig. 195) is but $1/n$ of the motion of the wheel, and the jar and vibration caused by a roughness in the track is correspondingly reduced.

The details of applying these principles are varied, but in general it is done as follows:

(a) American and ten wheeled types. Drivers on each side form a system. The center-bearing pilot-truck is the third point of support. The method is illustrated in Fig. 196.

(b) Mogul and consolidation types. The front pair of drivers is connected with the two-wheeled pilot-truck (as illustrated in Fig. 197) to form one system. The remaining drivers on each side are each formed into a system.

The device of equalizers is an American invention. Until recently it has not been used on foreign locomotives. The necessity for its use becomes less as the track is maintained with greater perfection and is more free from sharp curves. A locomotive not equipped with this device would deteriorate very rapidly on the comparatively rough tracks which are usually found on light-traffic roads. It is still an open ques-

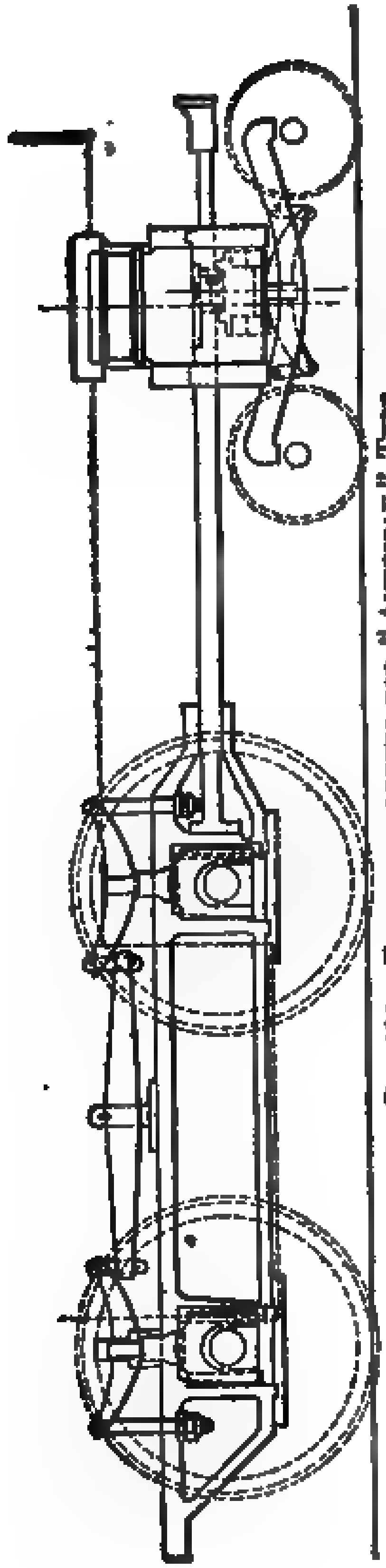


FIG. 190.—EQUALIZING-LEVERS FOR "AMERICAN" TYPE.

tion to what extent the neglect of this device is responsible for the statistical fact that average freight-train loads on foreign

trains are less in proportion to the weight on the drivers than is the case with American practice. The recent increasing use of this device on foreign heavy freight locomotives is perhaps an acknowledgment of this principle.

413. Counterbalancing. At very high velocities the centrifugal force developed by the weight of the rotating parts becomes a quantity which cannot be safely neglected. These rotating parts include the crank-pin, the crank-pin boss, the side rod, and that part of the weight of the connecting-rod which may be considered as rotating about the center of the crank-driver. As a numerical illustration, a driving-wheel 62″ in diameter, running 60 miles per hour, will revolve 325 times per minute. The weights are:

Crank-pin..........................	110 lbs.
" boss.....................	150 "
One-half side rod..................	240 "
Back end of connecting-rod.........	190 "
Total..........................	690 lbs.

If the stroke is 24″, the radius of rotation is 12″, or 1 foot. Then

$$\frac{Gv^2}{gr}=\frac{690\times 4\pi^2 1^2\times 325^2}{32.2\times 1\times 60^2}=24821 \text{ lbs.},$$

which is half as much again as the weight on a driver, 16000 lbs. Therefore if *no* counterbalancing were used, the pressure between the drivers and the rail would always be less (at any velocity) when the crank-pin was at its highest point. At a velocity of about 48 miles per hour the pressure would become zero, and at higher velocities the wheel would actually be thrown from the rail. As an additional objection, when the crank-pin was at the lowest point, the rail pressure would be increased (velocity 60 miles per hour) from 16000 lbs. to nearly 41000 lbs., an objectionably high pressure. These injurious effects are neutralized by "counterbalancing." Since all of the above-mentioned weights can be considered as concentrated at the center of the crank-pin, if a sufficient weight is so placed in the drivers that the center of gravity of the eccentric weight is diametrically opposite to the crank-pin, this centrifugal force can be wholly balanced. This is done by filling up a portion of the space between the spokes. If the center of gravity of the counterbalancing weight is 20″ from the center, then, since the crank-pin radius is 12″, the required weight would be $690\times\frac{12}{20}=414$ lbs.

In addition to the effect of these revolving parts there is the effect of the sudden acceleration and retardation of the reciprocating parts. In the engine above considered the weights of these reciprocating parts will be:

Front end of connecting-rod........	150 lbs.
Cross-head..........................	174 "
Piston and piston-rod..............	300 "
Total.........................	624 lbs.

Assume as before that the reciprocating parts may be considered as concentrated at one point, the point P of the diagram in Fig. 198. Since the motion of P is horizontal only, the force required to overcome its inertia at any point will exactly equal the *horizontal component* of the force required to overcome the inertia of an *equal* weight at S revolving in a circular path. Then evidently the horizontal component of the force required to keep W in the circular path will exactly balance the force required to overcome the inertia of P. Of course $W=P$. But a smaller weight W', whose weight is inversely proportional to its radius of rotation, will evidently accomplish the same result. In the above numerical case, if the center of gravity of the counterweights is 20″ from the center, the required weight to completely counterbalance the reciprocating parts would be $624 \times \frac{12}{20} = 374.4$ lbs. This counterweight need not be all placed on the driver carrying the main crank-pin, but can be (and is) distributed among all the drivers. Suppose it were divided between the two drivers in the above case. At 60 miles per hour such a counterweight would produce an additional pressure of 11211 lbs. when the counterweight was down, or a lifting force of the same amount when the counterweight was up. Although this is not sufficient to lift the driver from the rail, it would produce an objectionably high pressure on the rail (over 27000 lbs.), thus inducing just what it was desired to avoid on account of the eccentric rotating parts. Therefore a compromise must be made. Only a portion (one half to three fourths) of the weight of the reciprocating parts is balanced. Since the effect of the rotating

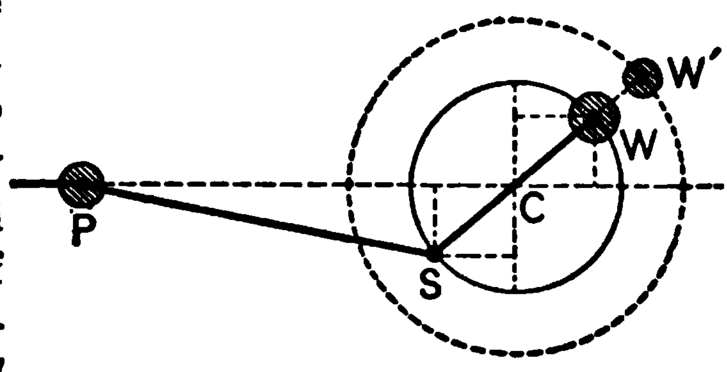


FIG. 198.—ACTION OF COUNTERBALANCE.

weights is to cause variable pressure on the rail, while the effect of the reciprocating parts is to cause a horizontal wobbling or "nosing" of the locomotive, it is impossible to balance both. Enough counterweight is introduced to partially neutralize the effect of the reciprocating parts, still leaving some tendency to horizontal wobbling, while the counterweights which were introduced to reduce the wobbling cause some variation of pressure. By using hollow piston-rods of steel, ribbed cross-heads, and connecting- and side-rods with an I section, the weight of the reciprocating parts may be greatly lessened without reducing their strength, and with a decrease in weight the effect of the unbalanced reciprocating parts and of the "excess balance" (that used to balance the reciprocating parts) is largely reduced.

Current practice is somewhat variable on three features:

(*a*) The proportion of the weight of the connecting-rod which should be considered as revolving weight.

(*b*) The proportion of the total reciprocating weight that should be balanced.

(*c*) The distribution among the drivers of the counterweight to balance the reciprocating parts.

An exact theoretical analysis of (*a*) shows that it is a function of the weights and dimensions of the reciprocating parts. The weight which may be considered as revolving equals *

$$W_1\left(\frac{r^2+k^2-rd\left(1+\frac{r}{l}\right)}{l^2-r^2}\right)+W_2\frac{r^2}{l^2-r^2},$$

in which r = radius of the crank, l = length of connecting-rod, k = distance of center of gyration from wrist-pin, d = distance of center of gravity from wrist-pin, W_1 = weight of connecting-rod in pounds, and W_2 = weight of piston, piston-rod, and cross-head in pounds; all dimensions in feet. An application of this formula will show that for the dimensions of usual practice, from 51 to 57% of the weight of the connecting-rod should be considered as revolving weight.

The principal rules which have been formulated for counterbalancing may be stated as follows:

1. Each wheel should be balanced correctly for the revolving parts connected with it.

* R. A. Parke, in R. R. Gazette, Feb. 23, 1894.

2. *In addition*, introduce counterbalance sufficient for 50% of the weight of the reciprocating parts for ordinary engines, increasing this to 75% when the reciprocating parts are excessively heavy (as in compound locomotives) or when the engine is light and unable to withstand much lateral strain or when the wheel-base is short.

3. Consider the weight of the connecting-rod as $\frac{1}{2}$ revolving and $\frac{1}{2}$ reciprocating when it is over 8 feet long; when shorter than 8 feet, consider $\frac{6}{10}$ of the weight as revolving and $\frac{4}{10}$ as reciprocating.

4. The part of the weight of the connecting-rod considered as revolving should be entirely balanced in the crank-driver wheel

5. The "excess balance" should be divided equally among the drivers.

6. Place the counterbalance as near the rim of the wheel as possible and also as near the outside of the wheel as possible in order that the center of gravity shall be as near as possible opposite the center of gravity of the rods, etc., which are all outside of even the plane of the face of the wheel.

In Fig. 199 is shown a section of a locomotive driver with the cavities in the casting for the accommodation of the lead which is used for the counterbalance weight. Incidentally several other features and dimensions are shown in the illustration.

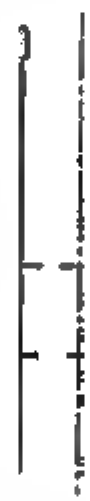


FIG. 199.—SECTION OF LOCOMOTIVE-DRIVER.

414. Mutual relations of the boiler power, tractive power, and cylinder power for various types. The design of a locomotive includes three *distinct* features which are varied in their mutual relations according to the work which the engine is expected to do.

(a) **The boiler power.** This is limited by the rate at which steam may be generated in a boiler of admissible size and weight. Engines which are designed to haul very fast trains which are comparatively light must be equipped with very large grates and heating surfaces so that steam may be developed with great rapidity in order to keep up with the very rapid consumption.

Engines for very heavy freight work are run at very much lower velocity and at a lower piston speed in spite of the fact that more strokes are required to cover a given distance and the demand on the boiler for *rapid* steam production is not as great as with high-speed passenger-engines. The capacity of a boiler to produce steam is therefore limited by the limiting weight of the general type of engine required. Although improvements may be and have been made in the design of fire-boxes so as to increase the steam-producing capacity without adding proportionately to the weight, yet there is a more or less definite limit to the boiler power of an engine of given weight.

(b) **The tractive power.** This is limited by the possible driver adhesion. The absolute limit of tractive adhesion between a steel-tired wheel and a steel rail is about one-third of the pressure, but not more than one-fourth of the weight on the drivers can be depended on for adhesion and wet rails will often reduce this to one fifth and even less. The tractive power is therefore absolutely limited by the practicable weight of the engine. In some designs, when the maximum tractive power is desired, not only is the entire weight of the boiler and running gear thrown on the drivers, but even the tank and fuel-box are loaded on. Such designs are generally employed in switching-engines (or on engines designed for use on abnormally heavy mountain grades) in which the maximum tractive power is required, but in which there is no great tax on the boiler for *rapid* steam production (the speed being always very low), and the boiler and fire-box, which furnish the great bulk of the weight of an engine, are therefore comparatively light, and the requisite weight for traction must, therefore, be obtained by loading the drivers as much as possible. On the other hand, engines of the highest speed cannot possibly produce steam fast enough to maintain the required speed unless the load be cut down to a comparatively small amount. The tractive power required for this comparatively small load will be but a small part of the weight of the engine, and therefore engines of this class have but a small proportion of their weight on the drivers; generally have but two driving-axles and sometimes but one.

(c) **Cylinder power.** The running gear forms a mechanism which is simply a means of transforming the energy of the boiler into tractive force and its power is unlimited, within the practical conditions of the problem. The power of the running

gear depends on the steam pressure, on the area of the piston, on the diameter of the drivers, and on the ratio of crank-pin radius to wheel radius, or of stroke to driver diameter. It is always possible to increase one or more of these elements by a relatively small increase of expenditure until the cylinders are able to make the drivers slip, assuming a sufficiently great resistance. Since the power of the engine is limited by the power of its weakest feature, and since the running gear is the most easily controlled feature, the power of the running gear (or the "cylinder power") is always made somewhat excessive on all well-designed engines. It indicates a badly designed engine if it is stalled and unable to move its drivers, the steam pressure being normal. If it is attempted to use a freight-engine on *fast* passenger service, it will probably fail to attain the desired speed on account of the steam pressure falling. The tractive power and cylinder power are superabundant, but the boiler cannot make steam as fast as it is needed for high speed, especially when the drivers are small. The practical result would be a comparatively low speed kept up with a forced fire. If it is attempted to use a high-speed passenger-engine on heavy freight service, the logical result is a slipping of the drivers until the load is reduced. The boiler power and cylinder power are ample, but the weight on the drivers is so small that the tractive power is only sufficient to draw a comparatively small load.

These relations between boiler, cylinder, and tractive power are illustrated in the following comparative figures referring to a fast passenger-engine, a heavy freight-engine, and a switching-engine. The weights of the passenger- and freight-engines are about the same, but the passenger-engine has only 74% *

Kind.	Cylinders.	Total W ght.	Wt. on Driv'rs	Heating Surface, sq. ft.	Grate area sq. ft.	Steam Pressure in Boiler.	$\frac{\text{Stroke.}}{\text{Diam. Driver.}}$
Fast passenger.	19″×24″	126700	81500	1831.8	26.2	180	$\frac{24}{78}=.31$
Heavy freight.	20″×24″	128700	112600	1498.3	31.5	140	$\frac{24}{50}=.48$
Switcher.....	19″×24″	109000	109000	1498.0	22.8	160	$\frac{24}{50}=.48$

* Computed from Eq. 137.

of the tractive power of the freight. But the passenger-engine has 22% more heating-surface and can generate steam much faster; it makes less than two-thirds as many strokes in covering a given distance, but it runs at perhaps twice the speed and probably consumes steam much faster. The switch-engine is lighter in total weight, but the tractive power is a little greater than the freight and much greater than the passenger-engine. While the heating-surfaces of the freight- and switching engines are practically identical, the grate area of the switcher is much less; its speed is always low and there is but little necessity for rapid steam development.

While these figures show the general tendency for the relative proportions, and in this respect may be considered as typical, there are large variations. The recent enormous increase in the dead weight of passenger-trains has necessitated greater tractive power. This has been provided sometimes by using the "Pacific" type, which combines rapid steaming capacity and great tractive power. On the other hand, the demand for fast-freight service, and the possibility of safely operating such trains by the use of air-brakes, has required that heavy freight-engines shall be run at comparatively high speeds, and that requires the rapid production of steam, large grate areas and heating surfaces. But in spite of these variations, the normal standard for passenger service is a four-driver engine carrying about two-thirds of the weight of the engine on the drivers, which are very large; the normal standard for freight work is an 8-driver engine with perhaps 90% of the weight on the drivers, which are small, but which must have the pony truck for such speed as it uses; and finally the normal standard for switching service has all the weight on the drivers and has comparatively low steam-producing capacity.

415. Life of Locomotives. The life of locomotives (as a whole) may be taken as about 800000 miles or about 22 to 24 years. While its life should be and is considered as the period between its construction and its final consignment to the scrap pile, parts of the locomotive may have been renewed more than once. The boiler and fire-box are especially subject to renewal. The mileage life is much longer than formerly. This is due partly to better design and partly to the custom of drawing the fires less frequently and thereby avoiding some of the destructive strains caused by extreme alternations of

heat and cold. Recent statistics give the average annual mileage on twenty-three leading roads to be 41000 miles.

CARS.

416. Capacity and size of cars. The capacity of freight-cars has been enormously increased of late years. In 1870 the usual live-load capacity for a box-car was about 20000 lbs. In 1916, out of 58299 box cars owned by the Pennsylvania R. R., 32923 or 56% had a capacity of 100000 or over; 49597 or 85% had a capacity 70000 or over; only 555, less than 1%, had a capacity of less than 60000 lbs., and the most of these were refrigerator cars or cars for special service. The Norfolk & Western R. R. had (in 1916), 750 gondola drop-bottom coal cars, each with a nominal capacity of 180000 lbs.; their length is 46 feet 10¾ inches, and the extreme width 10 feet 4½ inches. These cars are carried on six-wheel trucks. The usual width of freight-cars is about 9 to 10 feet, while parlor-cars and sleepers are generally 10 feet wide and sometimes 11 feet. The highest point of a train is usually the smokestack of the locomotive, which is generally 15 feet above the rails and occasionally over 16 feet. A sleeping-car usually has the highest point of the car about 14 feet above the rails. Box-cars are usually about 8 feet high (above the sills), with a total height of 13 to 14 feet. Some furniture and automobile cars, whose unit live load per cubic foot of space is not high, have a total height of over 15 feet. The *average* length of freight cars, as required in the design of freight yards, is now considered to be 42 feet; the allowance for each car was formerly 40 feet. The P. R. R. standards vary between 38 feet 1 inch and 44 feet 6 inches in length. Day coaches have an extreme length varying from 45 to 80 feet. An 80-foot all-steel coach weighs about 118000 lbs. and has a seating capacity of 88. Allowing the high average weight of 150 lbs., the maximum live load would be 13200 lbs., a little over 11% of the dead load, which shows that the tractive force required to haul the car will be almost constant, whether the car is full or empty. A dining-car may weigh 150000 lbs. and a sleeper even more. The weight of the 25 or 30 passengers it *may* carry is hardly worth considering in comparison.

417. Stresses to which car-frames are subjected. A car is structurally a truss, supported at points at some distance from the ends and subjected to transverse stress. There is,

therefore, a change of flexure at two points between the trucks. Besides this stress the floor is subjected to compression when the cars are suddenly stopped and to tension when in ordinary motion, the tension being greater as the train resistance is geater and as the car is nearer the engine. The shocks, jars, and sudden strains to which the car-frames are subjected are very much harder on them than the mere static strains due to their maximum loads if the loads were quiescent. Consequently any calculations based on the static loads are practically valueless, except as a very rough guide, and previous experience must be relied on in designing car bodies. As evidence of the increasing demand for strength in car-frames, it has been recently observed that freight-cars, built some years ago and built almost entirely of wood, are requiring repairs of wooden parts which have been *crushed* in service, the wood being perfectly sound as regards decay.

418. The use of metal. The use of metal in car construction

FIG. 201.

is very rapidly increasing. The demand for greater strength in car-frames has grown until the wooden framing has become so heavy that it is found possible to make steel frames and trucks at a small additional cost, the steel frames being twice as strong and yet reducing the dead weight of the car about 5000 lbs., a consideration of no small value, especially on roads having heavy grades. Another reason for the increasing use of metal is the great reduction in the price of rolled or pressed

100,000-LB. BOX CAR.

STEEL COAL CAR.

WOODEN BOX CAR; STEEL FRAME.

FIG. 200.—SOME HEAVY FREIGHT CARS.

(*To face page* 456.)

steel, while the cost of wood is possibly higher than before. The advocates of the use of steel advise steel floors, sides, etc. For box-cars a wooden floor has advantages. For ore and coal-cars an all-metal construction has advantages. (Fig. 200.) In Germany, where steel frames have been almost exclusively in use for many years, they have not yet been able to determine the normal age limit of such frames; none have yet *worn* out. The life is estimated at 50 to 80 years

Brake-beams are also best made of metal rather than wood, as was formerly done. Metal brake-beams are generally used on cars having air-brakes, as a wooden beam must be excessively large and heavy in order to have sufficient rigidity.

Truck-frames (see Fig. 201), which were formerly made principally of wood, are now largely made of pressed steel. It makes a reduction in weight of about 3000 lbs. per car. The increased durability is still an uncertain quantity.

419. Draft gear. The enormous increase in the weight and live load capacities of rolling stock have necessitated a corresponding development in draft gear. Even within recent years, "coal-jimmies," carrying a few tons have been made up into trains by dropping a chain of three big links over hooks on the ends of the cars. But the great stresses due to present loadings would tear such hooks from the cars or tear the cars apart if such cars were used in the make-up of long heavy trains as now operated. The next stage in the development of draft gear was the invention of the "spring coupler," by which the energy due to a sudden tensile jerk or the impact of compression may be absorbed by heavy springs and gradually imparted to the car body. Such devices, for which there are many designs, seemed to answer the purpose for cars of 25 to 40 tons capacity. The use of 100,000-pound steel cars soon proved the inadequacy of even spring couplers. The friction-draft gear was then invented. The general principle of such a gear is that, when acting at or near its maximum capacity, it harmlessly transforms into heat the excessive energy developed by jerks or compression. There are several different designs of such gear, but the general principle underlying all of them may be illustrated by a description of the Westinghouse draft gear. The gear employs springs which have sufficient stiffness to act as ordinary spring-couplers for the ordinary pushing and pulling of train operations. Sections of the gear are shown in Fig. 202,

Fig. 202.

while the method of its application to the framing of a car of the pressed steel type is shown in Fig. 203, *a* and *b*. When the draft gear is in tension the coupler, which is rigidly attached to *B*, is drawn to the left, drawing the follower *Z* with it. Compression is then exerted through the gear mechanism to the follower *A* which, being restrained by the shoulders *RR*, against which it presses, causes the gear to absorb the compression. The coil-spring *C* forces the eight wedges *n* against the eight corresponding segments *E*. The great compression of these surfaces against the outer shell produces a friction which retards the compression of the gear. The total possible movement of the gear, as determined by an official test, was 2.42 inches, when the maximum stress was 180,000 pounds. The work done in producing this stress amounted to 18,399 foot-pounds. Of this total energy 16,666 foot-pounds, or over 90%, represents the amount of energy absorbed and dissipated as heat by the frictional gear. The remaining 10% is given back by the recoil. The main release spring *K* is used for returning the segments and friction strips to their normal position after the force to close them has been removed. It also gives additional capacity to the entire mechanism. The auxiliary spring *L* releases the wedge *D*, while the release pin *M* releases the pressure of the auxiliary spring *L* against the wedge during frictional operation. If we omit from the above design the frictional features and consider only the two followers *A* and *Z*, separated by the springs *C* and *K*, acting as one spring, we have the essential elements of a spring-draft gear. In fact, this gear acts exactly like a spring-draft gear for all ordinary service, the frictional device only acting during severe tension and compression.

420. Gauge of wheels and form of wheel-tread.—In Fig. 204 is shown the standard adopted by the Master Car Builders' Association at their twentieth annual convention. Note the normal position of the gauge-line on the wheel-tread. In Fig. 118, § 267, the relation of rail to wheel-tread is shown on a smaller scale. It should be noted that there is no definite position where the wheel-flange is absolutely "chock-a-block" against the rail. As the pressure increases the wheel mounts a little higher on the rail until a point is soon reached when the resistance is too great for it to mount still higher. By this means is avoided the shock of unyielding impact when the car

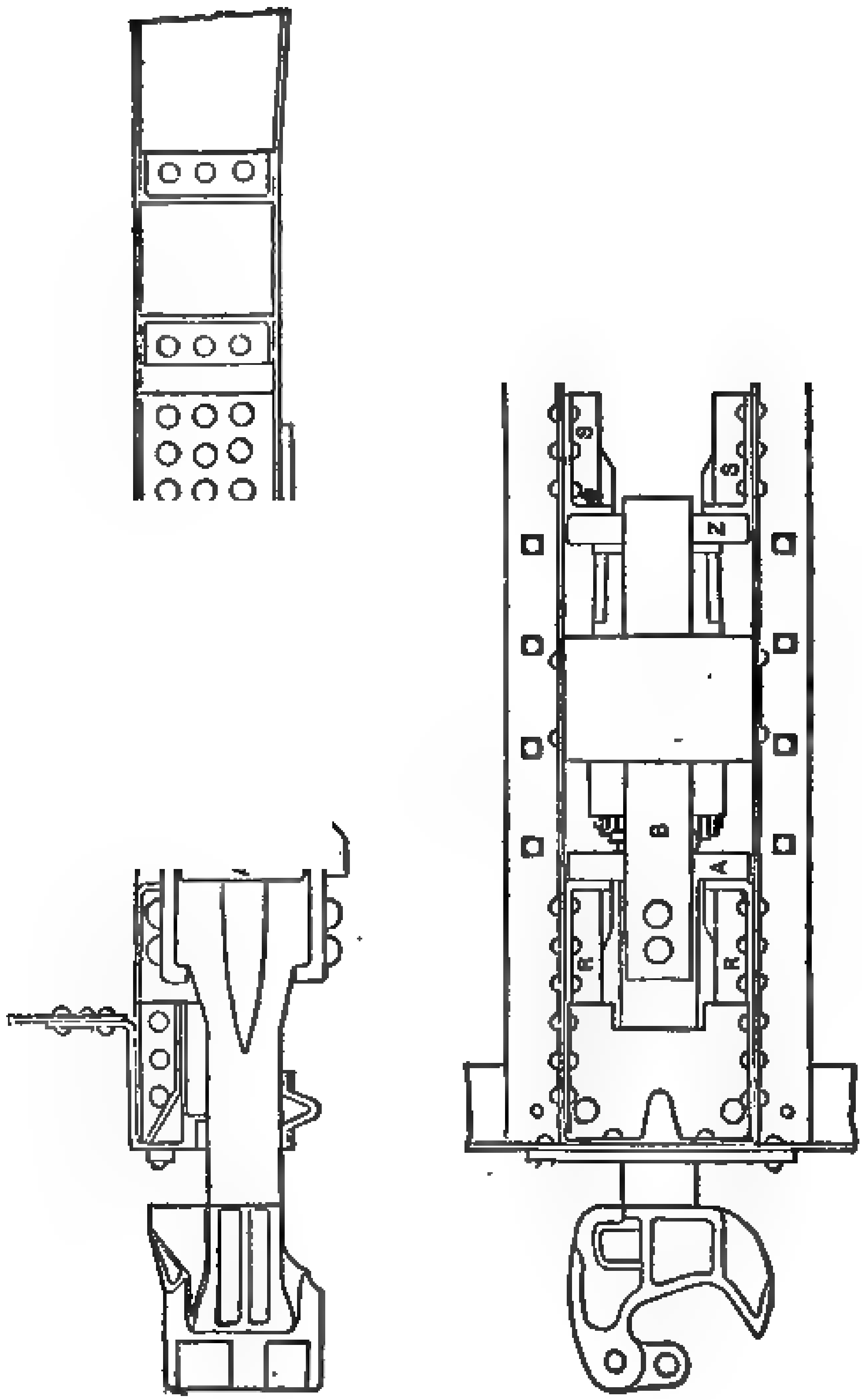


Fig. 203.

sways from side to side. When the gauge between the inner faces of the wheels is greater or less than the limits given in the figure, the interchange rules of the Master Car Builders' Association authorize a road to refuse to accept a car from another road for transportation. At junction points of railroads inspectors are detailed to see that this rule (as well as many others) is complied with in respect to all cars offered for transfer.

TRAIN-BRAKES.

421. Introduction. Owing to the very general misapprehension that exists regarding the nature and intensity of the action of brakes, a complete analysis of the problem is considered justifiable. This misapprehension is illustrated by the common notion (and even practice) that the effectiveness of braking a car is proportional to the brake pressure, and therefore a brakeman is frequently seen using a bar to obtain a greater leverage on the brake-wheel and using his utmost strength to obtain the maximum pull on the brake-chain while the car is skidding along with locked wheels.

When a vehicle is moving on a track with a considerable velocity, the mass of the vehicle possesses kinetic energy of translation and the wheels possess kinetic energy of rotation. To stop the vehicle, this energy must be destroyed. The rotary kinetic energy will vary from about 4 to 8% of the kinetic energy of translation, according to the car loading (see § 435). On steam railroads brake action is obtained by pressing brake-shoes against car-wheel treads. As the brake-shoe pressure increases, the brake-shoes retard with increasing force the rotary action of the wheels. As long as the wheels do not slip or "skid" on the rails, the adhesion of the rails forces them to rotate with a circumferential velocity equal to the train velocity. The retarding action of the brake-shoe checks first the rotative kinetic energy (which is small), and the remainder develops a *tendency* for the wheel to slip on the rail. Since the rotative kinetic energy is such a small percentage of the total, it will hereafter be ignored, except as specifically stated, and it will be assumed for simplicity that the only work of the brakes is to overcome the kinetic energy of translation. The possible effect of grade in assisting or preventing retardation, and the effect of all other track resist-

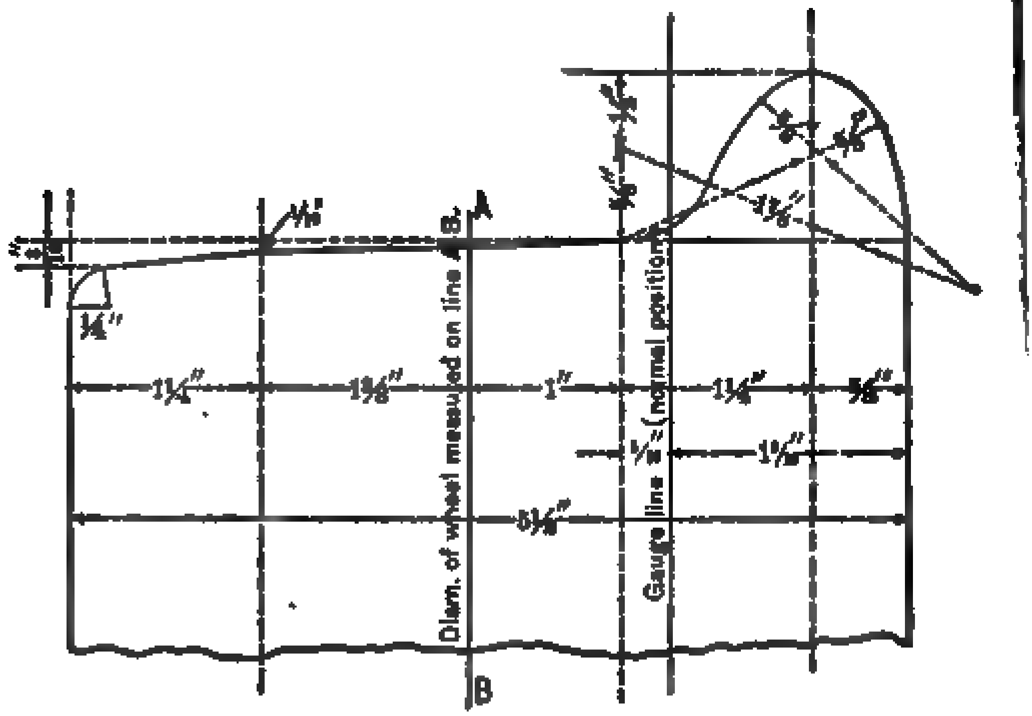


FIG. 204.—M. C. B. STANDARD WHEEL-TREAD AND AXLE.

ances, is also ignored. The amount of the developed force which retards the train movement is limited to the possible adhesion or *static* friction between the wheel and the rail. When the friction between the brake-shoe and the wheel exceeds the adhesion between the wheel and the rail, the wheel skids, and then the friction between the wheel and the rail at once drops to a much less quantity. It must therefore be remembered at the outset that the retarding action of brake-shoes on wheels as a means of stopping a train is absolutely limited by the possible static friction between the braked wheels and the rails.

422. Laws of friction as applied to this problem. Much of the misapprehension regarding this problem arises from a very common and widespread misstatement of the general laws of friction. It is frequently stated that friction is independent of the velocity and of the unit of pressure. The first of these so-called laws is not even approximately true. A very exhaustive series of tests were made by Capt. Douglas Galton on the Brighton Railway in England in 1878 and 1879, and by M. George Marié on the Paris and Lyons Railway in 1879, with trains which were specially fitted with train-brakes and with dynagraphs of various kinds to measure the action of the brakes. Experience proved that variations in the condition of the rails (wet or dry), and numerous irregularities incident to measuring the forces acting on a heavy body moving with a high velocity, were such as to give somewhat discordant results, even when the conditions were made as nearly identical as possible. But the tests were carried so far and so persistently that the general laws stated below were demonstrated beyond question, and even the numerical constants were determined as closely as they may be practically utilized. These laws may be briefly stated as follows:

(*a*) The coefficient of friction between cast-iron brake-blocks and steel tires is about .3 when the wheels are "just moving"; it drops to about .16 when the velocity is about 30 miles per hour, and is less than .10 when the velocity is 60 miles per hour. These figures fluctuate considerably with the condition of the rails, wet or dry.

(*b*) The coefficient of friction is greatest when the brakes are first applied; it then reduces very rapidly, decreasing nearly one third after the brakes have been applied 10 seconds,

and dropping to nearly one half in the course of 20 seconds. Although the general truth of this law was established beyond question, the tests to demonstrate the law of the variation of friction with time of application were too few to determine accurately the numerical constants.

(*c*) The friction of skidded wheels on rails is always very much less than the adhesion when the wheel is rolling on the rail—sometimes less than one third as much.

(*d*) An analysis of the tests all pointed to a law that the friction developed does *not* increase as rapidly as the *intensity* of pressure increases, but this may hardly be considered as an established law.

(*e*) The adhesion between the wheel and the rail appears to be independent of velocity. The adhesion here means the force that must be developed before the wheel will slip on the rail.

The practical effect of these laws is shown by the following observed phenomena:

(*a*) When the brakes are first applied (the velocity being very high), a brake pressure far in excess of the weight on the wheel (even three or four times as much) may be applied without skidding the wheel. This is partly due to the fact that the wheel has a very high rotative kinetic energy (which varies as the square of the velocity, and which must be overcome first), but it is chiefly due to the fact that the coefficient of friction at the higher velocity is very small (at 60 miles per hour it is about .07), while the adhesion between the wheel and the rail is independent of the velocity.

(*b*) As the velocity decreases the brake pressure must be decreased or the wheels will skid. Although the friction decreases with the time required to stop and increases with the reduction of speed, and these two effects tend to neutralize each other, yet unless the stop is very slow, the increase in friction due to reduction of speed is much greater than the decrease due to time, and therefore the brake pressure must not be greater than the weight on the wheel, unless momentarily while the speed is still very high.

(*c*) The adhesion between wheels and rails varies from .20 to .25 and over when the rail is dry. When wet and slippery it may fall to .18 or even .15. The use of sand will always raise it above .20, and on a dry rail, when the sand is not blown away by wind, it may raise it to .35 or even .40.

(*d*) Experiments were made with an automatic valve by which the brake-shoe pressure against the wheel should be reduced as the friction increased, but since (1) the essential requirement is that the friction produced by the brake-shoes shall not exceed the adhesion between rail and wheel, and since (2) the rail-wheel adhesion is a very variable quantity, depending on whether the rail is wet or dry, it has been found impracticable to use such a valve, and that the best plan is to leave it to the engineer to vary the pressure, if necessary, by the use of the brake-valve.

MECHANISM OF BRAKES.

423. **Hand-brakes.** The old style of brakes consists of brake-shoes of some type which are pressed against the wheel-treads by means of a brake-beam, which is operated by means of a hand-windlass and chain operating a set of levers. It is desirable that brakes shall not be set so tightly that the wheels shall be locked, and then slide over the track, producing flat places on them, which are very destructive to the rolling-stock and track afterward, on account of the impact occasioned at each revolution. With air-brakes the maximum pressure of the brake-shoes can be quite carefully regulated, and they are so designed that the maximum pressure exerted by any pair of brake-shoes on the wheels of any axle shall not exceed a certain per cent. of the weight carried by that axle when the car is *empty*, 90% being the figure usually adopted for passenger-cars and 70% for freight-cars. Consider the case of a freight-car of 100000 lbs. capacity, weighing 33100 lbs., or 8275 lbs. on an axle, and equipped with a hand-brake which operates the levers and brake-beams, which are sketched in Fig. 205. The dead weight on an axle is 8275 lbs.; 70% of this is 5792 lbs., which is the maximum allowable pressure per brake-beam, or 2896 lbs. per brake-shoe. With the dimensions shown, such a pressure will be produced by a pull of about 1158 lbs. on the brake-chain. The power gained by the brake-wheel is not equal to the ratio of the brake-wheel diameter to the diameter of the shaft, about which the brake-chain winds, which is about 16 to 1½. The ratio of the circumference of the brake-wheel to the length of chain wound up by one complete turn would be a closer figure. The loss of effi-

ciency in such a clumsy mechanism also reduces the effective ratio. Assuming the *effective* ratio as 6:1 it would require a pull of 193 lbs. at the circumference of the brake-wheel to exert 1158 lbs. pull on the brake-chain, or 5792 lbs. pressure on the wheels at B, and even this will not lock the wheels when the car is empty, much less when it is loaded. Note that the pressures at A and B are unequal. This is somewhat objectionable, but it is unavoidable with this simple form of brake-beam. More complicated forms to avoid this are sometimes used. Hand-brakes are, of course, cheapest in first cost, and even with the best of automatic brakes, additional mechanism to operate the brakes by hand in an emergency is always provided, but their slow operation when a quick stop is desired makes it exceedingly dangerous to attempt to run a train at high speed unless some automatic brake directly under the control of the engineer is at hand. The great increase in the

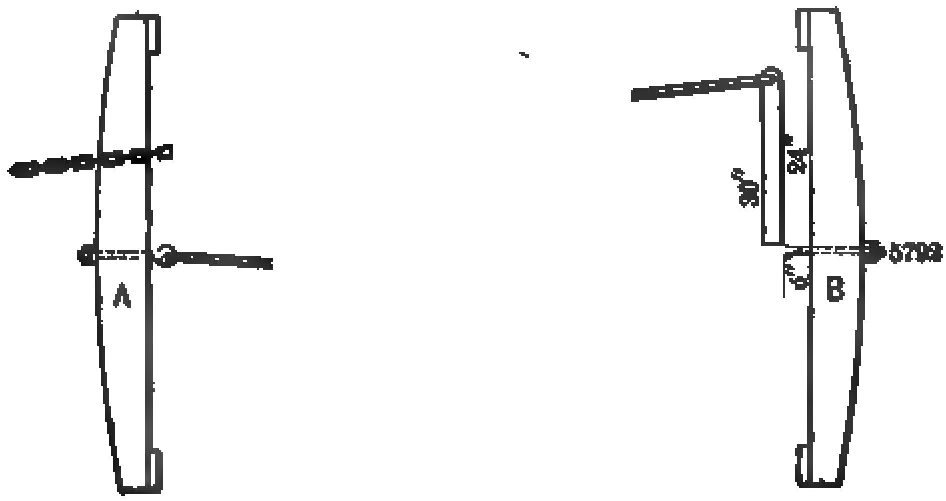


Fig. 205.—Sketch of Mechanism of Hand-brake.

average velocity of trains during recent years has only been rendered possible by the invention of automatic brakes.

424. "Straight" air-brakes. The essential constructive features of this form of brake are (1) an air-pump on the engine, operated by steam, which compresses air into a reservoir on the engine; (2) a "brake-pipe" running from the reservoir to the rear of the engine and pipes running under each car, the pipes having flexible connections at the ends of the cars and engine; (3) a cylinder and piston under each car which

operates the brakes by a system of levers, the cylinder being connected to the brake-pipe. The reservoir on the engine holds compressed air at about 45 lbs. pressure. To operate the brakes, a valve on the engine is opened which allows the compressed air to flow from the reservoir through the brake-pipe to each cylinder, moving the piston, which thereby moves the levers and applies the brakes. The *defects* of this system are many: (1) With a long train, considerable time is required for the air to flow from the reservoir on the engine to the rear cars, and for an emergency-stop even this delay would often be fatal; (2) if the train breaks in two, the rear portion is not provided with power for operating the brakes, and a dangerous collision would often be the result; (3) if an air-pipe coupling bursts under any car, the whole system becomes absolutely helpless, and as such a thing might happen during some emergency, the accident would then be especially fatal.

This form of brake has almost, if not entirely, passed out of use. It is here briefly described in order to show the logical development of the form which is now in almost universal use, the automatic.

425. Automatic air-brakes. The above defects have been overcome by a method which may be briefly stated as follows: A reservoir for compressed air is placed under each car and the tender; whenever the pressure in these reservoirs is reduced for any reason, it is automatically replenished from the main reservoir on the engine; whenever the pressure in the brake-pipe is reduced for any cause (opening a valve at any point of its length, parting of the train, or bursting of a pipe or coupler), valves are automatically moved under each car to operate the piston and put on the brakes. *All* the brakes on the train are thus applied almost simultaneously. If the train breaks in two, *both* sections will at once have *all* the brakes applied automatically; if a coupling or pipe bursts, the brakes are at once applied and attention is thereby attracted to the defect; if an emergency should arise, such that the conductor desires to stop the train instantly without even taking time to signal to the engineer, he can do so by opening a valve placed on each car, which admits air to the train-pipe, which will set the brakes on the whole train, and the engineer, being able to discover instantly what had occurred, would shut off steam and do whatever else was necessary to stop the train as quickly as pos-

sible. The most important and essential detail of this system is the "automatic triple valve" placed under each car. Quoting from the Westinghouse Air-brake Company's Instruction Book, "A moderate reduction of air pressure in the train-pipe causes the greater pressure remaining stored in the auxiliary reservoir to force the piston of the triple valve and its slide-valve to a position which will allow the air in the auxiliary reservoir to pass directly into the brake-cylinder and apply the brake. A sudden or violent reduction of the air in the train-pipe produces the same effect, and in addition causes supplemental valves in the triple valve to be opened, permitting the pressure from the train-pipe to also enter the brake-cylinder, augmenting the pressure derived from the auxiliary reservoir about 20%, producing practically instantaneous action of the brakes to their highest efficiency throughout the entire train. When the pressure in the brake-pipe is again restored to an amount in excess of that remaining in the auxiliary reservoir, the piston- and slide-valves are forced in the opposite direction to their normal position, opening communication from the train-pipe to the auxiliary reservoir, and permitting the air in the brake-cylinder to escape to the atmosphere, thus releasing the brakes. If the engineer wishes to apply the brake, he moves the handle of the engineer's brake-valve to the right, which first closes a port, retaining the pressure in the main reservoir, and then permits a portion of the air in the train-pipe to escape. To release the brakes, he moves the handle to the extreme left, which allows the air in the main reservoir to flow freely into the brake-pipe, restoring the pressure therein."

426. Tests to measure the efficiency of brakes. Let v represent the velocity of a train in feet per second; W, its weight; F, the retarding force due to the brakes; d, the distance in feet required to make a stop; and g, the acceleration of gravity (32.16 feet per square second); then the kinetic energy possessed by the train (disregarding for the present the rotative kinetic energy of the wheels) $=\frac{Wv^2}{2g}$. The work done in stopping the train $=Fd$. $\therefore Fd=\frac{Wv^2}{2g}$. The ratio of the retarding force to the weight,

$$\frac{F}{W}=\frac{v^2}{2gd}=.0155\frac{v^2}{d}.$$

In order to compare tests made under varying conditions, the ratio $F \div W$ should be corrected for the effect of grade ($+$ or $-$), if any, and also for the proportion of the weight of the train which is on *braked* wheels. For example, a train weighed 146076 lbs., the proportion on braked wheels was 67%, speed 60 feet per second, length of stop 450 feet, track level. Substituting these values in the above formula, we find $(F \div W) = .124$. This value is really unduly favorable, since the ordinary track resistance helps to stop the train. This has a value of from 6 to 20 lbs. per ton, averaging say 10 lbs. per ton during the stop, or .005 of the weight. Since the effect of this is small and is nearly constant for all trains, it may be ignored in comparative tests. The grade in this case was level, and therefore grade had no effect. But since only 67% of the weight was on braked wheels, the ratio, on the basis of *all* the wheels braked, or of the weight reduced to that actually on the braked wheels, is $0.124 \div .67 = 0.185$. This was called a "good" stop, although as high a ratio as 0.200 has been obtained.

427. **Brake-shoes.** Brake-shoes were formerly made of wrought iron, but when it was discovered that cast-iron shoes would answer the purpose, the use of wrought-iron shoes was abandoned, since the cast-iron shoes are so much cheaper. A cheap practice is to form the brake-shoe and its head in one piece, which is cheaper in first cost, but when the wearing-surface is too far gone for further use, the whole casting must be renewed. The "Christie" shoe, adopted by the Master Car Builders' Association as standard, has a separate shoe which is fastened to the head by means of a wrought-iron key. The shoe is beveled $\frac{1}{4}''$ in a width of $3\frac{3}{8}''$ to fit the coned wheel. This is a greater bevel than the standard coning of a car-wheel. It is perhaps done to allow for some bending of the brake-beam and also so that the maximum pressure (and wear) should come on the outside of the tread, rather than next to the flange, where it might tend to produce sharp flanges. By concentrating the brake-shoe wear on the outer side of the tread, the wear on the tread is more nearly equalized, since the rail wears the wheel-tread chiefly near the flange. This same idea is developed still further in the "flange-shoes," which have a curved form to fit the wheel-flange and which bear on the wheel on the flange and on the outside of the tread. It is

claimed that by this means the standard form of the tread is better preserved than when the wear is entirely on the tread. The Congdon brake-shoe is one of a type in which wrought-iron pieces are inserted in the face of a cast-iron shoe. It is claimed that these increase the life of the shoe.

CHAPTER XVI.

TRAIN RESISTANCE.

428. Classification of the various forms. The various resistances which must be overcome by the power of the locomotive may be classified as follows:

(*a*) *Resistances internal to the locomotive*, which include friction of the valve-gear, piston- and connecting-rods, journal friction of the drivers; also all the loss due to radiation, condensation, friction of the steam in the passages, etc. In short, these resistances are the sum-total of the losses by which the power at the circumference of the drivers is less than the power developed by the boiler.

(*b*) *Velocity resistances*, which include the atmospheric resistances on the ends and sides; oscillation and concussion resistances, due to uneven track, etc.

(*c*) *Wheel resistances*, which include the rolling friction between the wheels and the rails of *all* the wheels (including the drivers); also the journal friction of all the axles, except those of the drivers.

(*d*) *Grade and curve resistances*, which include those resistances which are due to grade and to curves, and which are not found on a straight and level track.

(*e*) *Brake resistances.* As shown later, brakes consume power and to the extent of their use increase the energy to be developed by the locomotive.

(*f*) *Inertia resistances.* The resistance due to inertia is not generally considered as a train resistance because the energy which is stored up in the train as kinetic energy may be utilized in overcoming future resistances. But in a discussion of the demands on the tractive power of the engine, one of the chief items is the energy required to *rapidly* give to a starting train its normal velocity. This is especially true of suburban trains, which must acquire speed very quickly in order that

their general average speed between termini may be even reasonably fast.

429. Resistance internal to the locomotive. These are resistances which do not tax the adhesion of the drivers to the rails, and hence are frequently considered as not being a part of the train resistance properly so called. If the engine were considered as lifted from the rails and made to drive a belt placed around the drivers, then all the power that reached the belt would be the power that is ordinarily available for adhesion, while the remainder would be that consumed internally by the engine. The power developed by an engine may be obtained by taking indicator diagrams which show the actual steam pressure in a cylinder at any part of a stroke. From such a diagram the average steam pressure is easily obtained, and this average pressure, multiplied by the length of the stroke and by the net area of the piston, gives the energy developed by one half-stroke of one piston. Four times this product divided by 550 times the time in seconds required for one stroke gives the "indicated horse-power" Even this calculation gives merely the power behind the piston, which is several per cent. greater than the power which reaches the circumference of the drivers, owing to the friction of the piston, piston-rod, cross-head, connecting-rod bearings, and driving-wheel journals. (See § 411, Chapter XV.) By measuring the amount of water used and turned into steam, and by noting the boiler pressure, the energy possessed by the steam used is readily computed. The indicator diagrams will show the amount of steam that has been effective in producing power at the cylinders. The steam accounted for by the diagrams will ordinarily amount to 80 or 85% of the steam developed by the boiler, and the other 15 or 20% represents the loss of energy due to radiation, condensation, etc.

Locomotive resistance has been estimated and tabulated by a Committee of the Amer. Rwy. Eng. Assoc. and the results are given in Table XXIX, which is taken from the Manual of that Association. As a numerical illustration, what is the computed resistance for a Mikado locomotive of which the total weight of engine and tender is 315,000 lbs. of which 153,200 lbs. is carried on the drivers, at a velocity of 6 miles per hour? In this case, Item $A = (18.7 \times 76.6) + (80 \times 4) = 1432$ lbs. The weight carried on the engine and tender trucks $= 315{,}000 - 153{,}200 = 161{,}800$

=80.9 tons. Item B=(2.6×80.9)+(20×6)=330 lbs. Item C is comparatively insignificant at this low velocity. From the table, we read 9 lbs. Then the sum of A, B, and C=1771 lbs., which must be subtracted from a computed tractive effort to obtain the estimated draw-bar pull.

TABLE XXIX. LOCOMOTIVE RESISTANCES.

Total Locomotive Resistance $=A+B+C$, in which

A = resistance between cylinder and rim of drivers, and in pounds $=18.7T+80N$

in which T = tons weight on drivers, and N = number of driving axles;

B = resistance of engine and tender trucks, and in pounds $=2.6T+20N$

in which T = tons weight on engine and tender trucks and N = number of truck axles;

C = head end or "air" resistance, and in pounds $=.002V^2A$

in which V = velocity in miles per hour, and A = end area of locomotive.

On the basis that the end area averages 125 square feet, the formula becomes $C=0.25V^2$. The number of pounds air resistance for various velocities is as given below.

Vel.	Res.	Vel.	Res.	Vel.	Res.	Vel.	Res.	Vel.	Res.	Vel.	Res.
1	0.25	8	16.00	15	56	22	121	29	210	36	324
2	1.00	9	20.25	16	64	23	132	30	225	37	342
3	2.25	10	25.00	17	72	24	144	31	240	38	361
4	4.00	11	30	18	81	25	156	32	256	39	380
5	6.25	12	36	19	90	26	169	33	272	40	400
6	9.00	13	42	20	100	27	182	34	289	50	625
7	12.25	14	49	21	110	28	196	35	306	60	900

Draw-bar pull on level tangent equals the cylinder tractive power less the sum of the engine resistances.

At low speeds, the adhesion of the drivers should be considered and available draw-bar pull should never be estimated greater than 30% of weight on drivers at starting with use of sand, 25% of weight on drivers at running speeds.

Taken from Table 7 in "Economics" section of Manual of the Amer. Rwy. Eng. Assoc., 1915 edition.

430. Velocity resistance. (*a*) *Atmospheric.* This consists of the head and tail resistances and the side resistance. The head

and tail resistances are nearly constant for all trains of given velocity, varying but slightly with the varying cross-sections of engines and cars. The side resistance varies with the length of the train and the character of the cars, box-cars or flats, etc. Vestibuling cars has a considerable effect in reducing this side resistance by preventing much of the eddying of air-currents between the cars, although this is one of the least of the advantages of vestibuling. Atmospheric resistance is generally assumed to vary as the square of the velocity, and although this may be nearly true, it has been experimentally demonstrated to be at least inaccurate. Values for head resistance are given in Table XXIX, which are probably accurate enough for all practical purposes, especially at ordinary freight train velocities. A freight-train composed partly of flat-cars and partly of box-cars will encounter considerably more atmospheric resistance than one made exclusively of either kind, other things being equal. The definite information on this subject is very unsatisfactory, but this is possibly due to the fact that it is of little practical importance to know just how much such resistance amounts to.

(*b*) *Oscillatory and concussive.* These resistances are considered to vary as the square of the velocity. Probably this is nearly, if not quite, correct on the general principle that such resistances are a succession of impacts and the force of impacts varies as the square of the velocity. These impacts are due to the defects of the track, and even though it were possible to make a precise determination of the amount of this resistance in any particular case, the value obtained would only be true for that particular piece of track and for the particular degree of excellence or defect which the track *then* possessed. The general improvement of track maintenance during late years has had a large influence in increasing the possible train-load by decreasing the train resistance. The expenditure of money to improve track will give a road a large advantage over a competing road with a poorer track, by reducing train resistance, and thus reducing the cost of handling traffic.

431. **Wheel resistances.** (*a*) *Rolling friction of the wheels.* To determine experimentally the rolling friction of wheels, apart from all journal friction, is a very difficult matter and has never been satisfactorily accomplished. Theory as well as practice shows that the higher and the more perfect the

elasticity of the wheel and the surface, the less will be the rolling friction. But the determination, if made, would be of theoretical interest only.

The combined effect of rolling friction and journal friction is determinable with comparative ease. From the nature of the case no great reduction of the rolling friction by any device is possible. It is only a very insignificant part of the total train resistance.

(*b*) *Journal friction of the axles.* This form of resistance has been studied quite extensively by means of the measurement of the force required to turn an axle in its bearings under various conditions of pressure, speed, extent of lubrication, and temperature. The following laws have been fairly well established: (1) The coefficient of friction increases as the pressure diminishes; (2) it is higher at very slow speeds, gradually diminishing to a minimum at a speed corresponding to a train velocity of about 10 miles per hour, then slowly increasing with the speed; it is very dependent on the perfection of the lubrication, it being reduced to one sixth or one tenth, when the axle is lubricated by a bath of oil rather than by a mere pad or wad of waste on one side of the journal; (3) it is much lower at higher temperature, and *vice versa.* The practical effect of these laws is shown by the observed facts that (1) loaded cars have a less resistance per ton than unloaded cars, the figures being, for speeds of about 10 miles per hour, approximately:

For passenger- and loaded freight-cars..	4	lbs.	per	ton
" empty freight-cars...............	8	"	"	"
" street-cars.......................	10	"	"	"
" freight-trucks without load........	14	"	"	"

(2) When starting a train, the resistances are about 20 lbs. per ton, notwithstanding the fact that the velocity resistances are practically zero; at about 2 miles per hour it will drop to 10 lbs. per ton and above 10 miles per hour it may drop to 4 lbs. per ton if the cars are in good condition. (3) The resistance could probably be materially lowered if some practicable form of journal-box could be devised which would give a more perfect lubrication. (4) It is observed that freight-train loads must be cut down in winter by about 10 or 15% of the loads that the same engine can haul over the same track in summer. This is due partly to the extra roughness and inelasticity of the

track in winter, and partly to increased radiation from the engine wasting some energy, but this will not account for all of the loss, and the effect, which is probably due largely to the lower temperature of the journal-boxes, is very marked and costly. It has been suggested that a jacketing of the journal-boxes, which would prevent rapid radiation of heat and enable them to retain some of the heat developed by friction, would result in a saving amply repaying the cost of the device.

Roller journals for cars have been frequently suggested, and experiments have been made with them. It is found that they are very effective at low velocities, greatly reducing the starting resistance, which is very high with the ordinary forms of journals. But the advantages disappear as the velocity increases. The advantages also decrease as the load is increased, so that with heavily loaded cars the gain is small. The excess of cost for construction and maintenance has been found to be more than the gain from power saved.

432. Grade resistance. The amount of this may be computed with mathematical exactness. Assume that the ball or cylinder (see Fig. 206) is being drawn up the plane. If W

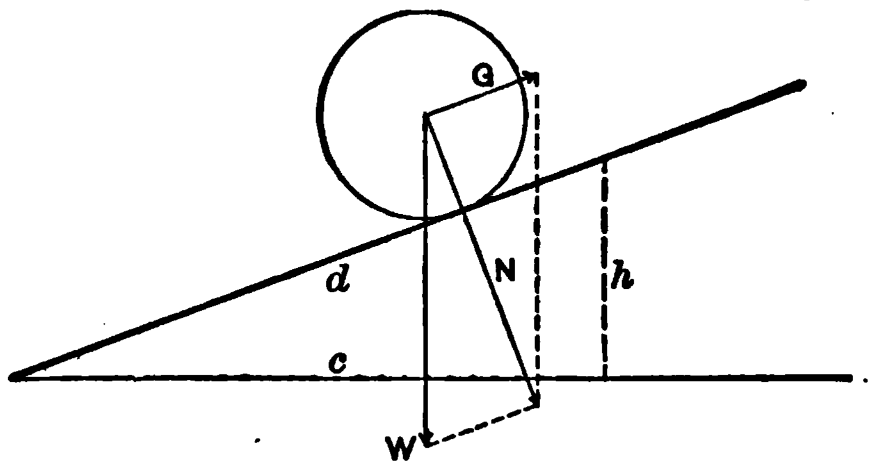


FIG. 206.

is the weight, N the normal pressure against the rail, and G the force required to hold it or to draw it up the plane with uniform velocity, the rolling resistances being considered zero or considered as provided for by other forces, then

$$G:W::h:d, \quad \text{or} \quad G=\frac{Wh}{d};$$

but for all ordinary railroad grades, $d=c$ to within a tenth of 1%, i.e., $G=\frac{Wh}{c}=W\times$ rate of grade. In order that the student may appreciate the exact amount of this approximation the percentage of slope distance to its horizontal projection is given in the following tabular form:

Grade in per cent.	1	2	3	4	5
$\frac{\text{Slope dist.}}{\text{hor. dist.}} \times 100$........	100.005	100.020	100.045	100.080	100.125

Grade in per cent.	6	7	8	9	10
$\frac{\text{Slope dist.}}{\text{hor. dist.}} \times 100$........	100.180	100.245	100.319	100.404	100.499

This shows also the error on various grades of measuring with the tape on the ground rather than held horizontally. Since almost all railroad grades are less than 2% (where the error is but .02 of 1%), and anything in excess of 4% is unheard of for normal construction, the error in the approximation is generally too small for practical consideration.

If the rate of grade is 1 : 100, $G = W \times \frac{1}{100}$, i.e., $G = 20$ lbs. per ton; ∴ for *any* per cent. of grade, $G = (20 \times$ per cent. of grade) pounds per ton. When moving up a grade this force G is to be overcome in addition to all the other resistances. When moving down a grade, the force G assists the motion and may be more than sufficient to move the train at its highest allowable velocity. The force required to move a train on a level track at ordinary freight-train speeds (say 20 miles per hour) is about 7 lbs. per ton. A down grade of $\frac{7}{20}$ of 1% will furnish the same power; therefore on a down grade of 0.35%, a freight-train would move indefinitely at about 20 miles per hour. If the grade were higher and the train were allowed to gain speed freely, the speed would increase until the resistance at that speed would equal W times the rate of grade, when the velocity would become uniform and remain so as long as the conditions were constant. If this speed was higher than a safe permissible speed, brakes must be applied and power wasted. The fact that one terminal of a road is considerably higher than the other does not necessarily imply that the extra power needed to overcome the difference of elevation is a total waste of energy, especially if the maximum grades are so low that brakes will never need to be applied to reduce a dangerously high velocity. for although more power must be

used in ascending the grades, there is a considerable saving of power in descending the grades. The amount of this saving will be discussed more fully in Chapter XXIII.

433. Curve resistance. Some of the principal laws will be here given without elaboration. A more detailed discussion will be given in Chapter XXII.

(*a*) While the total curve resistance increases as the degree of curve increases, the resistance *per degree of curve* is much greater for easy curves than for sharp curves; *e.g.*, the resistance on the excessively sharp curves (radius 90 feet) of the elevated roads of New York City is very much less *per degree of curve* than that on curves of 1° to 5°. (*b*) Curve resistance increases with the velocity. (*c*) The total resistance on a curve depends on the central angle rather than on the radius; i.e., two curves of the same central angle but of different radius would cause about the same total curve resistance. This is partly explained by the fact that the longitudinal slipping will be the same in each case. (See § 395, Chapter XV.) In each case also the trucks must be twisted around and the wheels slipped laterally on the rails by the same amount $\Delta°$. (See § 396, Chapter XV.)

434. Brake resistances. If a down grade is excessively steep so that brakes must be applied to prevent the train acquiring a dangerous velocity, the energy consumed is hopelessly lost without any compensation. When trains are required to make frequent stops and yet maintain a high average speed, considerable power is consumed by the application of brakes in stopping. All the energy which is thus turned into heat is hopelessly lost, and in addition a very considerable amount of steam is drawn from the boiler to operate the air-brakes, which consume the power already developed. It can be easily demonstrated that engines drawing trains in suburban service, making frequent stops, and yet developing high speed between stops, will consume a very large proportion of the total power developed by the use of brakes. Note the double loss. The brakes consume power already developed and stored in the train as kinetic or potential energy, while the operation of the brakes requires additional steam power from the engine.

435. Inertia resistance. The two forms of train resistance which under some circumstances are the greatest resistances to be overcome by the engine are the grade and inertia resist-

ances, and fortunately both of these resistances may be computed with mathematical precision. The problem may be stated as follows: What constant force P (in addition to the forces required to overcome the various frictional resistances, etc.) will be required to impart to a body a velocity of v feet per second in a distance of s feet? The required number of foot-pounds of energy is evidently Ps. But this work imparts a kinetic energy which may be expressed by $\frac{Wv^2}{2g}$. Equating these values, we have $Ps=\frac{Wv^2}{2g}$, or

$$P=\frac{Wv^2}{2gs}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (104)$$

The force required to increase the velocity from v_1 to v_2 may likewise be stated as $P=\frac{W}{2gs}(v_2^2-v_1^2)$. Substituting in the formula the values $W=2000$ lbs. (one ton), $g=32.16$, and $s=5280$ feet (one mile), we have

$$P=.00588(v_2^2-v_1^2).$$

Multiplying by $(5280\div3600)^2$ to change the unit of velocity to miles per hour, we have

$$P=.01267(V_2^2-V_1^2).$$

But this formula must be modified on account of the rotative kinetic energy which must be imparted to the wheels of the cars. The precise additional percentage depends on the particular design of the cars and their loading and also on the design of the locomotive. Consider as an example a box-car, 60000 lbs. capacity, weighing 33000 lbs. The wheels have a diameter of 36″ and their radius of gyration is about 13″. Each wheel weighs 700 lbs. The rotative kinetic energy of each wheel is 4877 ft.-lbs. when the velocity is 20 miles per hour, and for the eight wheels it is 39016 ft.-lbs. For greater precision (really needless) we may add 192 ft.-lbs. as the rotative kinetic energy of the axles. When the car is fully loaded (weight 93000 lbs.) the kinetic energy of translation is 1,244,340 ft.-lbs.; when empty (weight 33000 lbs.) the energy is 441540 ft.-lbs. The rotative kinetic energy thus adds (for this particular car) 3.15% (when the car is loaded) and 8.9% (when the car is empty) to the kinetic energy of translation. The kinetic

energy which is similarly added, owing to the rotation of the wheels and axles of the locomotive, might be similarly computed. For one type of locomotive it has been figured at about 8%. The variations in design, and particularly the fluctuations of loading, render useless any great precision in these computations. For a train of "empties" the figure would be high, probably 8 to 9%; for a fully loaded train it will not much exceed 3%. Wellington considered that 6% is a good average value to use (actually used 6.14% for "ease of computation"), but considering (*a*) the increasing proportion of live load to dead load in modern car design, (*b*) the greater care now used to make up *full* train-loads, and (*c*) the fact that *full* train-loads are the critical loads, it would appear that 5% is a better average for the conditions of modern practice. Even this figure allows something for the higher percentage for the locomotive and something for a few empties in the train. Therefore, adding 5% to the coefficient in the above equation, we have the true equation

$$P \doteq .0133(V_2^2 - V_1^2), \quad . \quad . \quad . \quad . \quad . \quad (105)$$

in which V_2 and V_1 are the higher and lower velocities respectively, in *miles per hour*, and P is the force required *per ton* to impart that difference of velocity in a distance of *one mile*. If more convenient, the formula may be used thus:

$$P_1 = \frac{70}{s}(V_2^2 - V_1^2), \quad . \quad . \quad . \quad . \quad . \quad (106)$$

in which s is the distance in feet and P_1 is the corresponding force.

As a numerical illustration, the force required per ton to impart a kinetic energy due to a velocity of 20 miles per hour in a distance of 1000 feet will equal

$$P_1 = \frac{70(400-0)}{1000} = 28 \text{ lbs.},$$

which is the equivalent (see § 432) of a 1.4% grade. Since the velocity enters the formula as V^2, while the distance enters only in the first power, it follows that it will require *four* times

the force to produce twice the velocity in the same distance, or that with the *same* force it will require four times the distance to attain twice the velocity.

As another numerical illustration, if a train is to increase its speed from 15 miles per hour to 60 miles per hour in a distance of 2000 feet, the force required (in addition to all the other resistances) will be

$$P_1 = \frac{70.224(3600-225)}{2000} = 118.50 \text{ lbs. per ton.}$$

This is equivalent to a 5.9% grade and shows at once that it would be impossible unless there were a very heavy down grade, or that the train was very light and the engine very powerful.

436. **Dynamometer tests.** These are made by putting a "dynamometer-car" between the engine and the cars to be tested. Suitable mechanism makes an automatic record of the force which is transmitted through the dynamometer at any instant, and also a record of the velocity at any instant. One of the practical difficulties is the accurate determination of the velocity at any instant when the velocity is fluctuating. When the velocity is decreasing, the kinetic energy of the train is being turned into work and the force transmitted through the dynamometer is less than the amount of the resistance which is actually being overcome. On the other hand, when the velocity is increasing, the dynamometer indicates a larger force than that required to overcome the resistances, but the excess force is being stored up in the train as kinetic energy. Grade has a similar effect, and the force indicated by the dynamometer may be greater or less than that required at the given velocity on a level by the force which is derived from, or is turned into, potential energy. Therefore the resistance indicated by the dynamometer of a train will not be that on a level track at uniform velocity, unless the track is actually level and the velocity really uniform.

Dynamometer tests under other circumstances are therefore of no value unless it is possible to determine the true velocity at any instant and its rate of change, and also to determine the grade. Of course, the grade is easily found. An allowance for an increase or decrease of kinetic or potential energy must therefore be made before it is possible to

know how much force is being spent on the ordinary resistances.

437. Gravity or "drop" tests. Dynamometer tests require the use of a dynamometer which is capable of measuring a force of several thousands of pounds, and which therefore cannot determine such values with a close percentage of accuracy, especially if the force is small. A drop test utilizes the force of gravity which may be measured with mathematical accuracy. The general method is to select a stretch of track which has a uniform grade of about 0.7% and which is preferably straight for two or three miles. On such a grade cars with running gear in good condition may be started by a push. The velocity will gradually increase until at some velocity, depending on the resistances encountered, the cars will move uniformly. The only work requiring extreme care with this method is the determination of the velocity. If the velocity is fluctuating, as it is during the time when it is of the greatest importance to know the velocity, it is not sufficient to determine the time required to run some long measured distance, for the average velocity thus obtained would probably differ

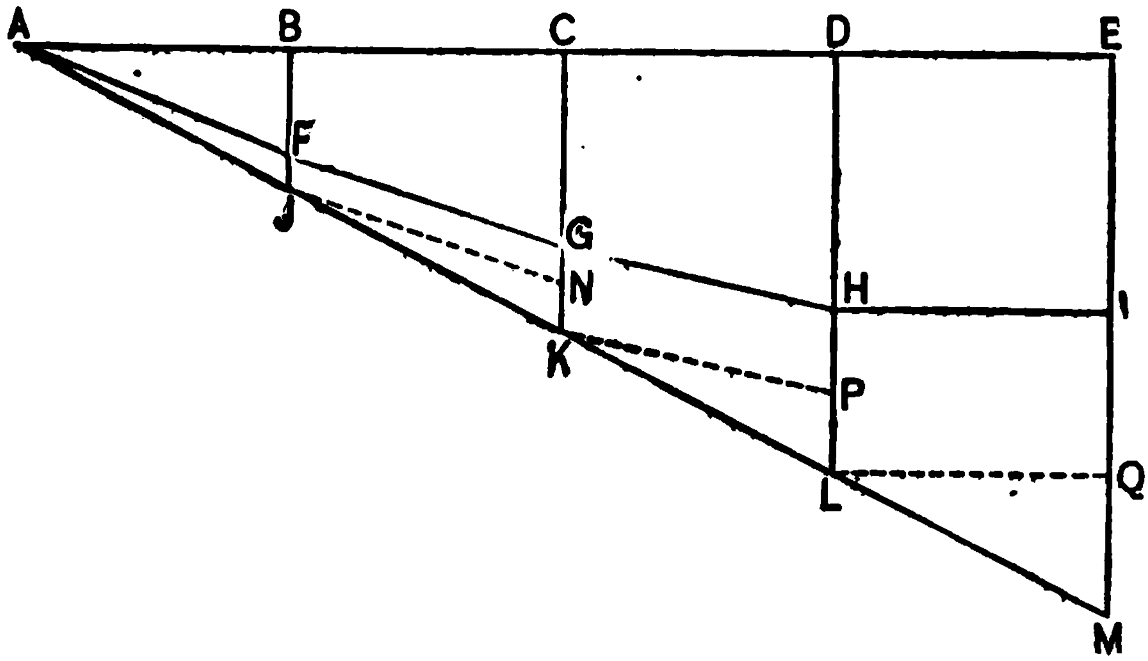


FIG. 207.—LOSS IN VELOCITY-HEAD.

considerably from the velocity at the beginning and end of that space. If the train consists of five cars or more, the velocity may be determined electrically (as described by Wellington in his "Economic Location," etc., p. 793 *et seq.*) from the automatic record made on a chronograph of the passage of the first wheel and the last, the chronograph also recording auto-

matically the ticks of a clock beating seconds. From this the exact time of the passage of the first and last wheels of the train of cars may be determined to the tenth or twentieth of a second.

Velocity-head. From theoretical mechanics we know that if a body descends through any path by the action of gravity, and is unaffected by friction, its velocity at any point in the direction of the path of motion is $V=\sqrt{2gh}$. If the body is retarded by resistances, its velocity at any point will be less than this. If AM, Fig. 207, represents any grade (exaggerated of course), then BJ, CK, etc., represent the actual fall at any point. Let BF represent the fall h_1, determined from $h_1=\frac{v_1^2}{2g}$, in which v_1 is the actual observed velocity at J. Then JF = the velocity-head consumed by the resistances between A and J. If the train continues to K, the corresponding h_2 is CG; the remaining fall GK consists of GN ($=JF$, which is the velocity-head lost back of J) and NK, the velocity-head lost between J and K. At some velocity (V_n) on any grade, the velocity will not further increase and the line $AFGHI$ will then be horizontal and at a distance $(h_n) = EI$ below $A \ldots E$. The grade AM is the "grade of repose" for that velocity (V_n); i.e., it is the grade that would just permit the train to move indefinitely at the velocity V_n. The broken line $AFGHI$ should really be a curve, and the grade of repose at any point is the angle between AM and the tangent to that curve at the given point. The "grade of repose" by its definition gives the total resistance of the train at the particular velocity, or multiplying the grade of repose in per cent. by 20 gives the pounds per ton of resistance. Thus being able to determine the total resistance in pounds per ton at any velocity, the variation of total resistance with velocity may be determined, and then by varying the resistances, using different kinds of cars, empty and loaded, box-cars and flats, the resistances of the different kinds at various velocities may be determined.

438. Formulæ for train resistance. These are generally given in one of the forms

$$\left.\begin{aligned} R&=aV+c, \quad . \quad . \quad . \quad (1)\\ R&=bV^2+c, \quad . \quad . \quad . \quad (2)\\ R&=aV+bV^2+c, \quad . \quad . \quad (3)\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (107).$$

in which R is the resistance in pounds per ton, a and b are coefficients to be determined, V is the velocity in miles per hour, and c is a constant, also to be determined. These formulæ disregard grade and curve resistances, inertia resistance and the active resistance (or assistance) of *wind*, as distinct from mere atmospheric resistance. In short, they are supposed to give the resistance of a train moving at a uniform velocity over a straight and level track, there being no appreciable wind.

The various formulæ are sometimes based directly on experiments made by the proposer of the formula; sometimes they are deduced from a mere study of the results of one or more series of tests made by others. Unfortunately for either method, no one investigator has ever been able to make tests which are so thorough and made under such a wide range of conditions that his results may be considered as conclusive, while a student of the tests of others is handicapped by a lack of knowledge of precise conditions, which, if fully understood, would perhaps permit some reconciliation of the very discordant figures which are reported. As already intimated, the condition of the rolling stock, the unit weight on the axles, the lubrication of the axles, the length of the train in relation to its weight and the condition of the track, which involves the weight of rail, spacing and size of ties, tamping of ties, etc., all have their influence in modifying the apparent resistance. There is also good reason to believe that the effect of grade, curvature, and changing velocity has not been properly allowed for in deducing many of the formulæ. In view of all these considerations, it may be considered as demonstrated that no one formula, and especially a simple formula, will represent the resistance for all conditions. But, since some of the calculations of railroad economics are absolutely dependent on the law of tractive resistance, some law must be deduced with sufficient accuracy for the purpose. Fortunately several of the formulæ are amply accurate for such purposes. A report of a committee of the A. R. E. & M. W. Assoc. (1907) quoted sixty-one different formulæ which have been suggested. Some of these are chiefly of historical value, since they were deduced from tests made many years ago with track and rolling stock very dissimilar from those in use at the present time. Such formulæ will therefore be omitted. For convenience of comparison, all formulæ will be changed (if necessary) from the original statement of them so that they give the

resistance per ton of 2000 pounds. The coefficients of V and V^2 will be given decimally. Other notation occasionally used is as follows:

t = weight of train in tons of 2000 pounds;
L = length of train in feet;
n = number of cars in train;
A = area of front of train in square feet.

(*a*) Formulæ of the first class: $R=aV+c$. Among those most commonly used are the following:

Engineering News,	$R=0.25V+2.0$	(108)
Baldwin locomotive,	$R=0.17V+3.0$	(109)
New York Central,	$R=0.11V+1.8$, ...	(110)
Henderson,	$R=0.25V+\frac{50n}{t}+0.5$.	(111)

Although Henderson's formula is in a class by itself, on account of the extra term, and although it is not applicable to general use, when the character of the trains cannot be estimated, it is perhaps more accurate than the others. It is apparently not intended for use at very low velocities.

(*b*) Formulæ of the second class: $R=bV^2+c$:

Crawford,	$R=0.00214V^2+2.5$	(112)
Wolff,	$R=0.00357V^2+2.7$	(113)
Henderson,	$R=0.00461V^2+3.0$	(114)
Forney,	$R=0.00585V^2+4.0$	(115)

$$\text{Wel-ling-ton}\left\{\begin{array}{l} R=0.0056V^2+\dfrac{.57V^2}{t}+3.9 \text{ (for loaded flat cars)} \\ R=0.0075V^2+\dfrac{.64V^2}{t}+3.9 \text{ (for loaded box cars)} \\ R=0.0083V^2+\dfrac{.57V^2}{t}+6.0 \text{ (for empty flat cars)} \\ R=0.0106V^2+\dfrac{.64V^2}{t}+6.0 \text{ (for empty box cars)} \end{array}\right\}, \quad (116)$$

Notice in formulæ (150) the additional journal resistance (indicated by the constant term) for unloaded cars. The second

term evidently indicates the atmospheric resistance. The first term allows for the oscillatory resistances. Assuming the constant term and the coefficients to have been correctly determined, these formulæ should be better than the others, since a choice of formulæ can be made depending on the conditions. A train consisting partly of box-cars and partly of flat-cars will have a higher resistance than is shown by any of the above formulæ (and *not* a mean value), on account of the increased atmospheric resistance acting on the irregular form of the train.

(*c*) Formulæ of the third class: $R=aV+bV^2+c$:

W. N. Smith, $$R=0.17V+\frac{0.0025AV^2}{t}+3.0; \quad . \quad . \quad . \quad . \quad (117)$$

Von Borries, $$R=0.04V+0.0016V^2+3.0; \quad . \quad . \quad . \quad . \quad (118)$$

Lundie, $$R=0.24V+\frac{4.8V^2}{t}+4.0; \quad . \quad . \quad . \quad . \quad (119)$$

Sprague, $$R=0.17V+\frac{0.333V^2}{t}+4.0. \quad . \quad . \quad . \quad . \quad (120)$$

Although several formulæ have been proposed which involve the area of the front of the train in order to allow more definitely for the atmospheric resistance, only one of these (117) has been quoted. In applying this formula, the proper value to choose for A is somewhat indefinite, since the shape of the front of the train will make a considerable difference in the atmospheric resistance encountered. The area is called 125 square feet in § 429. In the comparison of the formulæ given below, A will be assumed as 125 square feet. In order to compare these resistances, the values of R for the various speeds of 10, 20, 30, 40, 50 and 60 miles per hour will be computed by these formulæ on the basis of a train of twelve cars, having a length of 480 feet, and a weight of 600 tons. Therefore in applying the formula, $t=600$, $L=480$, $n=12$, and $A=125$. In order to apply formula (116) to this case, it will be assumed that this train consists of loaded box-cars, and therefore we must apply the second of that group of formulæ. Computing the resistance according to these several formulæ, we may tabulate the results as given below:

Formula.	Velocity in miles per hour.					
	10	20	30	40	50	60
108	2700	4200	5700	7200	8700	10200
109	2800	3800	4800	5800	6800	7800
110	1747	2413	3080	3747	4413	5080
111	2400	3900	5400	6900	8400	9900
112	1628	2014	2656	3554	4710	6122
113	1834	2477	3548	5047	6975	9331
114	2077	2906	4289	6226	8715	11746
115	2751	3804	5559	8116	11175	15036
116	2854	4396	6966	10564	15188	20844
117	2845	3940	5085	6280	7525	8820
118	2136	2664	3384	4296	5400	6696
119	4320	7200	11040	15840	19440	28080
120	3453	4573	5760	7013	8333	9720

Although there is a fair agreement among the results for ordinary velocities, it should be said, in fairness to the proposers of the various formulæ, that some of them evidently were not designed for use at high velocities such as 60 miles per hour.

Another method of comparing formulæ is to plot them on cross-section paper, using velocities as abscissæ and resistances as ordinates. For general use this method may only be applied to formulæ which do not involve the weight, length or area of the train nor the number of cars. All of the above formulæ have thus been plotted on Plate IX, with the exception of Nos. 111, 116, 117, 119 and 120.

439. American Railway Engineering Association Formula. The Economics Committee of the Amer. Rwy. Eng. Assoc., after considering all published formulæ, on the basis of some elaborate tests with freight trains, developed the fact that the resistance of freight trains is so nearly constant between the velocities of 7 and 35 miles per hour that it may be so considered in comparative calculations. A formula was then developed which is independent of velocity and which has the form, following the previous notation,

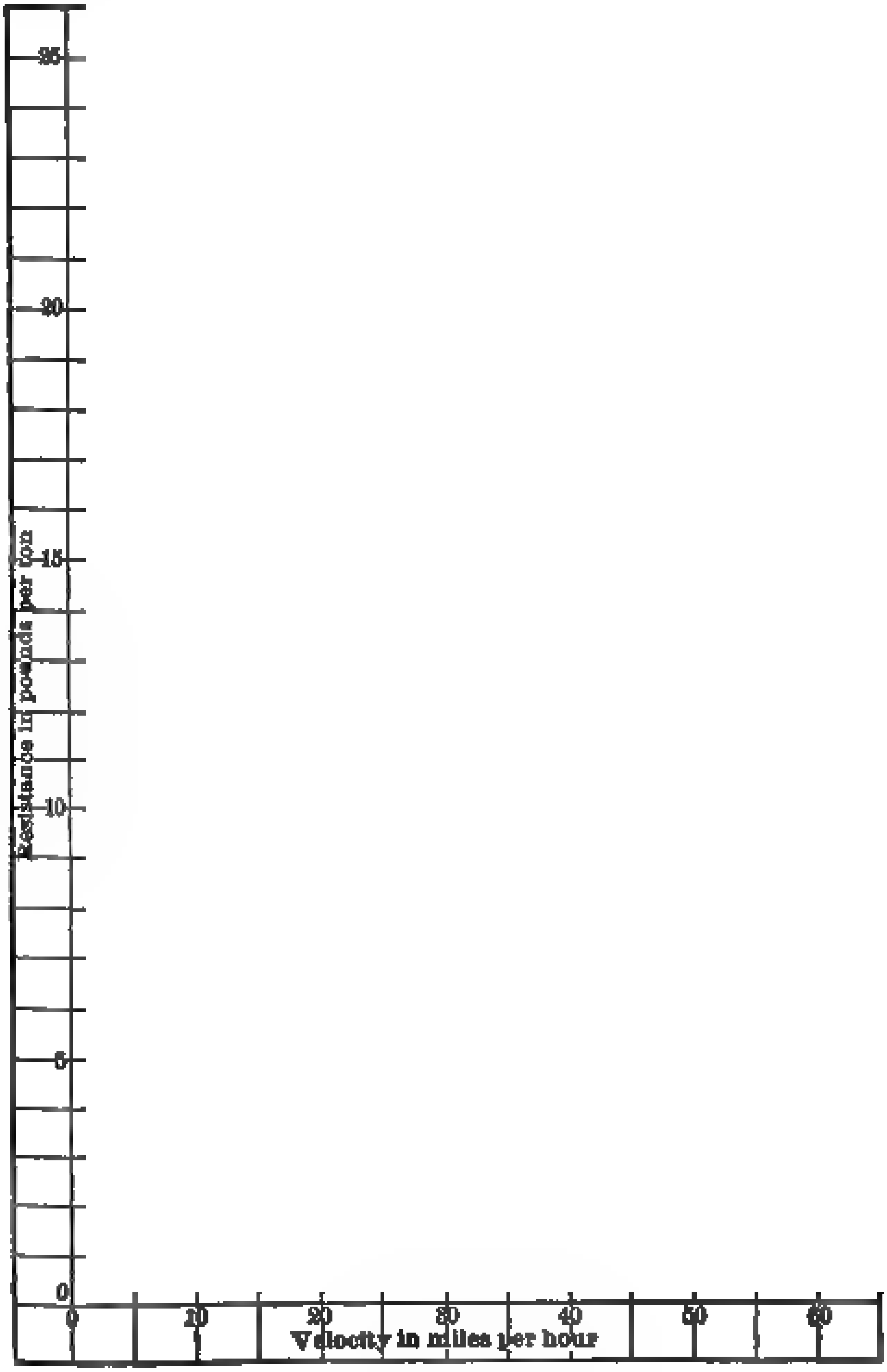
$$R = at + bn,$$

in which a and b have several values, depending on the temperature. These formulæ may be grouped as follows:

Rating	Temp. (F)	Formula
A	above 35°	$R = 2.2t + 122n$
B	35° to 20°	$R = 3.0t + 137n$
C	20° to 0°	$R = 4.0t + 153n$
D	below 0°	$R = 5.4t + 171n$

. . . . (121)

Applying the data of the above numerical case—12 cars, 600 tons, $t = 600$ and $n = 12$, we would have $R = 2784$ lbs. for A rating or for a temperature above 35° F. This means an average resistance of 4.64 lbs. per ton. If we draw in Plate IX, a horizontal line at the height 4.64 from the 7 vertical to the 35 vertical, it will represent the velocity curve for this train. The line, which is straight and not curved, intersects every curve shown in that diagram. And so, although the formula is utterly different from those previously given, there is a rough agreement at freight-train velocities.


Resistance in pounds per ton
25
20
15
10
5
0
0
10
20
30
40
50
60
Velocity in miles per hour

CHAPTER XVII

COST OF RAILROADS.

440. General considerations. Although there are many elements in the cost of railroads which are roughly constant per mile of road, yet the published reports of the cost of railroads differ very widely. The variation in the figures is due to several causes. (*a*) Economy requires that a road shall be operated and placed on an earning basis as soon as possible. Therefore the reported cost of a road during the first few years of its existence is somewhat less than that reported later. This is well illustrated when a long series of consecutive reports from an old-established road is available; nearly every year there will be shown an addition to the previous figures. And this is as it should be. The magnificent road-beds of some old roads cannot be the creation of a single season. It takes many years to produce such settled perfect structures. (*b*) A large part of the variation is due to a neglect to charge up "permanent" improvements" as additions to the cost of the road. For the first few years of the life of a road a great deal of work is done which is in reality a completion of the work of construction, and yet the cost of it is buried under the item "maintenance of way." For example, a long wooden trestle is replaced by an earth embankment and a culvert. Since the original trestle is to be considered a temporary structure, the *excess* of the cost of the permanent structure over that of the temporary structure should evidently be considered as an addition to the cost of the road. But if the filling-in was done slowly, a few train-loads at a time, and the work scattered over many years, the cost of operating the "mud-train" has perhaps been buried under "maintenance" charges. (*c*) The reports from which many of the following figures were taken have not always analyzed the items of cost with the same detail as has been here attempted, and to that is probably due many of the variations and apparent discrepancies.

The various items of cost will be classified as follows:

1. Preliminary financiering.
2. Surveys and engineering expenses.
3. Land and land damages.
4. Clearing and grubbing.
5. Earthwork.
6. Bridges, trestles, and culverts
7. Trackwork.
8. Buildings and miscellaneous structures.
9. Interest on construction.
10. Telegraph line.

441. Item 1. PRELIMINARY FINANCIERING. The cost of this preliminary work is exceedingly variable. The work includes the clerical and legal work of organization, printing, engraving of stocks and bonds, and (sometimes the most expensive of all) the securing of a charter. This sometimes requires special legislative enactments, or may sometimes be secured from a State railroad commission. It has been estimated that about 2% of the railway capital of Great Britain has been spent in Parliamentary expenses over the charters. These expenses are usually but a small percentage of the total cost of the enterprise, but for important lines the gross cost is large, while the amount of money thus spent by organizations which have never succeeded in constructing their roads is, in the aggregate, an enormous amount, although it is of course not ascertainable by any investigator.

Another occasional feature of the financing of a road must be kept in mind. The promoters of a railroad enterprise frequently endeavor to limit their own personal expenditures to the purely preliminary expenses as mentioned above. The project, after having been surveyed, mapped, and written up in a glowing "prospectus," is submitted to capitalists, in the endeavor to have them furnish money for construction, the money to be secured by bonds. If the project will stand it, the amount of the bond issue is made sufficient to pay the entire cost of the road, even with a discount of perhaps 15%. The bond issue may also provide for a very generous commission to the broker who is the intermediary between the promoters and the capitalists. The bond issue may even provide for repaying the promoters for their preliminary expenses. Frequently a considerable proportion of the capital stock goes to the capitalists

who take the bonds, the promoters retaining only such proportion as may be agreed upon. In such a case, the capital stock is "pure velvet," and costs nothing. Its future value, whatever it may be, is so much clear profit. The effect of such a financial policy is to burden the project with a capitalization which is far in excess of the actual cost of constructing the road. Comparatively few projects will stand such over-capitalization. The apparent financial failure of many railroads, which have gone into the hands of receivers is due to their inability to make returns on an over-capitalization rather than because they could not earn enough to pay the legitimate cost of their construction. These features of financiering are really foreign to the engineer's work, but he should know that many projects which would return a handsome profit on an investment amounting only to the legitimate cost, will be rejected by capitalists because it is apparent that there is not enough "velvet" in it.

442. Item 2. Surveys and Engineering Expenses. The comparison of a large number of itemized reports on the cost of construction shows that the cost of the "engineering" will average about 2% of the total cost of construction. This includes the cost of surveys and the cost of laying out and superintending the constructive work. The cost of mere surveying up to the time when construction actually commences has been variously quoted at $60, $75, and even $300 per mile. The lower figures generally refer to the hasty, ill-considered work which was formerly common and which has resulted in so much badly located road, much of which has been reconstructed, when improvements are practicable. See the introductory paragraphs of Chapter I. Except when the topography limits the location to one very obvious route, a thorough survey may cost about $300 per mile. In the estimate given at the end of this chapter the cost of "engineering and office expenses" is given at 5% of the cost of the construction work. The item then includes the cost of the very considerable amount of clerical work and superintendence incident to the expenditure of such a large sum of money.

443. Item 3. Land and Land Damages. The cost of this item varies from the extreme, in which not only the land for right-of-way but also grants of public land adjoining the road are given to the corporation as a subsidy, to the other extreme

where the right-of-way can only be obtained at exorbitant prices. The width required is variable, depending on the width that may be needed for deep cuts or high fills, or the extra land required for yards, stations, etc. A strip of land 1 mile long and 8.25 feet wide contains precisely 1 acre. An average width of 4 rods (66 feet), therefore, requires 8 acres per mile. On the Boston & Albany Railroad the expenditure assigned to "land and land damages" averages over $25000 per mile. Of course this includes some especially expensive land for terminals and stations in large cities. Less than $300 per mile was assigned to this item by an unimportant 18-mile road.

444. Item 4. CLEARING AND GRUBBING. The cost of this may vary from zero to 100% for miles at a time, but as an average figure it may be taken as about 3 acres per mile at a cost of say $50 per acre. The possibility of obtaining valuable timber, which may be utilized for trestles, ties, or otherwise, and the value of which may not only repay the cost of clearing and grubbing, but also some of the cost of the land, should not be forgotten.

445. Item 5. EARTHWORK. This item also includes rock-work. The methods of estimating the cost of earthwork and rockwork have been discussed in Chapter III. The percentage of this item to the total cost is very variable. On a western prairie it might not be more than 5 to 10%. On a road through the mountains it will run up to 20 or 25%, and even more. The item also includes tunneling, which on some roads is a heavy item.

446. Item 6. BRIDGES, TRESTLES, AND CULVERTS. This item will usually amount to 5 or 6% of the total cost of the road. In special cases, where extensive trestling is necessary, or several large bridges are required, the percentage will be much higher. On the other hand, a road whose route avoids the watercourses may have very little except minor culverts. On the Boston & Albany the cost is given as $5860 per mile; on the Adirondack Railroad, $2845 per mile. Considering their relative character (double and single track), these figures are relatively what we might expect.

447. Item 7. Trackwork. This item will be considered as including everything above subgrade, except as otherwise itemized.

(a) **Ballast.** With an average width, for single track, of 10 feet and an average depth of 15 inches, 2444 cubic yards of ballast will be required. The Pennsylvania Railroad estimate is 2500 yards of gravel per mile of single track. At an estimate of 60 c. per yard, this costs $1500 per mile. Broken-stone ballast must be filled out over the ends of the ties and therefore more is required; 2800 cubic yards of broken stone at $1.25 per yard in place will cost $3500 per mile.

(b) **Ties.** Ties cost anywhere from 80 c. down to 35 c. and even 25 c. At an average figure of 50 c., 2640 ties per mile will cost $1320 per mile of single track. The cheaper ties are usually smaller and more must be used per mile, and this tends to compensate the difference in cost.

The following tabular form is convenient for reference:

TABLE XXX.—NUMBER OF CROSS-TIES PER MILE.

Number per 33′ rail.	Average spacing center to center.	Number per mile.
22	18.0 inches	3520
21	18.9 "	3360
20	19.8 "	3200
19	20.9 "	3040
18	22.0 "	2880
17	23.3 "	2720
16	24.75 "	2560
15	26.4 "	2400
14	28.3 "	2240
13	30.5 "	2080

(c) **Rails.** The total weight of the rails used per mile may best be seen by the tabular form.

A convenient and useful rule to remember is that the number of *long* tons (2240 lbs.) per mile of single track equals the weight of the rail per yard times $\frac{11}{7}$. The rule is exact. For example, there are 3520 yards of rail in a mile of single track; at 70 lbs. per yard this equals 246400 lbs., or 110 long tons (exactly); but $70 \times \frac{11}{7} = 110$.

Any calculation of the required weight of rail for a given weight of rolling-stock necessarily depends on the assumptions which are made regarding the support which the rails receive from the ties. This depends not only on the width and spacing of the ties (which are determinable), but also on the support which the ties receive from the ballast, which is not only very uncertain but variable. No general rule can therefore claim

TABLE XXXI.—TONS PER MILE (WITH COST) OF RAILS OF VARIOUS WEIGHTS.

Weight in lbs. per yd.	Tons (2240 lb.) per mile of single track.	Cost at $26 per ton.	Cost at $30 per ton.	Weight in lbs. per yd.	Tons (2240 lb.) per mile of single track.	Cost at $26 per ton.	Cost at $30 per ton.
8	12.571	$326.86	$377.14	65	102.143	$2655.71	$3064.29
10	15.714	408.57	471.43	66	103.714	2696.57	3111.43
12	18.857	490.29	565.71	67	105.286	2737.43	3158.59
14	22.000	572.00	660.00	68	106.857	2778.29	3205.79
16	25.143	653.71	754.20	70	110.000	2860.00	3300.00
20	31.429	817.14	942.86	71	111.571	2900.86	3347.14
25	39.286	1021.43	1178.57	72	113.143	2941.71	3394.29
30	47.143	1225.71	1414.29	73	114.714	2982.57	3441.43
35	55.000	1430.00	1650.00	75	117.857	3064.29	3535.71
40	62.857	1634.29	1885.71	78	122.571	3186.86	3677.14
45	70.714	1838.57	2121.43	80	125.714	3268.57	3771.43
48	75.429	1961.14	2262.86	82	128.857	3350.29	3865.71
50	78.571	2042.86	2357.14	85	133.571	3472.86	4007.14
52	81.714	2124.57	2451.43	88	138.286	3595.43	4148.57
56	88.000	2288.00	2640.00	90	141.429	3677.14	4242.86
57	89.571	2328.86	2687.14	92	144.571	3758.86	4337.14
60	94.286	2451.43	2828.57	95	149.286	3881.43	4478.57
61	95.857	2492.29	2875.71	98	154.000	4004.00	4620.00
63	99.000	2574.00	2970.00	100	157.143	4085.71	4714.29

About two per cent. (2%) extra should be allowed for waste in cutting.

any degree of precision, but the following is given by the Baldwin Locomotive Works: "**Each ten pounds weight per yard of ordinary steel rail, properly supported by cross-ties (not less than 14 per 30-foot rail), is capable of sustaining a safe load per wheel of 3000 pounds.**" For example, a Mikado locomotive with 153200 lbs. on 8 drivers has a load of 19150 lbs. per wheel. This divided by 3000 gives 6.38. According to the rule, the rails for such a locomotive should weigh at least 63.8 lbs. per yard.

On the basis of 33-foot lengths, and 10% shorter lengths, varying by even feet down to 27 feet (see § 274, 8), the average length, assuming an equal number each of the shorter length rails would be 32.65 feet. Calculating similarly for 30-ft. rails, with 10% shorts to 24 feet, the average length would be 29.65 feet. 60-ft. rails, used extensively for electric roads, with 10% shorts to 40 feet, will have average length of 58.95 feet.

(d) **Splice-bars, track-bolts, and spikes.** These are usually sold by the pound, except the patented forms of rail-joints, which are sold by the pair. In any case they are subject to market fluctuations in price. As an approximate value the following prices are quoted: Splice-bars, 1.35 cents per pound;

track-bolts, 2.4 cents; spikes, 1.75 cents. The weight of the splice-bars will depend on the precise pattern adopted—its cross-section and length.

In Table XXXII are quoted from a catalogue of the Illinois Steel Co. the weights per foot of sections of angle-bars which they recommend for various weights of rail and which are designed to fit standard A. S. C. E. rail sections of those weights. The net weight of the angle-bars may be approximated by subtracting about 2.5% to 4% from the gross weight to allow for the bolt-holes. A deduction of 2.5% is usually about right for the heavier sections. Their recommendations regarding lengths of angle-bars do not include those for rails heavier than 50 pounds per yard. On the basis of a length of 23 inches for four-hole splices and of 33 inches for six-hole splices, the weights of splice-bars have been computed for the several styles of splices for heavier rails, allowing 2.5% for the holes. The lengths recommended for track bolts are those which will allow about ½ inch for the nutlock and for margin, except for the lighter rails.

TABLE XXXII.—SPLICE-BARS FOR VARIOUS WEIGHTS OF RAILS.

Weight of rail.	Length of angle-bar.	Weight per foot.	Weight of pair.	Proper size of track-bolt.	Proper size of spikes.
30	21″	4 49	15 1	2½″ ×	4[illegible]
35	21″	4.7	15.9	2¾″ ×	4[illegible]
40	21″	5 54	18.8	3 ″ ×	5[illegible]
45	21″	6.3	21.5	3 ″ ×	5[illegible]
50	21″	6 97	23.4	3¼″ ×	5[illegible]
55	24″	7.5	29 2	3½″ ×	5[illegible]
60	24″	8.4	32.8	3½″ ×	5[illegible]
65	24″	9.2	35.9	4 ″ ×	5[illegible]
	32″	9 6	49 9	4¼″ ×	5[illegible]
70	24″	9.0	35.1	4 ″ ×	5[illegible]
	32″	10 0	52 0	4 ″ ×	5[illegible]
75	24″	10.68	42.6	4¼″ ×	5[illegible]
	32″	11 9	61.[illegible]	4 ″ ×	5[illegible]
80	24″	10.61	42 3	4¼″ ×	5[illegible]
	32″	14 [illegible]	76.2	4½″ ×	5[illegible]
85	32″	12 4	64.5	4½″ ×	5[illegible] or [illegible]″
90	32″	13.5	70.2	4½″ ×	5[illegible] or [illegible]″
95	32″	14.7	76 4	4½″ ×	5[illegible] or [illegible]″
100	32″	15.78	82.[illegible]	4½″ ×	5[illegible] or [illegible]″

(e) **Track-laying.** Much depends on the force of men employed and the use of systematic methods; $528 per mile is the estimate employed by the Pennsylvania Railroad. $500 per mile is the estimate given in § 451.

TABLE XXXIII.—RAILROAD SPIKES.

Size measured under head.	Average number per keg of 200 pounds	Ties 24″ between centers, 4 spikes per tie, number per mile.		Suitable weight of rail.
		Pounds.	Kegs.	
$5\frac{1}{2}''\times\frac{5}{8}''$	275	7680	38.40	90 to 100
$5\frac{1}{2}''\times\frac{9}{16}''$	375	5632	28.16	45 " 100
$5''\times\frac{9}{16}''$	400	5280	26.40	40 " 56
$5''\times\frac{1}{2}''$	450	4692	23.46	40
$4\frac{1}{2}''\times\frac{1}{2}''$	530	3984	19.92	35
$4''\times\frac{1}{2}''$	600	3520	17.60	30
$4\frac{1}{2}''\times\frac{7}{16}''$	680	3104	15.52	25 to 30

TABLE XXXIV.—TRACK-BOLTS.

Average number in a keg of 200 pounds.

Size of bolt.	Square nut.	Hexagonal nut.	Suitable rail.
$3''\times\frac{5}{8}''$	366	395	40 pound
$3''\times\frac{3}{4}''$	250	270	
$3\frac{1}{4}''\times\frac{3}{4}''$	243	261	
$3\frac{1}{2}''\times\frac{3}{4}''$	236	253	50
$3\frac{3}{4}''\times\frac{3}{4}''$	229	244	55 to 60
$4''\times\frac{3}{4}''$	222	236	65 " 70
$4\frac{1}{4}''\times\frac{3}{4}''$	215	228	75
$3\frac{1}{2}''\times\frac{7}{8}''$	170	180	
$3\frac{3}{4}''\times\frac{7}{8}''$	165	175	
$4''\times\frac{7}{8}''$	161	170	
$4\frac{1}{4}''\times\frac{7}{8}''$	157	165	80
$4\frac{1}{2}''\times\frac{7}{8}''$	153	160	85
$4\frac{3}{4}''\times\frac{7}{8}''$	149	156	90

TABLE XXXV.—RAIL-JOINTS AND TRACK-BOLTS. NUMBER PER MILE OF TRACK.

Length of rail. Feet.	Average length of rail. Feet.	Number of rails or complete joints.	Number of bolts.	
			4-bolt.	6-bolt.
All 30	30	352	1408	2112
30–24	29.65	356.2	1425	2137
All 33	33	320	1280	1920
33–27	32.65	323.4	1294	1941
All 60	60	176	704	1056
60–40	58.95	179.1	717	1075

448. Item 8. Buildings and Miscellaneous Structures. Except for rough and preliminary estimates, these items must be individually estimated according to the circumstances. The subitems include depots, engine-houses, repair-shops, water-stations, section- and tool-houses, besides a large variety of smaller buildings. The structures include turn-tables, cattle-guards, fencing, road-crossings, overhead bridges, etc. The detailed estimate, given in § 451, illustrates the cost of these smaller items.

449. Item 9. Interest on Construction. The amount of capital that must be spent on a railroad before it has begun to earn anything is so very large that the interest on the cost during the period of construction is a very considerable item. The amount that must be charged to this head depends on the current rate of money on the time required for construction and on the ability of the capitalists to retain their capital where it will be earning something until it is actually needed to pay the company's obligations. Of course, it is not necessary to have the entire capital needed for construction on hand when construction commences. Assuming money to be worth 6%, that the work of construction will require one year, that the money may be retained where it will earn something for an average period of six months after construction commences, or, in other words, it will be out of circulation six months before the road is opened for traffic and begins to earn its way, then we may charge 3% on the total cost of construction.

450. Item 10. TELEGRAPH LINES. This evidently depends on the scale of the road and the magnitude of the business to be operated. In the following estimate it is given as $200 per mile, which evidently is intended to apply to the business of a small road.

451. Detailed estimate of the cost of a line of road. The following estimate was given in the *Engineering News* of Dec. 27, 1900, of the cost of the Duluth, St. Cloud, Glencoe & Mankato Railroad, 157.2 miles long.

The estimate is exactly as copied from the *Engineering News.* There are some numerical discrepancies. Item 26 should evidently be based on the sum of the first 25 items, and item 27 on the sum of the first 26. The figures in parentheses ( ) are deduced from the figures given.

1.	Right-of-way: 1905.3 acres (12.12 acres per mile) @ $100 per acre		$190530
2.	Clearing and grubbing. 144 acres (0.916 acre per mile) @ $50 per acre		7200
3.	Earth excavation. 1907590 cu. yds. (12135 cu. yds. per mile) @ 15 c.		286138
4.	Rock excavation. 5100 cu. yds. (32.44 cu. yds. per mile) @ 80 c.		4080
5.	Wooden-box culverts. 508300 ft. B.M. @ $30 per M.	$15249	
	Iron-pipe culverts. 879840 lbs. @ 3c. per lb.	26395	41644
6.	Pile trestling: 4600 lin. ft. @ 35 c. per lin. ft.	1610	
	Timber trestling. 509300 ft. B.M. @ $30 per M	15279	16889
7.	Bridge masonry: 5520 cu. yds. @ $8 per cu. yd	44160	
	Bridges, iron, 100 spans, 2000000 lbs. @ 4 c. per lb.	80000	124160
8.	Cattle-guards		8750
9.	Ties (2640 per mile). 419813 (159.02 miles) @ 35 c.		146935
10.	Rails (70 lbs. per yd.): 110 tons per mile, 17492.2 tons (159.02 miles @$26		384797
11.	Rail sidings (70 lbs. per yd.): 110 tons per mile, 3300 tons (30 miles @ $26		85800
12.	Switch timbers and ties		3300
13.	Spikes: 5920 lbs. per mile, 1107040 (187 m.) @ 1.75. c. per lb.		19373
14.	Splice-bars. 2635776 lbs. @ 1.35 c. per lb.		35583
15.	Track-bolts (2 to joint (?)): 188458.3 lbs. @ 2.4 c. per lb.		4520
16.	Track-laying 187.2 miles @ $500 per mile		93600
17.	Ballasting: 2152 cu. yds. per mile, 402854 (187.2 m.) @ 60 c.		241712
18.	Turn-out and switch furnishings		6450
19.	Road-crossings, 68040 ft. B.M. @ $30 per M		2041
20.	Section and tool-houses, 16 @ $800		12800
21.	Water-stations		15000
22.	Turn-tables, 6 @ $800		4800
23.	Depots, grounds, and repair-shops		78000
24.	Terminal grounds and special land damages		150000
25.	Fencing, 314 miles ($150 per mile)		47100
26.	Engineering and office expenses (5% of $1984458)		99222
27.	Interest on construction (3% of $2083680)		62510
28.	Rolling-stock ($5000 per mile)		786000
29.	Telegraph line: 157 miles @ $200 per mile		31400
			$3060340

Average cost per mile ready for operation, $19467.

Approximate cost of 130 miles from St. Cloud to Duluth, estimated at $23000 per mile.

Approximate cost of entire line from Albert Lea to Duluth, 287.2 miles, $6050340 ($21060 per mile).

CHAPTER XVIII.

THE POWER OF A LOCOMOTIVE.

452. Pounds of steam produced. The power that can be developed by a locomotive depends very greatly on the quality of the coal burned and the design of the locomotive must correspond to the general kind or quality of coal to be used. A British thermal unit (symbolized as B.t.u.), is the quantity of heat required to raise the temperature of 1 lb. of pure water 1° F., when the water is at or near its maximum density at 39.1° F. When it is said that a certain grade of coal has 14000 B.t.u. it means that the heat in 1 lb. of that coal will raise the temperature of 14000 lbs. of water 1°, or, approximately, 100 lbs. of water 140°. But, although it only requires 180.9 heat units to heat water from 32° to 212°, it requires 965.7 more heat units to change it from water at 212° to steam at 212°. It requires only 53.6 more heat units to change it from steam at 212° to steam at 387.6° or with a pressure of 200 lbs. per square inch.

A study of locomotive tests made at the St. Louis Exposition resulted in the compilation of Table XXXVI, which is copied from the Proceedings of the American Railway Engineering Association, and is now included as Table I, in the "Economics" section of their Manual. It was found that the steam produced per square foot of heating surface is very nearly proportional to the coal burned per square foot of heating surface. The results are purposely made about 5% below the results obtained in the St. Louis tests to allow for ordinary working conditions.

453. Numerical example. The theory developed in this chapter will be illustrated numerically by applying it to a Mikado type of locomotive whose dimensions are as follows:

Cylinder............	diam. 22″	Weight, driving wheels.	153,200 lbs.
Cylinder............	stroke 28″	engine alone........	196,100 lbs.
Driving wheel........	diam. 57″	engine and tender....	315,000 lbs.
Boiler pressure.......	185 lbs.	Heating surface, fire-box	
Fire-box............	length $102\frac{1}{4}$″	and tubes..........	2565 sq. ft.
Fire-box............	width $65\frac{1}{4}$″	superheating surface.	550 sq. ft.
Grate area............	46.8 sq. ft.		

TABLE XXXVI.—AVERAGE EVAPORATION IN LOCOMOTIVE BOILERS BURNING BITUMINOUS AND SIMILAR COALS OF VARIOUS QUALITIES, AND FOR VARIOUS QUANTITIES CONSUMED PER SQUARE FOOT OF HEATING SURFACE PER HOUR.

(Based on feed water at 60° Fahrenheit, and boiler pressure 200 pounds)

Coal per square foot of heating surface per hour (lb.)	Steam per pound of coal of given thermal value (lb.)					
	15,000 B.t.u.	14,000 B.t.u.	13,000 B.t.u.	12,000 B.t.u.	11,000 B.t.u.	10,000 B.t.u.
0.8	7.86	7.34	6.81	6.29	5.76	5.24
0.9	7.58	7.07	6.57	6.06	5.56	5.05
1.0	7.31	6.82	6.34	5.85	5.36	4.87
1.1	7.06	6.59	6.12	5.65	5.18	4.71
1.2	6.82	6.37	5.91	5.46	5.00	4.55
1.3	6.59	6.15	5.71	5.27	4.83	4.39
1.4	6.37	5.95	5.52	5.10	4.67	4.25
1.5	6.17	5.76	5.35	4.94	4.52	4.11
1.6	5.97	5.57	5.18	4.78	4.38	3.98
1.7	5.79	5.40	5.02	4.63	4.25	3.86
1.8	5.61	5.24	4.86	4.49	4.12	3.74
1.9	5.44	5.08	4.71	4.35	3.99	3.63
2.0	5.27	4.92	4.57	4.22	3.86	3.51
2.1	5.12	4.78	4.44	4.10	3.75	3.41
2.2	4.97	4.64	4.31	3.98	3.64	3.31
2.3	4.83	4.51	4.19	3.86	3.54	3.22
2.4	4.69	4.38	4.07	3.75	3.44	3.13
2.5	4.56	4.26	3.95	3.65	3.34	3.04
2.6	4.44	4.14	3.84	3.55	3.25	2.96
2.7	4.32	4.03	3.74	3.46	3.17	2.88
2.8	4.21	3.93	3.64	3.37	3.09	2.80
2.9	4.10	3.83	3.55	3.28	3.01	2.73
3.0	3.99	3.73	3.46	3.19	2.93	2.66

The quantity of steam evaporated for intermediate quantities or qualities of coal can be found by interpolation.

On bad-water districts deduct the following from tabular quantities:

For each $\frac{1}{16}$ inch of accumulated scale.......... 10 per cent
For each grain per U. S. gallon of foaming salts in the average feed water.................. 1 per cent

Assume that this locomotive is using coal whose air-dried mine samples tested 13000 B.t.u.; then the average run-of-car coal would have about 90% of this or 11700 B.t.u. On the basis that a fireman can handle 4000 lbs. of coal per hour and maintain such work throughout his run, the coal may be fed at the rate of $(4000 \div 2565) = 1.56$ lbs. per hour per square foot of heating surface. Interpolating in Table XXXVI for 1.56 and 11700 we find that the pounds of steam per pound of coal would be 4.72. The tests at St. Louis showed that a reduction in

boiler pressure increased very slightly the amount of steam produced, but that this amount was only 0.5% greater when the pressure was 160 lbs. instead of 200 lbs. The effect of variation of pressure can therefore be ordinarily ignored. In this case it might add 0.2% or make the figure 4.73. Considering that a superheater adds from 15 to 25% to the efficiency, we will assume the average of 20% and say that 0.80 lb. of the superheated steam produced may be considered as having the same volume and pressure as 1 lb. of saturated steam. Then the amount of steam developed by 1 lb. of coal would be the equivalent of $4.73 \div 0.80 = 5.91$ lbs. Then the equivalent amount of steam developed per hour equals $5.91 \times 4000 = 23640$ lbs.

454. **Weight of steam per stroke at full cut-off.** This may be computed most easily by utilizing Table XXXVII, which is also taken (but somewhat amplified), from the Proceedings of the American Railway Engineering Association, and is now included as Table 2 in the "Economics" section of their Manual. The weight of steam per foot of stroke for 22 ins. diameter and 185 lbs. gauge pressure is 1.161 lbs. and for a stroke of 28 ins. ($2\frac{1}{3}$ ft.) it is 2.709 lbs. For a complete revolution of the drivers it is $4 \times 2.709 = 10.836$ lbs. Since the engine can develop the equivalent of 23640 lbs. of steam per hour and will use 10.836 lbs. at one revolution, it can run at a speed of $23640 \div 10.836 = 2182$ revolutions per hour, or 36.36 revolutions per minute, at full stroke and maintain full boiler pressure. The drivers are 57 ins. in diameter and, therefore, have a circumference of $(57 \div 12) \times 3.1416 = 14.923$ ft. The maximum engine speed for full stroke is $36.36 \times 14.923 = 542.6$ ft. per minute. Multiplying by 60 and dividing by 5280, or dividing by 88, we have 6.167 miles per hour as the maximum speed at which full stroke can be maintained, which is the value M for these conditions.

455. **Pounds of steam and per cent. of cut-off for multiples of M velocity.** In Table XXXVIII, also taken from the Proceedings of the American Railway Engineering Association and now included at Table 4 in the "Economics" section of the Manual, are given the pounds of steam per indicated horse-power hour for simple and for compound locomotives for various velocities, which are multiples of M, the maximum velocity at which the locomotive can use steam at full stroke and yet the boiler can maintain steam at full pressure. The table is computed on the basis of 200 lbs. gauge pressure, but factors are

TABLE XXXVII.—WEIGHT OF STEAM USED IN ONE FOOT OF STROKE IN LOCOMOTIVE CYLINDERS.

(Cylinder diameter is for high-pressure cylinders in compound locomotives)

Diameter of cylinder (inches)	Weight of steam per foot of stroke for various gauge pressures.						
	220 lbs. per sq. in. (lb.)	210 lbs. per sq. in. (lb.)	200 lbs. per sq. in. (lb.)	190 lbs. per sq. in. (lb.)	180 lbs. per sq. in. (lb.)	170 lbs. per sq. in. (lb.)	160 lbs. per sq. in. (lb.)
12	0.405	0.389	0.370	0.354	0.337	0.321	0.304
13	0.475	0.456	0.435	0.415	0.396	0.376	0.357
14	0.551	0.529	0.504	0.482	0.459	0.436	0.414
15	0.633	0.607	0.579	0.553	0.527	0.501	0.476
15½	0.675	0.649	0.618	0.590	0.562	0.535	0.508
16	0.720	0.691	0.658	0.629	0.599	0.570	0.541
17	0.812	0.780	0.744	0.710	0.676	0.643	0.611
18	0.911	0.875	0.834	0.796	0.759	0.722	0.685
18½	0.962	0.924	0.881	0.841	0.801	0.762	0.724
19	1.015	0.975	0.928	0.887	0.845	0.804	0.763
19½	1.069	1.027	0.978	0.934	0.890	0.847	0.804
20	1.125	1.080	1.029	0.983	0.936	0.891	0.836
20½	1.181	1.134	1.081	1.032	0.984	0.936	0.888
21	1.240	1.191	1.134	1.083	1.032	0.982	0.932
22	1.361	1.307	1.245	1.189	1.133	1.078	1 023
23	1.487	1.428	1.361	1.300	1.238	1.178	1.118
24	1.620	1.555	1.482	1.416	1.348	1.283	1.218
25	1.758	1.688	1.608	1.536	1.462	1.392	1.322
26	1.901	1.825	1.739	1.661	1.582	1.506	1.430
27	2.050	1.968	1.875	1.792	1.706	1.624	1.542
28	2.204	2.117	2.017	1.926	1.835	1.745	1.657

For weight of steam used per revolution of drivers at full cut-off: Multiply the tabular quantity by four times the length of stroke in feet for simple and four-cylinder compounds. For two-cylinder compounds multiply by two times the length of stroke.

given for other pressures. For example, continuing the above numerical problem, the pounds of steam per i.h.p.-hour, for a simple locomotive, at *M* velocity, and at 200 lbs. pressure, taken from Table XXXVIII, is 38.30; for 185 lbs. pressure we must multiply by the factor 1.0095, which makes the quantity 38.66. Dividing this into 23640, the steam produced per hour, we have 611.5, the i.h.p. at *M* velocity. Multiplying this by 33000, the foot-pounds per minute in one horse-power, and dividing by 542.6, the velocity in feet per minute, we have 37190, the cylinder tractive power in pounds, when burning 4000 lbs. of coal per hour and running at 6.167 m.p.h.

TABLE XXXVIII.—MAXIMUM CUT-OFF AND POUNDS OF STEAM PER I.H.P.-HOUR FOR VARIOUS MULTIPLES OF M.

(M is maximum velocity in miles per hour at full cut-off, with boiler pressure at 200 pounds per square inch)

Velocity	Cut-off per cent	Pounds steam per I.H.P.-hour		Velocity	Cut-off per cent	Pounds steam per I.H.P.-hour	
		Simple	Compound			Simple	Compound
1.0 M	Full	38.30	25.80	2.9 M	38.5	24.37	21.04
1.1 "	94.4	36.46	24.36	3.0 "	37.0	24.22	21.21
1.2 "	89.1	34.89	23.24	3.2 "	34.2	24.00	21.57
1.3 "	84.3	33.56	22.35	3.4 "	31.8	23.85	21.93
1.4 "	79.7	32.41	21.65	3.6 "	29.8	23.80	22.27
1.5 "	75.4	31.40	21.14	3.8 "	28.0	23.80	22.57
1.6 "	71.4	30.49	20.77	4.0 "	26.4	23.87	22.85
1.7 "	67.7	29.67	20.52	4.25 "	24.7	24.05	23.22
1.8 "	64.3	28.93	20.40	4.50 "	23.3	24.24	23.56
1.9 "	61.0	28.25	20.40	4.75 "	22.1	24.44	23.85
2.0 "	58.0	27.62	20.40	5.0 "	21.1	24.64	24.15
2.1 "	55.2	27.05	20.40	5.5 "	19.5	24.98	24.70
2.2 "	52.6	26.52	20.40	6.0 "	18.4	25.20	
2.3 "	50.1	26.06	20.40	6.5 "	17.6	25.45	
2.4 "	47.8	25.67	20.40	7.0 "	17.1	25.60	
2.5 "	45.7	25.32	20.47	7.5 "	16.7	25.70	
2.6 "	43.7	25.02	20.60	8.0 "	16.4	25.80	
2.7 "	41.8	24.76	20.73	9.0 "	16.1	25.90	
2.8 "	40.1	24.54	20.88				

For steam per i.h.p.-hour for other boiler pressure take the following percentages of values given in table:

160 lb., 103.0%	180 lb., 101.3%.	210 lb., 99.5%
170 lb., 102.1%	190 lb., 100.6%	200 lb., 99.2%

456. Draw-bar Pull. To obtain the draw-bar pull we must deduct the engine resistance. These have already been discussed in § 429 and the numerical value of the resistance of this same locomotive has been there computed to be about 1771 lbs. Subtracting this from 37190 we have 35419 lbs., the estimated draw-bar pull for that speed and coal consumption.

457. Effect of increasing the rate of coal consumption. To note the effect of increasing the rate of coal consumption, the problem may be again worked through on the basis that the rate of coal consumption is increased, even temporarily, from 4000 lbs. to 5000 lbs. per hour. The steam developed per pound of coal is reduced from 5.91 to 5.23, but the total steam produced per hour is increased from 23640 to 26150. The increased capacity comes through a loss of efficiency. The increased steam

production raises the velocity at which full stroke may be maintained from 6.167 m.p.h to 6.820 m.p.h and the i.h.p. from 611.5 to 676.4. But the computed cylinder tractive power is practically identical, the numerical computation of 37190 being only changed to 37189. But these cylinder tractive powers are each computed for the "*M*" velocities, the maximum velocities at which full stroke can be maintained, and "*M*" is higher with increased coal consumption. For a real comparison, the figures must be reduced to the same velocity, e.g., the working velocity of 10 m.p.h. 10÷6.167=1.621, the multiple for the original problem. For 5000 lbs. of coal per hour, *M* velocity is

TABLE XXXIX*.—PER CENT CYLINDER TRACTIVE POWER FOR VARIOUS MULTIPLES OF *M*.

(*M* is maximum velocity in miles per hour at which boiler pressure can be maintained with full cut-off)

Velocity	Per cent (Compound)	Per cent (Simple)	Velocity	Per cent (Compound)	Per cent (Simple)	Velocity	Per cent (Compound)	Per cent (Simple)
Start	135.00	106.00	3.6 *M*	32.40	44.75	6.4 *M*		23.59
0.5 *M*	103.00	103.00	3.7 "	31.25	43.56	6.5 "		23.18
1.0 "	100.00	100.00	3.8 "	30.10	42.39	6.6 "		22.79
1.1 "	96.28	95.57	3.9 "	29.14	41.24	6.7 "		22.42
1.2 "	92.55	91.53	4.0 "	28.24	40.10	6.8 "		22.06
1.3 "	88.83	87.83	4.1 "	27.38	39.00	6.9 "		21.71
1.4 "	85.12	84.46	4.2 "	26.56	37.96	7.0 "		21.38
1.5 "	81.40	81.37	4.3 "	25.77	36.97	7.1 "		21.06
1.6 "	77.68	78.55	4.4 "	25.03	36.03	7.2 "		20.75
1.7 "	73.96	75.97	4.5 "	24.34	35.13	7.3 "		20.45
1.8 "	70.25	73.60	4.6 "	23.69	34.26	7.4 "		20.16
1.9 "	66.54	71.41	4.7 "	23.07	33.41	7.5 "		19.88
2.0 "	63.21	69.37	4.8 "	22.48	32.59	7.6 "		19.61
2.1 "	60.20	67.47	4.9 "	21.92	31.82	7.7 "		19.34
2.2 "	57.48	65.67	5.0 "	21.38	31.11	7.8 "		19.08
2.3 "	54.97	63.94	5.1 "	20.87	30.42	7.9 "		18.82
2.4 "	52.68	62.22	5.2 "	20.37	29.75	8.0 "		18.57
2.5 "	50.42	60.55	5.3 "	19.89	29.10	8.1 "		18.33
2.6 "	48.16	58.92	5.4 "	19.43	28.48	8.2 "		18.09
2.7 "	46.08	57.33	5.5 "	18.99	27.87	8.3 "		17.86
2.8 "	44.10	55.78	5.6 "		27.33	8.4 "		17.64
2.9 "	42.29	54.26	5.7 "		26.81	8.5 "		17.43
3.0 "	40.57	52.78	5.8 "		26.30	8.6 "		17.22
3.1 "	38.95	51.33	5.9 "		25.81	8.7 "		17.01
3.2 "	37.42	49.91	6.0 "		25.34	8.8 "		16.82
3.3 "	35.98	48.55	6.1 "		24.88	8.9 "		16.63
3.4 "	34.66	47.24	6.2 "		24.44	9.0 "		16.45
3.5 "	33.53	45.97	6.3 "		24.01			

* Table 5 in "Economics" Section of Manual of American Railway Engineering Association.

6.820 m.p.h., and the multiple is 1.466. From Table XXXIX we find that the percentages of cylinder tractive power for simple engines for these two multiples of M are 78.01 and 82.42, respectively. The higher value is 105.7% of the lower, which shows that, in this case, adding 25% to the rate of coal consumption adds only 5.7 to the cylinder tractive power at 10 m.p.h.

458. Effect of using a better quality of coal. As another instructive variation of the same problem, assume that the coal has effective B.t.u. of 13000, instead of only 11700. It will be found that steam will be produced more rapidly, the M velocity is 6.867 m.p.h. and the horsepower at that velocity is 680.3, but the cylinder power is computed to be 37191 lbs., which is again almost identical with the previous values, although the M velocity is still higher. The multiple for 10 m.p.h. is 1.456 and by Table XXXIX the per cent. of cylinder tractive power is 82.73, which is an increase of 6% over 78.01%, showing that the increase in effective B.t.u. from 11700 to 13000 adds 6% to the cylinder tractive power at 10 m.p.h.

459. Check with approximate rule. Applying Eq. 103 to the above data on the basis that the "effective steam pressure" is 85% of the gauge pressure (185) or 157 lbs., we will have

$$\text{Tractive force} = \frac{22^2 \times 157 \times 28}{57} = 37327 \text{ lbs.}$$

This agrees with the more precise value (37190) computed above to within one-half of one per cent. This rule is more simple as a method of obtaining merely the maximum tractive power at slow velocities, but the previous method, although longer, is preferable, since it computes the critical velocity M, and also the tractive force at higher velocities.

460. Tractive Force at Higher Velocities. At higher velocities than M, the cylinder power falls off quite rapidly, since the steam is cut off at part stroke and is used expansively. The proper per cent of cut-off for any given velocity and the number of pounds of steam per i.h.p. are shown in Table XXXVIII, in which is give the per cent of cylinder tractive power for multiples of M. The table shows, for example, that, for simple engines, the cylinder tractive power is 69.37% of its value for full stroke when the velocity is $2M$ and that when the velocity is increased to $5M$ the tractive power is reduced to 31.11%.

Applying this to the above numerical problem, when $M = 6.167$ m.p.h., the cylinder tractive power is reduced to 31.11% of 37190, or 11570 lbs., but, since the velocity is five times as great, the horse-power developed is $31.11\% \times 5 = 1.55$ times as great. It should be noted that Table XXXIX shows a slight excess of tractive power (6% when starting), for the simple engine. This is due to the fact that with very low velocities the cylinder pressure more nearly equals the full boiler pressure and there is not the usual reduction of about 15%. Also, compound locomotives are operated with all the cylinders using full-pressure steam, which increases their effectiveness at starting about 35%, although at some loss in economy of steam due to compounding. But since the starting resistances are so much greater than the resistances above 5 miles per hour, the extra assistance is very timely.

Any competent locomotive designer will, of course, make a design such that there is a proper relation between cylinder power and tractive adhesion. In the above case, 106% of 37190 = 39421 lbs., which is 25.7% of the weight on the drivers, and this is just about the ratio of adhesion which may be expected.

Velocity.		Cylinder tractive. power		Locomotive resistance pounds.	Draw-bar pull. pounds
Multiples of M.	Miles per hour.	Per cent.	Pounds.		
0.0	0.000	106.00	39421	1762	37659
1.0	6.167	100.00	37190	1771	35419
1.2	7.400	91.53	34040	1776	32264
1.5	9.250	81.37	30261	1783	28478
2.0	12.334	69.37	25799	1800	23999
3.0	18.501	52.78	19629	1847	17782
4.0	24.668	40.10	14913	1913	13000
5.0	30.835	31.11	11570	1999	9571
6.0	37.002	25.34	9424	2104	7320

A graphical illustration of the variation in tractive power and velocity may be obtained by computing first and setting down in tabular form the multiple values of M (6.167); the percentages taken from Table XXXIX, for each multiple of M; the products of each percentage times the tractive force (37190), for M velocity; the locomotive resistance, from Table XXIX, for each velocity; and the net draw-bar pull for each velocity. These several values for cylinder tractive power and for draw-bar pull may be plotted as shown in Fig. 208.

The student should realize that the above values represent the maximum draw-bar pull which the locomotive *can* produce, provided the fire-box is fed with 4000 lbs. of coal per hour. These draw-bar pulls as given will overcome the resistance of a train of some definite weight, at uniform speed, along a straight level track, at the several velocities given. A less weight of train will be drawn somewhat faster; or, it will travel at the same speed by using less coal or by throttling the steam and, perhaps, wasting it at the blow-off. A heavier train could not maintain such speed. While the values given are approximately correct, a variation in the quality of the coal, or in the condition of the

FIG. 208.—TRACTIVE POWER, MIKADO LOCOMOTIVE.

track, or in the firing, or in the management by the engineman, will alter the results materially, and they should not be relied on to give an accurate measure of what can and will be accomplished at all times. But the method is useful and dependable in comparing two types of engines, or, for comparing the operating results of light trains at faster speed or heavier trains at slower speed, using the same engine, or, as shown later, of comparing the operating results of using a certain type of engine on two grades and thus estimating the value of reducing the higher grade.

461. Effect of Grade on Tractive Power. The effect of grade on tractive power is best shown by some numerical computations whose results are plotted in Fig. 209. The cylinder tractive power was computed for three engines of greatly different total weight and power, but which had driving-axle loads nearly identical (about 50750 lbs.), and, therefore, by the Baldwin

Locomotive Works rule, given in § 268, could all be operated on the same kind of track. Using the rule, $\frac{1}{6} \times 50750 \div 300 = 84.5$, which means that the rails should weigh at least 85 lbs. per yard. Making computations for these locomotives, using 12000 B.t.u. coal, similar to those already detailed in §§ 453 *et seq.*, it was found that the cylinder tractive powers of the Pacific, Mikado, and Mallet locomotives were 29718, 33575, 49095 lbs., respectively, when the velocity was uniformly 10 m.p.h. and the locomotives each burned 4000 lbs. of coal per hour. The several engine resistances at 10 m.p.h. are easily computed from Table XXIX and are tabulated below.

Engine characteristics (At velocity $V = 10$ m.p.h.)	Pacific 4–6–2 (lb.)	Mikado 2–8–2 (lb.)	Mallet 2–8–8–2 (lb.)
Cylinder tractive power.......	29,718	33,575	49,095
Engine resistance on level.....	2,205	2,648	4,864
Draw-bar pull on level........	27,513	30,927	44,231
Draw-bar pull on 3% grade....	15,213	18,207	25,631

The net values, or the draw-bar pulls, are plotted on the left-hand vertical line of Fig. 209, and in each case are the left-hand ends of the solid lines which show the tractive powers of the locomotives. On a 3% grade the grade resistances for the locomotives equal 60 lbs. per ton, and are 12300, 12720 and 18600 lbs., respectively. This reduces the effective draw-bar pull approximately 40% in each case. Since this reduction varies uniformly with the grade, we may plot the three values, 15213, 18207 and 25631, on the 3% vertical line and draw straight lines which represent in each case the tractive power of the locomotive at 10 m.p.h. and on any grade within that range.

Assume trains of cars, all averaging 50 tons per car and varying from 10 cars weighing 500 tons to 50 cars weighing 2500 tons. The resistances at 10 m.p.h on a level grade are given by Eq. 121, and may be plotted on the left-hand vertical line of Fig. 209. Grade adds resistance proportional to the grade. For example, on a 0.7% grade the grade resistance per ton is 14 lbs. and for 2500 tons is 35000 lbs. Adding this to 11580, the tractive resistance, we have 46580, which we plot on the 0.7% vertical line. It is indicated by a small circle. Joining the two points gives the resistance line for 2500 tons hauled at 10 m.p.h. The circles on the other lines indicate similar computations. The inter-

sections of these resistance lines with the lines of tractive power indicate the relative power of each locomotive. For example, the 1000-ton train can be hauled by the Pacific locomotive at 10 m.p.h. up a 0.96% grade, but a Mikado can do the same on a 1.1% grade, while the Mallet can do it on a 1.52% grade.

Drawbar pull - pounds

Per cent of grade

FIG. 209.—CURVES SHOWING EFFECT OF GRADE ON TRACTIVE POWER.

All of these calculations were made on the basis of burning 4000 lbs. of coal per hour, which, as before stated, is the practical limit of what an ordinary fireman can be expected to do for an extended run.

The description of the Mallet locomotive (built by the Baldwin Locomotive Works), stated that its tractive power is 91000 lbs. A computation of its cylinder tractive power at M velocity, using 12000 B.t.u. coal, shows it to be 95389 lbs. Subtracting the engine resistance (4843 lbs.), we would have 90546 lbs., which is a very fair check, especially as the Baldwin Locomotive Works method of calculation is different.

462. Acceleration-speed curves. The time required for an engine of given weight and power to haul a train of known weight and resistance over a track with known grades and curvature is an important and necessary matter for an engineer to compute, since the saving in time has such a value as to justify constructive or operating changes which will reduce that time. Fig. 208 shows that the draw-bar pull is very much greater at very low velocities than at the moderate speed of even 15 m.p.h. In spite of the increased resistance at these low velocities the margin of power left for acceleration is also greater and the "speed curve" is really a curve and not a straight line. Its general form may be most easily developed by a numerical example, especially as each case has its own special curve.

Illustrative Example. The Mikado locomotive, whose characteristics have already been investigated in §§ 453 *et seq.*, has draw-bar pulls at various velocities as shown in the tabular form in § 460, to which frequent reference must be made in this demonstration. Assume that this locomotive starts from rest on a 0.4% upgrade, hauling a train of 14 cars, each weighing 50 tons, and a caboose weighing 10 tons. Then the normal level tractive resistance, by Eq. 121, equals

$$R = (2.2 \times 710) + (122 \times 15) = 3392 \text{ lbs.}$$

The grade resistance of the cars will be $20 \times 0.4 \times 710 = 5680$ lbs. The extra starting resistance will be considered as 6 lbs. per ton, or 4260 lbs. These three items total 13332 lbs. The average draw-bar pull of the locomotive at velocities between zero and M velocity, which is 6.167 m.p.h., is $\frac{1}{2}(37659 + 35419) = 36539$ lbs., but this must be diminished in this case by $20 \times 0.4 \times 157.5 = 1260$ lbs. for grade and by $157.5 \times 6 = 945$ lbs. for starting resistance, leaving a net draw-bar pull of 34334 lbs., excluding the force required for the acceleration of the locomotive. The net force available for acceleration of both the locomotive and the train is $34334 - 13332 = 21002$ lbs., or prorated, is $21002 \div (157.5 + 710) = 24.21$ lbs. per ton. Transposing Eq. 106, with $V_1 = O$, $V_2 = 6.167$, and $P = 24.21$ lbs., we have $s = 70(38.03 - 0) \div 24.21 = 110$ feet, the distance required to attain a velocity of 6.167 m.p.h.

While the velocity is increasing from 1.0 M to 1.2 M, the mean draw-bar pull is $\frac{1}{2}(35419 + 32264) - 1260 = 32582$ lbs., less the accelerative resistance of the locomotive. Subtracting the

tractive and grade resistances of the cars, we have 32582 − 3392 − 5680 = 23510 lbs. Note that there is no longer any starting resistance. The accelerative force in pounds per ton is 23510 ÷ 867.5 = 27.10. The distance s required to increase the velocity from 6.167 m.p.h. to 7.400 m.p.h., is 70(54.76 − 38.03) ÷ 27.10 = 43 feet. Similarly the distances required to increase the velocity from 1.2 M to 1.5 M, from 1.5 M to $2M$, etc., are computed as in the accompanying tabular form.

The corresponding distances and velocities have been plotted in Fig. 210. The velocity of 10 m.p.h. is acquired in a little over 300 feet, but it requires 500 feet to acquire a velocity of 12.33 m.p.h. and about 16000 feet to raise it to 29 m.p.h. The force, in pounds per ton, available for acceleration, is maximum at low velocities, after the extra starting resistance is overcome. As the margin per ton for acceleration becomes less and less, the greater is the distance required to increase the velocity 1 mile per hour—especially through the last increments—up to the velocity at which the net draw-bar pull exactly equals the total car resistance and the velocity becomes uniform, which is later computed to be 4.78 M. There is an approximation in using *average* draw-bar pulls between the different velocities at which the draw-bar pull has been definitely computed, but the computed distances are practically correct up to 4 M velocity or 24.67 m.p.h. But the computation for the distance required to increase the velocity from 4 M up to 4.78 M is far less accurate if the average draw-bar pull is used. The effective pull at 4 M velocity equals 13000 − 1260 = 11740, less the accelerative resistance of the locomotive. The tractive and grade resistance of the cars at this velocity is 3392 + 5680 = 9072. This leaves 11740 − 9072 = 2668 lbs. available for acceleration of both locomotive and cars. The reduction in tractive force between 4 M velocity and 5 M velocity (see § 460), is 13000 − 9571 = 3429 lbs. By proportionate interpolation we would then say that the excess force available for acceleration would be exhausted at (2668 ÷ 3429) = .78 of the interval, or at a velocity of 4.78 M, or 29.48 m.p.h. The mean accelerative force is one-half of 2668, or 1334 lbs., which is 1.53 lbs. per ton of train. The distance, by an inversion of Eq. 106, is computed to be 11925 feet. Owing to the approximate equality of working force and resistance and the momentary variations in both, the precise point where the acceleration would cease and the velocity would

DATA AND COMPUTATIONS FOR ACCELERATION AND RETARDATON CURVES.

	Velocities.				Tractive Forces.						Distances.		Time.
	Feet per sec.	Range, miles per hour.		Mean, feet per sec.	Mean draw-bar pull, level, lbs.	Loco-motive resist-ance, grade plus start* lbs.	Actual draw-bar pull, average, lbs.	Car re-sistance tractive grade, plus start* lbs.	Differ-ence ef-fective for ac-celera-tion or retarda-tion. lbs.	Net force per ton, lbs.	Accel-eration, or re-tarda-tion, feet	Total from start] feet.	sec.
Acceleration......	0.00	0.00	6.167	4.52	36539	*2205	34334	*13332	21002	24.21	110	110	24
	9.04	6.167	7.40	9.95	33842	1260	32582	9072	23510	27.10	43	153	4
	10.86	7.40	9.25	12.22	30371	1260	29111	9072	20039	23.10	93	246	8
	13.57	9.25	12.33	15.83	26239	1260	24979	9072	15907	18.34	254	500	16
	18.09	12.33	18.50	22.61	20891	1260	19631	9072	10559	12.17	1094	1594	48
	27.13	18.50	24.67	31.66	15391	1260	14131	9072	5059	5.83	3196	4790	101
	36.18	24.67	29.48	39.71	11666	1260	10406	9072	1334	1.53	11925	16715	300
Retardation......	43.24	29.48	24.67	39.71	11662	3780	7882	20432	12550	14.46	1262	1262	32
	36.18	24.67	18.50	31.66	15391	3780	11611	20432	8821	10.17	1832	3094	58
	27.13	18.50	12.33	22.61	20891	3780	17111	20432	3321	3.83	3477	6571	154
	18.09	12.33	12.21	17.99	24106	3780	20326	20432	106	0.122	1681	8252	93

* The extra starting resistance only applies to the first item.

actually become uniform would be be very uncertain. Fortunately the inaccuracy is of little or no practical importance and for the purposes of our calculations we may call this last interval 11925 feet, assuming that the grade is as long as 16715 feet or 3.1 miles. If the 0.4% grade continued indefinitely the train would travel at this uniform velocity as long as the locomotive operated on the basis assumed for this problem. Note that Fig. 210 would have to be extended to nearly three times its

DISTANCES IN FEET

Fig. 210.

present length before the time curve would reach and become tangent to the " line of uniform velocity."

463. **Retardation-speed curves.** When, on account of grade resistance, the total of tractive and grade resistance is greater than the draw-bar pull, there is retardation.

Illustrative Example. Continuing the numerical problem of § 462, assume that, while moving up the 0.4% grade at a velocity of 4.78 M, or 29.48 m.p.h., the train reaches a grade of $+1.2\%$. The grade resistance of the cars will be $20\times1.2\times710=17040$ lbs. The tractive resistance will be 3392 lbs., as before, making a total of 20432 lbs. Interpolating in the tabular form in § 460 for the draw-bar pull at 4.78 M velocity, we find 10325; at 4 M it is 13000 and the mean is 11662; but from this must be subtracted $20\times1.2\times157.5=3780$ for grade resistance of the locomotive, leaving 7882 lbs. for the net draw-bar pull. The retarding force is $20432-7882=12550$; or in pounds per ton of train, is $12{,}550\div867.5=14.46$. As before, using an inversion of

Eq. 106, $s=(29.48^2-24.67^2)70 \div 14.46=1262$ feet, the distance at which the velocity would reduce to 4 M. As before, the other quantities may be computed and recorded, with less danger of confusion and error, by tabulating them, as given in § 462.

The mean velocity, when retarding from 4.78 M to 4.0 M, reduced to feet per second, is as before 39.71 feet per second, and dividing this into the distance, 1262 feet, gives 32, the time in seconds. The quantities for the reduction in velocity from 4 M to 3 M and from 3 M to 2 M are computed similarly. The level draw-bar pull for 1.5 M is 28478 (see § 460), and by subtracting 3780, we get 24698 lbs. the actual net pull on the grade. Similarly, the actual pull at 2 M is 20219 lbs. The increase from 20219 to 20432 is $\frac{213}{4479}=4.7\%$ of the interval from 20219 to 24698 and $4.7\% \times .5=.02$; therefore, the actual draw-bar pull just equals the resistance at $2.00-.02=1.98M$, or 12.21 m.p.h. The deficiency of draw-bar pull at $2.0\,M=20{,}432-20219=213$ lbs. At 1.98 M the deficiency is zero and, therefore, the mean deficiency is one-half of 213, or 106. Dividing this by 867.5, we have 0.122, which is the value of P in Eq. 106. Then

$$s=(152.01-149.08)70 \div 0.122=1681 \text{ ft.}$$

Velocities in miles per hour can be readily converted into velocities in feet per second by multiplying by 1.4667. Averaging the two velocities at the beginning and the end of each period gives the mean velocity; and dividing each of these into the distance for that period gives the time in seconds.

464. Drifting. The tractive resistance of the cars of the problem just worked out is 3392 lbs.; the locomotive resistance at 20 m.p.h. is 1862 lbs., or a total of 5254 lbs. Variation in velocity will affect this but little. Dividing by 867.5, the total weight in tons, we have 6.06 lbs., the resistance per ton, from which the equivalent rate of grade is $6.06 \div 20=.303\%$. This means practically that when this train is running *down* a grade which is over .303% it will run by gravity and steam may be shut off. If the grade is much greater than .303% the acceleration on the downgrade may become so great, if the grade is very long, that the velocity may become objectionably high.

Illustrative Example. Assume that the limiting safe velocity for freight trains, considering the condition of track and rolling

stock, is 35 m.p.h.; assume that the train we have been consider- reaches a 0.4% downgrade at a velocity of 15 m.p.h. How far down the grade will it run with steam shut off, before the speed reaches 35 m.p.h. and brakes must be applied? There is no question here of variable tractive power since the only motive power is gravity. The resistance is nearly independent of velocity and we will here assume it to be so and utilize Table XLII. At 15 m.p.h. the train has a velocity head of 7.90 feet. At 35 m.p.h. the velocity head is 43.01 feet. The train can, therefore, drop down the grade a vertical height of $43.01-7.90=35.11$ feet before the velocity reaches 35 m.p.h. On a 0.4% grade the distance required for such a fall is $35.11 \div .004 = 8777$ feet. The problem in § 462 assumed that the 0.4% grade is 16715 feet or more, and this shows what will happen to the trains moving in the opposite direction.

But it must not be thought that there is no loss of energy during drifting. Even though no steam is used in the cylinders, some is frequently wasted at the safety valve and more is used in operating brakes and in maintaining the brake air-reservoir at full pressure. But the greatest loss of heat is that due to radiation, especially in winter, in spite of all the jacketing devices to retain heat. Although the results of the numerous tests which have been made are quite variable, the following approximate averages may be used: The loss due to radiation while standing may be figured at 120 lbs. of coal per hour per 1000 square feet of heating surface; while drifting the loss will increase to 220 lbs. per hour. The amount of coal used for firing up will be about 510. This is based on the use of 12000 B.t.u. coal. The better the coal, the less will be used.

Illustrative Example. The Mikado locomotive we have been considering has 2565 square feet of heating surface. It will then require about $2.565 \times 510 = 1308$ lbs. of coal to fire up. While drifting down the grade, referred to above, a distance of 8777 feet, the average velocity is $\frac{1}{2}(15+35) = 25$ m.p.h. $= 36.67$ ft. per sec. and the required time is $8777 \div 36.67 = 239$ seconds $= 3$ min. 59 sec. $= .066$ hour. The coal used while drifting down this short run would be

$$220 \times 2.565 \times .066 = 37 \text{ lbs.}$$

At this point brakes would need to be applied and the time spent in drifting beyond this point must be computed as an item

in the total time spent on the run and also to compute the total amount of coal consumed while drifting. Although this item of 37 lbs. is relatively very small, its method of computation is typical of the computation of the several items to make up the total of coal consumed during a trip.

465. **Review of computed power of one locomotive.** It was assumed that it started on a +0.4% grade with a load of 15 cars weighing 710 tons. After moving 16715 feet (assuming that the grade was that long), and doing it in 493 seconds, or 8 minutes 13 seconds, the train acquired a velocity of 29.48 m.p.h. and the power of the locomotive would then be sufficient, when burning 4000 lbs. of coal per hour, to keep it moving up such a grade indefinitely at that velocity. In case the grade were not as long as 16715 feet, it would be necessary to compute the velocity where the rate of grade changed and make that the basis for the computation on the succeeding grade. But, assuming that the grade were as long as 16715 feet, or more, and that the velocity of 29.48 m.p.h. had been acquired, and that the train had run at that speed for some distance—although this does not modify the problem—the train is assumed to reach a still steeper grade +1.2%. The velocity then begins to decrease and in a total distance of 8252 feet and a total time of 337 seconds, or 5 minutes 37 seconds, the velocity is reduced to 12.21 m.p.h., at which velocity the locomotive is able to make steam fast enough to overcome the higher resistance on the steeper grade. From that point on, assuming that the 1.2% grade is longer than 8252 feet, the train would continue for the remaining length of that grade at the velocity of 12.21 m.p.h.

As before stated, precision in the above results depends on many factors (such as B.t.u. of coal used, or the actual consumption in pounds per hour), which are somewhat variable. Sometimes the variation of these factors from the values used above is known; sometimes it is unknown and then the accuracy of the results is correspondingly uncertain. But whether accurately known or not, when this method is used, employing the best values for the factors which are obtainable, the method shows a valuable *comparison* of two proposed alinements or grades. In such a comparison, any error in the factors will affect both results nearly, if not quite, equally, and the comparative results will still be substantially correct.

466. Selection of route. The preceding articles may be utilized in comparing two routes. If one of the lines is already in operation, the engineer has the great advantage of being able to determine by test exactly what results may be obtained on that line and what factors should be used in computations. It is then only necessary to compute the quantities for the proposed new line. When both lines are "on paper" there is less certainty as to the accuracy of the results, except that the line which is shown to be most advantageous will probably continue to be most advantageous even if the uncertain factors used in the comparison are somewhat changed. Using the methods outlined in §§ 462 to 464, there will be computed the behavior of an assumed type of locomotive, hauling one or more types of train load, and passing over tracks having definite grades and lengths. The effect of curves may be disregarded provided that the grades were properly compensated during original construction, and then the rate of grade for the entire length of straight and curved track may be taken as the rate on the straight track. If the rate of grade is actually uniform, even through the curves, then the lengths of curved track must be computed separately and on the basis of a rate of grade equal to the actual rate plus an allowance of .035% for each degree of curve. The behavior of a train from starting to stopping must be computed, making due allowance for each change in condition which will affect the hauling power of the locomotive. The locomotive is assumed to be working at the limit of its steaming capacity, except when drifting with steam shut off on a down grade, or when brakes are applied, either to prevent objectionably high velocity on a down grade or to make a stop. The action of brakes during a service stop (as distinguished from an emergency stop), may be considered as a retarding force varying from 10% to 20% of the train weight. Unfortunately brake action is so variable, being directly under the control of the locomotive engineer and varying from zero to the full braking power, that any computation of energy used in operating them or of the effect of the brakes is impracticable except on the basis of arbitrary assumptions such as the requirement that the brakes are used in such a way that a train will be retarded at a specified rate. The performance of the locomotive over the entire division, the total time required, its velocity in critical places, etc., can be computed. In §§ 462 and 463 it

was shown that the locomotive considered could haul the particular train considered up a 0.4% grade at a velocity of 29.48 m.p.h. and maintain such speed indefinitely; also that it could haul the same train up a 1.2% grade at 12.21 m.p.h. and maintain its velocity indefinitely. This of course,, means that a much heavier train could be hauled up the 0.4% grade and that a somewhat heavier train could be hauled up the 1.2% grade without being stalled, although the velocities in each case would be reduced. There are an infinite number of combinations, but there are usually some considerations which narrow the choice. Even after construction is complete these tables may be utilized in a study of the most economical combination of type of locomotive and amount of train load for the track conditions as they may exist.

467. Rating of locomotives. The maximum power of a locomotive on any grade at M velocity is measured by its "rating."

Let P = the tractive power of the locomotive, measured at the rim of the drivers;
E = Weight of engine and tender, in pounds;
W = Weight of cars behind tender, in pounds;
r = rate of grade, or the ratio of vertical to horizontal;
a = a constant, which as determined by tests = 2.2 lbs. per ton or .0011 lb. per pound of train;
b = a constant, which as determined by tests = 122 lbs. per ton. a and b are the same constants as are used in § 439.
n = number of cars in train.

Then $P=(E+W)\ (r+a)+bn.$

Transforming,

$$\frac{P}{r+a}-E=W+n\frac{b}{r+a}. \quad . \quad . \quad . \quad . \quad . \qquad (122)$$

The right-hand side of this equation is called the "rating," A, and is the weight of the train behind the tender plus the number of cars times a quantity made up of two constants and the rate of grade. This quantity is independent of any special engine or train values and may be tabulated for various rates of grade, as given in Table XL.

Examples. The Mikado locomotive considered in §§ 453, *et seq.*, has a tractive power, measured at the rim of the drivers,

TABLE XL. LOCOMOTIVE RATING DISCOUNTS.

VALUES OF $C \div (R+K)$ FOR VARIOUS GRADES.

(In tons per car)

Grade R (per cent)	Tons per car $C \div (R+K)$	Grade R (per cent)	Tons per car $C \div (R+K)$	Grade R (per cent)	Tons per car $C \div (R+K)$	Grade R (per cent)	Tons per car $C \div (R+K)$	Grade R (per cent)	Tons per car $C \div (R+K)$
Level	55	0.5	10.0	1.0	5.5	1.5	3.8	2.0	2.88
0.1	29	0.6	8.5	1.1	5.0	1.6	3.6	2.1	2.75
0.2	20	0.7	7.5	1.2	4.6	1.7	3.4	2.2	2.63
0.3	14	0.8	6.7	1.3	4.3	1.8	3.2	2.3	2.52
0.4	12	0.9	6.0	1.4	4.0	1.9	3.0	2.4	2.42

at M velocity, or 6.167 m.p.h., of $37190-1432=35758$ lbs., which equals P; 1432 is the locomotive resistance between cylinder and rim of drivers, see § 429. The weight of engine and tender is 315000 lbs. What is its rating on a 1.2% grade? The value of r for a 1.2% grade $=.012$; $a=.0011$ lb. per pound. Then

$$A=\frac{P}{r+a}-E=\frac{35758}{.012+.0011}-315000=2{,}414{,}000 \text{ lbs.}=1207 \text{ tons},$$

which is the rating for that locomotive for a 1.2% grade. But this does not mean 1207 tons of cars. Placing this equal to the right-hand side of Eq. 122, we have

$$1207=W+n\frac{b}{r+a}.$$

The value of $\frac{r+a}{b}$ for a 1.2% grade is given in Table XL as 4.6.

Then $$W=1207-4.6n,$$

which shows that the weight of train depends on the number of cars. Assume that $n=16$. Then $W=1133.4$ and the average weight per car is 70.8 tons. Assume that the cars are all "empties," weighing 18 tons each; then $W=18n$, and

$$n=1207\div(18+4.6)=53.4,$$

which must be interpreted as 53 empty cars.

In the above examples the pulling power P is determined on the basis of the locomotive working at the maximum velocity M at

which it can maintain full stroke. See § 455. This represents practically the maximum power of the locomotive. The velocity *M* is usually from 4 to 7 miles per hour and is as low as should be allowed on maximum grades, since an attempt to utilize a slightly higher tractive force at a somewhat lower velocity would probably result in stalling the train if an unexpected resistance in the track slightly increased the normal resistance.

CHAPTER XIX.

THE PROMOTION OF RAILROAD PROJECTS.

468. Method of formation of railroad corporations. Many business enterprises, especially the smaller ones, are financed entirely by the use of money which is put into them directly in the form of stock or mere partnership interest. A railroad enterprise is frequently floated with a comparatively small financial expenditure on the part of the original promoters. The promoters become convinced that a railroad between *A* and *B*, passing through the intermediate towns of *C* and *D*, with others of less importance, will be a paying investment. They organize a company, have surveys made, obtain a charter, and then, being still better able (on account of the additional information obtained) to exploit the financial advantages of their scheme, they issue a prospectus and invite subscriptions to bonds. Sometimes a portion of these bonds are guaranteed, principal and interest, or perhaps the principal alone, by townships or by the national government. The cost of this preliminary work, although large in gross amount if the road is extensive, is yet but an insignificant proportion of the total amount involved. The proportionate amount that *can* be raised by means of bonds varies with the circumstances. In the early history of railroad building, when a road was projected into a new country where the traffic possibilities were great and there was absolutely no competition, the financial success of the enterprise would seem so assured that no difficulty would be experienced in raising from the sale of bonds all the money necessary to construct and equip the road. But the promoters (or stockholders) must furnish all money for the preliminary expenses, and must make up all deficiencies between the proceeds of the sale of the bonds and the capital needed for construction.

"In theory, stocks represent the property of the responsible owners of the road, and bonds are an encumbrance on that

property. According to this theory, a railroad enterprise should begin with an issue of stock somewhere near the value of the property to be created and no more bonds should be issued than are absolutely necessary to complete the enterprise. Now it is not denied that there are instances in which this theory is followed out. In New England, for example, as well as in some of the Southern States, there are a few roads represented wholly by stock or very lightly mortgaged. But this theory does not conform to the general history of railway construction in the United States, nor is it supported by the figures that appear in the summary. The truth is, railroads are built on borrowed capital, and the amount of stock that is issued represents in the majority of cases the difference between the actual cost of the undertaking and the confidence of the public expressed by the amount of bonds it is willing to absorb in the ultimate success of the venture." *

"The same general law obtains and has always obtained throughout the world, that such properties (as railways) are always built on borrowed money up to the limit of what is regarded as the positive and certain minimum value. The risk only—the dubious margin which is dependent upon sagacity, skill, and good management—is assumed and held by the company proper who control and manage the property." †

469. **The two classes of financial interests—the security and profits of each.** From the above it may be seen that stocks, bonds, car-trust obligations, and even current liabilities represent railroad capital. The issue of the bonds "was one means of collecting the capital necessary to create the property against which the mortgage lies." The variation between these interests lies chiefly in the security and profits of each. The current liabilities are either discharged or, as frequently happens, they accumulate until they are funded and thus become a definite part of the railroad capital.

The growth of this tendency is shown in the following tabular form (see next page):

The bonded interest has greater security than the stock, but less profit. The interest on the bonds must be paid before any money can be disbursed as dividends. If the bond interest

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* Henry C. Adams, Statistician, U. S. Int. Con. Commission.

† A. M. Wellington, Economic Theory of Railway Location

Capitalization of Railroads in the United States.	June 30, 1888.		June 30, 1898.		June 30, 1912.	
	Amount, millions.	Per cent.	Amount, millions.	Per cent.	Amount, millions.	Per cent.
Stocks	3864	47.5	5311	44.6	8622	43.7
Funded debt	3869	47.6	5510	46.3	11130	56.3
Current liabilities, etc.	396	4.9	1087	9.1		

is not paid, a receivership, and perhaps a foreclosure and sale of the road, is a probability, and in such case the stockholder's interests are frequently wiped out altogether. The bondholder's real profit is frequently very different from his nominal profit. He sometimes buys the bonds at a very considerable discount, which modifies the rate which the interest received bears to the amount really invested. Even the bondholder's security may suffer if his mortgage is a second (or fifth) mortgage, and the foreclosure sale fails to net sufficient to satisfy all previous claims.

On the other hand, the stockholder, who may have paid in but a small proportion of his subscription, *may*, if the venture is successful, receive a dividend which equals 50 or 100% of the money actually paid in, or, as before stated, his entire holdings may be entirely wiped out by a foreclosure sale. When the road is a great success and the dividends very large, additional issues of stock are generally made, which are distributed to the stockholders in proportion to their holdings, either gratuitously or at rates which give the stockholders a large advantage over outsiders. This is the process known as "watering." While it may sometimes be considered as a legitimate "salting down" of profits, it is frequently a cover for dishonest manipulation of the money market.

For the twelve years between 1887 and 1899 about *two thirds* of all the railroad stock in the United States paid *no* dividends, while of those that paid dividends the average rate varied from 4.96 to 5.74%. The year from June 30, 1898, to June 30, 1899, was the most prosperous year of the group, and yet nearly 60% of all railroad stock paid *no* dividend, and the average rate paid by those which paid at all was 4.96%. The total amount distributed in dividends was greater than ever before, but the average rate is the least of the above group because many roads, which had passed their dividends for many previous

years, distinguished themselves by declaring a dividend, even though small. During that same period but 13.35% of the stock paid over 6% interest. The total dividends paid amounted to but 2.01% of all the capital stock, while investments ordinarily are expected to yield from 4 to 6% (or more) according to the risk. Of course the effect of "watering" stock is to decrease the nominal rate of dividends, but there is no dodging the fact that, watered or not, even in that year of "good times," about 60% of all the stock paid *no* dividends. Unfortunately there are no accurate statistics showing how much of the stock of railroads represents actual paid-in capital and how much is "water." The great complication of railroad finances and the dishonest manipulation to which the finances of some railroads have been subjected would render such a computation practically worthless and hopelessly unreliable now.

During the year ending June 30, 1898 (which may in general be considered as a sample), 15.82% of the funded debt paid no interest. About one third of the funded debt paid between 4 and 5% interest, which is about the average which is paid.

The income from railroads (both interest on bonds and dividends on stock) may be shown graphically by diagrams, such as are given in the annual reports of the Interstate Commerce Commission. They show that while railroad investments are occasionally very profitable, the average return is less than that of ordinary investments *to the investors*. The *indirect* value of railroads in building up a section of country is almost incalculable and is worth many times the cost of the roads. It is a discouraging fact that very few railroads (old enough to have a history) have escaped the experience of a receivership, with the usual financial loss to the then stockholders. But there is probably not a railroad in existence which, however much a financial failure in itself, has not profited the community more than its cost.

470. The small margin between profit and loss to projectors. When a railroad is built entirely from the funds furnished by its promoters (or from the sale of stock) it will generally be a paying investment, although the rate of payment may be very small. The percentage of receipts that is demanded for actual operating expenses is usually about 67%. The remainder will usually pay a reasonable interest on the total capital involved. But the operating expenses are frequently 90 and even 100% of

the gross receipts. In such cases even the bondholders do not get their due and the stockholders have absolutely nothing. Therefore the stockholder's interest is very speculative. A comparatively small change in the business done (as is illustrated numerically in § 472) will not only wipe out altogether the dividend—taken from the last small percentage of the total receipts and which may equal 50% or more of the capital stock *actually paid in*—but it may even endanger the bondholders' security and cause them to foreclose their mortgage. In such a case the stockholders' interest is usually entirely lost. It does not alter the essential character of the above-stated relations that the stockholders sometimes protect themselves somewhat by buying bonds. By so doing they simply decrease their risk and also decrease the possible profit that might result from the investment of a given total amount of capital.

471. Extent to which a railroad is a monopoly. It is a popular fallacy that a railroad, when not subject to the direct competition of another road, has an absolute monopoly—that it controls "all the traffic there is" and that its income will be practically independent of the facilities afforded to the public. The growth of railroad traffic, like the use of the so-called necessities or luxuries of life, depends entirely on the supply and the cost (in money or effort) to obtain it. A large part of railroad traffic belongs to the unnecessary class—such as traveling for pleasure. Such traffic is very largely affected by mere matters of convenience, such as well-built stations, convenient terminals, smooth track, etc. The freight traffic is very largely dependent on the possibility of delivering manufactured articles or produce at the markets so that the *total* cost of production and transportation shall not exceed the total cost in that same market of similar articles obtained elsewhere. The creation of facilities so that a factory or mine may successfully compete with other factories or mines will develop such traffic. The receipts from such a traffic may render it possible to still further develop facilities which will in return encourage further business. On the other hand, even the partial withdrawal of such facilities may render it impossible for the factory or mine to compete successfully with rivals; the traffic furnished by them is completely cut off and the railroad (and indirectly the whole community) suffers correspondingly. The "strictly necessary" traffic is thus so small that few railroads could pay

their operating expenses from it. The dividends of a road come from the last comparatively small percentage of its revenue, and such revenue comes from the "unnecessary" traffic which must be coaxed and which is so easily affected by apparently insignificant "conveniences."

472. Profit resulting from an increase in business done; loss resulting from a decrease. In a subsequent chapter it will be shown that a large portion of the operating expenses are independent of small fluctuations in the business done and that the operating expenses are roughly two thirds of the gross revenue. Assume that by changes in the alinement the business obtained has been increased (or diminished) 10%. Assume for simplicity that the operating expenses on the revised track are the same as on the route originally planned; also that the cost of the track is the same and hence the fixed charges are assumed to be constant for all the cases considered. Assume the fixed charges to be 28%. The additional business, when carried in cars otherwise but partly filled will hardly increase the operating expenses by a measurable amount. When extra cars or extra trains are required, the cost will increase up to about 60% of the average cost per train mile. We may say that 10% increase may in general be carried at a rate of 40% of the average cost of the traffic. A reduction of 10% in traffic may be assumed to reduce expenses a similar amount. The effect of the change in business will therefore be as follows:

	Business increased 10%.	Business decreased 10%.
Operating exp. = 67	67(1 + 10% × 40%) = 69.68	67(1 − 10% × 40%) = 64.32
Fixed charges = 28	.... 28.00	.... 28.00
95	97.68	92.32
Total income... 100	Income............110.00	Income............ 90.00
Available for dividends....... 5	Available for dividends.......... 12.32	Deficit............ 2.32

In the one case the increase in business, which may often be obtained by judicious changes in the alinement or even by better management without changing the alinement, more than doubles the amount available for dividends. In the other case the profits are gone, and there is an absolute deficit. The above is a numerical illustration of the argument, previously

stated, of the small margin between profit and loss to the original projectors.

473. Estimation of probable volume of traffic and of probable growth. Since traffic and traffic facilities are mutually interdependent and since a large part of the normal traffic is merely potential until the road is built, it follows that the traffic of a road will not attain its normal volume until a considerable time after it is opened for operation. But the estimation even of this normal volume is a very uncertain problem. The estimate may be approached in three ways:

1st. The actual gross revenue derived by all the railroads in that section of the country (as determined by State or U. S. Gov. reports) may be divided by the total population of the section and thus the average annual expenditure per head of population may be determined. A determination of this value for each one of a series of years will give an idea of the normal rate of growth of the traffic. Multiplying this annual contribution by the population which may be considered as tributary gives *a valuation* of the possible traffic. Such an estimate is unreliable (*a*) because the *average* annual contribution may not fit that particular locality, (*b*) because it is very difficult to correctly estimate the number of the true tributary population especially when other railroads encroach more or less into the territory. Since a rough value of this sort may be readily determined, it has its value as a check, if for nothing else.

2d. The actual revenue obtained by some road whose circumstances are as nearly as possible identical with the road to be considered may be computed. The weak point consists in the assumption that the character of the two roads is identical or in incorrectly estimating the allowance to be made for observed differences. The method of course has its value as a check.

3d. A laborious calculation may be made from an actual study of the route—determining the possible output of all factories, mines, etc., the amount of farm produce and of lumber that might be shipped, with an estimate of probable passenger traffic based on that of like towns similarly situated. This method is the best when it is properly done, but there is always the danger of leaving out sources of income—both existent and that to be developed by traffic facilities, or, on the other hand, of overestimating the value of expected traffic. In the

following tabular form are shown the population, gross receipts, receipts per head of population, mileage, earnings per mile of line operated, and mileage per 10,000 of population for the whole United States. It should be noted that the values are only *averages*, that individual variations are large, and that only a very rough dependence may be placed on them as applied to any particular case.

Year.	Population (estimated).	Gross receipts.	Receipts per head of population.	Mileage†	Earnings per mile of line operated.	Mileage per 10,000 population.‡
1888...	60,100,000	$910,621,220	$15.15	136,884	$6653	24.94
1889...	61,450,000	964,816,129	15.81	153,385	6290	25.67
1890...	*62,801,571	1051,877,632	16.75	156,404	6725	26.05
1891...	64,150,000	1096,761,395	17.10	161,275	6801	26.28
1892...	65,500,000	1171,407,343	17.89	162,397	7213	26.19
1893...	68,850,000	1220,751,874	18.26	169,780	7190	26.40
1894...	68,200,000	1073,361,797	15.74	175,691	6109	26.20
1895...	69,550,000	1075,371,462	15.46	177,746	6050	25.97
1896...	70,900,000	1150,169,376	16.22	181,983	6320	25.78
1897...	72,350,000	1122,089,773	15.53	183,284	6122	25.53
1898...	73,600,000	1247,325,621	16.95	184,648	6755	25.32
1899...	74,950,000	1313,610,118	17.53	187,535	7005	25.25
1900...	*76,295,220	1487,044,814	19.49	192,556	7722	25.44
1901...	77,863,000	1588,526,037	20.47	195,562	8123	25.52
1902...	79,431,000	1726,380,267	21.88	200,155	8625	25.76
1903...	80,998,000	1900,846,907	23.70	205,314	9258	26.03
1904...	82,566,000	1975,174,091	24.23	212,243	9306	26.34
1905...	84,134,000	2082,482,406	25.15	216,974	9508	26.44
1906...	85,701,000	2325,765,167	27.65	222,340	10460	26.78
1907...	87,279,000	2589,105,578	29.63	227,455	11383	26.38
1908...	88,837,000	2393,805,989	26.95	231,540	10338	26.30
1909...	90,405,000	2418,677,538	26.71	234,800	10301	26.20
1910...	*91,972,266	2750,667,435	29.91	238,609	11528	26.14
1911...	93,572,266	2789,761,669	29.81	244,476	11411	26.10
1912...	95,172,266	2842,695,382	29.87	247,981	11463	25.93

* Actual. † Excludes a small percentage not reporting "gross receipts." ‡ Actual mileage.

The probable growth in traffic, after the traffic has once attained its normal volume, is a small but almost certain quantity. In the above tabular form this is indicated by the gradual growth in "receipts per head of population" from 1897 to 1907. Then the sudden drop due to the panic of 1907 is clearly indicated, and also the gradual growth in the last few years. Even in England, where the population has been nearly stationary for many years, the growth though small is unmistakable. On the other hand the growth in some of the Western States

has been very large. For example, the gross earnings per head of population in the State of Iowa increased from $1.42 in 1862 to $10.00 in 1870, and to $19.46 in 1884.

There will seldom be any justification in building to accommodate a larger business than what is "in sight." Even if it could be anticipated with certainty that a large increase in business would come in ten years, there are many reasons why it would be unwise to build on a scale larger than that required for the business to be immediately handled. Even though it may cost more in the future to provide the added accommodations (*e.g.* larger terminals, engine-houses, etc.), the extra expense will be nearly if not quite offset by the interest saved by avoiding the larger outlay for a period of years which may often prove much longer than was expected. A still more important reason is the avoidance of uselessly sinking money at a time when every cent may be needed to insure the success of the enterprise as a whole.

474. Probable number of trains per day. Increase with growth of traffic. The number of passenger trains per day cannot be determined by dividing the total number of passengers estimated to be carried per day by the capacity of the cars that can be hauled by one engine. There are many small railroads, running three or four passenger trains per day each way, which do not carry as many passengers all told as are carried on one heavy train of a trunk line. But because the bulk of the passenger traffic, especially on such light-traffic roads, is "unnecessary" traffic (see § 471) and must be encouraged and coaxed, the trains must be run much more frequently than mere capacity requires. The minimum number of passenger trains per day on even the lightest-traffic road should be two. These need not necessarily be passenger trains exclusively. They may be mixed trains.

The number required for freight service may be kept more nearly according to the actual tonnage to be moved. At least one local freight will be required, and this is apt to be considerably within the capacity of the engine. Some very light-traffic roads have little else than local freight to handle, and on such there is less chance of economical management. Roads with heavy traffic can load up each engine quite accurately according to its hauling capacity and the resulting economy is great. Fluctuations in traffic are readily allowed for by adding on or drop-

ping off one or more trains. Passenger trains must be run on regular schedule, full or empty. Freight trains are run by train-despatcher's orders. A few freight trains per day may be run on a nominal schedule, but all others will be run as extras. The criterion for an increase in the number of passenger trains is impossible to define by set rules. Since it should always come before it is absolutely demanded by the train capacity being overtaxed, it may be said in general terms that a train should be added when it is believed that the consequent increase in facilities will cause an increase in traffic the value of which will equal or exceed the added expense of the extra train.

475. Effect on traffic of an increase in facilities. The term facilities here includes everything which facilitates the transport of articles from the door of the producer to the door of the consumer. As pointed out before, in many cases of freight transport, the reduction of facilities below a certain point will mean the entire loss of such traffic owing to local inability to successfully compete with more favored localities. Sometimes owing to a lack of facilities a railroad company feels compelled to pay the cartage or to make a corresponding reduction on what would normally be the freight rate. In competitive freight business such a method of procedure is a virtual necessity in order to retain even a respectable share of the business. Even though the railroad has no direct competitor, it must if possible enable its customers to meet their competitors on even terms. In passenger business the effect of facilities is perhaps even more marked. The pleasure travel will be largely cut down if not destroyed.

476. Loss caused by inconvenient terminals and by stations far removed from business centers. This is but a special case of the subject discussed just in the preceding paragraph. The competition once existing between the West Shore and the New York Central was hopeless for the West Shore from the start. The possession of a terminal at the Grand Central Station gave the New York Central an advantage over the West Shore with its inconvenient terminal at Weehawken which could not be compensated by any obtainable advantage by the West Shore. This is especially true of the passenger business. The through freight business passing through or terminating at New York is handled so generally by means of floats that the disadvantage in this respect is not so great. The

enormous expenditure (roughly $10,000,000) made by the Pennsylvania R. R., on the Broad Street Station (and its approaches) in Philadelphia, a large part of which was made in crossing the Schuylkill River and running to City Hall Square, rather than retain their terminal in West Philadelphia, is an illustration of the policy of a great road on such a question. The fact that the original plan and expenditure has been very largely increased since the first construction proves that the management has not only approved the original large outlay, but saw the wisdom of making a very large increase in the expenditure.

The construction of great terminals is comparatively infrequent and seldom concerns the majority of engineers. But an engineer has frequently to consider the question of the location of a way station with reference to the business center of the town. The following points may (or may not) have to be considered, and the real question consists in striking a proper balance between conflicting considerations.

(1) During the early history of a railroad enterprise it is especially needful to avoid or at least postpone all expenditures which are not demonstrably justifiable.

(2) The ideal place for a railroad station is a location immediately contiguous to the business center of the town. The location of the station even one fourth of a mile from this may result in a loss of business. Increase this distance to one mile and the loss is very serious. Increase it to five miles and the loss approaches 100%.

(3) The cost of the ideal location and the necessary right of way may be a very large sum of money for the new enterprise. On the other hand the increase in property values and in the general prosperity of the town, caused by the railroad itself, will so enhance the value of a more convenient location that its cost at some future time will generally be extravagant if not absolutely prohibitory. The original location is therefore under ordinary conditions a finality.

(4) To some extent the railroad will cause a movement of the business center toward it, especially in the establishment of new business, factories, etc., but the disadvantages caused to business already established is permanent.

(5) In any attempt to compute the loss resulting from a location at a given distance from the business center it must be

recognized that each problem is distinct in itself and that any change or growth in the business of the town changes the amount of this loss.

The argument for locating the station at some distance from the center of the town may be based on (*a*) the cost of right of way, thus involving the question of a large initial outlay, (*b*) the cost of very expensive construction (*e.g.* bridges), again involving a large initial outlay, (*c*) the avoidance of excessive grade into and out of the town. It sometimes happens that a railroad is following a line which would naturally cause it to pass at a considerable elevation above (rarely below) the town. In this case there is to be considered not only the possible greater initial cost, but the even more important increase in operating cost due to the introduction of a very heavy grade. The loss of business due to inconvenient location can only be guessed at. Wellington says that at a distance of one mile the loss would average 25%, with upper and lower limits of 10 and 40%, depending on the keenness of the competition and other modifying circumstances. For each additional mile reduce 25% of the preceding value. While such estimates are grossly approximate, yet with the aid of sound judgment they are better than nothing and may be used to check gross errors.

477. General principles which should govern the expenditure of money for railroad purposes. It will be shown later that the elimination of grade, curvature, and distance have a positive money value; that the reduction of ruling grade is of far greater value; that the creation of facilities for the handling of a large traffic is of the highest importance and yet the added cost of these improvements is sometimes a large percentage of the cost of *some road* over which it would be physically possible to run trains between the termini.

The subsequent chapters will be largely devoted to a discussion of the value of these details, but the general principles governing the expenditure of money for such purposes may be stated as follows:

1. No money should be spent (beyond the unavoidable minimum) unless it may be shown that the addition is in itself a profitable investment. The additional sum may not wreck the enterprise and it may add something to the value of the road, but unless it adds more than the improvement costs it is not justifiable.

2. If it may be positively demonstrated that an improvement will be more valuable to the road than its cost, it should certainly be made even if the required capital is obtained with difficulty. This is all the more necessary if the neglect to do so will permanently hamper the road with an operating disadvantage which will only grow worse as the traffic increases.

3. This last principle has two exceptions: (*a*) the cost of the improvement may wreck the whole enterprise and cause a total loss to the original investors. For, unless the original promoters can build the road and operate it until its stock has a market value and the road is beyond immediate danger of a receivership, they are apt to lose the most if not all of their investment; (*b*) an improvement which is very costly although unquestionably wise may often be postponed by means of a cheap temporary construction. Cases in point are found at many of the changes of alinement of the Pennsylvania R. R., the N. Y., N. H. & H. R. R., and many others. While some of the cases indicate faulty original construction, at many of the places the original construction was wise, considering the then scanty traffic, and now the improvement is wise considering the great traffic.

478. Study of railroad economics—its nature and limitations. The multiplicity of the elements involved in most problems in railroad construction preclude the possibility of a solution which is demonstrably perfect. Barring out the comparatively few cases in this country where it is difficult to obtain *any* practicable location, it may be said that a comparatively low order of talent will suffice to locate anywhere a railroad over which it is physically possible to run trains. It may be very badly located for obtaining business, the ruling grades may be excessive, the alinement may be very bad, and the road may be a hopeless financial failure, and yet trains can be run. Among the infinite number of possible locations of the road, the engineer must determine the route which will give the best railroad property for the least expenditure of money—the road whose earning capacity is so great that after paying the operating expenses and interest on the bonds the surplus available for dividends or improvements is a maximum.

An unfortunate part of the problem is that even the blunders are not always readily apparent nor their magnitude. A defective dam or bridge will give way and every one realizes the

failure, but a badly located railroad affects chiefly the finances of the enterprise by a series of leaks which are only perceptible and demonstrable by an expert, and even he can only say that certain changes would probably have a certain financial value.

479. Outline of the engineer's duties. The engineer must realize at the outset the nature and value of the conflicting interests which are involved in variable amount in each possible route.

(a) **The maximum of business must be obtained,** and yet it *may* happen that some of the business may only be obtained by an extravagant expenditure in building the line or by building a line very expensive to operate.

(b) **The ruling grades should be kept low,** and yet this *may* require a sacrifice in business obtained and also *may* cost more than it is worth.

(c) **The alinement should be made as favorable as possible;** favorable alinement reduces the future operating expenses, but it may require a very large immediate outlay.

(d) **The total cost** must be kept within the amount at which the earnings will make it a profitable investment.

(e) **The road must be completed and operated** until the "normal" traffic is obtained and the road is self-supporting without exhausting the capital obtainable by the projectors; for no matter how valuable the property may ultimately become, the projectors will lose nearly, if not quite, all they have invested if they lose control of the enterprise before it becomes a paying investment.

Each new route suggested makes a new combination of the above conflicting elements. The engineer must select a route by first eliminating all lines which are manifestly impracticable and then gradually narrowing the choice to the best routes whose advantages are so nearly equal that a closer detailed comparison is necessary.

The ruling grade and the details of alinement have a large influence on the operating expenses. A large part of this course of instruction therefore consists of a study of operating expenses under average normal conditions, and then a study of the effect on operating expenses of given changes in the alinement.

CHAPTER XX.

OPERATING EXPENSES.

480. Distribution of gross revenue. When a railroad comprises but one single property, owned and operated by itself, the distribution of the gross revenue is a comparatively simple matter. The operating expenses then absorb about two thirds of the gross revenue; the fixed charges (chiefly the interest on the bonds) require about 25 or 30% more, leaving perhaps 3 to 8% (more or less) available for dividends. The report on the Fitchburg R. R. for 1898 shows the following:

Operating expenses	$5,083,571	69.1%
Fixed charges	1,567,640	21.3%
Available for dividends, surplus, or permanent improvements	708,259	9.6%
Total revenue	$7,359,470	100.0%

But the financial statements of a large majority of the railroad corporations are by no means so simple. The great consolidations and reorganizations of recent years have been effected by an exceedingly complicated system of leases and sub-leases, purchases, "mergers," etc., whose forms are various. Railroads in their corporate capacity frequently own stocks and bonds of other corporations (railroad properties and otherwise) and receive, as part of their income, the dividends (or bond interest) from the investments.

The Interstate Commerce Commission annually makes a report of the income and profit-and-loss account of all the railroads of the United States, considered as one system. For example, the statement for the year 1912 includes the following items. Operating revenues from rail operations $2,842,695,382; operating expenses due to rail operations $1,972,415,776, which is 69.4%. Interest on funded debt used up 13.9% of the revenues, and taxes 4.2%. There were other miscellaneous incomes and expenditures which caused a net loss of another 2.0%

of revenue, leaving 10.5% or $299,361,208 which were issued as dividends. These dividends are about 3.4% of the outstanding stock. The percentage to the amount of money actually paid for the stock is unknown and unknowable.

481. Operating expenses per train-mile. The uniformity in the average operating expenses per train mile for light-traffic and heavy-traffic roads and for long and short roads is very remarkable. This is illustrated by a comparison of figures for ten heavy traffic roads and ten small roads selected *at random*, except that each had a mileage of less than 100 miles.

OPERATING EXPENSES PER TRAIN-MILE ON LARGE AND SMALL ROADS (1904 AND 1910).

	Mileage.		Operating expenses per train-mile.		Ratio expenses to earnings per cent.	
	1904.	1910.	1904.	1910.	1904.	1910.
Whole United States ...	220,112	240,439	1.314	1.489	67.79	66 29
Canadian Pacific	8,332	10,271	1 320	1.504	68 72	65 41
C., B. & Q.	8,326	9,040	1 313	1.710	64.35	71 71
Chicago & Northwestern	7,412	7,629	1 136	1 306	66 61	70.31
Southern Railway	7,197	7,050	1.048	1.234	70 30	67 43
C., R. I. & P	6,761	7,396	1 199	1 344	72 90	73.07
Northern Pacific	5,619	6,189	1.302	1 824	52 26	61.71
A., T. & S. F	5,031	7,460	1.305	1 626	60 05	64 33
Great Northern	4,489	7,147	1.464	1.808	49.72	60.53
Illinois Central	4,374	4,551	1.107	1.409	70 02	74.84
Atlantic Coast Line	4,229	4,491	0.984	1.213	58.95	62.44
Average of ten	.....	.....	1.227	1.498	63.39	67.18
Montpelier & Wells River	44	50	1 169	1.430	80 73	75 08
Somerset Railway Co.*	42	94	0.802	1.314	59.37	76.65
Huntingdon & Broadtop Mountain	66	70	0.950	2 052	52.10	96 40
Lehigh & New England	96	170	0.793	2.045	69 80	62.84
Ligonier Valley	11	16	1.427	1.480	69 33	49 15
Newburgh, Dutchess & Connecticut †	59	....	0.922	....	85.09	....
Susquehanna & New York	55	80	1.368	1.028	78.47	77.81
Detroit & Charlevoix	51	51	1.424	1.010	67.52	99.53
Harriman & Northeastern *	20	20	2.162	1.733	79.26	63.79
Galveston, Houston & Henderson	50	50	1.556	1.759	47.27	70.37
Average of ten (or nine)	....	.....	1.257	1.539	68.89	74.61

* Subsidiary road since 1904.
† Merged since 1904; separate figures not available.

The fluctuations of the average cost per train-mile for several years past may be noted from the following tabular form:

AVERAGE COST PER TRAIN-MILE (FOR WHOLE U. S.) IN CENTS.

Year.	Cents.	Year.	Cents.	Year.	Cents.	Year.	Cents.
1890	96.006	1896	93.838	1902	117.960	1908	147.340
1891	95.707	1897	92.918	1903	126.604	1909	143.370
1892	96.580	1898	95.635	1904	131.375	1910	148.865
1893	97.272	1899	98.390	1905	132.140	1911	154.338
1894	93.478	1900	107.288	1906	137.060	1912	159.077
1895	91.829	1901	112.292	1907	146.993		

The enforced economies after the panic of 1893 are well shown. The reduction generally took the form of a lowering of the standards of maintenance of way and of maintenance of equipment. The marked advance since 1895 is partly due to the necessity for restoring the roads to proper conditions, replenishing worn-out equipment and providing additional equipment to handle the greatly increased volume of business. The recent advance is chiefly due to the increase in wages and the generally increased cost of supplies.

It may be noted from the I. C. C. reports that the cases where the operating expenses per train-mile and the ratio of expenses to earnings vary very greatly from the average are almost invariably those of the very small roads or of "junction roads" where the operating conditions are abnormal. For example, one little road, with a total length of 13 miles and total annual operating expenses of $5342, spent but 22¼c. per train-mile, which precisely exhausted its earnings. This precise equality of earnings and expenses suggests jugglery in the bookkeeping. As another abnormal case, a road 44 miles long spent $3.81 per train-mile, which was nearly *fourteen* times its earnings. In another case a road 13 miles long earned $7.76 per train-mile and spent $6.03 (78%) on operating expenses, but the fixed charges were abnormal and the earnings were less than half the sum of the operating expenses and fixed charges. The *normal* case, even for the small road, is that the cost per train-mile and the ratio of operating expenses to earnings will agree fairly well with the average, and when there is a marked difference it is generally due to some abnormal conditions of expenses or of earning capacity.

482. Reasons for uniformity in expenses per train-mile. The chief reason is that, although on the heavy-traffic road everything is kept up on a finer scale, better roadbed, heavier rails, better rolling stock, more employees, better buildings,

rails, better rolling stock, more employees, better buildings, stations, and terminals, etc., yet the number of trains is so much greater that the divisor is just enough larger to make the average cost about constant. This is but a general statement of a fact which will be discussed in detail under the different items of expense.

483. Detailed classification of expenses with ratios to the total expense. The Interstate Commerce Commission now publishes each year a classification with detailed summation for the cost of each item. These summations are made up from reports furnished by railroads which have (in the reports recently made) represented over 99% of the total traffic handled. In the annexed tabular form (Table XLI) are shown the percentages which each item bears to the total. The railroads have been divided into two classes, "large" and "small," as indicated below. Large roads report on 116 items which are combined and condensed with 44 items for small roads.

"Large roads" are those with mileage greater than 250 miles, or those with operating revenues greater than $1,000,000. Roads subsidiary to "large roads" are also included in this class.

"Small roads" are those with mileage less than 250 miles and also with operating revenues less than $1,000,000.

484. Amounts and percentages of the various items. The I. C. C. report for the year ending June 30, 1909, was the first to include the distribution of expenses according to the present classification. The items as given are reliable and may be utilized, as far as any such computations are to be depended on, in estimating future expenses. The chief purpose of this discussion is to point out those elements of the cost of operating trains which may be affected by such changes of location as an engineer is able to make. There are some items of expense with which the engineer has not the slightest concern, nor will they be altered by any change in alinement or constructive detail which he may make. In the following discussion such items will be passed over with a brief discussion of the sub-items included.

MAINTENANCE OF WAY AND STRUCTURES.

485. Items 2 to 5. Track material. The relative cost of ballast, ties, rails and other track material, as shown by com-

TABLE XLI.—ANALYSIS OF OPERATING EXPENSES OF ALL "LARGE"* RAILROADS IN THE UNITED STATES FOR YEAR ENDING JUNE 30, 1912, SHOWING PERCENTAGE OF EACH ITEM TO TOTAL AND COST IN CENTS PER TRAIN-MILE.

Item. No.	Account.	Total Amount (thousands)	Per cent of total Expenses	Cents per Train-Mile.
	Maintenance of Way and Structures.			
1	Superintendence	$18,789	0.990	1.58
2	Ballast	7,157	0.377	.60
3	Ties	55,463	2.921	4.65
4	Rails	16,438	.866	1.38
5	Other track material	17,346	.914	1.45
6	Roadway and track	129,397	6.815	10.84
7	Removal of snow, sand, and ice	6,920	.364	.58
8	Tunnels	1,141	.060	.10
9	Bridges, trestles, and culverts	27,712	1.460	2.32
10–12	Crossings, all; fences; snow structures	8,066	.425	.68
13–15	Signals, telegraph, electrical power transmission	13,681	.720	1.14
16, 17	Buildings, grounds, docks, wharves	35,389	1.864	2.96
18	Roadway tools and supplies	4,480	.236	.38
19	Injuries to persons	1,989	.105	.17
20, 21	Stationery, printing and other expenses	1,038	.054	.09
22, 23	Joint tracks, etc. (net balance)	3,463	.182	.29
		$348,471	18.353	29.20
	Maintenance of Equipment.			
24	Superintendence	$13,175	.694	1.10
	Repairs, renewals and depreciation:			
25–30	Locomotives, steam and electric	175,889	9.263	14.74
31–33	Cars, passenger	38,968	2.052	3.26
34–36	Cars, freight	183,968	9.690	15.41
37–39	Equipment, electrical, car	318	.017	.03
40–42	Equipment, floating	1,333	.071	.11
43–45	Equipment, work	6,128	.322	.51
46	Equipment, shop (machinery and tools)	10,418	.548	.87
47	Equipment, power plant	268	.014	.02
48	Injuries to persons	1,818	.096	.15
49, 50	Stationery, printing and other expenses	4,036	.213	.34
51, 52	Joint equipment, at terminals (net balance)	676	.036	.06
		$436,995	23.016	36.61
	Traffic Expenses.			
53–60	Agencies; advertising; fast freight lines; etc.	$59,047	3.110	4.95

* The "large" roads here reported represent 88% of the total mileage.

paring either the gross amounts or the percentages in Table XLI, is suggestive and instructive. The fact that ties cost considerably more than all other track material combined shows

TABLE XLI. (*Continued*).—ANALYSIS OF OPERATING EXPENSES OF ALL "LARGE" RAILROADS IN THE UNITED STATES FOR YEAR ENDING JUNE 30, 1912, SHOWING PERCENTAGE OF EACH ITEM TO TOTAL AND COST IN CENTS PER TRAIN-MILE.

Item No.	Account.	Total Amount (thousands).	Per cent of total expenses.	Cents per train-mile.
	Transportation Expenses.			
61, 62	Superintendence and train dispatching	$40,743	2.146	3.41
63	Station employees	133,877	7.051	11.22
64–66	Weighing; car service association; coal and ore docks	15,949	.839	1.33
67–70	Yards (wages, expenses, supplies)	76,069	4.007	6.37
71–76	Yard locomotives (enginemen, fuel, water, lubricants, supplies)	74,370	3.917	6.23
77, 78 104, 105	Operating joint tracks, terminals, yards, and facilities (net balance)	10,430	.550	.88
79, 80	Motormen and road enginemen	120,966	6.371	10.14
81	Road locomotives, engine-house expenses	33,951	1.788	2.84
82	Road locomotives, fuel	194,142	10.225	16.27
83	Road locomotives, water	12,482	.657	1.04
84, 85	Road locomotives, lubricants, and other supplies	7,430	.392	.62
86, 87	Operating power plants, purchased power	1,797	.095	.15
88	Road trainmen	128,339	6.759	10.75
89	Train supplies and expenses	34,462	1.815	2.89
90–92	Interlockers, signals, flagmen, draw-bridges	17,831	.939	1.49
93	Clearing wrecks	5,167	.272	.43
94–98	Telegraph, floating equipment, stationery, miscellaneous	20,009	1.054	1.68
99–103	Loss and damage to property, personal injuries	56,838	2.994	4.76
		$984,852	51.871	82.51
	General Expenses.			
106–116	Salaries of general officers, clerks, etc.; law, insurance, pensions, miscellaneous	69,297	3.650	5.81
	Total operating expenses	$1,898,662	100.000	159.08

the importance of any possible saving in tie renewals. It is also significant that the relative importance of ties has increased in the last few years, and that the relative increase has not been due to a reduction in the cost of other track material. Apparently the lengthening of the average life of ties, due to preservative processes, the use of tie-plates, and greater care to avoid the premature withdrawal from the track of ties which

are still serviceable, has not kept pace with the increase in the average cost per tie. The cost of rails has advanced because of (*a*) the very general adoption of heavier rails; (*b*) the almost universal substitution of more expensive open-hearth steel for Bessemer, on account of greater reliability and durability, and (*c*) the increase in cost of all steel products.

486. Item 6. Roadway and track. This item is three-eighths of the total cost of maintenance of way and structures. It consists chiefly of the wages of trackmen. There has been an almost steady increase in the daily wages of section foremen and other trackmen since 1900, as shown below:

	1900	1901	1902	1903	1904	1905	1906
Section foremen......	1.68	1.71	1.72	1.78	1.78	1.79	1.80
Other trackmen......	1.22	1.23	1.25	1.31	1.33	1.32	1.36
No. of trackmen per 100 miles..........	118	122	140	147	136	143	155

	1907	1903	1909	1910	1911	1912
Section foremen......	1.90	1.95	1.96	1.99	2.07	2.09
Other trackmen......	1.46	1.45	1.38	1.47	1.50	1.50
No. of trackmen per 100 miles..........	162	130	136	157	147	143

The average number of section foremen per 100 miles of line has remained almost constant at 18. Although there have been fluctuations in the number of "other trackmen" required per 100 miles of line, there has been in general a very substantial increase. These two causes combined (increased number and increased wages) have had a great influence in producing the regular and steady increase in the average cost of a train-mile, as shown in § 481.

487. Items 8 to 15. Maintenance of track structures. As a matter of economics, the locating engineer has little or no concern with the cost of maintaining track structures. If he is comparing two proposed routes it would be seldom that they would be so different that he would be justified in attempting to compute a train-mile difference in cost of operation, based on differences in these items. Of course, one proposed line might call for one or more tunnels which the alternate line might not have, and the annual cost of maintaining the tunnels would increase the cost of operation. Such a case would justify special considera-

tion. So far as the maintenance of small bridges and culverts are concerned it would usually be sufficiently accurate to consider that a proposed change of line, involving perhaps several miles of road, would require substantially the same number of bridges and culverts, and therefore that the cost of maintaining them would be the same by either line. The error involved in such an assumption would usually be insignificant, unless there was a very large and material difference in the two lines in this respect. Under such conditions special computations should be made. The items total less than 3% for small roads and still less for large roads.

MAINTENANCE OF EQUIPMENT.

488. Items 25 to 27. Repairs, renewals and depreciation of steam and electric locomotives. The item is of interest to the locating engineer because he must appreciate the effect on locomotive repairs and renewals of an addition to distance. A large part of the repairs of locomotives are due to the wear of wheels, which is largely caused by curvature. Therefore the value of any reduction of curvature is a matter of importance, and this will be considered in Chapter XXII. A considerable portion of the deterioration of a locomotive is due to grade, and the economic advantages of reductions of grade will be considered in Chapter XXIII.

This item includes the expenses of work whose effect is supposed to last for an indefinite period. It does *not* include the expense of cleaning out boilers, packing cylinders, etc., which occurs regularly and which is charged to items 72 or 81. It does include all current repairs, general overhauling, and even the replacement of old and worn-out locomotives by new ones to the extent of keeping up the original standard and number. Of course additions beyond this should be considered as so much increase in the original capital investment. As a locomotive becomes older the *annual* repair charge becomes a larger percentage on the first cost, and it may become as much as one-fourth and even one-third of the first cost. When a locomotive is in this condition it is usually consigned to the scrap-pile; the annual cost for maintenance becomes too large an item for its annual mileage. The effect on expenses of increasing the weight of engines is too complicated a problem to be solved accurately, but

certain elements of it may be readily computed. While the cost of repairs is greater for the heavier engines, the increase is only about one-half as fast as the increase in weight—some of the subitems not being increased at all.

TRANSPORTATION.

489. Items 71 to 76. Yard-engine expenses. By comparing these items with the corresponding items (80 to 85) for road engines, it may be seen that the total expenses assignable to yard engines are about 20% of those of road engines; the relative fuel charge for 1912 was 15.6%. The number of switching locomotives in the United States in 1912 was 9529 or 15.3% of the total number, 62,262. The relative charge for wages of enginemen was 26.2%. This higher proportionate charge is probably due to the fact that the wages for yard enginemen must necessarily be on a per diem basis, but the wages of road enginemen are generally on a mileage basis, as explained later. On the other hand the mileage of a yard engine is usually comparatively low, and the coal consumed will be correspondingly, although not proportionately, low. It must also be remembered that these figures are exclusive of the work and equipment of switching and terminal companies.

490. Item 80. Road enginemen. This item requires 6% of the total operating expenses. The enginemen are usually paid on a mileage basis, or by the trip, except on very small railroads. On very short roads, where a train crew may make two, three, or even four complete round trips per day, they may readily be paid by the day, so many round trips being considered as a day's work, but on roads of great length, where all trains, and especially freight-trains, are run day and night, weekday and Sunday, all trainmen are necessarily paid by the trip. The pay for a trip is figured on a mileage basis except that a trip is usually considered to have a minimum length of 100 miles or 10 hours of time. Eight hours was fixed as standard by the "Adamson" law, in 1916. All extra time is called "overtime" and is paid for at an extra rate. The basis of train wages is too complicated for any brief discussion. Even the basis is constantly changing, the only uniform feature being a steady increase.

The increase in the average wages paid to enginemen and firemen since 1900 is plainly shown by the following figures:

INCREASE IN DAILY WAGES, FROM 1900 TO 1912.

	1900	1901	1902	1903	1904	1905	1906
	$	$	$	$	$	$	$
Enginemen..........	3.75	3.78	3.84	4.01	4.10	4.12	4.12
Firemen.............	2.14	2.16	2.20	2.28	2.35	2.38	2.42

	1907	1908	1909	1910	1911	1912
	$	$	$	$	$	$
Enginemen..........	4.30	4.45	4.44	4.55	4.79	5.00
Firemen.............	2.54	2.64	2.67	2.74	2.94	3.02

491. Item 82. Fuel for road locomotives. This item includes every subitem of the entire cost of the fuel until it is placed in the engine-tender. The cost therefore includes not only the first cost at the point of delivery to the road, but also the expense of hauling it over the road from the point of delivery to the various coaling-stations and the cost of operating the coal-pockets from which it is loaded on to the tenders. Even though the cost may be fairly regular for any one road, the cost for different roads is exceedingly variable. There has been an almost steady increase in the *percentage* of the cost of this item per train-mile since 1897. Items 73 and 82 amounted to nearly 12% of the total operating expenses in 1912, and required an actual expenditure of nearly $225,000,000. It is the largest item in the whole cost of railroad operation. Although some roads, which traverse coal-regions and perhaps actually own the coal-mines, are able to obtain their coal for a cost which may be charged up as $1 per ton or less, there are many roads which are far removed from coal-fields which have to pay $3 or $4 per ton, on account of the excessive distance over which the coal must be hauled. Unfortunately the figures published by the Interstate Commerce Commission do not show the variations in the percentage of this item in different localities. A surprisingly large percentage of the fuel consumed is not utilized in drawing a train along the road. A portion of this percentage is used in firing-up. A portion is wasted when the engine is standing still, which is a considerable proportion of the whole time. The policy of banking fires instead of drawing them reduces the injury resulting from great fluctuations in temperature, but in a general way we may say that there is but little, if any, saving in fuel by banking the fires, and therefore we may consider that

almost a fire-box full of coal is wasted whether the fires are banked or drawn. As given in § 464, the fuel used by a locomotive in firing-up may be estimated as 510 lbs. per 1000 square feet of heating surface, based on using 12000 B.t.u. coal. But even the amount of coal required to produce the required steam-pressure in the boiler from cold water does not represent the total loss. The train-dispatcher, in his anxiety that engines shall be ready when needed, will sometimes order out the locomotives which remain somewhere in the yard, perhaps exposed to cold weather, and blow off steam for several hours before they make an actual start. This loss has been estimated as 120 lbs. per hour per 1000 square feet of heating surface, but it would evidently be far greater on a windy winter day than on a calm summer day. A freight-train, especially on a single-track road, will usually spend several hours during the day on sidings, and when a single-track road is being run to the limit of its capacity, or when the management is not good, the time will be still greater. It is estimated that the amount lost through a 2½-inch safety-valve in one minute would represent the consumption of 15 pounds of coal, which would be sufficient to haul 100 tons on a mile of track with easy grades. Again we see that the amount thus lost is exceedingly variable and almost non-computable, although as a rough estimate the amount has been placed at from 3 to 6% of the total. Another very large subitem of loss of useful energy is that occasioned by stopping and starting. A train running 30 miles per hour has enough kinetic energy to move it on a straight level track for more than two miles. Therefore, every time a train running at 30 miles per hour is stopped, enough energy is consumed by the brakes to run it about two miles. There is a double loss, not only due to the fact of the loss of energy, but also because the power of the locomotive has been consumed in operating the brakes. When the train is again started, this kinetic energy must be restored to the train in addition to the ordinary resistances which are even greater, on account of the greater resistance at very low velocities. Of course, the proportion of fuel thus consumed depends on the frequency of the stops. It was demonstrated by some tests on the Manhattan Elevated Road in New York City, where the stops average one in every three-eighths of a mile, that this cause alone would account for the consumption of nearly three-fourths of the fuel. On ordinary railroads

the proportion, of course, will not be nearly so great, but there is reason to believe that 10 to 20% is not excessive as an average figure.

492. Item 88. Road trainmen. This item includes the wages of conductors and "other trainmen." As in the case of all other employees, the average daily wages have advanced since 1900 as shown below:

AVERAGE DAILY WAGES OF CONDUCTORS AND OTHER TRAINMEN, 1900 TO 1912.

	1900	1901	1902	1903	1904	1905	1906
	$	$	$	$	$	$	$
Conductors..........	3.17	3.17	3.21	3.38	3.50	3.50	3.51
Other trainmen......	1.96	2.00	2.04	2.17	2.27	2.31	2.35

	1907	1908	1909	1910	1911	1912
	$	$	$	$	$	$
Conductors..........	3.69	3.81	3.81	3.91	4.16	4.29
Other trainmen......	2.54	2.60	2.59	2.69	2.88	2.96

These figures are of vital importance from an economic standpoint, since they show a constant tendency to increase and thereby raise the average cost of a train-mile. And as there is no present indication of any limit to this increase, all economic calculations which attempt to predict future expenses, even for a few years in advance, must allow for these and other increased expenses.

493. Item 89. Train supplies and expenses. These items, which average about 1.8%, include the large list of consumable supplies such as lubricating oil, illuminating-oil or gas, ice, fuel for heating, cleaning materials, etc., which are used on the cars and not on the locomotives. The consumption of some of these articles is chiefly a matter of time. In other cases it is a function of mileage. The effect of changes which an engineer may make on this item will be considered when estimating the effect of the changes.

494. Items 93, 99 to 103. Clearing wrecks, loss, damage and injuries to persons and property. These expenses are fortuitous and bear no absolute relation either to the number of miles of road or the number of train-miles. While they depend largely on the standards of discipline on the road, even the best of roads have to pay some small proportion of their earnings to these

items. While we might expect that a road with heavy traffic would have a larger proportion of train accidents than a road of light traffic, it is usually true that on the heavy-traffic roads the precautions taken are such that they are usually freer from accidents than the light-traffic roads. During recent years there has been a very perceptible increase in the percentages of these items, particularly in the compensations paid for "injuries to persons." The increase in this item coincides with the increase already noted in the number of passengers killed during recent years. The possible relation between curvature and accidents has already been discussed, but otherwise the locating engineer has no concern with these items.

495. Items 104, 105. Operating joint tracks and facilities, Dr. and Cr. A large part of these debit and credit charges are those for car per diem and mileage charges. This is a charge paid by one road to another for the use of cars, which are chiefly freight-cars. To save the rehandling of freight at junctions, the policy of running freight-cars from one road to another is very extensively adopted. Since the foreign road receives its mileage proportion of the freight charge, it justly pays to the road owning the car at a rate which is supposed to represent the value of the use of the freight-car for the number of miles traveled. The foreign road then loads up the freight-car with freight consigned to some point on the home road and sends it back, paying mileage for the distance traveled on the foreign road, a proportional freight charge having been received for that service. All of these movements of freight-cars are reported to a car association, which, by a clearing-house arrangement, settles the debit and credit accounts of the various roads with each other. Such is the simple theory. In practice the cars are not sent back to the home road at once, but wander off according to the local demand. As long as a strict account is kept of the movements of every car, and as long as the home road is paid the charge which really covers the value of lost service, no harm is done to the home road, except that sometimes, when business has suddenly increased, the home road cannot get enough cars to handle its own business. The value of the car is then abnormally above its ordinary value, and the home road suffers for lack of the rolling stock which belongs to it. Formerly such charges were paid strictly according to the mileage. This developed the intolerable condition that loaded cars would be

run onto a siding and left there for several days, simply because it was not convenient to the consignee to unload the car immediately. On the mileage basis the car would be earning nothing, and, since the road on which the car then was had no particular interest in the car, the car was allowed to stand to suit the convenience of the consignee. To correct this evil a system of per diem charges has been developed, so that a railroad has to pay a per diem charge for every foreign car on its lines. To reduce this charge as much as possible, the railroads compel consignees, under penalty of heavy demurrage charges, to unload cars promptly. The running of freight-cars on foreign lines is now settled almost exclusively on the per diem basis, but the running of passenger-cars over other lines, as is done on account of the advantages of through-car service, as well as the running of Pullmans and other special cars, is still paid for on the mileage basis. To the extent to which this charge is settled on the mileage basis, any change in distance which the engineer may be able to effect in the length of the road will have its influence on this item, but when the freight-car business, which comprises by far the larger part of the running of cars over foreign lines, is settled on the per diem basis no changes in alinement which the engineer may make will affect the item appreciably.

Switching Charges. Where two or more railroads intersect there will be a considerable amount of shifting of cars, chiefly freight-cars, from one road to the other. This shifting at any one junction may be done entirely by the engines of one road or perhaps by those of both roads. A portion of the expense of this work is charged up against the other road by the road which does the work. The total amount of this work is carefully accounted for by a clearing-house arrangement, and the balance is charged up against the road which has done the least work. The item is very small, is fairly uniform year by year, and is seldom, if ever, affected by changes of alinement.

Other Items. All of the remaining items, as stated in Table XLI, are of no concern to the locating engineer. They are either general expenses, such as the salaries of general officers, insurance or law expenses, or are special items, such as advertising or the operation of marine equipment which will not be changed by any variations in distance, curvature, or grades which a locating engineer may make. There is therefore no need for their further discussion here.

CHAPTER XXI.

DISTANCE.

496. Relation of distance to rates and expenses. Rates are usually based on distance traveled, on the apparent hypotheses that each additional mile of distance adds its proportional amount not only to the service rendered but also to the expense of rendering it. Neither hypothesis is true. The value of the service of transporting a passenger or a ton of freight from A to B is a more or less uncertain gross amount depending on the necessities of the case and independent of the exact distance. Except for that very small part of passenger traffic which is undertaken for the mere pleasure of traveling, the general object to be attained in either passenger or freight traffic is the transportation from A to B, however it is attained. A mile greater distance does not improve the service rendered; in fact, it consumes valuable time of the passengers and perhaps deteriorates the freight. From the standpoint of service rendered, the railroad which adopts a more costly construction and thereby saves a mile or more in the route between two places is thereby fairly entitled to additional compensation rather than have it cut down as it would be by a strict mileage rate. The actual value of the service rendered may therefore vary from an insignificant amount which is less than any reasonable charge (which therefore discourages such traffic) and its value in cases of necessity—a value which can hardly be measured in money. If the passenger charge between New York and Philadelphia were raised to \$5, \$10, or even \$20, there would still be some passengers who would pay it and go, because *to them* it would be worth \$5, \$10, or \$20, or even more. Therefore, when they pay \$2.25 they are not paying what the service is worth to them. The service rendered cannot therefore be made a measure of the charge, nor is the service rendered proportional to the miles of distance.

The idea that the cost of transportation is proportional to

the distance is much more prevalent and is in some respects more justifiable, but it is still far from true. This is especially true of passenger service. The extra cost of transporting a single passenger is but little more than the cost of printing his ticket. Once aboard the train, it makes but little difference to the railroad whether he travels one mile or a hundred. Of course there are certain very large expenses due to the passenger traffic which must be paid for by a tariff which is rightfully demanded, but such expenses have but little relation to the cost of an additional mile or so of distance inserted between stations. The same is true to a slightly less degree of the freight traffic. As shown later, the items of expense in the total cost of a train-mile, which are directly affected by a small increase in distance, are but a small proportion of the total cost.

497. The conditions other than distance that affect the cost; reasons why rates are usually based on distance. Curvature and minor grades have a considerable influence on the cost of transportation, as will be shown in detail in succeeding chapters, but they are never considered in making rates. Ruling grades have a very large influence on the cost, but they are likewise disregarded in making rates. An accurate measure of the effect of these elements is difficult and complicated and would not be appreciated by the general public. Mere distance is easily calculated; the public is satisfied with such a method of calculation; and the railroads therefore adopt a tariff which pays expenses and profits even though the charges are not in accordance with the expenses or the service rendered.

EFFECT OF DISTANCE ON RECEIPTS.

498. Classification of traffic. There are various methods of classifying traffic, according to the use it is intended to make of the classification. The method here adopted will have reference to its competitive or non-competitive character and also to the method of division of the receipts on through traffic. Traffic may be classified first as "through" and "local"—through traffic being that traveling over two (or more) lines, no matter how short or non-competitive it may be; "local" traffic is that confined entirely to one road. A fivefold classification is however necessary—which is:

A. Non-competitive local—on one road with no choice of routes

B. Non-competitive through—on two (or more) roads, but with no choice.

C. Competitive local—a choice of two (or more) routes, but the entire haul may be made on the home road.

D. Competitive through—direct competition between two or more routes each passing over two or more lines.

E. Semi-competitive through—a non-competitive haul on the home road and a competitive haul on foreign roads.

There are other possible combinations, but they all reduce to one of the above forms so far as their essential effect is concerned.

499. Method of division of through rates between the roads run over. Through rates are divided between the roads run over in proportion to the mileage. There may be terminal charges and possibly other more or less arbitrary deductions to be taken from the total amount received, but when the final division is made the remainder is divided according to the mileage. On account of this method of division and also because non-competitive rates are always fixed according to the distance, there results the unusual feature that, unlike curvature and grade, there is a compensating advantage in increased distance, which applies to all the above kinds of traffic except one (competitive local), and that the compensation is sometimes sufficient to make the added distance an actual source of profit. It has been estimated that the cost of hauling a train an additional mile is only 33 to 49% of the average cost. Therefore in all non-competitive business (local or through) where the rate is according to the distance, there is an actual profit in all such added distance. In competitive local business, in which the rate is fixed by competition and has practically no relation to distance, any additional distance is dead loss. In competitive through business the profit or loss depends on the distances involved. This may best be demonstrated by examples.

500. Effect of a change in the length of the home road on its receipts from through competitive traffic. Suppose the home road is 100 miles long and the foreign road is 150 miles long. Then the home road will receive $\frac{100}{100+150}=40\%$ of the through rate.

Suppose the home road is lengthened 5 miles; then it will

receive $\frac{105}{105+150}=41.176\%$ of the through rate. The traffic being competitive, the rate will be a fixed quantity regardless of this change of distance. By the first plan the rate received is 0.4% per mile; adding 5 miles, the rate for the original 100 miles may be considered the same as before; and that the additional 5 miles receive 1.176%, or 0.235% per mile. This is 59% of the original rate per mile, and since this is more than the cost per mile for the additional distance, the added distance is evidently in this case a source of distinct profit. On the other hand, if the line is shortened 5 miles, it may be similarly shown that not only are the receipts lessened, but that the saving in operating expenses by the shorter distance is less than the reduction in receipts.

A second example will be considered to illustrate another phase. Suppose the home road is 200 miles long and the foreign road is 50 miles long. In this case the home road will receive $\frac{200}{200+50}=80\%$ of the through rate. Suppose the home road is lengthened 5 miles; then it will receive $\frac{205}{205+50}=80.392\%$ of the through rate. By the first plan the rate received is 0.400% per mile; adding 5 miles, there is a surplus of 0.392, or 0.0784 per mile, which is but 19.6% of the original rate. At this rate the extra distance evidently is not profitable, although it is not a dead loss—there is some compensation.

501. The most advantageous conditions for roads forming part of a through competitive route. From the above it may be seen that when a road is but a short link in a long competitive through route, an addition to its length will increase its receipts and increase them more than the addition to the operating expenses.

As the proportionate length of the home road increases the less will this advantage become, until at some proportion an increase in distance will just pay for itself. As the proportionate length grows greater the advantage becomes a disadvantage until, when the competitive haul is entirely on the home road, any increase in distance becomes a net loss without any compensation. It is therefore advantageous for a road to be a short link in a long competitive route; an increase in that link

will be financially advantageous; if the total length is less than that of the competing line, the advantage is still greater, for then the rate received per mile will be greater.

502. Effect of the variations in the length of haul and the classes of the business actually done. The above distances refer to particular lengths of haul and are not necessarily the total lengths of the road. Each station on the road has traffic relations with an indefinite number of traffic points all over the country. The traffic between each station on the road and any other station in the country between which traffic may pass therefore furnishes a new combination, the effect of which will be an element in the total effect of a change of distance. In consequence of this, any *exact* solution of such a problem becomes impracticable, but a sufficiently accurate solution for all practical purposes is frequently obtainable. For it frequently happens that the great bulk of a road's business is non-competitive, or, on the other hand, it may be competitive-through, and that the proportion of one or more definite kinds of traffic is so large as to overshadow the other miscellaneous traffic. In such cases an approximate but sufficiently accurate solution is possible.

503. General conclusions regarding a change in distance. (a) In *all* non-competitive business (local and through) the added distance is actually profitable. Sometimes practically all of the business of the road is non-competitive; a considerable proportion of it is always non-competitive.

(b) When the competitive local business is very large and the competitive through business has a very large average home haul compared with the foreign haul, the added distance is a source of loss. Such situations are unusual and are generally confined to trunk lines.

(c) The above may be still further condensed to the general conclusion that there is always *some* compensation for the added cost of operating an added length of line and that it frequently is a source of actual profit.

(d) There is, however, a limitation which should not be lost sight of. The above argument may be carried to the logical conclusion that, if added distance is profitable, the engineer should purposely lengthen the line. But added distance means added operating expenses. A sufficient tariff to meet these is a

traffic. It is contrary to public policy to burden a community with an avoidable expense. But, on the other hand, a railroad is not a charitable organization, but a money-making enterprise, and cannot be expected to unduly load up its first cost in order that subsequent operating expenses may be unduly cheapened and the tariff unduly lowered. A common reason for increased distance is the saving of the first cost of a very expensive although shorter line.

(e) Finally, although there is a considerable and uncompensated loss resulting from curvature and grade which will justify a considerable expenditure to avoid them, there is by no means as much justification to incur additional expenditure to avoid distance. Of course needless lengthening should be avoided. A moderate expenditure to shorten the line may be justifiable, but large expenditures to decrease distance are never justifiable except when the great bulk of the traffic is exceedingly heavy and is competitive.

504. Justification of decreasing distance to save time. It should be recalled that the changes which an engineer may make which are physically or financially possible will ordinarily have but little effect on the time required for a trip. The time which can thus be saved will have practically no value for the freight business—at least any value which would justify changing the route. When there is a large directly competitive passenger traffic between two cities (*e.g.* New York to Philadelphia) a difference of even 10 minutes in the time required for a run might have considerable financial importance, but such cases are comparatively rare. It may therefore be concluded that the value of the time saved by shortening distance will not ordinarily be a justification for increased expense to accomplish it.

505. Effect of change of distance on the business done. The above discussion is based on the assumption that the business done is unaffected by any proposed change in distance. If a proposed reduction in distance involves a loss of business obtained, it is almost certainly unwise. But if by increasing the distance the original cost of the road is decreased (because the construction is of less expensive character), and if the receipts are greater, and are increased still more by an increase in business done, then the change is probably wise. While it is almost impossible in a subject of such complexity to give a general

rule, the following is generally safe: Adopt a route of such length that the annual traffic per mile of line is a maximum. This statement may be improved by allowing the element of original cost to enter and say, adopt a route of such length that the annual traffic per mile of line divided by the average cost per mile is a maximum. Even in the above the operating cost per mile, as affected by the curvature and grades on the various routes, does not enter, but any attempt to formulate a general rule which would allow for variable operating expenses would evidently be too complicated for practical application.

CHAPTER XXII.

CURVATURE.

506. General objections to curvature. In the popular mind curvature is one of the most objectionable features of railroad alinement. The cause of this is plain. The objectionable qualities are on the surface, and are apparent to the non-technical mind. They may be itemized as follows:

1. Curvature increases **operating expenses** by increasing (a) the required tractive force, (b) the wear and tear of roadbed and track, (c) the wear and tear of equipment, and (d) the required number of track-walkers and watchmen.

2. It may affect the **operation of trains** (a) by limiting the length of trains, and (b) by preventing the use of the heaviest types of engines.

3. It may affect **travel** (a) by the difficulty of making time, (b) on account of rough riding, and (c) on account of the apprehension of danger.

4. There is actually an increased **danger** of collision, derailment, or other form of accident.

Some of these objections are quite definite and their true value may be computed. Others are more general and vague and are usually exaggerated. These objections will be discussed in inverse order.

507. Financial value of the danger of accident due to curvature. At the outset it should be realized that in general the problem is *not* one of curvature *vs.* no curvature, but simply sharp curvature *vs.* easier curvature (the central angle remaining the same), or a greater or less percentage of elimination of the degrees of central angle. A straight road between termini is in general a financial (if not a physical) impossibility. The practical question is then, how much is the financial value of such diminution of danger that may result from such eliminations of curvature as an engineer is able to make?

In the year 1898 there were 2228 railroad accidents reported by the *Railroad Gazette*, whose lists of all accidents worth reporting are very complete. Of these a very large proportion clearly had no relation whatever to curvature. But suppose we assume that 50% (or 1114 accidents) were directly caused by curvature. Since there are approximately 200,000 curves on the railroads of the country, there was on the average an accident for every 179 curves during the year. Therefore we may say, according to the theory of probabilities, that the chances are even that an accident may happen on any particular curve in 179 years. This assumes all curves to be equally dangerous, which is not true, but we may temporarily consider it to be true. If, at the time of the construction of the road, $1.00 were placed at compound interest at 5% for 179 years, it would produce in that time $620.89 for each dollar saved, wherewith to pay all damages, while the amount necessary to eliminate that curvature, even if it were possible, would probably be several thousand dollars. The number of passengers carried one mile for one killed in 1898–99 was 61,051,580. If a passenger were to ride continuously at the rate of sixty miles per hour, day and night, year after year, he would need to ride for more than 116 years before he had covered such a mileage, and even then the probabilities of his death being due to curvature or to such a reduction of curvature as an engineer might accomplish are very small. Of course particular curves are often, for special reasons, a source of danger and justify the employment of special watchmen. They would also justify very large expenditures for their elimination if possible. But as a general proposition it is evidently impossible to assign a definite money value to the danger of a serious accident happening on a particular curve which has no special elements of danger.

Another element of safety on curved track is that trait of human nature to exercise greater care where the danger is more apparent. Many accidents are on record which have been caused by a carelessness of locomotive engineers on a *straight* track when the extra watchfulness usually observed on a curved track would have avoided them.

508. Effect of curvature on travel. (a) Difficulty in making time. The growing use of transition curves has largely eliminated the necessity for reducing speed on curves, and even when the speed is reduced it is done so easily and quickly by means

of air-brakes that but little time is lost. If two parallel lines were competing sharply for passenger traffic, the handicap of sharp curvature on one road and easy curvature on the other might have a considerable financial value, but ordinarily the *mere reduction* of time due to sharp curvature will not have any computable financial value.

(b) On account of rough riding. Again, this is much reduced by the use of transition curves. Some roads suffer from a general reputation for crookedness, but in such cases the excessive curvature is practically unavoidable. This cause probably does have some effect in influencing competitive passenger traffic.

(c) On account of the apprehension of danger. This doubtless has its influence in deterring travel. The amount of its influence is hardly computable. When the track is in good condition and transition curves are used so that the riding is smooth, even the apprehension of danger will largely disappear.

Travel is doubtless more or less affected by curvature, but it is impossible to say how much. Nevertheless the engineer should not ordinarily give this item any financial weight whatsoever. Freight traffic (two thirds of the total) is unaffected by it. It chiefly affects that limited class of sharply competitive passenger traffic—a traffic of which most roads have not a trace.

509. Effect on operation of trains. (a) Limiting the length of trains. When curvature actually limits the length of trains, as is sometimes true, the objection is valid and serious. But this can generally be avoided. If a curve occurs on a ruling grade without a reduction of the grade sufficient to compensate for the curvature, then the resistance on that curve will be a maximum and that curve will limit the trains to even a less weight than that which may be hauled on the ruling grade. In such cases the unquestionably correct policy is to "compensate for curvature," as explained later (see §§510, 511), and not allow such an objection to exist. It is *possible* for curvature to limit the length of trains even without the effect of grade. On the Hudson River R. R. the total net fall from Albany to New York is so small that it has practically no influence in determining grade. On the other hand, a considerable portion of the route follows a steep rocky river bank which is so crooked that much curvature is unavoidable and very sharp curvature

can only be avoided by very large expenditure. As a consequence sharp curvature has been used and the resistance on the curves is far greater than that of any fluctuations of grade which it was necessary to use. Or, at least, a comparatively small expenditure would suffice to cut down any grade so that its resistance would be less than that of some curve which could not be avoided except at an enormous cost. And as a result, since the length of trains is really limited by curvature, minor grades of .0.3 to 0.5% have been freely introduced which might be removed at comparatively small expense The above case is very unusual. Low grades are usually associated with generally level country where curvature is easily avoided—as in the Camden and Atlantic R. R. Even in the extreme case of the Hudson River road the maximum curvature is only equivalent to a comparatively low ruling grade.

(b) **Preventing the use of the heaviest types of engines.** The validity of this objection depends somewhat on the degree of curvature and the detailed construction of the engine. While some types of engines might have difficulty on curves of extremely short radius, yet the objection is ordinarily invalid. This will best be appreciated when it is recalled that the "Consolidation" type was originally designed for use on the sharp curvature of the mountain divisions of the Lehigh Valley R. R., and that the type has been found so satisfactory that it has been extensively employed elsewhere. It should also be remembered that during the Civil War an immense traffic daily passed over a hastily constructed trestle near Petersburg, Va., the track having a radius of 50 feet. As a result of a test made at Renovo on the Philadelphia and Erie R. R. by Mr. Isaac Dripps, Gen. Mast. Mech., in 1875,* it was claimed that a Consolidation engine encountered less resistance per ton than one of the "American" type. Whether the test was strictly reliable or not, it certainly demonstrated that there was no trouble in using these heavy engines on very sharp curvature, and we may therefore consider that, except in the most extreme cases, this objection has no force whatsoever.

* Seventh An. Rep. Am. Mast. Mech. Assn.

COMPENSATION FOR CURVATURE.

510. Reasons for compensation. The effect of curvature on a grade is to increase the resistance by an amount which is equivalent to a material addition to that grade. On minor grades the addition is of little importance, but when the grade is nearly or quite the ruling grade of the road, then the additional resistance induced by a curve will make that curve a place of maximum resistance and the real maximum will be a "virtual grade" somewhat higher than the nominal maximum. If, in Fig. 211,

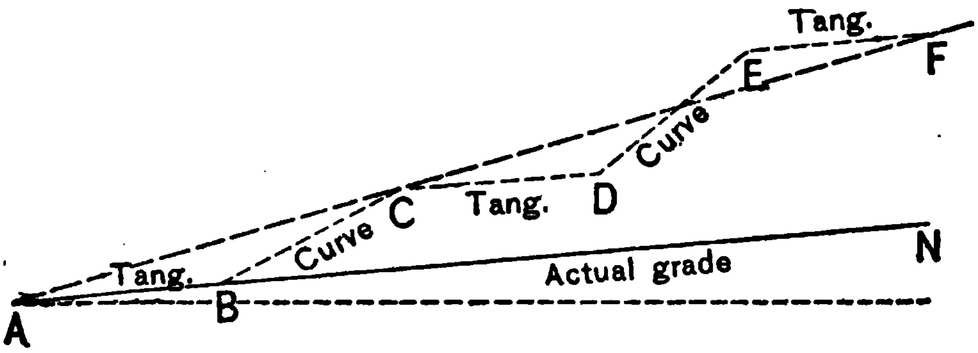

FIG. 211.

AN represents an actual *uniform* grade consisting of tangents and curves, the "virtual grade" on curves at *BC* and *DE* may be represented by *BC* and *DE*. If *BC* and *DE* are very long, or if a stop becomes necessary on the curve, then the full disadvantage of the curve becomes developed. If the whole grade may be operated without stoppage, then, as elaborated further in the next chapter, the whole grade may be operated as if equal to the average grade, *AF*, which is better than *BC*, although much worse than *AN*. The process of "compensation" consists in reducing the grade on every curve by such an amount that the actual resistance on each curve, due to both curvature and grade, shall precisely equal the resistance on the tangent. The practical effect of such reduction is that the "virtual" grade is kept constant, while the nominal grade fluctuates.

One effect of this is that (see Fig. 212) instead of accomplishing the vertical rise from *A* to *G* (i.e., *HG*) in the horizontal distance *AH*, it requires the horizontal distance *AK*. Such an addition to the horizontal distance can usually be obtained by proper development, and it should always be done on a ruling

grade. Of course it is possible that it will cost more to accomplish this than it is worth, but the engineer should be sure of this before allowing this virtual increase of the grade.

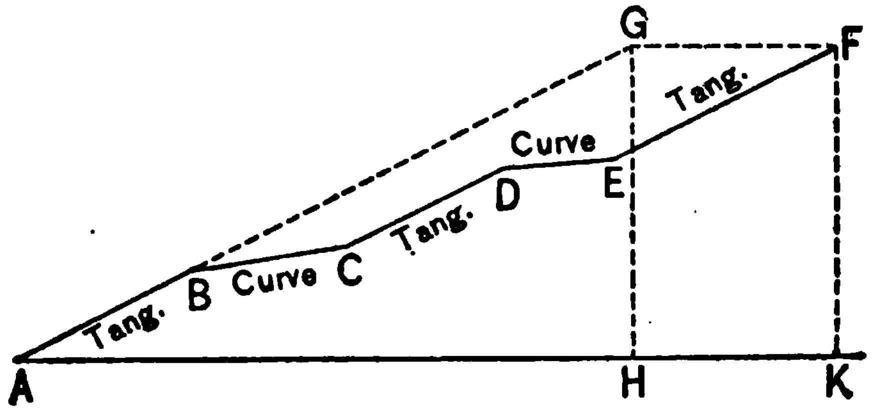

FIG. 212.

European engineers early realized the significance of unreduced curvature and the folly of laying out a uniform ruling grade regardless of the curvature encountered. Curve compensation is now quite generally allowed for in this country, but thousands of miles have been laid out without any compensation. A very common limitation of curvature and grade has been the alliterative figures 6° curvature and 60 feet per mile of grade, either singly or in combination. Assuming that the resistance on a 6° curve is equivalent to a 0.3% grade (15.84 feet per mile), then a 6° curve occurring on a 60-foot grade would develop more resistance than a 75-foot grade on a tangent. The "mountain cut-off" of the Lehigh Valley Railroad near Wilkesbarre is a fine example of a heavy grade compensated for curvature, and yet so laid out that the virtual grade is uniform from bottom to top, a distance of several miles.

511. The proper rate of compensation. This evidently is the rate of grade of which the resistance just equals the resistance due to the curve. But such resistance is variable. It is greater as the velocity is lower; it is generally about 2 lbs. per ton (equivalent to a 0.1% grade) per degree of curve when starting a train. On this account, the compensation for a curve which occurs at a known stopping-place for the heaviest trains should be 0.1% per degree of curve. The resistance is not even strictly proportional to the degree of curvature, although it is usually considered to be so. In fact most formulæ for curve resistance are based on such a relation. But if the experimentally determined resistances for low curvatures are applied to the excessive curvature of the New York Elevated road, for example, the

rules become ridiculous. On this account the compensation *per degree of curve* may be made less on a sharp curve than on an easy curve. The compensation actually required for very fast trains is less than for slow trains, say 0.02 or 0.03% per degree of curve; but since the comparatively slow and heavy freight trains are the trains which are chiefly limited by ruling grade, the compensation must be made with respect to those trains. From 0.04 to 0.05% per degree is the rate of compensation most usually employed for average conditions. Curves which occur *below* a known stopping-place for *all* trains need not be compensated, for the extra resistance of the curve will be simply utilized in place of brakes to stop the train. If a curve occurs just *above* a stopping-place, it is very serious and should be amply compensated. Of course the down-grade traffic need not be considered.

It sometimes happens that the ordinary rate of compensation will consume so much of the vertical height (especially if the curvature is excessive) that a steeper through grade must be adopted than was first computed, and then the trains might stall on the tangents rather than on the curves. In such cases a slight reduction in the rate of compensation might be justifiable.

The following rules have been approved by the Amer. Rwy. Eng. Assoc.

1. Compensate .03% per degree (*a*) when the length of curve is less than half the length of the longest train; (*b*) when a curve occurs within the first 20 feet of rise of a grade; (*c*) when curvature is in no sense limiting.

2. Compensate .035% per degree (*a*) when curves are between one-half and three-quarters as long as the longest train; (*b*) when the curve occurs between 20 feet and 40 feet of rise from the bottom of the grade.

3. Compensate .04% per degree (*a*) where the curve is habitually operated at low speed; (*b*) where the length of the curve is longer than three-quarters of the length of the longest train; (*c*) where elevation is excessive for freight trains; (*d*) at all places where curvature is likely to be limited.

4. Compensate .05% per degree wherever the loss of elevation can be spared.

512. The limitations of maximum curvature. What is the maximum degree of curvature which should be allowed on any

road? It has been shown that sharp curvature does not prevent the use of the heaviest types of engines, and although a sharp curve unquestionably increases operating expenses, the increase is but one of degree with hardly any definite limit. The general character of the country and the gross capital available (or the probable earnings) are generally the true criterions.

A portion of the road from Denver to Leadville, Col., is an example of the necessity of considering sharp curvature. The traffic that might be expected on the line was so meagre and yet the general character of the country was so forbidding that a road built according to the usual standards would have cost very much more than the traffic could possibly pay for. The line as adopted cost about $20,000 per mile, and yet in a stretch of 11.2 miles there are about 127 curves. One is a 25° 20′ curve, twenty-four are 24° curves, twenty-five are 20° curves, and seventy-two are sharper than 10°. If 10° had been made the limit (a rather high limit according to usual ideas), it is probable that the line would have been found impracticable (except with prohibitive grades) unless four or five times as much per mile had been spent on it, and this would have ruined the project financially.

For many years the main-line traffic of the Baltimore and Ohio R. R. has passed over a 300-foot curve (19° 10′) and a 400-foot curve (14° 22′) at Harper's Ferry. A few years ago some reduction was made in this by means of a tunnel, but the fact that such a road thought it wise to construct and operate such curves (and such illustrations on the heaviest-traffic roads are quite common) shows how foolish it is for an engineer to sacrifice money or (which is much more common) sacrifice gradients in order to reduce the *rate* of curvature on a road which at its best is but a second- or third-class road.

Of course such belittling of the effects of curvature may be (and sometimes is) carried to an extreme and cause an engineer to fail to give to curvature its due consideration. Degrees of central angle should always be reduced by all the ingenuity of the engineer, and should only be limited by the general relation between the financial and topographical conditions of the problem. Easy curvature is in general better than sharp curvature and should be adopted when it may be done at a small financial sacrifice, especially since it reduces distance generally and may even cut down the initial cost of that section of the

road. But large financial expenditures are rarely, if ever, justifiable where the net result is a mere increase in radius without a reduction in central angle. An analysis of the changes which have been so extensively made during late years on the Penn. R. R. and the N. Y., N. H. & H. R. R. will show invariably a reduction of distance, or of central angle, or both, and perhaps incidentally an increase in radius of curvature. There are but few, if any, cases where the sole object to be attained by the improvement is a mere increase in radius.

The requirements of standard M. C. B. car-couplers have virtually placed a limitation on the radius on account of the corners of adjacent cars striking each other on very sharp curves. This limitation has been crystallized into a rule on the P. R. R. that no curve, even that of a siding, can have a less radius than 175 feet, which is nearly the radius of a 33° curve. Of course only the most peremptory requirements of yard work would justify the employment of such a radius.

CHAPTER XXIII.

GRADE.

513. Two distinct effects of grade. The effects of grade on train expenses are of two distinct kinds; one possible effect is very costly and should be limited even at considerable expenditure; the other is of comparatively little importance, its cost being slight. As long as the length of the train is not limited, the occurrence of a grade on a road simply means that the engine is required to develop so many foot-pounds of work in raising the train so many feet of vertical height. For example, if a freight train weighing 600 tons (1,200,000 lbs.) climbs a hill 50 feet high, the engine performs an *additional* work of creating 60,000,000 foot-pounds of potential energy. If this height is surmounted in 2 miles and in 6 minutes of actual time (20 miles per hour), the *extra* work is 10,000,000 foot-pounds per minute, or about 303 horse-power. But the disadvantages of such a rise are always largely compensated. Except for the fact that one terminus of a road is generally higher than the other, every up grade is followed, more or less directly, by a down grade which is operated partly by the potential energy acquired during the previous climb. But when we consider the trains running in both directions even the difference of elevation of the termini is largely neutralized. If we could eliminate frictional resistances and particularly the use of brakes, the *net* effect of minor grades on the operation of minor grades in both directions would be zero. Whatever was lost on any up grade would be regained on a succeeding down grade, or at any rate on the return trip. On the very lowest grades (the limits of which are defined later) we may consider this to be literally true, viz., that nothing is lost by their presence; whatever is temporarily lost in climbing them is either immediately regained on a subsequent light down grade or is regained on the return trip. If a stop is required at the bottom of a sag, there is a net and uncompensated loss of energy.

On the other hand, if the length of trains is limited by the grade, it will require more trains to handle a given traffic. The receipts from the traffic are a definite sum. The cost of handling it will be nearly in proportion to the number of trains. Assume that by lowering the rate of ruling grade it becomes possible to handle such an increased number of cars with one engine that four engines can haul as many cars on the reduced grade as five engines could haul on the higher grade and at a cost but slightly more than four-fifths as much. The effect of this on dividends may readily be imagined.

514. Application to the movement of trains of the laws of accelerated motion. When a train starts from rest and acquires its normal velocity, it overcomes not only the usual tangent resistances (and perhaps curve and grade resistances), but it also performs work in storing into the train a vast fund of kinetic energy. This work is not lost, for every foot-pound of such energy may later be utilized in overcoming resistances, provided it is not wasted by the action of train-brakes. If for a moment we consider that a train runs without any friction, then, when running at a velocity of v feet per second, it possesses a kinetic energy which would raise it to a height h feet, when $h=\frac{v^2}{2g}$, in which g is the acceleration of gravity $=32.16$. Assuming that the engine is exerting just enough energy to overcome the frictional resistances, the train would climb a grade until the train was raised h feet above the point where its velocity was v. When it had climbed a height h' (less than h) it would have a velocity $v_1=\sqrt{2g(h-h')}$. As a numerical illustration, assume $v=30$ miles per hour $=44$ feet per second. Then $h=\frac{v^2}{2g}=30.1$ feet, and assuming that the engine was exerting just enough force to overcome the rolling resistances on a level, the kinetic energy in the train would carry it for two miles up a grade of 15 feet per mile, or half a mile up a grade of 60 feet per mile. When the train had climbed 20 feet, there would still be 10.1 feet left and its velocity would be $v_1=\sqrt{2g(10.1)}=25.49$ feet per second $=17.4$ miles per hour. These figures, however, must be slightly modified on account of the weight and the revolving action of the wheels, which form a considerable percentage of the total weight of the train. When train velocity is being

acquired, part of the work done is spent in imparting the energy of rotation to the driving-wheels and various truck-wheels of the train. Since these wheels run on the rails and must turn as the train moves, their rotative kinetic energy is just as effective—as far as it goes—in becoming transformed back into useful work. The proportion of this energy to the total kinetic energy has already been demonstrated (see Chapter XVI, § 435). The value of this correction is variable, but an average value of 5% has been adopted for use in the accompanying tabular form (Table XLII), in which is given the corrected "velocity head" corresponding to various velocities in miles per hour. The table is computed from the following formula:

$$\text{Velocity head} = \frac{v^2 \text{ in ft. per sec.}}{64.32} = \frac{2.151V^2 \text{in m. per h.}}{64.32} = 0.03344V^2$$

adding 5% for the rotative kinetic energy of the wheels, $\underline{0.00167V^2}$

The corrected velocity head therefore equals $0.03511V^2$

Part of the figures of Table XLII were obtained by interpolation and the final *hundredth* may be in error by one unit, but it may readily be shown that the final hundredth is of no practical importance. It is also true that the chief use made of this table is with velocities much less than 45 miles per hour. Corresponding figures may be obtained for higher velocities, if desired, by multiplying the figure for *half* the velocity by *four*.

515. Construction of a virtual profile. The following simple demonstration will be made on the basis that the ordinary tractive resistances and also the tractive force of the locomotive are independent of velocity. For a considerable range of velocity which includes the most common freight-train velocities the first assumption is practically true; the second assumption is so nearly true under certain possible operative conditions that it may serve as a preliminary to the more accurate solution. It may best be illustrated by considering a simple numerical example.

Assume that Fig. 213 shows the profile of a section of road and that the grade of AE is 0.40%, which is 21.12 feet per mile. Assume also that a freight engine is climbing up the grade at a uniform velocity of 20 miles per hour. But since the train is moving at 20 miles per hour it has a kinetic energy corresponding to a velocity of 14.05 feet (see Table XLII). At A it encounters a down-grade of 0.20 per cent, which is 1500 feet long. Although

AB has a down-grade of only 0.20%, its grade with respect to the up-grade of AE (0.40%) is 0.60%. Therefore B is 9.00 feet below B'. Since the work done by the engine would have carried the train up to the point B' with a velocity of 20 miles per hour, the *virtual* drop of 9 feet will increase the velocity head from 14.05 feet to 23.05 feet, which corresponds to the velocity of 25.6 miles per hour, and this will actually be the velocity of the train at the point B. At B the grade changes to a 1.0% up-grade for a distance of 2300 feet.

The approach of the grade BC to the grade $B'C$ is at the rate of $1.0-0.4=0.6\%$ and therefore, the point C will be reached in 1500 feet. In the remaining 800 feet the line will climb to D, which is 4.8 feet above D'. Although at B the train is moving at the rate of 25.6 miles per hour and the engine is working at such a rate that it will carry the train up a 0.4% grade, yet when climbing up a 1.0% grade t consumes its kinetic energy in vercoming the additional grade. Vhen it reaches C, it has lost e additional kinetic energy ich it gained from A to B, and t continues it loses even more. en it reaches D, it has lost 4.8 more and its velocity head luced to $14.05-4.8=9.25$ ft., h corresponds to a velocity .2 miles per hour. At D ade changes to $+0.1\%$.

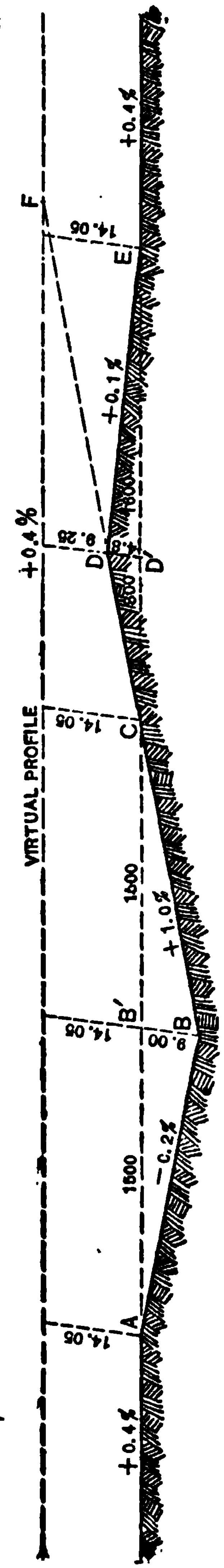

FIG. 213.—TYPICAL PROFILE OF ROAD SECTION.

TABLE XLII—VELOCITY HEAD (REPRESENTING THE KINETIC ENERGY) OF TRAINS MOVING AT VARIOUS VELOCITIES.

Vel. mi. hr.	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
5	0.88	0.91	0.95	0.99	1.02	1.06	1.10	1.14	1.18	1.22
6	1.26	1.31	1.35	1.40	1.44	1.48	1.53	1.58	1.62	1.67
7	1.72	1.77	1.82	1.87	1.92	1.97	2.03	2.08	2.14	2.19
8	2.25	2.30	2.36	2.42	2.48	2.54	2.60	2.66	2.72	2.78
9	2.85	2.91	2.97	3.04	3.10	3.17	3.24	3.30	3.37	3.44
10	3.51	3.58	3.65	3.72	3.79	3.87	3.95	4.02	4.10	4.17
11	4.25	4.33	4.41	4.49	4.57	4.65	4.73	4.81	4.89	4.97
12	5.06	5.15	5.23	5.32	5.41	5.50	5.58	5.67	5.75	5.84
13	5.93	6.02	6.12	6.21	6.31	6.40	6.50	6.59	6.69	6.78
14	6.88	6.98	7.08	7.19	7.29	7.39	7.49	7.60	7.70	7.80
15	7.90	8.00	8.11	8.22	8.33	8.44	8.55	8.66	8.77	8.88
16	8.99	9.10	9.21	9.32	9.43	9.55	9.67	9.79	9.91	10.03
17	10.15	10.27	10.39	10.51	10.63	10.75	10.87	10.99	11.12	11.25
18	11.38	11.50	11.63	11.76	11.89	12.02	12.15	12.28	12.41	12.55
19	12.68	12.81	12.95	13.08	13.22	13.35	13.49	13.63	13.77	13.91
20	14.05	14.19	14.33	14.47	14.61	14.75	14.89	15.04	15.19	15.34
21	15.49	15.64	15.79	15.94	16.09	16.24	16.39	16.54	16.69	16.84
22	17.00	17.15	17.30	17.46	17.62	17.78	17.94	18.10	18.26	18.42
23	18.58	18.74	18.90	19.06	19.22	19.38	19.55	19.72	19.89	20.06
24	20.23	20.40	20.57	20.74	20.91	21.08	21.25	21.42	21.59	21.77
25	21.95	22.12	22.30	22.48	22.66	22.84	23.02	23.20	23.38	23.56
26	23.74	23.92	24.10	24.28	24.46	24.65	24.84	25.03	25.22	25.41
27	25.60	25.79	25.98	26.17	26.36	26.55	26.74	26.93	27.13	27.33
28	27.53	27.73	27.93	28.13	28.33	28.53	28.73	28.93	29.13	29.33
29	29.53	29.73	29.93	30.13	30.34	30.55	30.76	30.97	31.18	31.39
30	31.60	31.81	32.02	32.23	32.44	32.65	32.86	33.08	33.30	33.52
31	33.74	33.96	34.18	34.40	34.62	34.84	35.06	35.28	35.50	35.72
32	35.95	36.17	36.39	36.62	36.85	37.08	37.31	37.54	37.77	38.00
33	38.23	38.46	38.69	38.92	39.15	39.38	39.62	39.86	40.10	40.34
34	40.58	40.82	41.06	41.30	41.54	41.78	42.02	42.26	42.51	42.76
35	43.01	43.26	43.51	43.76	44.01	44.26	44.51	44.76	45.01	45.26
36	45.51	45.76	46.01	46.26	46.52	46.78	47.04	47.30	47.56	47.82
37	48.08	48.34	48.60	48.86	49.12	49.38	49.64	49.91	50.18	50.45
38	50.72	50.99	51.26	51.53	51.80	52.07	52.34	52.61	52.88	53.15
39	53.42	53.69	53.96	54.23	54.51	54.79	55.07	55.35	55.63	55.91
40	56.19	56.47	56.75	57.03	57.31	57.59	57.87	58.16	58.45	58.74
41	59.03	59.32	59.61	59.90	60.19	60.48	60.77	61.06	61.35	61.64
42	61.94	62.23	62.52	62.82	63.12	63.42	63.72	64.02	64.32	64.62
43	64.92	65.22	65.52	65.82	66.12	66.43	66.74	67.05	67.36	67.67
44	67.98	68.29	68.60	68.91	69.22	69.53	69.84	70.15	70.46	70.78

Here we have the rather surprising condition that, although the grade is actually rising, it is virtually a down-grade under the given conditions, for the engine is working harder than is required to run up merely a 0.1% grade and hence will gain in velocity. At E, a distance of 1600 feet from D, it reaches what

would have been a uniform 0.4% grade from A to E and the grade continues at that rate. Although the train has actually climbed 1.6 feet from D to E, it has virtually fallen the 4.8 feet between D and D', and the velocity head has increased from its value of 9.25 feet at D to 14.05 feet, and its velocity is again 20 miles per hour. The upper line represents the "virtual profile," which may always be drawn by measuring off to the proper scale at every point an ordinate which is the velocity head at that point. Since the engine is working uniformly, the virtual profile is in this case a straight line.

As another case, assume that a train is climbing the grade AE and exerting a pull just sufficient to maintain a constant velocity up that grade. Then $A'B'$ (parallel to AB) is the virtual profile, AA' representing the velocity head. A stop being required at C, steam is shut off and brakes are applied at B, and the velocity head BB' reduces to zero at C.

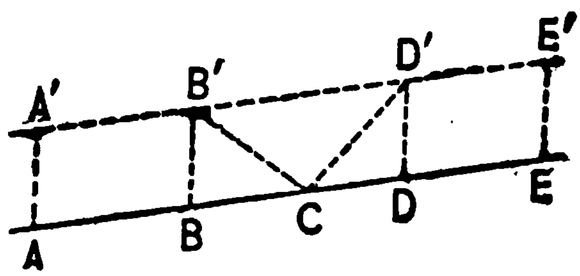

FIG. 214.

The train starts from C, and at D attains a velocity corresponding to the ordinate DD'. At D the throttle may be slightly closed so that the velocity will be uniform and the virtual grade is $D'E'$, parallel to DE.

From the above it may be seen that a virtual profile has the following properties:

(*a*) When the velocity is *uniform*, the virtual profile is parallel with the actual.

(*b*) When the velocity is increasing the profiles are separating; when decreasing the profiles are approaching.

(*c*) When the velocity is zero the profiles coincide.

(*d*) The virtual grade at any place is a measure of the work required of the engine beyond that required to overcome merely the tractive resistances. If it is horizontal it shows that the engine is doing nothing besides overcoming the tractive resistances. If it is upward and is uniform, as in Fig. 213, it shows that it is working uniformly and is storing in the train "potential" energy which may be utilized on the return trip if it is not utilized to overcome tractive resistance in moving down a succeeding down-grade. If it is downward, as from B' to C, Fig. 214, it shows that the train is giving up kinetic energy, probably consuming most of it in brakes, but utilizing some of it

to furnish the tractive power to run from B to C and also to overcome the grade from B to C.

516. Variation in draw-bar pull. The above demonstration has been made on the basis that the draw-bar pull is constant throughout. It is shown in Chapter XVIII that, when the engine is working at its full capacity the draw-bar pull decreases as the velocity increases, which is chiefly due to the fact that if we attempt to use full stroke at 2 M or 3 M velocity the steam will be so rapidly exhausted from the boiler that the pressure will fall. Therefore the valves are set to cut off so as to use the steam expansively but as this reduces the average pressure in the cylinder, then (see Eq. 103), the tractive power must be less. The reduction of tractive power for several multiples of M is shown in Table XXXIX.. For example, in the numerical problem given above, and assuming the use of the Mikado engine whose characteristics have already been computed, the velocity at $A = 20 \div 6.167 = 3.25\ M$ and the tractive power at this velocity is 49.23% of its power at M velocity. From the tabular form in § 460 the draw-bar pull at 3.25 M-velocity may be found by interpolation to be 16587 lbs. Similarly at B the velocity is *expected* to be 25.6 m.p.h. $= 4.15\ M$, and then the tractive power is 38.48% and the draw-bar pull only 12484 lbs., about 75% of the pull at A. But since the draw-bar pull is so much reduced the velocity evidently would not be increased the theoretical amount due to the virtual drop BB'. On the other hand, when the train reaches D, where the velocity is *supposed* to be 16.2 m.p.h. $= 2.62\ M$, the draw-bar pull would be 20144, which is over 121% of the normal pull at 3.25 M velocity. The average pull between B and D is 16314 or within 2% of the normal 16587. The average between A and E, assuming that the theoretical velocities at B and D were actually realized, would be about 2% below the assumed pull at A. The 3000-foot sag ABC will be passed in 90 seconds and no very great reduction in boiler power could take place in that time, especially if the fireman used extra care to maintain the pressure. Investigators have declared that tests of trains, with a dynamometer car between the tender and cars, have shown a practically uniform draw-bar pull, with an unchanged throttle and with velocities varying substantially on the principles indicated above. If the sag ABC is excessively long or deep the reduction of tractive force with increased velocity would be so great that the error of the method would be

too great for practical use. But experience has proven that for ordinary cases the method can be used with substantial accuracy.

517. Use, value, and possible misuse. The essential feature respecting grades is the demand on the locomotive. From the foregoing it may readily be seen that the ruling grade of a road is not necessarily the steepest nominal grade. When a grade may be operated by momentum, i.e., when every train has an opportunity to take "a run at the hill," it may become a very harmless grade and not limit the length of trains, while another grade, actually much less, which occurs at a stopping-place for the heaviest trains, will require such extra exertion to get trains started that it may be the worst place on the road. Therefore the true way to consider the value of the grade at any critical place on the road is to construct a virtual profile for that section of the road. The required length of such a profile is variable, but in general may be said to be limited by points on each side of the critical section at which the velocity is definite, as at a stopping-place (velocity zero), or a long heavy grade where it is the minimum permissible, say M miles per hour.

Since the velocities of different trains vary, each train will have its own virtual profile at any particular place. Fast passenger trains are less affected than slow freight trains. The requirement of high average speed necessitates the use of powerful engines, and grades which would stall a heavy freight will only cause a momentary and harmless reduction of speed of the fast passenger train.

A possible misuse of virtual profiles lies in the chance that a station or railroad grade crossing may be subsequently located on a heavy grade that was designed to be operated by momentum. But this should not be used as an argument against the employment of a virtual profile. The virtual profile shows the *actual state of the case* and only points out the necessity, if an unexpected requirement for a full stoppage of trains at a critical point has developed, of changing the location (if a station), or of changing the grade by regrading or by using an overhead crossing.

518. Undulatory grades. Advantages. Money can generally be saved by adopting an actual profile which is not strictly uniform—the matter of compensation for curvature being here

ignored. Its effect on the operation of trains is harmless provided the sag or hump is not too great. In Fig. 215 the undulatory grade may actually be operated as a uniform grade *AG*. The sag at *C* must be considered as a sag, even though *BC* is actually an up grade. But the engine is supposed to be working hard enough to carry a train at uniform velocity up a grade *AG*. Therefore it *gains* in velocity from *B* to *C*, and from *C* to *D* loses an equal amount. It may even be proven that the *time* re-

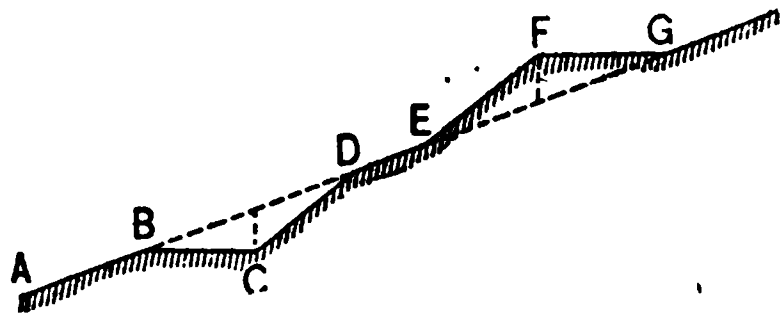

FIG. 215.

quired to pass the sag will be slightly less than the time required to run the uniform grade.

Disadvantages. The hump at *F* is dangerous in that, if the velocity at *E* is not equal to that corresponding to the extra velocity-head ordinate at *F*, the train will be stalled before reaching *F*. In practice there should be considerable margin. Any train should have a velocity of at least *M* (see § 455) in passing any summit. An extra heavy head wind, slippery rails, etc., may use up any smaller margin and stall the train. If the grade *AG* is a ruling grade, then *no* bump should be allowed under any circumstances. For the heaviest trains are supposed to be so made up that the engine will *just* haul them up the ruling grades—of course with some margin for safety. Any increase of this grade, however short, would probably stall the train.

Safe limits. Since over 99.4% of all freight cars are now equipped with train brakes and automatic couplers, there is not now the limitation which formerly existed about operating freight trains at high speeds, but it may frequently happen that it would be undesirable to run a freight train through a deep sag at such a velocity as would result from a free run and it would therefore become necessary to use brakes, which will add a distinct element of cost.

The term "safe limits" as used here, refers to the limits within

which a freight train may be safely operated without the application of brakes or varying the work of the engine. Of course much greater undulations are frequently necessary and are safely operated, but it should be remembered that they add a distinct element to the cost of operating trains and that they must not be considered as harmless or that they should be introduced unless really necessary.

RULING GRADES.

519. Definition. Ruling grades are those which limit the weight of the train of cars which may be hauled by one engine. The subject of "pusher grades" will be considered later. For the present it will suffice to say that on all well-designed roads the large majority of the grades on any one division are kept below some limit which is considered the ruling grade. If a heavier grade is absolutely necessary no special expense will be made to keep it below a rate where the resistance is twice (or possibly three times) the resistance on the ruling grade, and then the trains can be hauled unbroken up these few special grades with the help of one (or two) pusher engines. So far as limitation of train length is concerned, these pusher grades are no worse than the regular ruling grades and, except for the expense of operating the pusher engines (which is a separate matter), they are not appreciably more expensive than any ruling grade. As before stated, the engineer cannot alter very greatly the ruling grade of the road when the general route has been decided on. He may remove sags or humps, or he may lower the natural grade of the route by development in order to bring the grade within the adopted limit of ruling grade.

520. Choice of ruling grade. It is of course impracticable for an engine to drop off or pick up cars according to the grades which may be encountered along the line. A train load is made up at one terminus of a division and must run to the other terminus. Excluding from consideration any short but steep grades which may *always* be operated by momentum, and also all pusher grades, the maximum grade on that division is the ruling grade.

It will evidently be economy to reduce the few grades which naturally would be a little higher than the great majority of

others until such a large amount of grade is at some uniform limit that a reduction at all these places would cost more than it is worth. The precise determination of this limit is practically impossible, but an approximate value may be at once determined from a general survey of the route. The distance apart of consecutive control points (see § 18) into their difference of elevation is a first trial figure for the rate of the grade. If a grade even approximately uniform is impossible owing to the elevation of intervening ground, the worst place may be selected and the natural grade of that part of the route determined. If this grade is much steeper than the general run of the natural grades, it may be policy to reduce it by development or to boldly plan to operate that place as a pusher grade. The choice of possible grades thus has large limitations, and it justifies very close study to determine the best combination of grades and pusher grades. When the choice has narrowed down to two limits, the lower of which may be obtained by the expenditure of a definite extra sum, the choice may be readily computed, as will be developed.

521. Maximum train load on any grade. The Mikado locomotive, whose characteristics were analyzed in Chapter XVIII, has a net pulling power at the rim of the drivers, at *M* velocity, of 35758 lbs. which is 23.3% of 153,200, the weight on the drivers. This percentage is slightly over $\frac{9}{40}$. Increasing the percentage 6% on account of increased power at starting we have 24.7% or nearly $\frac{1}{4}$. On the other hand, wet, slippery rails may render the adhesion as low as $\frac{1}{5}$ and thus limit the actual drawing power. Although the real power of a locomotive depends on the velocity at which it seems desirable to run, the maximum tractive power at "*M*" velocity can always be approximately estimated as $\frac{1}{4}$ of the weight on the drivers. In Table XLIII are given the weights of several types of locomotives together with their tractive powers at three ratios of adhesion. These values are useful when the more elaborate method detailed in Chapter XVIII is not considered necessary.

The maximum train load on any grade depends on the character and number of the cars, as well as on their gross weight. The approximate resistance of cars is given by Eq. 121 as $R=2.2\ t+122\ n$. Applying this to a steel box-car weighing 40 tons net and loaded with 100,000 lbs., the resistance would be 310 lbs. or 3.55 lbs. per ton. Empty, the resistance would be 5.25 lbs. per

TABLE XLIII.—TRACTIVE POWER OF VARIOUS TYPES OF STANDARD-GAUGE LOCOMOTIVES AT VARIOUS RATES OF ADHESION.

ton. Applying the formula to a wooden box-car weighing 15 et and carrying 60000 lbs., the resistances for the car full npty would be 4.9 and 10.3 lbs. per ton, respectively. and 10 pounds per ton are the ordinary extremes. Al- resistances of less than 3 lbs. per ton have been ed for whole trains of heavy-loaded coal cars, there ally enough light-weight cars and empties in a train to e the average per ton resistance to perhaps 6 lbs. per ton.

The Mikado locomotive, referred to above, had a draw-bar pull on a level at M velocity (6.167 m.p.h.) of 35419 lbs. How much of a load could it draw up a 1.2% grade at M velocity? Assume that the cars have a weight and character such that the average resistance would be 6 lbs. per ton. The grade resistance of the locomotive is $315{,}000 \times .012 = 3780$, which subtracted from 35419 leaves 31639, the pull available for the cars. Then, calling T the tons weight of cars

$$31639 = 6T + (20 \times 1.2 \times T) = 30T,$$

and

$$T = 1054.$$

This allows only 6% margin for extra starting resistance if it should be necessary to stop and start on the grade, and makes no allowance for acceleration. It represents a limit, for the given condition, which would probably not be used.

522. Proportion of the traffic affected by the ruling grade. Some very light traffic roads are not so fortunate as to have a traffic which will be largely affected by the rate of the ruling grade. When passenger traffic is light, and when, for the sake of encouraging traffic, more frequent trains are run than are required from the standpoint of engine capacity, it may happen that no passenger trains are really limited by any grade on the road—i.e., an extra passenger car *could* be added if needed. The maximum grade then has no worse effect (for passenger trains) than to cause a harmless reduction of speed at a few points. The local freight business is frequently affected in practically the same way. All coal, mineral, or timber roads are affected by the rate of ruling grade as far as such traffic is concerned. Likewise the through business in general merchandise, especially of the heavy traffic roads, will generally be affected by the rate of ruling grade. Therefore in computing the effect of ruling grade, the total number of trains on the road should not ordinarily be considered, but only the trains to which cars are added, until the limit of the hauling power of the engine on the ruling grades is reached.

PUSHER GRADES.

523. General principles underlying the use of pusher engines. On nearly all roads there are some grades which are greatly in excess of the general average rate of grade, and these heavy grades cannot usually be materially reduced without an expenditure which is excessive and beyond the financial capacity of the road. If no pusher engines are used, the length of all heavy trains is limited by these grades. The financial value of the reduction of such ruling grades has already been shown. But in the operation of pusher grades there is incurred the additional cost of pusher-engine service, for a pusher engine must run *twice* over the grade for each train which is assisted. It is possible for this additional expense to equal or even exceed the advantage to be gained. In any case it means the adoption of the lesser of two evils, or the adoption of the more economical method. The work of overcoming the normal resistances of so many loaded cars over so many miles of track and of lifting so many tons up the gross differences of elevation of predetermined points of the line is approximately the same whatever the exact

route, and if the grades are so made that fewer engines working more constantly can accomplish the work as well as more engines which are not hard worked for a considerable proportion of the time, the economy is very apparent and unquestionable. Wellington expresses it concisely: "It is a truth of the first importance that the objection to high gradients is not the work which the engines have to do on them, but it is the work which they do *not* do when they thunder over the track with a light train behind them, from end to end of a division, in order that the needed power may be at hand at a few scattered points where alone it is needed."

524. **Balance of grades for pusher service.** Assume that both pusher and through engines are the Mikado engine with dimensions already given (§ 453), and that they will be operated at their most effective velocity, $M = 6.167$ m.p.h., and that the effective draw-bar pull of each is $37190 - 1771 = 35419$ lbs., less the locomotive grade resistance, which on a 1.9% grade is $20 \times 1.9 \times 157.5 = 5985$ lbs. The net draw-bar pull on this grade for each engine is, therefore, 29434 lbs. Assume that the train considered is made up of coal cars weighing 40000 lbs. net and carrying 100,000 lbs. each; also a caboose weighing 12 tons. Utilizing Eq. 121, the tractive resistance of a loaded coal car will be $2.2 \times 70 + 122 = 276$, and the grade resistance $20 \times 1.9 \times 70 = 2660$, making a total of 2936. The total for the caboose is $148 + 456 = 604$. The two engines have a net draw-bar pull of $2 \times 29434 = 58868$ lbs. Subtracting 604 for the caboose, there is left 58264 for coal cars. $58264 \div 2936 = 19.84$, the number of cars. Although the number of cars must, of course, be a whole number, the computation of the relative through and pusher grades requires that we use the fractional number. The tractive resistance of the 19.84 cars and caboose is $2.2\,[(19.84 \times 70) + 12] + (122 \times 20.84) = 5624$. The force available for grade is $35419 - 5624 = 29795$. The tonnage on the single engine grade is 157.5 (engine) plus $19.84 \times 70 = 1388.8$ (coal cars), plus 12 (caboose), or 1558.3 tons. $29795 \div 1558.3 = 19.12$ lbs. per ton, which is the grade resistance for a 0.956% grade. This means that the through grade can be made 0.956% and the corresponding pusher grade may be 1.9%. If the same problem is worked out on the basis of some other type of engine, which, perhaps, weighs considerably less, very nearly the same through grade to correspond with the pusher grade will be

obtained. The above combination of unit car weights must be worked as 19 coal cars and a caboose and have a considerable margin of unused power. A different combination of car weights would use up the power with less or no margin, but in any case the computation of the corresponding lower grade, or the computation of an allowable pusher grade on the basis of a given through grade, should be made by using a fractional number of cars.

Since the pusher engine service is intermittent, and since it is working at full power for much less than half the time, it is practicable for the fireman to feed coal faster than the standard of 4000 lbs. of coal per hour while going up the pusher grade. The above computation was made on the basis of power production at the 4000-lb. rate. In § 457, it is shown that increasing the rate of coal consumption increases the value of M, and conversely when the locomotive is run at a velocity less than M the tractive power is increased, although the increase is disproportionately small. Increasing the tractive power of the pusher engine will increase the number of cars, although probably not as much as one car. Then the increase in car number will increase the computed resistance and *decrease* the amount available for grade. This decreased amount is divided by an increased number of tons and the amount of available for grade per ton is less and the computed through grade is less. Considering the very slight and disproportionate difference made by increasing the rate of coal consumption beyond the 4000-lb. standard, it is, perhaps, wisest to make the ratio of the grades on the basis of engines of equal power.

525. Two-pusher grades. It may happen, although rarely, that three systems of ruling grades may be necessary on one division, which may be so balanced that one unbroken train is handled with equal facility on through grades with one engine, on one-pusher grades with two engines and on two-pusher grades with three engines. The relation of these three grades may be computed on the same principles as are used above.

526. Operation of pusher engines. The maximum efficiency in operating pusher engines is obtained when the pusher engine is kept constantly at work, and this is facilitated when the pusher grade is as long as possible, i.e., when the heavy grades and the great bulk of the difference of elevation to be surmounted is at one place. For example, a pusher grade of three miles fol-

lowed by a comparatively level stretch of three miles and then by another pusher grade of two miles cannot all be operated as cheaply as a continuous pusher grade of five miles. Either the two grades must be operated as a continuous grade of eight miles (sixteen pusher miles per trip) or else as two short pusher grades, in which case there would be a very great loss of time and a difficulty in so arranging the schedules that a train need not wait for a pusher or the pushers need not waste too much time in idleness waiting for trains. If the level stretch were imperative, the two grades would probably be operated as one, but an effort should be made to bring the grades together. It is not necessary to bring the trains to a stop to uncouple the pusher engine, but a stop is generally made for coupling on, and the actual cost in loss of energy and in wear and tear of stopping and starting a heavy train is as great as the cost of running an engine light for several miles.

There are two ways in which it is *possible* to economize in the use of pusher engines. (*a*) When the traffic of a road is so very light that a pusher engine will not be kept reasonably busy on the pusher grade it *may* be worth while to place a siding long enough for the longest trains both at top and bottom of the pusher grade and then take up the train in sections. Perhaps the worst objection to this method is the time lost while the engine runs the extra mileage, but with such very light traffic roads a little time more or less is of small consequence. On light traffic roads this method of surmounting a heavy grade will be occasionally adopted even if pushers are never used. If the traffic is fluctuating, the method has the advantage of only requiring such operation when it is needed and avoiding the purchase and operation of a pusher engine which has but little to do and which might be idle for a considerable proportion of the year. (*b*) The second possible method of economizing is only practicable when a pusher grade begins or ends at or near a station yard where switching-engines are required. In such cases there is a possible economy in utilizing the switching-engines as pushers, especially when the work in each class is small, and thus obtain a greater useful mileage. But such cases are special and generally imply small traffic.

A telegraph-station at top and bottom of a pusher grade is generally indispensable to effective and safe operation.

527. Length of a pusher grade. The virtual length of the

pusher grade, as indicated by the mileage of the pusher engine, is always somewhat in excess of the true length of the grade as shown on the profile, and sometimes the excess length is very great. If a station is located on a lower grade within a mile or so of the top or bottom of a pusher grade, it will ordinarily be advisable to couple or uncouple at or near the station, since the telegraph-station, switching, and signaling may be more economically operated at a regular station. If the extra engine is coupled on ahead of the through engine (as is sometimes required by law for passenger trains) the uncoupling at the top of the grade may be accomplished by running the assistant engine ahead at greater speed after it is uncoupled, and, after running it on a siding, clearing the track for the train. But this requires considerable extra track at the top of the grade. Therefore, when estimating the length of the pusher grade, the most desirable position for the terminal sidings must be studied and the length determined accordingly rather than by measuring the mere length of the grade on the profile. Of course these odd distances are always *excess*; the coupling or uncoupling should not be done while on the grade.

528. **The cost of pusher-engine service.** When we analyze the elements of cost, we will find that many of them are dependent only on time, while others are dependent upon mileage. Still others are dependent on both. Very much will depend on the constancy of the service, and this in turn depends on the train schedule and on a variety of local conditions which must be considered for each particular case. The effect of a pusher-engine on maintenance of way may be considered on the basis that an engine is responsible for one-half of the deterioration of maintenance of way and structures, and, therefore, one-half of the percentage of the first 19 items in Table XLI or 9.06% of the average cost of a train-mile will be considered as chargeable for each mile of pusher engine service. Although the cost of repairs and renewals of engines is evidently a function of the mileage, and would therefore be somewhat less for a pusher-engine which did little work than for an engine which was worked to the limit of its capacity, yet it is only safe to make the same allowance as for other engines. Other items of maintenance of equipment are evidently to be ignored. The item of wages of enginemen will evidently depend upon the system employed on the particular road. Whatever the precise system

TABLE XLIV.—COST FOR EACH MILE OF PUSHER-ENGINE SERVICE.

Item number.	Item (abbreviated).	Normal average.	Per cent affected.	Cost per engine mile, per cent.
1–19	Track material, labor, bridges....	18.12%	50	9.06
25–27	Steam locomotives..............	9.24	100	9.24
80, 81	Road enginemen and engine-house expenses.....................	8.12	100	8.12
82–85	Fuel and other engine supplies....	11.27	100	11.27
90, 91, 94	Signaling, flagmen, and telegraph..	1.21	100	1.21
		....	....	38.90

the general result is to pay the enginemen as much in wages as the average payment for regular service, and therefore the full allowance for Item 80 will be made. Similarly we must allow the full cost of the items for engine supplies. While the engine is doing its heavy work in climbing up the grade, the consumption of fuel and water is certainly greater than the average; but, on the other hand, on the return trip, when the engine is running light, it probably runs for a considerable portion of the distance actually without steam, and therefore the consumption of fuel and water will nearly, if not quite, average the consumption for an engine running up and down grade along the whole line. That portion of fuel consumption which is due to radiation, blowing-off steam, and the many other causes previously enumerated, will be the same regardless of the work done. We therefore allow 100% for all of these items of engine supplies. In general we must add 100% for Items 90, 91, and 94, the cost of switchmen and telegraphic service. While there might be cases where there would be no actual addition to the pay-rolls or the operating expenses on account of these items, we are not justified in general in neglecting to add the full quota for such service. Collecting these items we will have 38.90% of the average cost of a train-mile for the cost of each mile run by the pusher engine. On the basis that the average cost of a train mile is \$1.60, the cost of one mile of pusher engine service would be $.3890 \times \$1.60 = 62.24$ cents. Assume that the pusher engine grade is five miles long but that the engine actually runs 11 miles on a round trip and that it makes 5 round trips or 55 miles per day. Then the daily cost would be $.6224 \times 55 =$ \$34.23 per day. Probably \$25 to \$30 per day should be charged

up even if the mileage did not amount to as much, since many of the items in the cost of service are largely independent of mileage. On the other hand the pusher engine service renders unnecessary the extra trains which would have been required to handle the traffic with one engine over the steeper grades. The cost of these must be computed for each particular case.

BALANCE OF GRADES FOR UNEQUAL TRAFFIC.

529. **Nature of the subject.** It sometimes happens, as when a road runs into a mountainous country for the purpose of hauling therefrom the natural products of lumber or minerals, that the heavy grades are all in one direction—that the whole line consists of a more or less unbroken climb having perhaps a few comparatively level stretches, but no down grade (except possibly a slight sag) in the direction of the general up grade. With such lines this present topic has no concern. But the majority of railroads have termini at nearly the same level (500 feet in 500 miles has no practical effect on grade) and consist of up and down grades in nearly equal amounts and rates. The general rate of ruling grade is determined by the character of the country and the character and financial backing of the road to be built. It is always possible to reduce the grade at some point by "development" or in general by the expenditure of more money. It has been tacitly assumed in the previous discussions that when the ruling grade has been determined all grades in either direction are cut down to that limit. If the traffic in both directions were the same this would be the proper policy and sometimes is so. But it has developed, especially on the great east and west trunk lines, that the *weight* of the eastbound freight traffic is enormously greater than that of the westbound—that westbound trains consist very largely of "empties" and that an engine which could haul twenty loaded cars up a given grade in eastbound traffic could haul the same cars empty up a much higher grade when running west. As an illustration of the large disproportion which may exist, the eastbound ton-mileage on the P. R. R. between the years 1851 and 1885 was 3.7 times the westbound ton-mileage. Between the years 1876 and 1880 the ratio rose to more than 4.5 to 1. On such a basis it is as important and necessary to obtain, say, a 0.6% ruling grade against the eastbound traffic as to have,

say, a 1.0% grade against the westbound traffic. This is the basis of the following discussion. It now remains to estimate the probable ratio of the traffic in the two directions and from that to determine the proper "balance" of the opposite ruling grades.

530. Computation of the theoretical balance. Assume first, for simplicity, that the exact business in either direction is accurately known. A little thought will show the truth of the following statements.

1. The locomotive and passenger-car traffic in both directions is equal.

2. Except as a road may carry emigrants, the passenger traffic in both directions is equal. Of course there are innumerable individual instances in which the return trip is made by another route, but it is seldom if ever that there is any marked tendency to uniformity in this. Considering that a car load of, say, 50 passengers at 150 pounds apiece weigh but 7500 pounds, which is $\frac{1}{10}$ of the 75000 pounds which the car may weigh, even a considerable variation in the number of passengers will not appreciably affect the hauling of cars on grades. On parlor-cars and sleepers the ratio of live load to dead load (say 20 passengers, 3000 pounds, and the car, 125000 pounds) is even more insignificant. The effect of passenger traffic on balance of grades may therefore be disregarded.

3. Empty cars have a greater resistance *per ton* than loaded cars. Therefore in computing the hauling capacity of a locomotive hauling so many tons of "empties," a larger figure must be used for the ordinary tractive resistances—say four pounds per ton greater.

4. Owing to greater or less imperfections of management a small percentage of cars will run empty or but partly full in the direction of greatest traffic.

5. Freight having great bulk and weight (such as grain, lumber, coal, etc.) is run from the rural districts toward the cities and manufacturing districts.

6. The return traffic—manufactured products—although worth as much or more, do not weigh as much.

As a simple numerical illustration assume that the weight of the cars is $\frac{1}{3}$ and the live load $\frac{2}{3}$ of the total load when the cars are "full"—although not loaded to their absolute limit of capacity. Assume that the relative weight of live load

to be hauled in the other direction is but ⅓; assume that the grade against the heaviest traffic is 0.9%. Since the tractive resistance per ton is considerably greater in the case of unloaded cars than it is in the case of loaded cars, allowance must be made for this in calculating the train resistance. Mr. A. C. Dennis, of the Canadian Pacific Railway Company, has made some elaborate tests of train resistance for trains which were alternately loaded and empty, and found that the tractive resistance of loaded cars was very uniform at 4.7 pounds per ton when the weight of the empty cars was ⅓ of the total weight. He also found that the tractive resistance of empty cars was very uniform at 8.9 pounds per ton. Although the live load capacity of a box-car is usually considerably more than twice the weight of the empty car, it will probably coincide more nearly with actual running conditions to consider that the live load is just twice the dead load. Assume that these loads are being hauled by a consolidation engine with a total weight, including engine and tender, of 107 tons, of which 106000 pounds is on the drivers. We will assume that the tractive resistance of the locomotive is likewise 4.7 pounds per ton. On the 0.9% grade, the grade resistance will be 18 pounds per ton, and therefore the total resistance is 22.7 pounds per ton. Assume that this engine is working with a tractive adhesion of ¼; the tractive power at the circumference of the drivers will be ¼ of 106000 pounds, or 26500 pounds. Dividing this by 22.7, we obtain 1167 as the gross load of the train in tons. Subtracting the weight of the locomotive, 107 tons, we have 1060 tons as the weight of the loaded cars which could be hauled by this locomotive up a 0.9% grade, assuming an adhesion of ¼. Since the traffic in the other direction is but ⅓, we will assume that ⅔ of the return cars are empty. We then have 353 tons of loaded cars with a locomotive weighing 107 tons, and 236 tons of empty cars in the return train. The loaded cars with the locomotive will weigh 460 tons, and their tractive resistance will be 4.7 pounds per ton, or 2162 pounds. The 236 tons of empty cars will have a resistance of 8.9 pounds per ton, or a total tractive resistance of 2100 pounds. This makes a total of 4262 pounds of tractive resistance. Subtracting this from the 26500 of total adhesion of the drivers, we have left 22238 as the amount of pull available for grade. But the return train weighs 696 tons. Dividing this into 22238, we find that 32

pounds per ton is available for grade, which is the resistance on a 1.60% grade. Therefore, *under the above conditions*, a 0.9% grade against the heaviest traffic will correspond with a 1.60% grade against the lighter traffic.

Of course these figures will be slightly modified by variations in the assumptions as to the tractive resistance of loaded and unloaded cars, and more especially by variations in the ratio of live load to dead load in the two directions. Therefore no great accuracy can be claimed for the ratio of these two grades in opposite directions, nevertheless the above calculation shows unmistakably that under the given conditions, a very considerable variation in the rate of grade in opposite directions is not only justifiable, but a neglect to allow for it would be a great economic error.

531. Computation of relative traffic. Some of the principal elements have already been referred to, but in addition the following facts should be considered.

(*a*) The greatest disparity in traffic occurs through the handling of large amounts of coal, lumber, iron ore, grain, etc. On roads which handle but little of these articles or on which for local reasons coal is hauled one way and large shipments of grain the other way the disparity will be less and will perhaps be insignificant.

(*b*) A marked change in the development of the country may, and often does, cause a marked difference in the disparity of traffic. The heaviest traffic (in mere weight) is always toward manufacturing regions and away from agricultural regions. But when a region, from being purely agricultural or mineral, becomes largely manufacturing, or when a manufacturing region develops an industry which will cause a growth of heavy freight traffic from it, a marked change in the relative freight movement will be the result.

(*c*) Very great fluctuations in the relative traffic may be expected for prolonged intervals.

(*d*) An estimate of the relative traffic may be formed by the same general method used in computing the total traffic of the road (see § 473, Chap. XIX) or by noting the relative traffic on existing roads which may be assumed to have practically the same traffic as the proposed road will obtain.

CHAPTER XXIV.

THE IMPROVEMENT OF OLD LINES.

532. **Classification of improvements.** The improvements here considered are only those of alignment—horizontal and vertical. Strictly there is no definite limit, either in kind or magnitude, to the improvements which may be made. But since a railroad cannot ordinarily obtain money, even for improvements, to an amount greater than some small proportion of the previously invested capital, it becomes doubly necessary to expend such money to the greatest possible advantage. It has been previously shown that securing additional business and increasing the train load are the two most important factors in increasing dividends. After these, and of far less importance, come reductions of curvature, reductions of distance (frequently of doubtful policy, see Chap. XXI, § 503), and elimination of sags and humps. These various improvements will be briefly discussed.

(a) **Securing additional business.** It is not often possible by any small modification of alignment to materially increase the business of a road. The cases which do occur are usually those in which a gross error of judgment was committed during the original construction. For instance, in the early history of railroad construction many roads were largely aided by the towns through which the road passed, part of the money necessary for construction being raised by the sale of bonds, which were assumed or guaranteed and subsequently paid by the towns. Such aid was often demanded and exacted by the promoters. Instances are not unknown where a failure to come to an agreement has caused the promoters to deliberately pass by the town at a distance of some miles, to the mutual disadvantage of the road and the town. If the town subsequently grew in spite of this disadvantage, the *annual* loss of business might readily amount to more than the original sum in dispute.

Such an instance would be a legitimate opportunity for study of the advisability of re-location.

As another instance (the original location being justifiable) a railroad might have been located along the bank of a considerable river too wide to be crossed except at considerable expense. When originally constructed the enterprise would not justify the two extra bridges needed to reach the town. A growth in prosperity and in the business obtainable might subsequently make such extra expense a profitable investment.

(b) **Increasing the train load.** On account of its importance this will be separately considered in § 535 *et seq.*

(c) **Reduction in curvature and distance and the elimination of sags and humps.** Such improvements are constantly being made by all progressive roads. The need for such changes occurs in some cases because the original location was very faulty, the revised location being no more expensive than the original, and in other cases because the original location was the best that was then financially possible and because the present expanded business will justify a change.

(d) **Changing the location of stations or of passing sidings.** The station may sometimes be re-located so as to bring it nearer to the business center and thus increase the business done. But the principal reasons for re-locating stations or passing sidings is that starting trains may have an easier grade on which to overcome the additional resistances of starting. Such changes will be discussed in detail in § 537.

533. **Advantages of re-locations.** There are certain undoubted advantages possessed by the engineer who is endeavoring to improve an old line.

(*a*) The gross traffic to be handled is definitely known.

(*b*) The actual cost per train-mile for that road (which *may* differ very greatly from the average) is also known, and therefore the value of the proposed improvement can be more accurately determined.

(*c*) The actual performance of such locomotives as are used on the road may be studied at leisure and more reliable data may be obtained for the computations.

534. **Disadvantages of re-locations.** The disadvantages are generally more apparent and frequently appear practically insuperable—more so than they prove to be on closer inspection.

(*a*) It frequently means the abandonment of a greater or less length of old line and the construction of new line. At first thought it might seem as if a change of line such as would permit an increase of train-load of 50 or perhaps 100% could never be obtained, or at least that it could not be done except at an impracticable expense. On the contrary a change of 10% of the old line is frequently all that is necessary to reduce the grades so that the train-loads hauled by one engine may be nearly if not quite doubled. And when it is considered that the cost of a road to sub-grade is generally not more than one-third of the total cost of construction and equipment per mile, it becomes plain that an expenditure of but a small percentage of the original outlay, expended where it will do the most good, will often suffice to increase enormously the earning capacity.

(*b*) One of the most difficult matters is to convince the financial backers of the road that the proposed improvement will be justifiable. The cause is simple. The disadvantages of the original construction lie in the large increase of certain items of expense which are necessary to handle a given traffic. And yet the fact that the expenditures are larger than they need be are only apparent to the expert, and the fact that a saving may be made is considered to be largely a matter of opinion until it is demonstrated by actual trial. On the other hand the cost of the proposed changes is definite, and the very fact that the road has been uneconomically worked and is in a poor financial condition makes it difficult to obtain money for improvements.

(*c*) The legal right to abandon a section of operated line and thus reduce the value of some adjoining property has sometimes been successfully attacked. A common instance would be that of a factory which was located adjoining the right of way for convenience of transportation facilities. The abandonment of that section of the right of way would probably be fatal to the successful operation of the factory. The objection may be largely eliminated by the maintenance of the old right of way as a long siding (although the business of the factory might not be worth it), but it is not always so easy of solution, and this phase of the question must always be considered.

REDUCTION OF VIRTUAL GRADE.

535. Obtaining data for computations. As developed in the last chapter (§§ 515–517) the real object to be attained is the reduction of the *virtual grade*. The method of comparing grades under various assumed conditions was there discussed. When the road is still "on paper" some such method is all that is possible; but when the road is in actual operation the virtual grade of the road at various critical points, with the rolling stock actually in use, may be determined by a simple test and the effect of a proposed change may be reliably computed. Bearing in mind the general principle that the virtual grade line is the locus of points determined by adding to the actual grade profile ordinates equal to the velocity head of the train, it only becomes necessary to measure the velocity at various points. Since the velocity is *not* usually uniform, its precise determination at any instant is almost impossible, but it will generally be found to be sufficiently precise to assume the velocity to be uniform for a short distance, and then observe the time required to pass that short space. Suppose that an ordinary watch is used and the time taken to the nearest second. At 30 miles per hour, the velocity is 44 feet per second. To obtain the time to within 1%, the time would need to be 100 seconds and the space 4400 feet. But with variable velocity there would be too great error in assuming the velocity as uniform for 4400 feet or for the time of 100 seconds. Using a stop-watch registering fifths of a second, a 1% accuracy would require but 20 seconds and a space of 880 feet, at 30 miles per hour. Wellington suggests that the space be made 293 feet 4 inches, or $\frac{1}{18}$ of a mile; then the speed in miles per hour equals $200 \div s$, in which s is the time in seconds required to traverse the 293′ 4″. For instance, suppose the time required to pass the interval is 12.5 seconds. $\frac{1}{18}$ mile in 12.5 seconds = one mile in 225 seconds, or 16 miles per hour. But likewise $200 \div 12.5 = 16$, the required velocity. The following features should be noted when obtaining data for the computations:

(*a*) All critical grades on the road should be located and their profiles obtained—by a survey if necessary.

(*b*) At the bottom and top of all long grades (and perhaps at intermediate points if the grades are very long) spaces of known

length (preferably 293⅓ feet) should be measured off and marked by flags, painted boards, or any other serviceable targets.

(*c*) Provided with a stop-watch marking fifths of seconds the observer should ride on the trains affected by these grades and note the exact interval of time required to pass these spaces. If the space is 293⅓ feet, the velocity in miles per hour = 200 ÷ interval in seconds. In general,

$$V = \frac{\text{distance in feet} \times 3600}{\text{time in seconds} \times 5280}.$$

(*d*) Since these critical grades are those which require the greatest tax on the power of the locomotive, the conditions under which the locomotive is working must be known—i.e., the steam pressure, point of cut-off, and position of the throttle. Economy of coal consumption as well as efficient working at high speeds requires that steam be used expansively (using an early cut-off), and even that the throttle be partly closed; but when an engine is slowly climbing up a maximum grade with a full load it is not exerting its maximum tractive power unless it has its maximum steam pressure, wide-open throttle, and is cutting off nearly at full stroke. These data must therefore be obtained so as to know whether the engine is developing at a critical place all the tractive force of which it is capable. The condition of the track (wet and slippery or dry) and the approximate direction and force of the wind should be noted with sufficient accuracy to judge whether the test has been made under ordinary conditions rather than under conditions which are exceptionally favorable or unfavorable.

(*e*) The train-loading should be obtained as closely as possible. Of course the dead weight of the cars is easily found, and the records of the freight department will usually give the live load with all sufficient accuracy.

536. Use of the data obtained. A very brief inspection of the results, freed from refined calculations or uncertainties, will demonstrate the following truths:

(*a*) If, on a uniform grade, the velocity increases, it shows that, under those conditions of engine working, the load is less than the engine can handle on that grade

(*b*) If the velocity decreases, it shows that the load is greater than the engine can handle on an indefinite length of such

grade. It shows that such a grade is being operated by momentum. From the rate of decrease of velocity the maximum practicable length of such a grade (starting with a given velocity) may be easily computed.

(c) By combining results under different conditions of grade but with practically the same engine working, the tractive power of the engine may be determined (according to the principles previously demonstrated) for any grade and velocity. For example: On an examination of the profile of a division of a road the maximum grade was found to be 1.62% (85.54 feet per mile). At the bottom and near the top of this grade two lengths of 293′ 4″ are laid off. The distance between the centers of these lengths is 6000 feet. A freight train moving up the grade is timed at $9\frac{3}{8}$ seconds on the lower stretch and $7\frac{5}{8}$ seconds on the upper. These times correspond to $\frac{200}{9.4}$ and $\frac{200}{7.6}$ or 21.3 and 26.3 miles per hour respectively. It is at once observed that the velocity has increased and that the engine could draw even a heavier load up such a grade for an indefinite distance. How much heavier might the load be?

For simplicity we will assume that the conditions were normal, neither exceptionally favorable nor unfavorable, and that the engine was worked to its maximum capacity. The engine is a "consolidation" weighing 128700 pounds, with 112600 pounds on the drivers. The train-load behind the engine consists of ten loaded cars weighing 465 tons and eleven empties weighing 183 tons, thus making a total train-weight of 712 tons. Applying Eq. 106, we find that the *additional* force which the engine has actually exerted per ton in increasing the velocity from 21.3 to 26.3 miles per hour in a distance of 6000 feet is

$$P=\frac{70.224}{6000}(26.3^2-21.3^2)=2.78 \text{ pounds } \textit{per ton}$$

The grade resistance on a 1.62% grade is 32.4 pounds per ton. The average train resistance may be computed similarly to the method adopted in § 439:

$$\left.\begin{array}{r}465\\64\end{array}\right\}\text{ tons at } 4.7 \text{ pounds per ton} = 2486 \text{ pounds}$$

183	" " 8.9 " " "	=1629	"
712		4115	"

The average tractive resistance is therefore $4115 \div 712 = 5.78$ pounds per ton. Adding the grade resistance (32.4) we have a total train resistance of 38.18 pounds per ton. But, computing from the increase in velocity, the locomotive is evidently exerting a pull of 2.78 pounds per ton in excess of the computed required pull on that grade, or a total pull of 40.96 pounds per ton. Therefore the train load might have been increased proportionately and might have been made

$$712 \times \frac{2.78 + 38.18}{38.18} = 764 \text{ tons.}$$

This shows that 52 tons additional might have been loaded on to the train, or say, three more empties or one additional loaded car.

A pull of 40.96 pounds per ton means a total adhesion at the drivers of 29164 pounds, which is about 26% of the weight on the drivers—112600 pounds. This indicates average conditions as to traction, although better conditions than can be depended on for regular service.

The above calculation should of course be considered simply as a "single observation." The performance of the same engine on the same grade (as well as on many other grades) on succeeding days should also be noted. It may readily happen that variations in the condition of the track or of the handling of the engine may make considerable variation in the results of the several calculations, but when the work is properly done it is always possible to draw definite and very positive deductions.

537. Reducing the starting grade at stations. The resistance to starting a train is augmented from two causes: (*a*) the tractive resistances are usually about 20 pounds per ton instead of, say, 6 pounds, and (*b*) the inertia resistance must be overcome. The inertia resistance of a freight train (see § 435) which is expected to attain a velocity of 15 miles per hour in a distance of 1000 feet is (see Eq. 140)

$P = \frac{70.224}{1000}(15^2 - 0) = 15.8$ pounds per ton, which is the equivalent of a 0.79% grade. Adding this to a grade which nearly or quite equals the ruling grade, it virtually creates a new and higher ruling grade. Of course that additional force can be greatly reduced at the expense of slower acceleration, but even

this cannot be done indefinitely, and an acceleration to only 15 miles per hour in 1000 feet is as slow as should be allowed for. With perhaps 14 pounds per ton additional tractive resistance, we have about 30 pounds per ton additional—equiva-

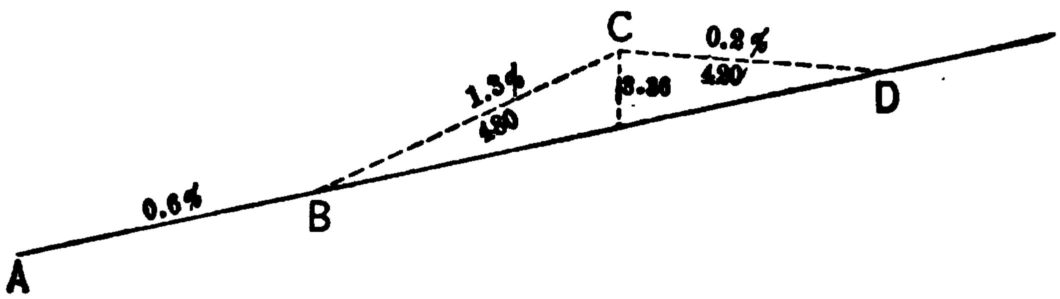

FIG. 216.

lent to a 1.5% grade. Instances are known where it has proven wise to create a *hump* (in what was otherwise a uniform grade) at a station. The effect of this on high-speed passenger trains moving *up* the grade would be merely to reduce their speed very slightly. No harm is done to trains moving *down* the grade. Freight trains moving *up* the grade and intending to stop at the station will merely have their velocity reduced as they approach the station and will actually save part of the wear and tear otherwise resulting from applying brakes. When the trains start they are assisted by the short down grade, just where they need assistance most. Even if the grade *CD* is still an up grade, the pull required at starting is *less* than that required on the uniform grade by an amount equal to 20 times the difference of the grade in per cent.

APPENDIX.

THE ADJUSTMENTS OF INSTRUMENTS.

THE accuracy of instrumental work may be vitiated by any one of a large number of inaccuracies in the geometrical relations of the parts of the instruments. Some of these relations are so apt to be altered by ordinary usage of the instrument that the makers have provided adjusting-screws so that the inaccuracies may be readily corrected. There are other possible defects, which, however, will seldom be found to exist, provided the instrument was properly made and has never been subjected to treatment sufficiently rough to distort it. Such defects, when found, can only be corrected by a competent instrument-maker or repairer.

A WARNING is necessary to those who would test the accuracy of instruments, and especially to those whose experience in such work is small. Lack of skill in handling an instrument will often indicate an apparent error of adjustment when the real error is very different or perhaps non-existent. It is always a safe plan when testing an adjustment to note the amount of the apparent error; then, beginning anew, make another independent determination of the amount of the error. When two or more *perfectly independent* determinations of such an error are made it will generally be found that they differ by an appreciable amount. The *differences* may be due in variable measure to careless inaccurate manipulation and to instrumental *defects* which are wholly independent of the particular test being made. Such careful determinations of the amounts of the errors are generally advisable in view of the next paragraph.

DO NOT DISTURB THE ADJUSTING-SCREWS ANY MORE THAN NECESSARY. Although metals are apparently rigid, they are really elastic and yielding. If some parts of a complicated mechanism, which is held together largely by friction, are subjected to greater internal stresses than other parts of the mech-

anism, the jarring resulting from handling will frequently cause a slight readjustment in the parts which will tend to more nearly equalize the internal stresses. Such action frequently occurs with the adjusting mechanism of instruments. One screw may be strained more than others. The friction of parts may prevent the opposing screw from *immediately* taking up an equal stress. Perhaps the adjustment appears perfect under these conditions Jarring diminishes the friction between the parts, and the unequal stresses tend to equalize. A motion takes place which, although microscopically minute, is sufficient to indicate an error of adjustment. A readjustment made by unskillful hands may not make the final adjustment any more perfect. The frequent shifting of adjusting-screws wears them badly, and when the screws are worn it is still more difficult to keep them from moving enough to vitiate the adjustments. It is therefore preferable in many cases to refrain from disturbing the adjusting-screws, especially as the accuracy of the work done is not *necessarily* affected by errors of adjustment, as may be illustrated:

(*a*) Certain operations are *absolutely* unaffected by certain errors of adjustment.

(*b*) Certain operations are so slightly affected by certain *small* errors of adjustment that their effect may properly be neglected.

(*c*) Certain errors of adjustment may be readily allowed for and neutralized so that no error results from the use of the unadjusted instrument. Illustrations of all these cases will be given under their proper heads.

ADJUSTMENTS OF THE TRANSIT.

1. *To have the plate-bubbles in the center of the tubes when the axis is vertical.* Clamp the upper plate and, with the lower clamp loose, swing the instrument so that the plate-bubbles are parallel to the lines of opposite leveling-screws. Level up until both bubbles are central. Swing the instrument 180°. If the bubbles again settle at the center, the adjustment is perfect. If either bubble does not settle in the center, move the leveling-screws until the bubble is *half-way* back to the center. Then, before touching the adjusting-screws, note carefully the position of the bubbles and observe whether the bubbles always settle at the *same* place in the tube, no matter to what position the in-

strument may be rotated. When the instrument is so leveled, the axis is truly vertical and the discrepancies between this constant position of the bubbles and the centers of the tubes measure the errors of adjustment. By means of the adjusting-screws bring each bubble to the center of the tube. If this is done so skillfully that the true level of the instrument is not disturbed, the bubbles should settle in the center for all positions of the instrument. Under unskillful hands, two or more such trials may be necessary.

When the plates are not horizontal, the measured angle is greater than the true horizontal angle by the difference between the measured angle and its projection on a horizontal plane. When this angle of inclination is small, the difference is insignificant. Therefore when the plate-bubbles are *very nearly* in adjustment, the error of measurement of horizontal angles may be far within the lowest unit of measurement used. A *small* error of adjustment of the plate-bubble *perpendicular* to the telescope will affect the horizontal angles by only a small proportion of the error, which will be perhaps imperceptible. Vertical angles will be affected by the same insignificant amount. A *small* error of adjustment of the plate-bubble *parallel* to the telescope will affect horizontal angles very slightly, but will affect vertical angles by the full amount of the error.

All error due to unadjusted plate-bubbles may be avoided by noting in what positions in the tubes the bubbles will remain fixed for all positions of azimuth and then keeping the bubbles adjusted to these positions. for the axis is then truly vertical. It will often save time to work in this way temporarily rather than to stop to make the adjustments. This should especially be done when accurate vertical angles are required.

When the bubbles are truly adjusted, they should remain stationary regardless of whether the telescope is revolved with the upper plate loose and the lower plate clamped or whether the whole instrument is revolved, the plates being clamped together. If there is any appreciable difference, it shows that the two vertical axes or "centers" of the plates are not concentric. This may be due to cheap and faulty construction or to the excessive wear that may be sometimes observed in an old instrument originally well made. In either case it can only be corrected by a maker.

2. *To make the revolving axis of the telescope perpendicular to the vertical axis of the instrument.* This is best tested by using a long plumb-line, so placed that the telescope must be pointed upward at an angle of about 45° to sight at the top of the plumb-line and downward about the same amount, if possible, to sight at the lower end. The vertical axis of the transit must be made truly vertical. Sight at the upper part of the line, clamping the horizontal plates. Swing the telescope down and see if the cross-wire again bisects the cord. If so, the adjustment is *probably* perfect (a conceivable exception will be

noted later); if not, raise or lower one end of the axis by means of the adjusting-screws, placed at the top of one of the standards, until the cross-wire will bisect the cord both at top and bottom. The plumb-bob may be steadied, if necessary, by hanging it in a pail of water. As many telescopes cannot be focused on an object nearer than 6 or 8 feet from the telescope, this method requires a long plumb-line swung from a high point, which may be inconvenient.

Another method is to set up the instrument about 10 feet from a high wall. After leveling, sight at some convenient mark high up on the wall. Swing the telescope down and make a mark (when working alone some convenient natural mark may generally be found) low down on the wall. Plunge the telescope and revolve the instrument about its vertical axis and again sight at the upper mark. Swing down to the lower mark. If the wire again bisects it, the adjustment is perfect. If not, fix a point *half-way* between the two positions of the lower mark. The plane of this point, the upper point, and the center of the instrument is truly vertical. Adjust the axis to these upper and lower points as when using the plumb-line.

3. *To make the line of collimation perpendicular to the revolving axis of the telescope.* With the instrument level and the telescope nearly horizontal point at some well-defined point at a distance of 200 feet or more. Plunge the telescope and establish a point in the opposite direction. Turn the whole instrument about the vertical axis until it again points at the first mark. Again plunge to "direct position" (*i.e.*, with the level-tube *under* the telescope). If the vertical cross-wire again points at the second mark, the adjustment is perfect. If not, the error is *one-fourth* of the distance between the two positions of the second mark. Loosen the capstan screw on one side of the telescope and tighten it on the other side until the vertical wire is set at the one-fourth mark. Turn the whole instrument by means of the tangent screw until the vertical wire is *midway* between the two positions of the second mark. Plunge the telescope. If the adjusting has been skillfully done, the cross-wire should come exactly to the first mark. As an "erecting eyepiece" reinverts an image already inverted, the ring carrying the cross-wires must be moved in the *same* direction as the *apparent* error in order to correct that error.

The necessity for the third adjustment lies principally in the practice of producing a line by plunging the telescope, but when this is required to be done with great accuracy it is always better to obtain the forward point by reversion (as described above for making the test) and take the *mean* of the two forward points. Horizontal and vertical angles are practically unaffected by *small* errors of this adjustment, unless, in the case of horizontal angles, the vertical angles to the points observed are very different.

Unnecessary motion of the adjusting-screws may sometimes be avoided by carefully establishing the forward point on line by repeated reversions of the instrument, and thus determining by repeated trials the exact amount of the error. *Differences* in the amount of error determined would be evidence of inaccuracy in manipulating the instrument, and would show that an adjustment based on the first trial would *probably* prove unsatisfactory.

The 2d and 3d adjustments are mutually dependent. If either adjustment is badly out, the other adjustment cannot be made except as follows:

(*a*) The second adjustment can be made regardless of the third when the lines to the high point and the low point make *equal* angles with the horizontal.

(*b*) The third adjustment can be made regardless of the second when the front and rear points are *on a level* with the instrument.

When both of these requirements are *nearly* fulfilled, and especially when the error of either adjustment is small, no trouble will be found in perfecting either adjustment on account of a small error in the other adjustment.

If the test for the second adjustment is made by means of the plumb-line and the vertical cross-wire intersects the line at all points as the telescope is raised or lowered, it not only demonstrates at once the accuracy of that adjustment, but also shows that the third adjustment is either perfect or has so small an error that it does not affect the second.

4. *To have the bubble of the telescope-level in the center of the tube when the line of collimation is horizontal.* The line of collimation should coincide with the optical axis of the telescope. If the object-glass and eyepiece have been properly centered, the previous adjustment will have brought the vertical cross-wire to the center of the field of view. The horizontal cross-wire should also be brought to the center of the field of view, and the bubble should be adjusted to it.

a. Peg method. Set up the transit at one end of a nearly level stretch of about 300 feet. Clamp the telescope with its bubble in the center. Drive a stake vertically under the eyepiece of the transit, and another about 300 feet away. Observe the height of the center of the eyepiece (the telescope being level) above the stake (calling it *a*); observe the reading of the rod when held on the other stake (calling it *b*); take the instrument to the other stake and set it up so that the eyepiece is

vertically over the stake, observing the height, c; take a reading on the first stake, calling it d. If this adjustment is perfect, then

$$a-d=b-c,$$
$$\text{or}\quad (a-d)-(b-c)=0.$$
$$\text{Call}\quad (a-d)-(b-c)=2m.$$

When m is positive, the line points downward;
" m " negative, " " " upward.

To adjust: if the line points *up*, sight the horizontal cross-wire (by moving the vertical tangent screw) at a point which is m lower, then adjust the bubble so that it is in the center.

By taking several independent values for a, b, c, and d, a mean value for m is obtained, which is more reliable and which may save much unnecessary working of the adjusting-screws.

b. Using an auxiliary level. When a carefully adjusted level is at hand, this adjustment may sometimes be more easily made by setting up the transit and level, so that their lines of collimation are as nearly as possible at the same height. If a point may be found which is half a mile or more away and which is on the horizontal cross-wire of the level, the horizontal cross-wire of the transit may be pointed directly at it, and the bubble adjusted accordingly. Any slight difference in the heights of the lines of collimation of the transit and level (say $\frac{1}{4}''$) may almost be disregarded at a distance of $\frac{1}{2}$ mile or more, or, if the difference of level would have an appreciable effect, even this may be practically eliminated by making an estimated allowance when sighting at the distant point. Or, if a distant point is not available, a level-rod with target may be used at a distance of (say) 300 feet, making allowance for the carefully determined difference of elevation of the two lines of collimation.

5. *Zero of vertical circle.* When the line of collimation is truly horizontal and the vertical axis is truly vertical, the reading of the vertical circle should be 0°. If the arc is adjustable, it should be brought to 0°. If it is not adjustable, the *index error* should be observed, so that it may be applied to all readings of vertical angles.

ADJUSTMENTS OF THE WYE LEVEL.

1. *To make the line of collimation coincide with the center of the rings.* Point the intersection of the cross-wires at some

well-defined point which is at a considerable distance. The instrument need not be level, which allows much greater liberty in choosing a convenient point. The vertical axis should be clamped, and the clips over the wyes should be loosened and raised. Rotate the telescope in the wyes. The intersection of the cross-wires should be continually on the point. If it is not, it requires adjustment. Rotate the telescope 180° and adjust *one-half* of the error by means of the capstan-headed screws that move the cross-wire ring. It should be remembered that, with an erecting telescope, on account of the inversion of the image, the ring should be moved in the direction of the *apparent* error. Adjust the other half of the error with the leveling-screws. Then rotate the telescope 90° from its usual position, sight accurately at the point, and then rotate 180° from that position and adjust any error as before. It may require several trials, but it is necessary to adjust the ring until the intersection of the cross-wires will remain on the point for any position of rotation.

If such a test is made on a very distant point and again on a point only 10 or 15 feet from the instrument, the adjustment may be found correct for one point and incorrect for the other. This indicates that the object-slide is improperly centered. Usually this defect can only be corrected by an instrument-maker. If the difference is very small it may be ignored, but the adjustment should then be made on a point which is at about the mean distance for usual practice—say 150 feet.

If the whole image appears to shift as the telescope is rotated, it indicates that the eyepiece is improperly adjusted. This defect is likewise usually corrected only by the maker. It does not interfere with instrumental accuracy, but it usually causes the intersection of the cross-wires to be eccentric with the field of view.

2. *To make the axis of the level-tube parallel to the line of collimation.* Raise the clips as far as possible. Swing the level so that it is parallel to a pair of opposite leveling-screws and clamp it. Bring the bubble to the middle of the tube by means of the leveling-screws. Take the telescope out of the wyes and replace it end for end, using *extreme care* that the wyes are not jarred by the action. If the bubble does not come to the center, correct *one-half* of the error by the vertical adjusting-screws at one end of the bubble. Correct the other half by the leveling-screws. Test the work by again changing the telescope end for end in the wyes.

Care should be taken while making this adjustment to see

that the level-tube is vertically under the telescope. With the bubble in the center of the tube, rotate the telescope in the wyes for a considerable angle each side of the vertical. If the first half of the adjustment has been made and the bubble moves, it shows that the axis of the wyes and the axis of the level-tube are not in the same vertical plane although both have been made horizontal. By moving one end of the level-tube *sidewise* by means of the horizontal screws at one end of the tube, the two axes may be brought into the same plane. As this adjustment is liable to disturb the other, both should be alternately tested until both requirements are complied with.

By these methods the axis of the bubble is made parallel to the axis of the wyes; and as this has been made parallel to the lines of collimation by means of the previous adjustment, the axis of the bubble is therefore parallel to the line of collimation.

3. *To make the line of collimation perpendicular to the vertical axis.* Level up so that the instrument is approximately level over both sets of leveling-screws. Then, after leveling carefully over one pair of screws, revolve the telescope 180°. If it is not level, adjust half of the error by means of the capstan-headed screw under one of the wyes, and the other half by the leveling-screws. Reverse again as a test.

When the first two adjustments have been accurately made, good leveling may always be done by bringing the bubble to the center by means of the leveling-screws, at every sight if necessary, even if the third adjustment is not made. Of course this third adjustment should be made as a matter of convenience, so that the line of collimation may be always level no matter in what direction it may be pointed, but it is not *necessary* to stop work to make this adjustment every time it is found to be defective.

ADJUSTMENTS OF THE DUMPY LEVEL.

1. *To make the axis of the level-tube perpendicular to the vertical axis.* Level up so that the instrument is approximately level over both sets of leveling-screws. Then, after leveling carefully over one pair of screws, revolve the telescope 180°. If it is not level, adjust *one-half* of the error by means of the adjusting-screws at one end of the bubble, and the other half by means of the leveling-screws. Reverse again as a test.

2. *To make the line of collimation perpendicular to the vertical axis.* The method of adjustment is identical with that for the transit (No. 4, pl. 505) except that the cross-wire must be

adjusted to agree with the level-bubble rather than *vice versa*, as is the case with the corresponding adjustment of the transit; i.e., with the level-bubble in the center, raise or lower the horizontal cross-wire until it points at the mark known to be on a level with the center of the instrument.

If the instrument has been well made and has not been distorted by rough usage, the cross-wires will intersect at the center of the field of view when adjusted as described. If they do not, it indicates an error which ordinarily can only be corrected by an instrument-maker. The error may be due to any one of several causes, which are

(*a*) faulty centering of object-slide;

(*b*) faulty centering of eyepiece;

(*c*) distortion of instrument so that the geometric axis of the telescope is not perpendicular to the vertical axis. If the error is only just perceptible, it will not probably cause any error in the work.

AZIMUTH.

The azimuth of a line on the surface of the earth is its angle with a true meridian through a point on the line. It is the true bearing as distinguished from "magnetic bearing." Federal law requires that all surveys of government lands shall be made by "Solar Observations" (rather than with the magnetic needle) so as to obtain true bearings.

Solar Azimuth may be obtained in two general ways, (*a*) by direct observation on the sun with an ordinary "complete" transit, provided with a colored glass shade, and (*b*) by the use of a "solar attachment" or a solar compass. The first method only requires as special equipment a colored glass shade costing but a few dollars, but it requires the separate solution of a formula for each observation made. Even the colored glass shade is not always necessary—as when the disc of the sun is just seen

through thin clouds and is not too bright to be observed with the naked eye. The "colored glass shade" may be merely a piece of colored glass fitted over the eye-piece, or the glass may be set into a frame very similar to the object glass cover and readily taken off and put on. In the latter case the glass must be "optically perfect," i.e., with the sides perfectly plane and parallel, so that there shall be no refraction of the image, or such glass as is used for the sun shade of a sextant.

The second method (*b*) does not require any calculation of a formula; the true meridian is given directly but it requires the use of a special instrument, whose adjustments must be made with great care or the resulting azimuth will often be in error by a *much larger* amount than the error in the adjustment. A proper appreciation of either method requires an understanding of certain astronomical relations.

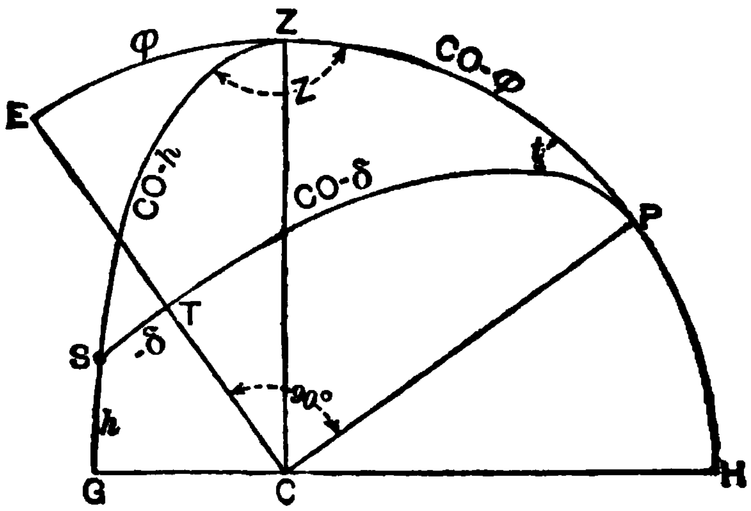

FIG. 1.

Fig. 1 represents the orthographic projection of the celestial sphere, projected on the plane of the meridian of the observer.

$HPZE$ represents the meridian of the observer.

Z = the zenith.

CP = the polar axis of the earth.

CE = the plane of the equator.

S = the position of the sun.

EZ = the latitude of the observer $= \phi$.

$ZP = 90° - \phi = \text{co } \phi$.

SG = the true altitude of the sun $= h$.

$SZ = 90° - h = \text{co } h$.

ST = the declination of the sun, north or south of the equator $= \delta$.

$SP = 90° - \delta = \text{co } \delta$.

The essential sign of δ must be considered. If the sun is south of the equator (as it is from about September 21 to March 21), δ is negative and if the declination is (say) S 20°, $\delta = -20°$. Then co $\delta = 90° - \delta = 90° - (-20°) = 110°$.

Z = the angle from the position of the sun to the true north = the spherical angle SZP.

Then, from spherical trigonometry, we have, in the spherical triangle SZP,

$$\text{Sin } \tfrac{1}{2} Z = \sqrt{\frac{\sin(S - \text{co } h) \sin(S - \text{co } \phi)}{\sin \text{co } h \sin \text{co } \phi}}$$

in which $S = \frac{1}{2}[\text{co } h + \text{co } \phi + \text{co } \delta]$.

The sun describes each day a path which is approximately parallel with the equator, the change in declination being very small during June and December and fastest when the sun is crossing the equator in March and September, the greatest rate of change being about 59 seconds of arc per hour. The declination of the sun must be known for the time of observation. This is obtainable from the Nautical Almanac or Ephemeris.

Example.—Declination for Philadelphia, Feb. 20, 1914, at 8:10 A. M., standard time, 75th meridian. Since "standard time" is a definite time interval from Greenwich mean local time, we may use it here regardless of precise longitude or mean local time, 8:10 A. M. on the 75° meridian is 1:10 P. M. mean time, at Greenwich. $1.17h \times 53''.64 = 62''.58 = 1'\ 2''.6$ and $-11°\ 7'\ 1''.1 + 0°\ 1'\ 2''.6 = -11°\ 5'\ 58''.5$ which is **south** declination.

Refraction. Refraction causes the sun to appear higher than it actually is. Therefore when the **altitude** of the sun is observed, the computed refraction should be **subtracted** from the **apparent** altitude to obtain the **true** altitude. The amount of the refraction is a very complicated function of the temperature and of the barometric pressure. For refined astronomical work, large refraction tables should be used, making due allowance for temperature and pressure, but for such work as may be done with an ordinary transit the values given in the following table will suffice.

Angular diameter of sun. The sun's angular diameter is about 0° 32′. With the comparatively high power telescopes now generally used on transits, this fills a large part of the field of view and it is impossible to accurately bisect such a large

MEAN REFRACTIONS—[BESSEL] TRUE FOR BAROMETER AT 29″.6, TEMP. 48° F.

Alt.	Refr.	Alt.	Refr.	Alt.	Refr.
0° 0′	34′ 54″	1° 30′	20′ 51″	5° 0′	9′ 46″
10	32 49	40	19 52	30	9 02
20	30 52	50	18 58	6 0	8 23
30	29 03	2 0	18 09	30	7 49
40	27 23	30	16 01	7 0	7 20
50	25 50	3 0	14 15	30	6 53
1° 0	24 25	30	12 48	8 0	6 30
10	23 07	4 0	11 39	30	6 08
20	21 56	30	10 40	9 0	5 49

Alt.	Refr.	Alt.	Refr.	Alt.	Refr.
9° 30′	5′ 32″	18°	2′ 56″	30°	1′ 40″
10 0	5 16	19	2 46	35	1 22
11 0	4 48	20	2 37	40	1 09
12 0	4 25	21	2 29	45	0 58
13 0	4 05	22	2 22	50	0 48
14 0	3 47	23	2 15	60	0 33
15 0	3 32	24	2 09	70	0 21
16 0	3 19	26	1 58	80	0 10
17 0	3 07	28	1 48	90	0 0

angular width especially as the apparent motion of the sun across the field of view is very rapid. It therefore becomes advisable (when sighting directly at the sun with the transit telescope) to sight the cross wires on the edges of the sun, as shown in Fig. 2, and make due allowance for the semi-diameter of the sun. The effect of this is to obtain an altitude which differs from the true altitude by the angular value of the semi-diameter. The observed azimuth differs from the true azimuth by the semi-diameter ÷ cos h. When the sun is at the horizon, cos $h = 1$, and the allowance equals the semi-diameter both for altitude and azimuth. For higher altitudes the allowance for azimuth is much larger than the semi-diameter, since the divisor (cos h) is small. If several observations are taken within a short interval, the *change* in this allowance for azimuth during this short interval may be too small for notice and one value may be sufficiently accurate for all the observations.

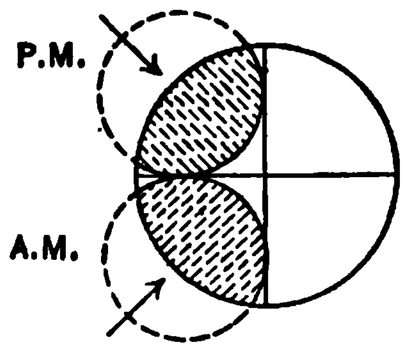

FIG. 2.

There is a slight variation in the semi-diameter as is shown in the accompanying tabular form, giving average values, which

may be used by interpolation, if a closer value than the nearest minute is desired.

Time.	Semi-diam. of the Sun in minutes of arc.
Jan. 1....	16′.30 (max)
April 1...	16 .03
July 1....	15 .76 (min)
Oct. 1....	16 .01

Latitude. If the latitude of the place of observation is not known to the nearest minute, it may readily be obtained by observing the altitude of the sun at culmination at noon. The horizontal cross wire should be sighted at the upper (or the lower) edge of the disc of the sun.

If d = angular diameter of sun r = refraction
ϕ = latitude δ = declination
h' = observed angle of elevation

$$\text{then } \phi = 90° - [h' - r - \delta \pm \tfrac{1}{2}d]$$

in which $\frac{1}{2}d$ is + for an observation on the lower edge,
and $\frac{1}{2}d$ is − for an observation on the upper edge.

Set up the transit several minutes before noon, taking sufficient time to level up with the utmost care. Set the horizontal cross wire on the upper (or lower) edge of the sun and with the tangent screw follow the motion of the sun. As the required angle is found at culmination, the motion of the telescope should cease when the highest altitude is obtained and the sun begins to descend.

Azimuth by an Observation with the transit telescope. Set up the transit at a convenient station from which an unobstructed view of the sun may be obtained at all times and from which a convenient permanent azimuth mark (e.g., a distant steeple or chimney) may be observed. Point at the azimuth mark with the horizontal plates reading zero. With the upper plate loose, point at the sun observing the time, altitude and the horizontal angle from the azimuth mark. Three or more such observations are generally advisable, especially as they are so easily and quickly taken and are such a valuable check on each other. A single observation may be vitiated by some inaccuracy or blunder

in manipulation or reading which would not be discovered unless more than one observation is taken, in which case the error would hardly be precisely repeated both in nature and amount. Finally, point at the azimuth mark to test whether the lower plate has slipped. The reading on the azimuth mark should be 0°.

Reducing the Observations. Compute the declinations for the given times of observation. If several observations are taken, it is generally best to compute the declinations for the times of the first and last observations and interpolate for the others. The observations may most readily be reduced by using a regular form as given below. The six observations quoted were taken in 15 minutes by one of the author's students.

Time	Apparent Altitude	α	h	δ	Z	$\frac{\text{Semi-diam.}}{\text{cos.ap. alt.}}$	True Azi. of Mark.
4:50	22° 48′.5	237° 41′	22° 30′.3	14° 45′.6	89° 16′.6	17′.2	213° 19′.6
4:53	22 12 .5	238 11	21 54 .3	45 .6	88 46 .6	17 .2	19 .6
4:55	21 44 .5	238 34	21 26 .2	45 .6	88 23 .3	17 .1	19 .8
4:58	21 19 .0	238 55	21 0 .7	45 .7	88 02 .4	17 .1	19 .7
5:00	20 49 .5	239 19 .5	20 31 .1	45 .7	87 38 .0	17 .0	19 .5
5:03	20 28 .0	239 38 .0	20 9 .5	14 45 .7	87 19 .9	17 .0	213 19 .1

Mean = 213°19′.55.

Observations taken Apr. 29, 1897: Semi-diam. of Sun 15′.9. Sun observed in lower left-hand corner.

α = horizontal angle to azimuth mark, the angle being measured to the right.

h = app. alt. − refraction − semi-diam. of sun; semi-diam. is + when sun is above hor. cross wire, − when below.

δ = declination, and Z = computed angle (as illustrated below).

True azimuth of mark $= 540° \pm \frac{\text{Semi-diam.}}{\text{cos. app. alt.}} \pm Z - \alpha$, in which

Z is + for A. M. and − for P. M. and the $\frac{\text{Semi-diam.}}{\text{cos. app. alt.}}$ is + when the sun is on the left of the middle wire (as above); $\frac{\text{Semi-diam.}}{\text{cos. app. alt.}}$ is − when the sun is on the right of the middle wire.

As a numerical specimen of the reduction:—App. decl. Greenwich mean noon Apr. 29, 1897, 14° 38′.0; hourly change +0′77; diff. of time between Greenwich and Philadelphia 5.0 hours; 5 P. M. at Philadelphia = 10 P. M. at Greenwich; therefore δ for 5 P. M. at Philadelphia = 14° 38′.0 + 10 × 0′.77 = 14° 45′.7. Using the equation

$$\sin \tfrac{1}{2} Z = \sqrt{\frac{\sin (s - \text{co } h) \sin (s - \text{co } \phi)}{\sin \text{co } h \sin \text{co } \phi}}$$

co h = 67° 29′.7	s − co h = 28° 53′.3, sin = 9.684041	
co ϕ = 50° 02 .0	s − co ϕ = 46° 21 .0, sin = 9.859480	
co δ = 75° 14 .4		9.543521
192° 46′.1	sin co h = 9.965599	
s = 96° 23 .0	sin co ϕ = 9.884466	
	9.850065	9.850065
		2.|9.693456
$\tfrac{1}{2}Z$ = 44° 38′.3; Z = 89° 16′.6		9.846728 = sin 44° 38′.3

$$\frac{\text{Semi-diam. Sun}}{\text{cos. app. alt.}} = \frac{15.9}{\cos 22° 48'} = 17'.2$$

$$\begin{array}{rr} & 540° + 17'.2 = 540° \ 17'.2 \\ -Z - \alpha = -89° \ 16'.6 - 237° \ 41' = & -326° \ 57'.6 \\ \hline & 213° \ 19'.6 = \text{true azimuth of mark.} \end{array}$$

The instrument used had a vertical circle reading 30″ directly and could be estimated to 15″.

EXPLANATORY NOTE ON THE USE OF THE TABLES

The logarithms here given are "five-place," but the last figure sometimes has a special mark over it (*e.g.*, $\bar{6}$) which indicates that one-half a unit in the last place should be *added.* For example

the value	includes all values between
·69586	·6958575000 + and ·6958624999...
·6958$\bar{6}$	·6958625000 + and ·6958674999...

The maximum error in any one value therefore does not exceed one-quarter of a fifth-place unit.

When adding or subtracting such logarithms allow a half-unit for such a sign. For example

·69586	·69586	·6958$\bar{6}$
·10841	·1084$\bar{1}$	·1084$\bar{1}$
·1294$\bar{7}$	·1294$\bar{7}$	·1294$\bar{7}$
·9337$\bar{4}$	·93375	·9337$\bar{5}$

All other logarithmic operations are performed as usual and are supposed to be understood by the student.

TABLE I.—RADII OF CURVES

Min	Radius.	Log R	Radius.	Log R	Radius.	Log R	Radius.	Log R	Min
0	∞	∞	5730 0	3 75813	2864 9	3 45711	1910 1	00	0
1	343775	5 53627	5636 7	75095	2841 3	45351	1899 6	[illegible]	1
2	171887	5 23524	5544 6	74380	2818 0	45001	1889 1	[illegible]	2
3	114592	5 059.1	5456 8	73694	2795 1	44649	1878 8	[illegible]	3
4	85944	4 93421	5371 8	730.0	2772 5	44297	1868 6	[illegible]	4
5	68755	4 83730	5288 9	72334	2750 4	43956	1858 5	[illegible]	5
6	57296	4 75812	5208 6	3 71673	2728 5	3 43608	1848 5	[illegible]	6
7	49111	69117	5131 0	71020	2707 0	43266	1838 6	[illegible]	7
8	42972	633.1	5055 5	70377	2685 9	42906	1828 8	[illegible]	8
9	38197	58203	4982 3	69743	2665 1	4257.	1819 1	[illegible]	9
10	34377	53627	4911 3	691.1	2644 6	42235	1809 6	[illegible]	10
11	31252	4 49488	4842 3	3 68507	2624 4	3 41901	1800 1	3 25529	11
12	28648	45709	4775 7	67903	2604 5	41572	1790 7	25303	12
13	26444	42233	4709 3	67306	2584 9	41243	1781 5	25077	13
14	24555	39014	4646 7	66708	2565 6	40916	1772 3	24853	14
15	22918	36018	4583 3	66122	[illegible]	40581	1763 2	24629	15
16	21486	4 33215	4523 4	3 65547	2527 9	3 40270	1754 2	3 24407	16
17	20222	30582	4464 7	64979	2509 5	39950	1745 3	24186	17
18	19099	28100	4407 5	64419	2491 5	39642	1736 6	23967	18
19	18093	2575.	4351 7	63867	2473 4	39321	1727 9	23748	19
20	17189	23524	4297 2	633.1	2455 7	39017	1719 1	23530	20
21	16370	4 21405	4244 3	3 62780	2438 3	3 38708	1710 6	3 23314	21
22	15626	19384	4192 5	62247	2421 1	38401	1702 1	23098	22
23	14947	17454	4142 0	61720	2404 2	38097	[illegible] 7	22884	23
24	14324	15606	4092 7	61200	2387 5	37794	1685 4	22670	24
25	13751	13833	4044 5	60686	2371 0	37494	1677 2	22458	25
26	13222	4 12130	3997 5	3 60179	2354 8	3 37195	1669 1	3 22247	26
27	12732	1049.	3951 5	59677	2338 8	36899	1661 0	22037	27
28	12278	08911	3906 6	59180	2323 0	36604	1653 0	21827	28
29	11854	07387	3862 7	58690	2307 4	36312	1645 1	21619	29
30	11459	05915	3819 8	58204	2292 0	36021	1637 3	21412	30
31	11090	4 04491	3777 9	3 57724	2276 8	3 35733	1629 5	3 21206	31
32	10743	03113	3736 8	57250	2261 9	35446	1621 8	21000	32
33	10417	01779	3696 6	56780	2247 1	35162	1614 2	20796	33
34	10111	4 00478	3657 3	563.0	2232 5	34879	1606 7	20593	34
35	9822 3	3 99221	3618 8	55855	2218 1	34598	1599 2	20390	35
36	9549 3	3 97997	3581 1	3 55401	2203 9	3 34319	1591 8	3 20189	36
37	9291 3	96807	3544 2	54951	2189 9	34041	1584 5	19988	37
38	9046 7	95648	3508 0	54506	2176 0	33765	1577 2	19789	38
39	8814 8	9452.	3472 6	54066	2162 3	33491	1570 0	19590	39
40	8594 4	93421	3437 9	53630	2148 8	33219	1562 9	19392	40
41	8384 8	3 92348	3403 8	3 53197	2135 4	3 32948	1555 8	3 19195	41
42	8185 3	91302	3370 5	52769	2122 2	32680	1548 8	18999	42
43	7994 8	9028.	3337 7	52345	2109 2	32412	1541 9	18804	43
44	7813 1	89283	3305 7	51924	2096 4	32147	1535 0	18610	44
45	7639 5	88306	3274 2	51510	2083 7	31883	1528 2	18417	45
46	7473 4	3 87352	3243 3	3 51098	2071 1	3 31621	1521 4	3 18224	46
47	7314 4	86418	3213 0	5069.	2058 7	31360	1514 7	18032	47
48	7162 0	85503	3183 2	50287	2046 5	31101	1508 1	17842	48
49	7015 9	84608	3154 0	4988.	2034 4	30843	1501 5	17652	49
50	6875 5	8373.	3125 4	49490	2022 4	30587	1495 0	17462	50
51	6740 7	3 82871	3097 3	3 49097	2010 6	3 30333	1488 5	3 17274	51
52	6611 1	82027	3069 6	48707	1998 9	30079	1482 1	17087	52
53	6486 4	81200	3042 4	48321	1987 3	29827	1475 7	16900	53
54	6366 5	80388	3015 7	47939	1975 9	29577	1469 4	16714	54
55	6250 9	79591	2989 5	47559	1964 6	29328	1463 2	16529	55
56	6139 6	3 78809	2963 7	3 47183	1953 5	3 29081	1457 0	3 16344	56
57	6031 9	78040	2938 4	46811	1942 4	28835	1450 8	16161	57
58	5927 2	77285	2913 5	46441	1931 5	28590	1444 7	15978	58
59	5826 8	76543	2888 9	46075	1920 7	28347	1438 7	15796	59
60	5729 6	.75813	2864 9	.45711	1910 1	28105	1432 7	15615	60

Min.	Radius.	Log R	Radius.	Log R	Radius.	Log R	Radius.	Log R	Deg. Min.
0	1432 7	3 15615	1146 3	3 05929	955 37	2 98017	819 02	2 91330	0
1	1426 7	15424	1142 8	05784	952 72	97894	817 09	91226	1
2	1420 8	15234	1138 7	05640	950 09	97774	815 14	91122	2
3	1415 0	15076	1134 6	05497	947 48	97657	813 22	91021	3
4	1409 2	14897	1131 2	05354	944 88	97537	811 30	90918	4
5	1403 5	14720	1127 0	05211	942 29	97418	809 40	90816	5
6	1397 8	3 14543	1123 3	3 05069	939 72	2 97300	807 50	2 90714	6
7	1392 1	14367	1120 2	04928	937 16	97181	805 61	90613	7
8	1386 6	14191	1116 5	04787	934 62	97063	803 73	90511	8
9	1380 9	14017	1112 8	04646	932 09	96945	801 86	90410	9
10	1375 6	13843	1109 2	04506	929 57	96828	800 00	90309	10
11	1370 2	3 13669	1105 6	3 04366	927 07	2 96711	798 14	2 90208	11
12	1364 9	13497	1102 0	04227	924 58	96594	796 30	90107	12
13	1359 1	13325	1098 7	04088	922 10	96478	794 46	90007	13
14	1354 3	13154	1095 2	03949	919 64	96361	792 63	89907	14
15	1349 4	12983	1091 7	03811	917 19	96245	790 81	89807	15
16	1343 3	3 12813	1088 3	3 03674	914 75	2 96130	789 00	2 89708	16
17	1338 0	12644	1084 9	03537	912 33	96015	787 20	89609	17
18	1333 0	12475	1081 4	03400	909 92	95900	785 41	89510	18
19	1327 9	12307	1078 1	03264	907 52	95785	783 62	89410	19
20	1322 9	12140	1074 7	03128	905 13	95671	781 84	89312	20
21	1317 9	3 11974	1071 3	3 02993	902 76	2 95557	780 07	2 89213	21
22	1312 6	11808	1068 0	02857	900 40	95443	778 31	89115	22
23	1307 4	11642	1064 7	02723	898 05	95330	776 56	89017	23
24	1302 5	11477	1061 4	02588	895 71	95217	774 81	88919	24
25	1297 6	11313	1058 2	02455	893 39	95104	773 07	88821	25
26	1292 7	3 11150	1054 9	3 02322	891 08	2 94991	771 34	2 88724	26
27	1287 9	10987	1051 7	02189	888 78	94879	769 61	88627	27
28	1283 1	10825	1048 5	02056	886 49	94767	767 90	88530	28
29	1278 3	10663	1045 3	01924	884 21	94655	766 19	88433	29
30	1273 6	10502	1042 1	01793	881 95	94544	764 49	88337	30
31	1268 9	3 10341	1039 0	3 01661	879 69	2 94433	762 [illegible]	2 88241	31
32	1264 2	10182	1035 9	01530	877 45	94322	761 11	88145	32
33	1259 6	10022	1032 8	01400	875 22	94211	759 43	88049	33
34	1255 0	09864	1029 7	01270	873 00	94101	757 76	87953	34
35	1250 4	09705	1026 6	01140	870 79	93991	756 10	87858	35
36	1245 8	3 09548	1023 6	3 01010	868 60	2 93882	754 44	2 87763	36
37	1241 4	09391	1020 5	00882	866 41	93772	752 80	87668	37
38	1236 9	09234	1017 5	00753	864 24	93663	751 16	87573	38
39	1232 5	09078	1014 5	00625	862 07	93554	749 52	87478	39
40	1228 1	08923	1011 6	00497	859 92	93446	747 89	87384	40
41	1223 7	3 08768	1008 6	3 00370	857 78	2 93337	746 27	2 87290	41
42	1219 4	08614	1005 6	00243	855 65	93229	744 66	87196	42
43	1215 1	08461	1002 7	3 00117	853 53	93122	743 06	87102	43
44	1210 8	08308	999 78	2 99991	851 42	93014	741 46	87008	44
45	1206 6	08155	996 87	99865	849 32	92907	739 86	86915	45
46	1202 4	3 08003	993 99	2 99739	847 23	2 92800	738 28	2 86822	46
47	1198 2	07852	991 11	99614	845 15	92693	736 70	86729	47
48	1194 0	07701	988 26	99489	843 08	92587	735 13	86636	48
49	1189 9	07550	985 42	99365	841 02	92480	733 56	86544	49
50	1185 8	07400	982 64	99241	838 97	92374	732 01	86451	50
51	1181 7	3 07251	979 84	2 99117	836 93	2 92269	730 45	2 86359	51
52	1177 7	07102	977 06	98993	834 90	92163	728 91	86267	52
53	1173 6	06954	974 29	98869	832 89	92058	727 37	86175	53
54	1169 7	06806	971 54	98746	830 88	91953	725 84	86084	54
55	1165 7	06658	968 81	98624	828 88	91848	724 31	85993	55
56	1161 8	3 06511	966 09	2 98501	826 89	2 91744	722 79	2 85901	56
57	1157 9	06365	963 38	98380	824 91	91640	721 28	85810	57
58	1154 0	06219	960 70	98258	822 93	91536	719 77	85719	58
59	1150 1	06074	958 03	98137	820 97	91432	718 27	85629	59
60	1146 3	05929	955 37	98017	819 02	91330	716 78	85538	60

TABLE I.—RADII OF CURVES.

Min.	Radius.	Log R	Radius.	Log R	Radius.	Log R	Radius.	Log R	Min.
0	716 78	2 85538	637 27	2 80432	573 69	2 75867	521 67	2 71739	0
1	715 30	85448	636 10	80352	572 73	75795	520 88	71674	1
2	713 81	85358	634 93	80272	571 78	75723	520 10	71608	2
3	712 34	85268	633 76	80192	570 84	75651	519 32	71543	3
4	710 87	85178	632 60	80112	569 90	75579	518 54	71478	4
5	709 40	85089	631 44	80033	568 96	75508	517 79	71413	5
6	707 95	2 85000	630 29	2 79954	568 02	2 75436	516 99	2 71348	6
7	706 49	84911	629 14	79874	567 09	75365	516 21	71283	7
8	705 05	84822	627 99	79796	566 16	75293	515 44	71218	8
9	703 61	84733	626 85	79717	565 23	75222	514 68	71153	9
10	702 17	84644	625 71	79638	564 31	75151	513 91	71089	10
11	700 75	2 84556	624 58	2 79559	563 38	2 75080	513 15	2 71024	11
12	699 33	84468	623 45	79480	562 47	75009	512 39	70960	12
13	697 91	84380	622 32	79402	561 55	74938	511 63	70895	13
14	696 50	84292	621 20	79323	560 64	74868	510 87	70831	14
15	695 09	84204	620 09	79245	559 73	74798	510 11	70767	15
16	693 70	2 84117	618 97	2 79167	558 82	2 74727	509 36	2 70703	16
17	692 30	84029	617 87	79089	557 92	74657	508 61	70639	17
18	690 91	83942	616 76	79011	557 02	74587	507 86	70575	18
19	689 53	83855	615 66	78934	556 12	74517	507 12	70511	19
20	688 16	83768	614 56	78856	555 23	74447	506 38	70447	20
21	686 79	2 83682	613 47	2 78779	554 34	2 74377	505 64	2 70383	21
22	685 42	83595	612 38	78702	553 45	74308	504 90	70320	22
23	684 06	83509	611 30	78625	552 56	74238	504 16	70257	23
24	682 70	83423	610 21	78548	551 68	74169	503 42	70193	24
25	681 35	83337	609 14	78471	550 80	74099	502 69	70130	25
26	680 01	2 83251	608 06	2 78394	549 92	2 74030	501 96	2 70067	26
27	678 67	83165	606 99	78318	549 05	73961	501 23	70004	27
28	677 34	83080	605 93	78242	548 17	73892	500 51	69941	28
29	676 01	82995	604 86	78165	547 30	73823	499 78	69878	29
30	674 69	82910	603 80	78089	546 44	73754	499 06	69815	30
31	673 37	2 82825	602 75	2 78013	545 57	2 73685	498 34	2 69752	31
32	672 06	82740	601 70	77938	544 71	73617	497 62	69690	32
33	670 75	82656	600 65	77862	543 86	73548	496 91	69627	33
34	669 45	82571	599 61	77786	543 00	73480	496 19	69565	34
35	668 15	82487	598 57	77711	542 15	73412	495 48	69503	35
36	666 86	2 82403	597 53	2 77636	541 30	2 73343	494 77	2 69440	36
37	665 57	82319	596 50	77561	540 46	73275	494 07	69378	37
38	664 29	82235	595 47	77486	539 61	73207	493 36	69316	38
39	663 01	82152	594 44	77411	538 78	73140	492 66	69254	39
40	661 74	82068	593 42	77336	537 92	73072	491 96	69193	40
41	660 47	2 81985	592 40	2 77261	537 09	2 73004	491 26	2 69131	41
42	659 21	81902	591 39	77187	536 26	72937	490 56	69069	42
43	657 96	81819	590 37	77112	535 42	72869	489 86	69007	43
44	656.60	81736	589 36	77038	534 60	72802	489 17	68946	44
45	655.45	81653	588 36	76964	533 77	72735	488.48	68884	45
46	654 20	2 81571	587 36	2 76890	532 94	2 72668	487 79	2 68823	46
47	652 96	81488	586 36	76816	532 12	72601	487 10	68762	47
48	651 72	81406	585 36	76742	531 30	72534	486 42	68701	48
49	650 50	81324	584 37	76669	530 49	72467	485 73	68640	49
50	649 27	81242	583 38	76595	529 67	72401	485 05	68579	50
51	648 05	2 81161	582 40	2 76522	528 86	2 72334	484 37	2 68518	51
52	646 84	81079	581 42	76448	528 05	72267	483 69	68457	52
53	645 63	80998	580 44	76375	527 25	72201	483 02	68396	53
54	644 42	80917	579 47	76302	526 44	72135	482 34	68335	54
55	643 22	80836	578 49	76229	525 64	72069	481 67	68275	55
56	642 02	2 80755	577 53	2 76157	524 84	2 72003	481 00	2 68214	56
57	640 83	80674	576 56	76084	524 05	71937	480 33	68154	57
58	639 64	80593	575 60	76011	523 26	71871	479 67	68094	58
59	638 45	80513	574 64	75939	522 46	71805	479 00	68033	59
60	637 27	80432	573 69	75867	521 67	71739	478 34	67973	60

TABLE II.—TANGENTS, EXTERNAL DISTANCES, AND LONG CHORDS FOR A 1° CURVE.

TABLE II.—TANGENTS, EXTERNAL DISTANCES, AND LONG CHORDS FOR A 1° CURVE.

Δ	Tang. T.	Ext. Dist. E.	Long Chord LC.	Δ	Tang. T.	Ext. Dist. E.	Long Chord LC.	Δ	Tang. T.	Ext. Dist. E.	Long Chord LC.
31°	1589.0	216.25	3062.4	41°	2142.2	387.38	4013.1	51°	2732.9	618.39	4933.4
10′	1598.0	218.66	3078.4	10	2151.7	390.71	4028.7	10	2743.1	622.81	4948.4
20	1606.9	221.08	3094.5	20	2161.2	394.06	4044.3	20	2753.4	627.24	4963.4
30	1615.9	223.51	3110.5	30	2170.8	397.43	4059.9	30	2763.7	631.69	4978.4
40	1624.9	225.96	3126.6	40	2180.3	400.82	4075.5	40	2773.9	636.16	4993.4
50	1633.9	228.42	3142.6	50	2189.9	404.22	4091.1	50	2784.2	640.66	5008.4
32°	1643.0	230.90	3158.6	42°	2199.4	407.64	4106.6	52°	2794.5	645.17	5023.4
10	1652.0	233.39	3174.6	10	2209.0	411.07	4122.2	10	2804.9	649.70	5038.4
20	1661.0	235.90	3190.6	20	2218.6	414.52	4137.7	20	2815.2	654.25	5053.4
30	1670.0	238.43	3206.6	30	2228.1	417.99	4153.3	30	2825.6	658.83	5068.3
40	1679.1	240.96	3222.6	40	2237.7	421.48	4168.8	40	2835.9	663.42	5083.3
50	1688.1	243.52	3238.6	50	2247.3	424.98	4184.3	50	2846.3	668.03	5098.2
33°	1697.2	246.08	3254.6	43°	2257.0	428.50	4199.8	53°	2856.7	672.66	5113.1
10	1706.3	248.66	3270.6	10	2266.6	432.04	4215.3	10	2867.1	677.32	5128.0
20	1715.3	251.26	3286.6	20	2276.2	435.59	4230.8	20	2877.5	681.99	5142.9
30	1724.4	253.87	3302.5	30	2285.9	439.16	4246.3	30	2888.0	686.68	5157.8
40	1733.5	256.50	3318.5	40	2295.6	442.75	4261.8	40	2898.4	691.40	5172.7
50	1742.6	259.14	3334.4	50	2305.2	446.35	4277.3	50	2908.9	696.13	5187.6
34°	1751.7	261.80	3350.4	44°	2314.9	449.98	4292.7	54°	2919.4	700.89	5202.4
10	1760.8	264.47	3366.3	10	2324.6	453.62	4308.2	10	2929.9	705.66	5217.3
20	1770.0	267.16	3382.2	20	2334.3	457.27	4323.6	20	2940.4	710.46	5232.1
30	1779.1	269.86	3398.2	30	2344.1	460.95	4339.0	30	2951.0	715.28	5246.9
40	1788.2	272.58	3414.1	40	2353.8	464.64	4354.5	40	2961.5	720.11	5261.7
50	1797.4	275.31	3430.0	50	2363.5	468.35	4369.9	50	2972.1	724.97	5276.5
35°	1806.6	278.05	3445.9	45°	2373.3	472.08	4385.3	55°	2982.7	729.85	5291.3
10	1815.7	280.82	3461.8	10	2383.1	475.82	4400.7	10	2993.3	734.76	5306.1
20	1824.9	283.60	3477.7	20	2392.8	479.59	4416.1	20	3003.9	739.68	5320.9
30	1834.1	286.39	3493.5	30	2402.6	483.37	4431.4	30	3014.5	744.62	5335.6
40	1843.3	289.20	3509.4	40	2412.4	487.16	4446.8	40	3025.2	749.59	5350.4
50	1852.5	292.02	3525.3	50	2422.3	490.98	4462.2	50	3035.8	754.57	5365.1
36°	1861.7	294.86	3541.1	46°	2432.1	494.82	4477.5	56°	3046.5	759.58	5379.8
10	1870.9	297.72	3557.0	10	2441.9	498.67	4492.8	10	3057.2	764.61	5394.5
20	1880.1	300.59	3572.8	20	2451.8	502.54	4508.2	20	3067.9	769.66	5409.2
30	1889.4	303.47	3588.6	30	2461.7	506.42	4523.5	30	3078.7	774.73	5423.9
40	1898.6	306.37	3604.5	40	2471.5	510.33	4538.8	40	3089.4	779.83	5438.6
50	1907.9	309.29	3620.3	50	2481.4	514.25	4554.1	50	3100.2	784.94	5453.3
37°	1917.1	312.22	3636.1	47°	2491.3	518.20	4569.4	57°	3110.9	790.08	5467.9
10	1926.4	315.17	3651.9	10	2501.2	522.16	4584.7	10	3121.7	795.24	5482.5
20	1935.7	318.13	3667.7	20	2511.2	526.13	4599.9	20	3132.6	800.42	5497.2
30	1945.0	321.11	3683.5	30	2521.1	530.13	4615.2	30	3143.4	805.62	5511.8
40	1954.3	324.11	3699.3	40	2531.1	534.15	4630.4	40	3154.2	810.85	5526.4
50	1963.6	327.12	3715.0	50	2541.0	538.18	4645.7	50	3165.1	816.10	5541.0
38°	1972.9	330.15	3730.8	48°	2551.0	542.23	4660.9	58°	3176.0	821.37	5555.6
10	1982.2	333.19	3746.5	10	2561.0	546.30	4676.1	10	3186.9	826.66	5570.2
20	1991.[illegible]	336.25	3762.3	20	2571.0	550.39	4691.3	20	3197.8	831.98	5584.7
30	2000.[illegible]	339.32	3778.0	30	2581.0	554.50	4706.5	30	3208.8	837.31	5599.3
40	2010.2	342.41	3793.8	40	2591.1	558.63	4721.7	40	3219.7	842.67	5613.8
50	2019.6	345.52	3809.5	50	2601.1	562.77	4736.9	50	3230.7	848.06	5628.3
39°	2029.0	348.64	3825.2	49°	2611.2	566.94	4752.1	59°	3241.7	853.46	5642.8
10	2038.4	351.78	3840.9	10	2621.2	571.12	4767.3	10	3252.7	858.89	5657.3
20	2047.8	354.94	3856.6	20	2631.3	575.32	4782.4	20	3263.7	864.34	5671.8
30	2057.2	358.11	3872.3	30	2641.4	579.54	4797.5	30	3274.8	869.82	5686.3
40	2066.6	361.29	3888.0	40	2651.5	583.78	4812.7	40	3285.8	875.32	5700.8
50	2076.0	364.50	3903.6	50	2661.6	588.04	4827.8	50	3296.9	880.84	5715.2
40°	2085.4	367.72	3919.3	50°	2671.8	592.32	4842.9	60°	3308.0	886.38	5729.7
10	2094.9	370.95	3935.0	10	2681.9	596.62	4858.0	10	3319.1	891.95	5744.1
20	2104.3	374.20	3950.6	20	2692.1	600.93	4873.1	20	3330.3	897.54	5758.5
30	2113.8	377.47	3966.3	30	2702.3	605.27	4888.2	30	3341.4	903.15	5772.9
40	2123.3	380.76	3981.9	40	2712.5	609.62	4903.2	40	3352.6	908.79	5787.3
50	2132.7	384.06	3997.5	50	2722.7	614.00	4918.3	50	3363.8	914.45	5801.7
41°	2142.2	387.38	4013.1	51°	2732.9	618.39	4933.4	61°	3375.0	920.14	5816.0

Correction Table (always additive)

Δ	Degree of curve.											
	5°			10°			15°			20°		
	T	E	LC	T	E	LC	T	E	LC	T	E	LC
10°	.03	001	.06	.06	003	13	10	004	.17	.13	008	26
20	.06	005	.12	.13	011	.25	.19	017	38	.26	022	.51
30	.09	012	18	.19	025	37	.29	036	.56	.39	051	.75
40	.13	022	.24	26	046	49	.40	070	74	.53	093	1 00
50	.16	036	30	34	075	.61	.51	112	.92	.68	151	1 23
60	.20	054	35	42	111	72	63	168	1 09	84	225	1.[illegible]
70	.24	077	40	50	159	.83	.76	240	1 25	1 02	.321	1.[illegible]
80	.29	107	.45	60	220	98	.91	332	1.40	1.22	.455	1.[illegible]
90	.35	146	49	72	298	1.02	1 09	451	1.54	1.46	.608	2 04

[illegible]	Nominal length of sub chord.														
	10	20	30	40	45	50	55	60	65	70	75	80	85	90	95
[illegible]	009	009	009	011	011	012	012	012	012	011	010	009	007	005	003
[illegible]	005	009	012	015	016	017	018	018	017	016	015	012	011	008	004
[illegible]	006	012	017	021	022	023	024	024	023	022	020	016	015	011	006
[illegible]	008	016	022	027	029	030	031	031	030	029	027	023	019	014	008
[illegible]	010	020	028	035	037	038	039	039	039	037	034	030	024	019	010
[illegible]	012	024	033	043	046	048	049	049	048	046	042	037	030	022	012
[illegible]	015	029	042	052	055	058	059	059	058	055	051	044	038	026	016
[illegible]	019	035	050	062	066	069	070	070	069	066	060	053	045	031	017
[illegible]	021	041	059	072	077	080	082	083	081	077	071	062	051	037	020
[illegible]	025	048	068	084	090	094	096	096	094	089	083	072	059	043	023
[illegible]	028	055	079	097	103	108	110	110	108	103	094	083	068	049	027
[illegible]	032	063	089	109	117	122	125	126	122	116	107	094	077	056	030
[illegible]	036	071	100	123	127	138	141	141	138	131	120	106	087	063	034
[illegible]	041	079	113	139	146	155	159	158	155	147	135	119	097	070	038
[illegible]	045	088	125	154	165	172	176	177	173	164	151	133	106	079	042
[illegible]	050	098	139	171	183	191	195	196	191	182	167	147	120	087	047
[illegible]	056	108	153	189	202	211	215	216	211	200	184	162	132	096	052
[illegible]	061	118	168	207	221	231	237	237	231	220	202	177	145	105	057
[illegible]	067	129	184	226	242	253	259	259	253	241	221	194	159	115	063
[illegible]	073	141	201	247	264	275	282	282	276	262	241	211	173	125	068
[illegible]	079	153	218	268	286	299	304	306	299	284	261	229	188	136	074
[illegible]	085	166	236	290	310	324	331	331	324	308	283	248	203	147	080
[illegible]	092	179	254	313	334	349	357	357	349	332	305	268	219	159	086
[illegible]	099	192	273	337	360	375	384	384	376	357	328	288	236	171	093
[illegible]	107	207	293	361	386	403	413	412	403	383	352	309	253	183	099
[illegible]	114	221	314	387	413	431	441	442	432	410	377	331	271	196	106

TABLE III. SWITCH LEADS AND DISTANCES.

A. TRIGONOMETRICAL FUNCTIONS OF THE FROG ANGLES.

Frog No. (n)	Frog Angle (F).	Nat. sin F.	Nat. cos F.	Log sin F.	Log cos F.	Log cot F.	Log vers F.	Frog No. (n)
4	14° 15′ 00″	.24615	.96923	9.39120	9.98643	10.59522	8.48617	4
5	11 25 16	.19802	.98020	.29670	.99131	.69461	.29670	5
6	9 31 38	.16552	.98621	.21884	.99397	.77513	.13968	6
7	8 10 16	.14213	.98985	.15268	.99557	.84288	8.00588	7
8	7 09 10	.12452	.99222	.09527	.99660	.90137	7.89110	8
9	6 21 35	.11077	.99385	.04447	.99732	.95286	.79915	9
9½	6 01 32	.10497	.99448	9.02107	.99759	.97652	.74232	9½
10	5 43 29	.09976	.99501	8.99901	.99783	10.99892	.69787	10
11	5 12 18	.09072	.99588	.95770	.99820	11.04050	.61527	11
12	4 46 19	.08319	.99653	.92007	.99849	.07842	.53989	12
13	4 [illegible] 08	.07659	.99778	.88343	.99003	.17530	.46881	13
16	3 34 47	.06244	.99805	.79543	.99915	.20370	.29030	16
18	3 10 56	.05551	.99846	.74437	.99933	.25494	.18807	18
20	2 51 51	.04997	.99875	.69870	.99946	.30076	7.09661	20
24	2° 23′ 13″	.04165	.99913	8.61961	9.99962	11.38002	6.93834	24

B. THEORETICAL LEADS, USING STRAIGHT POINT-RAILS AND STRAIGHT FROG RAILS; GAUGE 4′ 8½″. See §§ 305 and 313.

Frog No.	Frog Bluntness.	Frog.			Switch.				Switch Dimensions.						
		Wing rail.		Heel Length.	Length.	Angle.			Radius.	Degree of Lead Curve.			Ac. pt. of sw. rail to ac. pt. frog.	Closure.	
														Str'ght Rail.	Curv'd Rail.
(n)		(W)		(K)	(S)	(α)			(r)	(D)			(L')		
	ft.	ft.	in.	ft.in.	ft.in.	°	′	″	ft.	°	′	″	ft.	ft.	ft.
4	0.17	3	2	5 4	11 0	2	36	19	112.26	52	53	56	37.22	22.88	23.29
5	0.21	3	7	6 5	11 0	2	36	19	183.22	31	40	24	42.98	28.19	28.55
6	0.25	4	0	7 0	11 0	2	36	19	273.95	21	01	58	48.36	33.11	33.38
7	0.29	4	5	8 1	16 6	1	44	11	364.88	15	47	19	62.23	41.02	41.24
8	0.33	4	9	8 9	16 6	1	44	11	488.71	11	44	40	67.80	46.22	46.42
9	0.37	6	0	10 0	16 6	1	44	11	616.27	9	18	27	72.61	49.74	49.92
9½	0.40	6	0	10 0	16 6	1	44	11	699.97	8	11	33	75.30	52.40	52.58
10	0.42	6	0	10 6	16 6	1	44	11	790.25	7	15	18	77.93	55.01	55.17
11	0.46	6	0	11 6	22 0	1	18	08	940.21	6	05	48	92.52	64.06	64.20
12	0.50	6	5	12 1	22 0	1	18	08	1136.34	5	02	38	97.75	68.83	68.96
15	0.62	7	8	14 10	33 0	0	52	05	1744.45	3	17	06	131.12	89.83	89.94
16	0.67	8	0	16 0	33 0	0	52	05	2005.98	2	51	24	136.62	94.95	95.05
18	0.75	8	10	17 8	33 0	0	52	05	2587.66	2	12	52	147.13	104.54	104.61
20	0.83	9	8	19 4	33 0	0	52	05	3262.98	1	45	22	157.18	113.68	113.76
24	1.00	11	4	23 2	33 0	0	52	05	4932.77	1	09	42	176.09	130.66	130.77

C. PRACTICAL LEADS, USING STRAIGHT POINT-RAILS AND STRAIGHT FROG RAILS; GAUGE 4′ 8½″; See §§ 305–307.

Frog No. (n)	Radius of center line. (r)	Degree of lead curve. (D)			Tangent adjacent to switch rail. (T_s)	Tangent adjacent to toe of frog. (T_f)	Actual point of switch rail to act. pt. of frog. (L')	Closure for straight rail.	Closure for curved rail.
	ft.	°	′	″	ft.	ft.	ft.		
4	110.69	53	42	24	1.03	0.00	37.94	1–23.60	1–24
5	174.34	33	19	57	0.00	0.82	42.47	1–27.68	1–28
6	265.39	21	43	04	0.00	0.66	47.98	1–32.73	1–33
7	362.08	15	52	29	0.00	0.19	62.10	1–13.89 1–27	1–14.11 1–27
8	487.48	11	46	27	0.30	0.00	67.98	1–16.40 1–30	1–16.60 1–30
9	605.18	9	28	42	0.00	0.57	72.28	1–16.41 1–33	1–16.59 1–33
9½	695.45	8	14	45	0.76	0.00	75.71	1–25.82 1–27	1–26 1–27
10	790.25	7	15	18	0.00	0.00	77.93	1–27 1–28	1–27.17 1–28
11	922.65	6	12	47	2.99	0.00	94.31	1–32.85 1–33	2–33
12	1098.73	5	12	59	5.33	0.00	100.80	1–23.88 2–24	3–24
15	1743.80	3	17	10	0.09	0.00	131.19	2–30 1–29.89	3–30
16	1993.24	2	52	29	1.56	0.00	137.57	1–29.90 2–33	1–30 2–33
18	2546.31	2	14	31	0.00	1.08	146.51	1–25.93 3–26	4–26
20	3257.26	1	45	32	0.44	0.00	157.42	1–26.92 2–27 1–33	3–27 1–33
24	4886.16	1	10	21	2.43	0.00	177.22	1–32.89 3–33	4–33

TABLE IV.—FUNCTIONS OF THE TEN-CHORD SPIRAL.

PART A.—Coefficients of a_1 for deflection angles to chord points.

Deflection angle to chord-point number.	Transit at chord-point number.										
	0 T. S.	1	2	3	4	5	6	7	8	9	10 S. C.
0 T. S.	0	2	8	18	32	50	72	98	128	162	200
1	1	0	5	14	27	44	65	90	119	152	189
2	4	4	0	8	20	36	56	80	108	140	176
3	9	10	7	0	11	26	45	68	95	126	161
4	16	18	16	10	0	14	32	54	80	110	144
5	25	28	27	22	13	0	17	38	63	92	125
6	36	40	40	36	28	16	0	20	44	72	104
7	49	54	55	52	45	34	19	0	23	50	81
8	64	70	72	70	64	54	40	22	0	26	56
9	81	88	91	90	85	76	63	46	25	0	29
10 S. C.	100	108	112	112	108	100	88	72	52	28	0

PART B.—Values of $\frac{U}{L}$ and $\frac{V}{L}$.

ϕ	$\frac{U}{L}$	$\frac{V}{L}$	ϕ	$\frac{U}{L}$	$\frac{V}{L}$
0°	.666 667	.333 333	23°	.672 423	.338 586
1	.666 678	.333 343	24	.672 943	.339 061
2	.666 710	.333 372	25	.673 486	.339 559
3	.666 763	.333 421	26	.674 054	.340 078
4	.666 838	.333 490	27	.674 645	.340 619
5	.666 935	.333 578	28	.675 261	.341 183
6	.667 053	.333 685	29	.675 901	.341 769
7	.667 193	.333 812	30	.676 566	.342 378
8	.667 354	.333 959	31	.677 256	.343 011
9	.667 537	.334 126	32	.677 971	.343 667
10	.667 742	.334 313	33	.678 712	.344 346
11	.667 968	.334 519	34	.679 478	.345 050
12	.668 216	.334 746	35	.680 270	.345 777
13	.668 487	.334 992	36	.681 089	.346 529
14	.668 779	.335 259	37	.681 935	.347 307
15	.669 094	.335 546	38	.682 808	.348 109
16	.669 431	.335 853	39	.683 708	.348 937
17	.669 790	.336 181	40	.684 636	.349 791
18	.670 172	.336 529	41	.685 592	.350 671
19	.670 576	.336 899	42	.686 577	.351 578
20	.671 003	.337 289	43	.687 590	.352 513
21	.671 453	.337 700	44	.688 633	.353 474
22	.671 926	.338 132	45	.689 706	.354 464

Table IV, of which Part C is condensed, was computed by the Track Committee of the American Railway Engineering Association and is taken from the Proceedings of the Association.

TABLE IV.—FUNCTIONS OF THE TEN-CHORD SPIRAL.

PART C.

Total spiral angle, ϕ	A	$\frac{C}{L}$	$\frac{X}{L}$	$\frac{Y}{L}$
0° 0′	0° 00′ 00″	1.000 000	1.000 000	.000 000
30	0 10 00	.999 997	.999 993	.002 909
1 0	0 20 00	.999 987	.999 970	.005 818
30	0 30 00	.999 970	.999 932	.008 726
2 0	0 40 00	.999 947	.999 879	.011 635
30	0 50 00	.999 916	.999 811	.014 542
3 0	1 00 00	.999 880	.999 727	.017 450
30	1 10 00	.999 836	.999 629	.020 357
4 00	1 20 00	.999 786	.999 515	.023 263
30	1 30 00	.999 729	.999 387	.026 169
5 00	1 40 00	.999 666	.999 243	.029 073
30	1 50 00	.999 596	.999 084	.031 977
6 00	1 59 59	.999 519	.998 910	.034 880
30	2 09 59	.999 435	.998 721	.037 781
7 00	2 19 59	.999 345	.998 517	.040 681
30	2 29 59	.999 248	.998 298	.043 581
8 00	2 39 58	.999 145	.998 063	.046 478
30	2 49 58	.999 035	.997 814	.049 374
9 00	2 59 58	.998 918	.997 549	.052 269
30	3 09 57	.998 794	.997 270	.055 162
10 00	3 19 57	.998 664	.996 975	.058 053
30	3 29 57	.998 527	.996 666	.060 942
11 00	3 39 56	.998 384	.996 341	.063 829
30	3 49 55	.998 233	.996 002	.066 714
12 00	3 59 55	.998 077	.995 647	.069 598
30	4 09 54	.997 913	.995 278	.072 478
13 00	4 19 53	.997 743	.994 893	.075 357
30	4 29 53	.997 566	.994 494	.078 233
14 00	4 39 52	.997 383	.994 079	.081 106
30	4 49 51	.997 192	.993 650	.083 977
15 00	4 59 50	.996 996	.993 206	.086 846
30	5 09 49	.996 792	.992 747	.089 711
16 00	5 19 48	.996 582	.992 273	.092 574
30	5 29 47	.996 366	.991 785	.095 433
17 00	5 39 45	.996 142	.991 281	.098 290
30	5 49 44	.995 912	.990 763	.101 143
18 00	5 59 43	.995 676	.990 230	.103 993
30	6 09 41	.995 432	.989 682	.106 840
19 00	6 19 40	.995 183	.989 120	.109 683
30	6 29 36	.994 926	.988 543	.112 523
20 00	6 39 36	.994 663	.987 951	.115 360
30	6 49 34	.994 393	.987 344	.118 192
21 00	6 59 32	.994 117	.986 723	.121 021
30	7 09 30	.993 834	.986 088	.123 846
22 00	7 19 28	.993 545	.985 437	.126 667
22° 30′	7° 29′ 26″	.993 248	.984 772	.129 483

Part C.—*Con.*

Total spiral angle, ϕ	A	$\frac{C}{L}$	$\frac{X}{L}$	$\frac{Y}{L}$
22° 30′	7° 29′ 26″	.993 248	.984 772	.129 483
23 00	7 39 24	.992 946	.984 093	.132 296
30	7 49 21	.992 636	.983 399	.135 105
24 00	7 59 19	.992 321	.982 691	.137 909
30	8 09 16	.991 998	.981 968	.140 708
25 00	8 19 14	.991 669	.981 231	.143 504
30	8 29 11	.991 333	.980 479	.146 294
26 00	8 39 08	.990 991	.979 714	.149 080
30	8 49 05	.990 642	.978 933	.151 861
27 00	8 59 02	.990 287	.978 139	.154 638
30	9 08 58	.989 925	.977 330	.157 409
28 00	9 18 55	.989 557	.976 508	.160 176
30	9 28 51	.989 182	.975 670	.162 937
29 00	9 38 48	.988 800	.974 819	.165 693
30	9 48 44	.988 412	.973 954	.168 444
30 00	9 58 40	.988 018	.973 074	.171 189
30	10 08 36	.987 617	.972 181	.173 929
31 00	10 18 32	.987 209	.971 273	.176 664
30	10 28 27	.986 795	.970 352	.179 392
32 00	10 38 23	.986 375	.969 417	.182 116
30	10 48 18	.985 948	.968 468	.184 833
33 00	10 58 13	.985 514	.967 504	.187 544
30	11 08 08	.985 074	.966 528	.190 250
34 00	11 18 03	.984 627	.965 537	.192 949
30	11 27 58	.984 174	.964 532	.195 643
35 00	11 37 53	.983 715	.963 515	.198 330
30	11 47 47	.983 249	.962 483	.201 010
36 00	11 57 41	.982 777	.961 438	.203 685
30	12 07 36	.982 298	.960 379	.206 353
37 00	12 17 30	.981 813	.959 306	.209 014
30	12 27 23	.981 321	.958 221	.211 669
38 00	12 37 17	.980 823	.957 121	.214 317
30	12 47 11	.980 318	.956 009	.216 959
39 00	12 57 04	.979 807	.954 883	.219 593
30	13 06 57	.979 290	.953 744	.222 221
40 00	13 16 50	.978 766	.952 591	.224 841
30	13 26 43	.978 236	.951 426	.227 455
41 00	13 36 35	.977 700	.950 247	.230 061
30	13 46 28	.977 157	.949 055	.232 660
42 00	13 56 20	.976 608	.947 850	.235 252
30	14 06 12	.976 053	.946 632	.237 836
43 00	14 16 04	.975 491	.945 402	.240 413
30	14 25 56	.974 923	.944 158	.242 982
44 00	14 35 47	.974 348	.942 901	.245 544
30	14 45 38	.973 768	.941 632	.248 098
45° 00′	14° 55′ 29″	.973 181	.940 350	.250 644

TABLE V.—LOGARITHMS OF NUMBERS.

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TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9
[illegible]	92 942	947	952	957	962	967	972	977	982	988
851	993	998	*003	*008	*013	*018	*023	*028	*034	*039
852	93 044	049	054	059	064	069	074	079	084	090
853	095	100	105	110	115	120	125	130	135	140
854	146	151	156	161	166	171	176	181	186	191
855	196	201	207	212	217	222	227	[illegible]	237	242
856	247	252	257	262	267	272	278	283	288	293
857	298	303	308	313	318	323	328	333	338	343
858	348	354	359	364	369	374	379	384	389	394
859	399	404	409	414	419	424	429	434	439	445
860	450	455	460	465	470	475	[illegible]	485	490	495
861	500	505	510	515	520	525	[illegible]	535	540	545
862	550	556	561	566	571	576	581	586	591	596
863	601	606	611	616	621	626	631	636	641	646
864	651	656	661	666	671	676	681	686	691	696
865	701	706	711	716	721	726	731	736	742	747
866	752	757	762	767	772	777	782	787	792	[illegible]
867	802	807	812	817	822	827	832	837	842	847
868	852	857	862	[illegible]	872	877	882	887	892	897
869	902	907	912	917	922	927	932	937	942	947
870	952	957	962	967	972	977	[illegible]	987	992	997
871	94 002	007	012	017	022	026	031	036	041	046
872	051	056	061	066	071	076	081	086	091	096
873	101	106	111	116	121	126	131	136	141	146
874	151	156	161	166	171	176	181	186	191	196
875	201	206	210	215	220	225	230	235	240	245
876	250	255	260	265	270	275	280	285	290	295
877	300	305	310	315	320	324	329	334	339	344
878	349	354	359	364	369	374	379	384	389	394
879	399	404	409	413	418	423	428	433	438	443
880	448	453	458	463	[illegible]	[illegible]	478	483	487	492
881	497	502	507	512	517	522	527	532	537	542
882	547	552	556	561	566	571	576	581	586	591
883	596	601	606	611	615	620	625	630	635	640
884	645	650	655	660	665	670	674	679	684	689
885	694	699	704	709	714	719	724	728	733	738
886	743	748	753	758	763	768	773	777	782	787
887	792	797	802	807	812	817	821	826	831	836
888	841	846	851	856	861	865	870	875	880	885
889	890	895	[illegible]	905	909	914	919	924	929	934
890	939	944	949	953	958	963	968	973	978	983
891	[illegible]	992	997	*002	*007	*012	*017	*022	*026	031
892	95 036	041	046	051	056	061	066	070	075	080
893	085	090	095	099	104	109	114	119	124	129
894	134	138	143	148	153	158	163	167	172	177
895	182	187	192	197	201	206	211	216	221	226
896	231	236	240	245	250	255	260	264	269	274
897	279	284	289	294	299	303	308	313	318	323
898	327	332	337	342	347	352	357	361	366	371
899	376	381	386	390	395	400	405	410	415	419
900	424	[illegible]	434	439	444	[illegible]	453	458	463	468
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	5½
.1	0.5
.2	1.1
.3	1.6
.4	2.2
.5	2.7
.6	3.3
.7	3.8
.8	4.4
.9	4.9

	5
.1	0.5
.2	1.0
.3	1.5
.4	2.0
.5	2.5
.6	3.0
.7	3.5
.8	4.0
.9	4.5

	4½
.1	0.4
.2	0.9
.3	1.3
.4	1.8
.5	2.2
.6	2.7
.7	3.1
.8	3.6
.9	4.0

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9
950	97 772	777	781	786	790	795	800	804	809	813
951	818	822	827	831	836	841	845	850	854	859
952	863	868	873	877	882	886	891	895	900	904
953	909	914	918	923	927	932	936	941	945	950
954	955	959	964	968	973	977	982	986	991	996
955	98 000	005	009	014	018	023	027	032	036	041
956	046	050	055	059	064	068	073	077	082	086
957	091	095	100	105	109	114	118	123	127	132
958	136	141	145	150	154	159	163	168	173	177
959	182	186	191	195	200	204	209	213	218	222
960	227	231	236	240	245	249	254	259	263	268
961	272	277	281	286	290	295	299	304	308	313
962	317	322	326	331	335	340	344	349	353	358
963	362	367	371	376	380	385	389	394	398	403
964	407	412	416	421	425	430	434	439	443	448
965	452	457	461	466	470	475	479	484	488	493
966	497	502	506	511	515	520	524	529	533	538
967	542	547	551	556	560	565	569	574	578	583
968	587	592	596	601	605	610	614	619	623	628
969	632	637	641	646	650	655	659	663	668	672
970	677	681	686	690	695	699	704	708	713	717
971	722	726	731	735	740	744	749	753	757	762
972	766	771	775	780	784	789	793	798	802	807
973	811	815	820	824	829	833	838	842	847	851
974	856	860	865	869	873	878	882	887	891	896
975	900	905	909	914	918	922	927	931	936	940
976	945	949	954	958	963	967	971	976	980	985
977	989	994	998	*003	*007	*011	*016	*020	*025	*029
978	99 034	038	043	047	051	056	060	065	069	074
979	078	082	087	091	096	100	105	109	113	118
980	122	127	131	136	140	145	149	153	158	162
981	167	171	176	180	184	189	193	198	202	206
982	211	215	220	224	229	233	237	242	246	251
983	255	260	264	268	273	277	282	286	290	295
984	299	304	308	312	317	321	326	330	335	339
985	343	348	352	357	361	365	370	374	379	383
986	387	392	396	401	405	409	414	418	423	427
987	431	436	440	445	449	453	458	462	467	471
988	475	480	484	489	493	497	502	506	511	515
989	519	524	528	533	537	541	546	550	554	559
990	563	568	572	576	581	585	590	594	598	603
991	607	611	616	620	625	629	633	638	642	647
992	651	655	660	664	668	673	677	682	686	690
993	695	699	703	708	712	717	721	725	730	734
994	738	743	747	751	756	760	765	769	773	778
995	782	786	791	795	800	804	808	813	817	821
996	826	830	834	839	843	847	852	856	861	865
997	869	874	878	882	887	891	895	900	904	908
998	913	917	922	926	930	935	939	943	948	952
999	956	961	965	969	974	978	982	987	991	995
1000	00 000	004	008	013	017	021	026	030	034	039
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	5
.1	0.5
.2	1.0
.3	1.5
.4	2.0
.5	2.5
.6	3.0
.7	3.5
.8	4.0
.9	4.5

	4
.1	0.4
.2	0.9
.3	1.3
.4	1.8
.5	2.2
.6	2.7
.7	3.1
.8	3.6
.9	4.0

	4
.1	0.4
.2	0.8
.3	1.2
.4	1.6
.5	2.0
.6	2.4
.7	2.8
.8	3.2
.9	3.6

TABLE V.—LOGARITHMS OF NUMBERS.

TABLE V.—LOGARITHMS OF NUMBERS.

Log sin ϕ = log ϕ'' + S. Log tan ϕ = log ϕ'' + T.

0°

log ϕ'' = log sin ϕ + S'. log ϕ'' = log tan ϕ + T'.

″	′	S	T	Log. Sin.	S′	T′	Log. Tan.
0	0	4.685 57	57	— ∞	5.314 42	42	— ∞
60	1	57	57	6.46 372	42	42	6.46 372
120	2	57	57	.76 475	42	42	.76 475
180	3	57	57	.94 084	42	42	.94 084
240	4	57	57	7.06 578	42	42	7.06 578
300	5	4.685 57	57	7.16 269	5.314 42	42	7.16 269
360	6	57	57	.24 187	42	42	.24 188
420	7	57	57	.30 882	42	42	.30 882
480	8	57	57	.36 681	42	42	.36 681
540	9	57	57	.41 797	42	42	.41 797
600	10	4.685 57	57	7.46 372	5.314 42	42	7.46 372
660	11	57	57	.50 512	42	42	.50 512
720	12	57	57	.54 290	42	42	.54 291
780	13	57	57	.57 767	42	42	.57 767
840	14	57	57	.60 985	42	42	.60 985
900	15	4.685 57	58	7.63 981	5.314 42	42	7.63 982
960	16	57	58	.66 784	42	42	.66 785
1020	17	57	58	.69 417	42	42	.69 418
1080	18	57	58	.71 899	42	42	.71 900
1140	19	57	58	.74 248	42	42	.74 248
1200	20	4.685 57	58	7.76 475	5.314 43	42	7.76 476
1260	21	57	58	.78 594	43	42	.78 595
1320	22	57	58	.80 614	43	42	.80 615
1380	23	57	58	.82 545	43	42	.82 546
1440	24	57	58	.84 393	43	42	.84 394
1500	25	4.685 57	58	7.86 166	5.314 43	41	7.86 167
1560	26	57	58	.87 869	43	41	.87 871
1620	27	57	58	.89 508	43	41	.89 510
1680	28	57	58	.91 088	43	41	.91 089
1740	29	57	58	.92 612	43	41	.92 613
1800	30	4.685 57	58	7.94 084	5.314 43	41	7.94 086
1860	31	57	58	.95 508	43	41	.95 510
1920	32	57	58	.96 887	43	41	.96 889
1980	33	57	59	.98 223	43	41	.98 225
2040	34	57	59	.99 520	43	41	.99 522
2100	35	4.685 56	59	8.00 778	5.314 43	41	8.00 781
2160	36	56	59	.02 002	43	41	.02 004
2220	37	56	59	.03 192	43	41	.03 194
2280	38	56	59	.04 350	43	40	.04 352
2340	39	56	59	.05 478	43	40	.05 481
2400	40	4.685 56	59	8.06 577	5.314 43	40	8.06 580
2460	41	56	59	.07 650	43	40	.07 653
2520	42	56	59	.08 696	43	40	.08 699
2580	43	56	60	.09 718	43	40	.09 721
2640	44	56	60	.10 716	43	40	.10 720
2700	45	4.685 56	60	8.11 692	5.314 44	40	8.11 696
2760	46	56	60	.12 647	44	40	.12 651
2820	47	56	60	.13 581	44	40	.13 585
2880	48	56	60	.14 495	44	39	.14 499
2940	49	56	60	.15 390	44	39	.15 395
3000	50	4.685 56	60	8.16 268	5.314 44	39	8.16 272
3060	51	56	60	.17 128	44	39	.17 133
3120	52	56	61	.17 971	44	39	.17 976
3180	53	56	61	.18 798	44	39	.18 803
3240	54	55	61	.19 610	44	39	.19 615
3300	55	4.685 55	61	8.20 407	5.314 44	39	8.20 412
3360	56	55	61	.21 189	44	38	.21 195
3420	57	55	61	.21 958	44	38	.21 964
3480	58	55	61	.22 713	44	38	.22 719
3540	59	55	62	.23 455	44	38	.23 462

Log sin ϕ = log ϕ'' + S.
Log tan ϕ = log ϕ'' + T.

1°

log ϕ'' = log sin ϕ + S'.
log ϕ'' = log tan ϕ + T'.

"	'	S	T	Log. Sin.	S'	T'	Log. Tan.
3600	0	4.685 55	62	8.24 185	5.314 44	38	8.24 192
3660	1	55	62	.24 903	45	38	.24 910
3720	2	55	62	.25 609	45	38	.25 616
3780	3	55	62	.26 304	45	37	.26 311
3840	4	55	62	.26 988	45	37	.26 995
3900	5	4.685 55	62	8.27 661	5.314 45	37	8.27 669
3960	6	55	63	.28 324	45	37	.28 332
4020	7	54	63	.28 977	45	37	.28 985
4080	8	54	63	.29 620	45	37	.29 629
4140	9	54	63	.30 254	45	36	.30 263
4200	10	4.685 54	63	8.30 879	5.314 45	36	8.30 888
4260	11	54	63	.31 495	45	36	.31 504
4320	12	54	64	.32 102	45	36	.32 112
4380	13	54	64	.32 701	46	36	.32 711
4440	14	54	64	.33 292	46	36	.33 302
4500	15	4.685 54	64	8.33 875	5.314 46	35	8.33 885
4560	16	54	64	.34 450	46	35	.34 461
4620	17	54	65	.35 018	46	35	.35 029
4680	18	54	65	.35 578	46	35	.35 589
4740	19	53	65	.36 131	46	35	.36 143
4800	20	4.685 53	65	8.36 677	5.314 46	34	8.36 689
4860	21	53	65	.37 217	46	34	.37 229
4920	22	53	65	.37 750	46	34	.37 762
4980	23	53	66	.38 276	46	34	.38 289
5040	24	53	66	.38 796	47	34	.38 809
5100	25	4.685 53	66	8.39 310	5.314 47	33	8.39 323
5160	26	53	66	.39 818	47	33	.39 831
5220	27	53	67	.40 320	47	33	.40 334
5280	28	52	67	.40 816	47	33	.40 830
5340	29	52	67	.41 307	47	33	.41 321
5400	30	4.685 52	67	8.41 792	5.314 47	32	8.41 807
5460	31	52	67	.42 271	47	32	.42 287
5520	32	52	68	.42 746	47	32	.42 762
5580	33	52	68	.43 215	48	32	.43 231
5640	34	52	68	.43 680	48	31	.43 696
5700	35	4.685 52	68	8.44 139	5.314 48	31	8.44 156
5760	36	52	69	.44 594	48	31	.44 611
5820	37	51	69	.45 044	48	31	.45 061
5880	38	51	69	.45 489	48	30	.45 507
5940	39	51	69	.45 930	48	30	.45 948
6000	40	4.685 51	69	8.46 366	5.314 48	30	8.46 385
6060	41	51	70	.46 798	49	30	.46 817
6120	42	51	70	.47 226	49	30	.47 245
6180	43	51	70	.47 650	49	29	.47 669
6240	44	51	70	.48 069	49	29	.48 089
6300	45	4.685 50	71	8.48 485	5.314 49	29	8.48 505
6360	46	50	71	.48 896	49	28	.48 917
6420	47	50	71	.49 304	49	28	.49 325
6480	48	50	72	.49 708	49	28	.49 729
6540	49	50	72	.50 108	50	28	.50 130
6600	50	4.685 50	72	8.50 504	5.314 50	27	8.50 526
6660	51	50	72	.50 897	50	27	.50 920
6720	52	50	73	.51 286	50	27	.51 310
6780	53	49	73	.51 672	50	27	.51 696
6840	54	49	73	.52 055	50	26	.52 079
6900	55	4.685 49	73	8.52 434	5.314 50	26	8.52 458
6960	56	49	74	.52 810	51	26	.52 835
7020	57	49	74	.53 183	51	25	.53 208
7080	58	49	74	.53 552	51	25	.53 578
7140	59	49	75	.53 918	51	25	.53 944

$\text{Log sin } \phi = \log \phi'' + S.$ $\text{Log tan } \phi = \log \phi'' + T.$

2°

$\log \phi'' = \log \sin \phi + S'.$ $\log \phi'' = \log \tan \phi + T'.$

″	′	S	T	Log. Sin.	S′	T′	Log. Tan.
7200	0	4.685 48	75	8.54 282	5.314 51	25	8.54 308
7260	1	48	75	.54 642	51	24	.54 669
7320	2	48	75	.54 999	51	24	.55 027
7380	3	48	76	.55 354	52	24	.55 381
7440	4	48	76	.55 705	52	23	.55 733
7500	5	4.685 48	76	8.56 054	5.314 52	23	8.56 083
7560	6	48	77	.56 400	52	23	.56 429
7620	7	47	77	.56 743	52	22	.56 772
7680	8	47	77	.57 083	52	22	.57 113
7740	9	47	78	.57 421	52	22	.57 452
7800	10	4.685 47	78	8.57 756	5.314 53	22	8.57 787
7860	11	47	78	.58 089	53	21	.58 121
7920	12	47	79	.58 419	53	21	.58 451
7980	13	46	79	.58 747	53	21	.58 779
8040	14	46	79	.59 072	53	20	.59 105
8100	15	4.685 46	80	8.59 395	5.314 53	20	8.59 428
8160	16	46	80	.59 715	54	20	.59 749
8220	17	46	80	.60 033	54	19	.60 067
8280	18	46	81	.60 349	54	19	.60 384
8340	19	45	81	.60 662	54	19	.60 698
8400	20	4.685 45	81	8.60 973	5.314 54	18	8.61 009
8460	21	45	82	.61 282	54	18	.61 319
8520	22	45	82	.61 589	55	18	.61 626
8580	23	45	82	.61 893	55	17	.61 931
8640	24	45	83	.62 196	55	17	.62 234
8700	25	4.685 44	83	8.62 496	5.314 55	16	8.62 535
8760	26	44	83	.62 795	55	16	.62 834
8820	27	44	84	.63 091	55	16	.63 131
8880	28	44	84	.63 385	56	15	.63 425
8940	29	44	84	.63 677	56	15	.63 718
9000	30	4.685 43	85	8.63 968	5.314 56	15	8.64 009
9060	31	43	85	.64 256	56	14	.64 298
9120	32	43	86	.64 543	56	14	.64 585
9180	33	43	86	.64 827	57	14	.64 870
9240	34	43	86	.65 110	57	13	.65 153
9300	35	4.685 43	87	8.65 391	5.314 57	13	8.65 435
9360	36	42	87	.65 670	57	12	.65 715
9420	37	42	87	.65 947	57	12	.65 993
9480	38	42	88	.66 223	58	12	.66 269
9540	39	42	88	.66 497	58	11	.66 543
9600	40	4.685 42	89	8.66 769	5.314 58	11	8.66 816
9660	41	41	89	.67 039	58	10	.67 087
9720	42	41	89	.67 308	58	10	.67 356
9780	43	41	90	.67 575	59	10	.67 624
9840	44	41	90	.67 840	59	09	.67 890
9900	45	4.685 41	91	8.68 104	5.314 59	09	8.68 154
9960	46	40	91	.68 366	59	08	.68 417
10020	47	40	91	.68 627	59	08	.68 678
10080	48	40	92	.68 886	60	08	.68 938
10140	49	40	92	.69 144	60	07	.69 196
10200	50	4.685 40	93	8.69 400	5.314 60	07	8.69 453
10260	51	39	93	.69 654	60	06	.69 708
10320	52	39	93	.69 907	60	06	.69 961
10380	53	39	94	.70 159	61	06	.70 214
10440	54	39	94	.70 409	61	05	.70 464
10500	55	4.685 38	95	8.70 657	5.314 61	05	8.70 714
10560	56	38	95	.70 905	61	04	.70 962
10620	57	38	96	.71 150	61	04	.71 208
10680	58	38	96	.71 395	62	03	.71 453
10740	59	38	97	.71 638	62	03	.71 697

′	Log. Sin.	D	Log. Tan.	Com. D.	Log. Cot.	Log. Cos.	
0	−∞		−∞		+∞	0.00 000	60
1	6.46 372	30103	6.46 372	30103	3.53 627	0.00 000	59
2	6.76 475	17609	6.76 475	17609	3.23 524	0.00 000	58
3	6.94 084	12494	6.94 084	12494	3.05 915	0.00 000	57
4	7.06 578	9691	7.06 578	9691	2.93 421	0.00 000	56
5	7.16 269	7918	7.16 269	7918	2.83 730	0.00 000	55
6	7.24 187	6695	7.24 188	6694	2.75 812	0.00 000	54
7	7.30 882	5799	7.30 882	5799	2.69 117	0.00 000	53
8	7.36 681	5115	7.36 681	5115	2.63 318	0.00 000	52
9	7.41 797	4575	7.41 797	4575	2.58 203	0.00 000	51
10	7.46 372	4139	7.46 372	4139	2.53 627	0.00 000	50
11	7.50 512	3778	7.50 512	3779	2.49 488	0.00 000	49
12	7.54 290	3476	7.54 291	3476	2.45 709	9.99 999	48
13	7.57 767	3218	7.57 767	3218	2.42 233	9.99 999	47
14	7.60 985	2996	7.60 985	2996	2.39 014	9.99 999	46
15	7.63 981	2803	7.63 982	2803	2.36 018	9.99 999	45
16	7.66 784	2633	7.66 785	2633	2.33 215	9.99 999	44
17	7.69 417	2482	7.69 418	2482	2.30 582	9.99 999	43
18	7.71 899	2348	7.71 900	2348	2.28 099	9.99 999	42
19	7.74 248	2227	7.74 248	2227	2.25 751	9.99 999	41
20	7.76 475	2119	7.76 476	2119	2.23 524	9.99 999	40
21	7.78 594	2020	7.78 595	2020	2.21 405	9.99 999	39
22	7.80 614	1930	7.80 615	1930	2.19 384	9.99 999	38
23	7.82 545	1848	7.82 546	1848	2.17 454	9.99 999	37
24	7.84 393	1772	7.84 394	1773	2.15 605	9.99 999	36
25	7.86 166	1703	7.86 167	1703	2.13 832	9.99 999	35
26	7.87 869	1639	7.87 871	1639	2.12 129	9.99 999	34
27	7.89 508	1579	7.89 510	1579	2.10 490	9.99 998	33
28	7.91 088	1524	7.91 089	1524	2.08 910	9.99 998	32
29	7.92 612	1472	7.92 613	1472	2.07 386	9.99 998	31
30	7.94 084	1424	7.94 086	1424	2.05 914	9.99 998	30
31	7.95 508	1379	7.95 510	1379	2.04 490	9.99 998	29
32	7.96 887	1336	7.96 889	1336	2.03 111	9.99 998	28
33	7.98 223	1266	7.98 225	1296	2.01 774	9.99 998	27
34	7.99 520	1258	7.99 522	1259	2.00 478	9.99 998	26
35	8.00 778	1223	8.00 781	1223	1.99 219	9.99 997	25
36	8.02 002	1190	8.02 004	1190	1.97 995	9.99 997	24
37	8.03 192	1158	8.03 194	1158	1.96 805	9.99 997	23
38	8.04 350	1128	8.04 352	1128	1.95 647	9.99 997	22
39	8.05 478	1099	8.05 481	1099	1.94 519	9.99 997	21
40	8.06 577	1072	8.06 580	1072	1.93 419	9.99 997	20
41	8.07 650	1046	8.07 653	1046	1.92 347	9.99 997	19
42	8.08 696	1022	8.08 699	1022	1.91 300	9.99 997	18
43	8.09 718	998	8.09 721	999	1.90 278	9.99 996	17
44	8.10 716	976	8.10 720	976	1.89 279	9.99 996	16
45	8.11 692	954	8.11 696	954	1.88 303	9.99 996	15
46	8.12 647	934	8.12 651	934	1.87 349	9.99 996	14
47	8.13 581	914	8.13 585	914	1.86 415	9.99 996	13
48	8.14 495	895	8.14 499	895	1.85 500	9.99 996	12
49	8.15 390	877	8.15 395	877	1.84 605	9.99 995	11
50	8.16 268	860	8.16 272	860	1.83 727	9.99 995	10
51	8.17 128	843	8.17 133	843	1.82 867	9.99 995	9
52	8.17 971	827	8.17 976	827	1.82 023	9.99 995	8
53	8.18 798	811	8.18 803	812	1.81 196	9.99 995	7
54	8.19 610	797	8.19 615	797	1.80 384	9.99 994	6
55	8.20 407	782	8.20 412	783	1.79 587	9.99 994	5
56	8.21 189	768	8.21 195	768	1.78 804	9.99 994	4
57	8.21 958	755	8.21 964	755	1.78 036	9.99 994	3
58	8.22 713	742	8.22 719	742	1.77 280	9.99 994	2
59	8.23 455	730	8.23 462	730	1.76 538	9.99 993	1
60	8.24 185		8.24 192		1.75 808	9.99 993	0
	Log. Cos.	D	Log. Cot.	Com. D.	Log. Tan.	Log. Sin.	′

′	Log. Sin.	D	Log. Tan.	Com. D.	Log. Cot.	Log. Cos.	
0	8.24 185	718	8.24 192	718	1.75 808	9.99 993	60
1	8.24 903	706	8.24 910	706	1.75 090	9.99 993	59
2	8.25 609	694	8.25 616	695	1.74 383	9.99 993	58
3	8.26 304	684	8.26 311	684	1.73 688	9.99 992	57
4	8.26 988	673	8.26 995	673	1.73 004	9.99 992	56
5	8.27 661	663	8.27 669	663	1.72 331	9.99 992	55
6	8.28 324	653	8.28 332	653	1.71 667	9.99 992	54
7	8.28 977	643	8.28 985	643	1.71 014	9.99 992	53
8	8.29 620	634	8.29 629	634	1.70 371	9.99 991	52
9	8.30 254	625	8.30 263	625	1.69 736	9.99 991	51
10	8.30 879	616	8.30 888	616	1.69 111	9.99 991	50
11	8.31 495	607	8.31 504	607	1.68 495	9.99 990	49
12	8.32 102	599	8.32 112	599	1.67 888	9.99 990	48
13	8.32 701	591	8.32 711	591	1.67 288	9.99 990	47
14	8.33 292	583	8.33 302	583	1.66 697	9.99 990	46
15	8.33 875	575	8.33 885	575	1.66 114	9.99 989	45
16	8.34 450	567	8.34 461	568	1.65 539	9.99 989	44
17	8.35 018	560	8.35 029	560	1.64 971	9.99 989	43
18	8.35 578	553	8.35 589	553	1.64 410	9.99 989	42
19	8.36 131	546	8.36 143	546	1.63 857	9.99 988	41
20	8.36 677	539	8.36 689	539	1.63 310	9.99 988	40
21	8.37 217	533	8.37 229	533	1.62 771	9.99 988	39
22	8.37 750	526	8.37 762	527	1.62 238	9.99 987	38
23	8.38 276	520	8.38 289	520	1.61 711	9.99 987	37
24	8.38 796	514	8.38 809	514	1.61 191	9.99 987	36
25	8.39 310	508	8.39 323	508	1.60 676	9.99 986	35
26	8.39 818	502	8.39 831	502	1.60 168	9.99 986	34
27	8.40 320	496	8.40 334	496	1.59 666	9.99 986	33
28	8.40 816	491	8.40 830	491	1.59 169	9.99 986	32
29	8.41 307	485	8.41 321	485	1.58 678	9.99 985	31
30	8.41 792	479	8.41 807	480	1.58 193	9.99 985	30
31	8.42 271	474	8.42 287	475	1.57 713	9.99 985	29
32	8.42 746	469	8.42 762	469	1.57 238	9.99 984	28
33	8.43 216	464	8.43 231	464	1.56 768	9.99 984	27
34	8.43 680	459	8.43 696	460	1.56 304	9.99 984	26
35	8.44 139	454	8.44 156	455	1.55 844	9.99 983	25
36	8.44 594	450	8.44 611	450	1.55 389	9.99 983	24
37	8.45 044	445	8.45 061	445	1.54 938	9.99 982	23
38	8.45 489	440	8.45 507	441	1.54 493	9.99 982	22
39	8.45 930	436	8.45 948	437	1.54 052	9.99 982	21
40	8.46 366	432	8.46 385	432	1.53 615	9.99 981	20
41	8.46 798	428	8.46 817	428	1.53 183	9.99 981	19
42	8.47 226	423	8.47 245	424	1.52 754	9.99 981	18
43	8.47 650	419	8.47 669	419	1.52 330	9.99 980	17
44	8.48 069	415	8.48 089	416	1.51 911	9.99 980	16
45	8.48 485	411	8.48 505	412	1.51 495	9.99 979	15
46	8.48 896	407	8.48 917	408	1.51 083	9.99 979	14
47	8.49 304	404	8.49 325	404	1.50 675	9.99 979	13
48	8.49 708	400	8.49 729	400	1.50 270	9.99 978	12
49	8.50 108	396	8.50 130	396	1.49 870	9.99 978	11
50	8.50 504	393	8.50 526	393	1.49 473	9.99 978	10
51	8.50 897	389	8.50 920	390	1.49 080	9.99 977	9
52	8.51 286	386	8.51 310	386	1.48 690	9.99 977	8
53	8.51 672	382	8.51 696	383	1.48 304	9.99 976	7
54	8.52 055	379	8.52 079	379	1.47 921	9.99 976	6
55	8.52 434	375	8.52 458	376	1.47 541	9.99 975	5
56	8.52 810	373	8.52 835	373	1.47 165	9.99 975	4
57	8.53 183	369	8.53 208	370	1.46 792	9.99 975	3
58	8.53 552	366	8.53 578	366	1.46 422	9.99 974	2
59	8.53 918	363	8.53 944	364	1.46 055	9.99 974	1
60	8.54 282		8.54 308		1.45 691	9.99 973	0
	Log. Cos.	D	Log. Cot.	Com. D.	Log. Tan.	Log. Sin.	′

′	Log. Sin.	D	Log. Tan.	Com. D.	Log. Cot.	Log. Cos.
0	8.54 282	360	8.54 308	360	1.45 691	9.99 973
1	8.54 642	357	8.54 669	358	1.45 331	9.99 973
2	8.54 999	354	8.55 027	354	1.44 973	9.99 972
3	8.55 354	351	8.55 381	352	1.44 618	9.99 972
4	8.55 705	348	8.55 733	349	1.44 266	9.99 971
5	8.56 054	346	8.56 083	346	1.43 917	9.99 971
6	8.56 400	343	8.56 429	343	1.43 571	9.99 971
7	8.56 743	340	8.56 772	341	1.43 227	9.99 970
8	8.57 083	338	8.57 113	338	1.42 886	9.99 970
9	8.57 421	335	8.57 452	335	1.42 548	9.99 969
10	8.57 756	332	8.57 787	333	1.42 212	9.99 969
11	8.58 089	330	8.58 121	330	1.41 879	9.99 968
12	8.58 419	327	8.58 451	328	1.41 548	9.99 968
13	8.58 747	325	8.58 779	326	1.41 220	9.99 967
14	8.59 072	323	8.59 105	323	1.40 895	9.99 967
15		320	8.59 428	320	1.40 571	9.99 966
16		318	8.59 749	318	1.40 251	9.99 966
17		316	8.60 067	316	1.39 932	9.99 965
18		313	8.60 384	314	1.39 616	9.99 965
19		311	8.60 698	311	1.39 302	9.99 964
20		309	8.61 009	309	1.38 990	9.99 964
21		306	8.61 319	307	1.38 681	9.99 963
22		304	8.61 626	305	1.38 374	9.99 963
23		302	8.61 931	303	1.38 068	9.99 962
24		300	8.62 234	300	1.37 765	9.99 962
25		298	8.62 535	299	1.37 465	9.99 961
26		296	8.62 834	297	1.37 166	9.99 961
27		294	8.63 131	294	1.36 869	9.99 960
28		292	8.63 425	293	1.36 574	9.99 959
29		290	8.63 718	291	1.36 281	9.99 959
30		288	8.64 009	288	1.35 990	9.99 958
31		286	8.64 298	287	1.35 702	9.99 958
32		284	8.64 585	285	1.35 414	9.99 957
33		282	8.64 870	283	1.35 129	9.99 957
34		281	8.65 153	281	1.34 846	9.99 956
35		279	8.65 435	280	1.34 565	9.99 956
36		277	8.65 715	278	1.34 285	9.99 955
37		275	8.65 993	276	1.34 007	9.99 954
38		274	8.66 269	274	1.33 731	9.99 954
39		272	8.66 543	272	1.33 456	9.99 953
40		270	8.66 816	271	1.33 184	9.99 953
41		268	8.67 087	269	1.32 913	9.99 952
42		267	8.67 356	267	1.32 643	9.99 952
43		265	8.67 624	266	1.32 376	9.99 951
44		264	8.67 890	264	1.32 110	9.99 950
45	8.68 104	262	8.68 154	263	1.31 845	9.99 950
46	8.68 366	260	8.68 417	261	1.31 583	9.99 949
47	8.68 627	259	8.68 678	259	1.31 321	9.99 949
48	8.68 886	257	8.68 938	258	1.31 062	9.99 948
49	8.69 144	256	8.69 196	256	1.30 803	9.99 947
50	8.69 400	254	8.69 453	255	1.30 547	9.99 947
51	8.69 654	253	8.69 708	253	1.30 292	9.99 946
52	8.69 907	251	8.69 961	252	1.30 038	9.99 946
53	8.70 159	250	8.70 214	250	1.29 786	9.99 945
54	8.70 409	248	8.70 464	249	1.29 535	9.99 944
55	8.70 657	247	8.70 714	248	1.29 286	9.99 943
56	8.70 905	245	8.70 962	246	1.29 038	9.99 943
57	8.71 150	244	8.71 208	245	1.28 791	9.99 942
58	8.71 395	243	8.71 453	243	1.28 546	9.99 942
59	8.71 638	241	8.71 697	242	1.28 303	9.99 941
60	8.71 880		8.71			9.99 940
	Log. Cos.	D	Log.			Log. Sin.

18°

′	Log Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log Cos.	
0	0 35 209	54	0 36 336	57	0 63 664	0 98 872	60
1	0 35 263	54	0 36 394	57	0 63 606	[illegible]	59
2	0 35 318	54	0 36 451	57	0 63 549	[illegible]	58
3	0 35 373	54	0 36 508	57	0 63 49	[illegible]	57
4	0 35 427	54	0 36 565	57	0 63 435	0 98 860	56
5	0 35 481	54	0 36 623	57	0 63 376	[illegible]	55
6	0 35 536	54	0 36 681	57	0 63 310	0 98 856	54
7	0 35 590	54	0 36 738	57	0 63 262	0 98 852	53
8	0 35 644	54	0 36 795	57	0 63 204	[illegible]	52
9	0 35 698	54	0 36 852	57	0 63 147	[illegible]	51
10	0 35 752	54	0 36 909	57	0 63 090	[illegible]	50
11	0 35 806	54	0 36 966	57	0 63 033	0 98 840	49
12	0 35 860	54	0 37 023	57	0 62 977	0 98 837	48
13	0 35 914	54	0 37 080	56	0 62 920	0 98 834	47
14	0 35 968	54	0 37 136	57	0 62 863	0 98 831	46
15	0 36 021	54	0 37 193	57	0 62 806	0 98 828	45
16	0 36 075	54	0 37 250	56	0 62 750	0 98 825	44
17	0 36 128	54	0 37 306	56	0 62 693	0 98 822	43
18	0 36 182	54	0 37 362	56	0 62 637	0 98 819	42
19	0 36 235	54	0 37 419	56	0 62 580	0 98 816	41
20	0 36 289	53	0 37 475	56	0 62 524	0 98 813	40
21	0 36 342	53	0 37 532	56	0 62 468	0 98 810	39
22	0 36 395	53	0 37 588	56	0 62 412	0 98 807	38
23	0 36 448	53	0 37 644	56	0 62 356	0 98 804	37
24	0 36 501	53	0 37 700	56	0 62 300	0 98 801	36
25	0 36 554	53	0 37 756	56	0 62 243	0 98 798	35
26	0 36 607	53	0 37 812	56	0 62 188	0 98 795	34
27	0 36 660	53	0 37 868	56	0 62 132	0 98 792	33
28	0 36 713	53	0 37 924	56	0 62 076	0 98 789	32
29	0 36 766	53	0 37 979	56	0 62 020	0 98 786	31
30	0 36 818	53	0 38 035	56	0 61 964	0 98 783	30
31	0 36 871	53	0 38 091	55	0 61 909	0 98 780	29
32	0 36 923	53	0 38 146	55	0 61 853	0 98 777	28
33	0 36 976	53	0 38 202	55	0 61 798	0 98 774	27
34	0 37 028	53	0 38 257	55	0 61 742	0 98 771	26
35	0 37 081	52	0 38 313	55	0 61 687	0 98 768	25
36	0 37 133	52	0 38 368	55	0 61 63	0 98 765	24
37	0 37 185	52	0 38 423	55	0 61 577	0 98 762	23
38	0 37 237	52	0 38 478	55	0 61 521	0 98 759	22
39	0 37 289	52	0 38 533	55	0 61 466	0 98 755	21
40	0 37 341	52	0 38 589	55	0 61 411	0 98 752	20
41	0 37 393	52	0 38 644	54	0 61 356	0 98 749	19
42	0 37 445	52	0 38 698	55	0 61 301	0 98 746	18
43	0 37 497	52	0 38 753	55	0 61 246	0 98 743	17
44	0 37 549	51	0 38 808	55	0 61 191	0 98 740	16
45	0 37 600	52	0 38 863	54	0 61 137	0 98 737	15
46	0 37 652	51	0 38 918	54	0 61 082	0 98 734	14
47	0 37 703	52	0 38 972	54	0 61 027	0 98 731	13
48	0 37 755	51	0 39 027	54	0 60 973	0 98 728	12
49	0 37 806	51	0 39 081	54	0 60 918	0 98 725	11
50	0 37 857	51	0 39 136	54	0 60 864	0 98 721	10
51	0 37 909	51	0 39 190	54	0 60 809	0 98 71	9
52	0 37 960	51	0 39 244	54	0 60 755	0 98 715	8
53	0 38 011	51	0 39 299	54	0 60 701	0 98 712	7
54	0 38 062	51	0 39 353	54	0 60 647	0 98 709	6
55	0 38 113	51	0 39 407	54	0 60 593	0 98 706	5
56	0 38 164	51	0 39 461	54	0 60 539	0 98 703	4
57	0 38 215	50	0 39 515	54	0 60 484	0 98 700	3
58	0 38 266	51	0 39 569	54	0 60 430	[illegible]	2
59	0 38 317	51	0 39 623	54	0 60 376	[illegible]	1
60	0 38 367	50	0 39 677	53	0 60 323	[illegible]	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	[illegible]	′

	57	57	56	56
6	5.7	5.7	5.6	5.6
7	6.7	6.6	6.5	6.5
8	7.6	7.6	7.5	7.4
9	8.6	8.5	8.4	8.4
10	9.5	9.5	9.4	9.3
20	19.0	19.0	18.8	18.7
30	28.5	28.5	28.2	28.0
40	38.0	38.0	37.6	37.3
50	47.5	47.5	47.1	46.6

	55	55	54	54
6	[illegible]	[illegible]	[illegible]	[illegible]
7	[illegible]	[illegible]	[illegible]	[illegible]
8	[illegible]	[illegible]	[illegible]	[illegible]
9	[illegible]	[illegible]	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]	[illegible]
20	18	18	18	18
30	27	27	27	27
40	37	36	36	36
50	46	46	45	45

	53	53	52	52
6	[illegible]	[illegible]	[illegible]	[illegible]
7	[illegible]	[illegible]	[illegible]	[illegible]
8	[illegible]	[illegible]	[illegible]	[illegible]
9	[illegible]	[illegible]	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]	[illegible]
20	17	17	17	17
30	26	26	26	26
40	35	35	35	34
50	44	44	43	43

	51	51	50
6	[illegible]	[illegible]	[illegible]
7	[illegible]	[illegible]	[illegible]
8	[illegible]	[illegible]	[illegible]
9	[illegible]	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]
20	17	17	16
30	25	25	25
40	34	34	33
50	42	42	42

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.38 367	50	9.39 677	54	0.60 323	9.98 690	3	60
1	9.38 418	50	9.39 731	53	0.60 269	9.98 687	3	59
2	9.38 469	50	9.39 785	54	0.60 215	9.98 684	3	58
3	9.38 519	50	9.39 839	53	0.60 161	9.98 681	3	57
4	9.38 570	50	9.39 892	53	0.60 108	9.98 678	3	56
5	9.38 620	50	9.39 945	53	0.60 055	9.98 674	3	55
6	9.38 670	50	9.39 999	53	0.60 001	9.98 671	3	54
7	9.38 720	50	9.40 052	53	0.59 948	9.98 668	3	53
8	9.38 771	50	9.40 106	53	0.59 894	9.98 665	3	52
9	9.38 821	50	9.40 159	53	0.59 841	9.98 662	3	51
10	9.38 871	50	9.40 212	53	0.59 788	9.98 659	3	50
11	9.38 921	50	9.40 266	53	0.59 734	9.98 656	3	49
12	9.38 971	50	9.40 319	53	0.59 681	9.98 652	3	48
13	9.39 021	50	9.40 372	53	0.59 628	[illegible]	3	47
14	9.39 071	50	9.40 425	53	0.59 575	9.98 646	3	46
15	9.39 120	49	9.40 478	53	0.59 522	9.98 643	3	45
16	9.39 170	50	9.40 531	52	0.59 469	9.98 639	3	44
17	9.39 220	49	9.40 584	52	0.59 416	9.98 636	3	43
18	9.39 269	49	9.40 636	53	0.59 364	9.98 633	3	42
19	9.39 319	49	9.40 689	52	0.59 311	9.98 630	3	41
20	9.39 368	49	9.40 742	53	0.59 258	9.98 626	3	40
21	9.39 418	49	9.40 795	52	0.59 205	9.98 623	3	39
22	9.39 467	49	9.40 847	52	0.59 153	9.98 620	3	38
23	9.39 516	49	9.40 900	52	0.59 100	9.98 617	3	37
24	9.39 566	49	9.40 952	52	0.59 048	9.98 613	3	36
25	9.39 615	49	9.41 004	52	0.58 995	9.98 610	3	35
26	9.39 664	49	9.41 057	52	0.58 943	9.98 607	3	34
27	9.39 713	49	9.41 109	52	0.58 891	9.98 604	3	33
28	9.39 762	49	9.41 161	52	0.58 839	9.98 600	3	32
29	9.39 811	49	9.41 213	52	0.58 786	9.98 597	3	31
30	9.39 860	49	9.41 266	52	0.58 734	9.98 594	3	30
31	9.39 909	49	9.41 318	52	0.58 682	9.98 591	3	29
32	9.39 958	49	9.41 370	52	0.58 630	9.98 587	3	28
33	9.40 006	49	9.41 422	52	0.58 578	9.98 584	3	27
34	9.40 055	49	9.41 474	52	0.58 526	9.98 581	3	26
35	9.40 103	49	9.41 525	51	0.58 474	9.98 578	3	25
36	9.40 152	49	9.41 577	52	0.58 422	9.98 574	3	24
37	9.40 200	49	9.41 629	52	0.58 370	9.98 571	3	23
38	9.40 249	49	9.41 681	51	0.58 319	[illegible]	3	22
39	9.40 297	49	9.41 732	51	0.58 267	9.98 564	3	21
40	9.40 346	48	9.41 784	51	0.58 216	9.98 561	3	20
41	9.40 394	48	9.41 836	52	0.58 164	9.98 558	3	19
42	9.40 442	48	9.41 887	51	0.58 113	9.98 554	3	18
43	9.40 490	48	9.41 938	51	0.58 061	9.98 551	3	17
44	9.40 538	48	9.41 990	51	0.58 010	9.98 548	3	16
45	9.40 586	48	9.42 041	51	0.57 959	9.98 544	3	15
46	9.40 634	48	9.42 092	51	0.57 907	9.98 541	3	14
47	9.40 682	48	9.42 144	51	0.57 856	9.98 538	3	13
48	9.40 730	48	9.42 195	51	0.57 805	9.98 534	3	12
49	9.40 777	47	9.42 246	51	0.57 753	9.98 531	3	11
50	9.40 825	48	9.42 297	51	0.57 703	9.98 528	3	10
51	9.40 873	47	9.42 348	51	0.57 651	9.98 524	3	9
52	9.40 920	47	9.42 399	51	0.57 600	9.98 521	3	8
53	9.40 968	47	9.42 450	51	0.57 549	9.98 518	3	7
54	9.41 015	47	9.42 501	50	0.57 499	9.98 514	3	6
55	9.41 063	47	9.42 552	51	0.57 448	9.98 511	3	5
56	9.41 110	47	9.42 602	50	0.57 397	9.98 508	3	4
57	9.41 158	47	9.42 653	51	0.57 346	9.98 504	3	3
58	9.41 205	47	9.42 704	50	0.57 296	9.98 501	3	2
59	9.41 252	47	9.42 754	50	0.57 245	9.98 498	3	1
60	9.41 299		9.42 805		0.57 195	9.98 494		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	54	53	53
6	5.4	5.3	5.3
7	6.3	6.2	6.2
8	7.2	7.1	7.1
9	8.1	8.0	8.0
10	9.0	8.8	8.8
20	18.0	17.7	17.7
30	27.0	26.5	26.5
40	36.0	35.3	35.3
50	45.0	44.2	44.2

	52	51	51
6	5.2	5.1	5.1
7	6.1	6.0	6.0
8	6.9	6.8	6.8
9	7.8	7.7	7.7
10	8.7	8.5	8.5
20	17.3	17.0	17.0
30	26.0	25.5	25.5
40	34.7	34.0	34.0
50	43.3	42.5	42.5

	50	49	49
6	5.0	4.9	4.9
7	5.8	5.7	5.7
8	6.7	6.5	6.5
9	7.5	7.4	7.4
10	8.3	8.2	8.2
20	16.7	16.3	16.3
30	25.0	24.5	24.5
40	33.3	32.7	32.7
50	41.7	40.8	40.8

	48	48	47	47
6	4.8	4.8	4.7	4.7
7	5.6	5.6	5.5	5.5
8	6.4	6.4	6.3	6.3
9	7.2	7.2	7.1	7.1
10	8.0	8.0	7.8	7.8
20	16.0	16.0	15.7	15.7
30	24.0	24.0	23.5	23.5
40	32.0	32.0	31.3	31.3
50	40.0	40.0	39.2	39.2

	3	3
6	0.3	0.3
7	0.4	0.4
8	0.4	0.4
9	0.5	0.5
10	0.5	0.5
20	1.0	1.0
30	1.5	1.5
40	2.0	2.0
50	2.5	2.5

P. P.

TABLE VII.—LOGARITHMIC SINES, COSINE
AND COTANGENTS.

15°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9 41 300	47	9 42 805	51	0 57 195	9 98 494	3	60
1	9 41 347	47	9 42 856	50	0 57 144	9 98 491	3	59
2	9 41 394	47	9 42 906	51	0 57 094	9 98 488	4	58
3	9 41 441	47	9 42 957	50	0 57 043	9 98 484	3	57
4	9 41 488	47	9 43 007	50	0 56 993	9 98 481	4	56
5	9 41 535	47	9 43 057	51	0 56 943	9 98 477	3	55
6	9 41 582	46	9 43 108	50	0 56 892	9 98 474	3	54
7	9 41 628	47	9 43 158	50	0 56 842	9 98 471	4	53
8	9 41 675	47	9 43 208	50	0 56 792	9 98 467	3	52
9	9 41 722	46	9 43 258	50	0 56 742	9 98 464	4	51
10	9 41 768	47	9 43 308	50	0 56 692	9 98 460	3	50
11	9 41 815	46	9 43 358	50	0 56 642	9 98 457	4	49
12	9 41 861	47	9 43 408	50	0 56 592	9 98 453	3	48
13	9 41 908	46	9 43 458	50	0 56 542	9 98 450	3	47
14	9 41 954	47	9 43 508	50	0 56 492	9 98 447	4	46
15	9 42 001	46	9 43 558	49	0 56 442	9 98 443	3	45
16	9 42 047	46	9 43 607	50	0 56 393	9 98 440	4	44
17	9 42 093	47	9 43 657	50	0 56 343	9 98 436	3	43
18	9 42 140	46	9 43 707	49	0 56 293	9 98 433	4	42
19	9 42 186	46	9 43 756	50	0 56 244	9 98 429	3	41
20	9 42 232	46	9 43 806	49	0 56 194	9 98 426	4	40
21	9 42 278	46	9 43 855	50	0 56 145	9 98 422	3	39
22	9 42 324	46	9 43 905	49	0 56 095	9 98 419	4	38
23	9 42 370	46	9 43 954	50	0 56 046	9 98 415	3	37
24	9 42 416	45	9 44 004	49	0 55 996	9 98 412	3	36
25	9 42 461	46	9 44 053	49	0 55 947	9 98 409	4	35
26	9 42 507	46	9 44 102	49	0 55 898	9 98 405	3	34
27	9 42 553	46	9 44 151	50	0 55 849	9 98 402	4	33
28	9 42 599	45	9 44 201	49	0 55 799	9 98 398	3	32
29	9 42 644	46	9 44 250	49	0 55 750	9 98 395	4	31
30	9 42 690	45	9 44 299	49	0 55 701	9 98 391	3	30
31	9 42 735	46	9 44 348	49	0 55 652	9 98 388	4	29
32	9 42 781	45	9 44 397	49	0 55 603	9 98 384	3	28
33	9 42 826	46	9 44 446	49	0 55 554	9 98 381	4	27
34	9 42 872	45	9 44 495	49	0 55 505	9 98 377	4	26
35	9 42 917	45	9 44 544	48	0 55 456	9 98 373	3	25
36	9 42 962	45	9 44 592	49	0 55 408	9 98 370	4	24
37	9 43 007	46	9 44 641	49	0 55 359	9 98 366	3	23
38	9 43 053	45	9 44 690	48	0 55 310	9 98 363	4	22
39	9 43 098	45	9 44 738	49	0 55 262	9 98 359	3	21
40	9 43 143	45	9 44 787	49	0 55 213	9 98 356	4	20
41	9 43 188	45	9 44 836	48	0 55 164	9 98 352	3	19
42	9 43 233	45	9 44 884	49	0 55 116	9 98 349	4	18
43	9 43 278	45	9 44 933	48	0 55 067	9 98 345	3	17
44	9 43 323	44	9 44 981	48	0 55 019	9 98 342	4	16
45	9 43 367	45	9 45 029	49	0 54 971	9 98 338	4	15
46	9 43 412	45	9 45 078	48	0 54 922	9 98 334	3	14
47	9 43 457	45	9 45 126	48	0 54 874	9 98 331	4	13
48	9 43 502	44	9 45 174	48	0 54 826	9 98 327	3	12
49	9 43 546	45	9 45 222	49	0 54 778	9 98 324	4	11
50	9 43 591	44	9 45 271	48	0 54 729	9 98 320	3	10
51	9 43 635	45	9 45 319	48	0 54 681	9 98 317	4	9
52	9 43 680	44	9 45 367	48	0 54 633	9 98 313	4	8
53	9 43 724	45	9 45 415	48	0 54 585	9 98 309	3	7
54	9 43 769	44	9 45 463	48	0 54 537	9 98 306	4	6
55	9 43 813	44	9 45 511	48	0 54 489	9 98 302	3	5
56	9 43 857	44	9 45 559	47	0 54 441	9 98 299	4	4
57	9 43 901	45	9 45 606	48	0 54 394	9 98 295	4	3
58	9 43 946	44	9 45 654	48	0 54 346	9 98 291	3	2
59	9 43 990	44	9 45 702	48	0 54 298	9 98 288	4	1
60	9 44 034		9 45 750		0 54 250	9 98 284		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

'	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.51 264	37	9.53 697	41	0.46 303	9.97 567	4	60
1	9.51 301	36	9.53 738	41	0.46 262	9.97 563	5	59
2	9.51 337	37	9.53 779	41	0.46 221	9.97 558	4	58
3	9.51 374	37	9.53 820	41	0.46 180	9.97 554	4	57
4	9.51 411	36	9.53 861	41	0.46 139	9.97 550	5	56
5	9.51 447	37	9.53 902	41	0.46 098	9.97 545	4	55
6	9.51 484	36	9.53 943	41	0.46 057	9.97 541	5	54
7	9.51 520	37	9.53 984	41	0.46 016	9.97 536	4	53
8	9.51 557	36	9.54 025	40	0.45 975	9.97 532	4	52
9	9.51 593	36	9.54 065	41	0.45 935	9.97 528	5	51
10	9.51 629	37	9.54 106	41	0.45 894	9.97 523	4	50
11	9.51 666	36	9.54 147	40	0.45 853	9.97 519	5	49
12	9.51 702	36	9.54 187	41	0.45 813	9.97 514	4	48
13	9.51 738	36	9.54 228	41	0.45 772	9.97 510	4	47
14	9.51 774	37	9.54 269	40	0.45 731	9.97 506	5	46
15	9.51 811	36	9.54 309	41	0.45 691	9.97 501	4	45
16	9.51 847	36	9.54 350	40	0.45 650	9.97 497	5	44
17	9.51 883	36	9.54 390	41	0.45 610	9.97 492	4	43
18	9.51 919	36	9.54 431	40	0.45 569	9.97 488	4	42
19	9.51 955	36	9.54 471	41	0.45 529	9.97 484	5	41
20	9.51 991	36	9.54 512	40	0.45 488	9.97 479	4	40
21	9.52 027	36	9.54 552	41	0.45 448	9.97 475	5	39
22	9.52 063	36	9.54 593	40	0.45 407	9.97 470	4	38
23	9.52 099	36	9.54 633	40	0.45 367	9.97 466	5	37
24	9.52 135	36	9.54 673	41	0.45 327	9.97 461	4	36
25	9.52 171	36	9.54 714	40	0.45 286	9.97 457	5	35
26	9.52 207	35	9.54 754	40	0.45 246	9.97 452	4	34
27	9.52 242	36	9.54 794	41	0.45 206	9.97 448	4	33
28	9.52 278	36	9.54 835	40	0.45 165	9.97 444	5	32
29	9.52 314	36	9.54 875	40	0.45 125	9.97 439	4	31
30	9.52 350	35	9.54 915	40	0.45 085	9.97 435	5	30
31	9.52 385	36	9.54 955	40	0.45 045	9.97 430	4	29
32	9.52 421	35	9.54 995	40	0.45 005	9.97 426	5	28
33	9.52 456	36	9.55 035	40	0.44 965	9.97 421	4	27
34	9.52 492	35	9.55 075	40	0.44 925	9.97 417	5	26
35	9.52 527	36	9.55 115	40	0.44 885	9.97 412	4	25
36	9.52 563	35	9.55 155	40	0.44 845	9.97 408	5	24
37	9.52 598	36	9.55 195	40	0.44 805	9.97 403	4	23
38	9.52 634	35	9.55 235	40	0.44 765	9.97 399	5	22
39	9.52 669	36	9.55 275	40	0.44 725	9.97 394	4	21
40	9.52 705	35	9.55 315	40	0.44 685	9.97 390	5	20
41	9.52 740	35	9.55 355	40	0.44 645	9.97 385	4	19
42	9.52 775	36	9.55 395	39	0.44 605	9.97 381	5	18
43	9.52 811	35	9.55 434	40	0.44 566	9.97 376	4	17
44	9.52 846	35	9.55 474	40	0.44 526	9.97 372	5	16
45	9.52 881	35	9.55 514	40	0.44 486	9.97 367	5	15
46	9.52 916	35	9.55 554	39	0.44 446	9.97 362	4	14
47	9.52 951	35	9.55 593	40	0.44 407	9.97 358	5	13
48	9.52 986	35	9.55 633	40	0.44 367	9.97 353	4	12
49	9.53 021	36	9.55 673	39	0.44 327	9.97 349	5	11
50	9.53 057	35	9.55 712	40	0.44 288	9.97 344	4	10
51	9.53 092	35	9.55 752	39	0.44 248	9.97 340	5	9
52	9.53 127	34	9.55 791	40	0.44 209	9.97 335	4	8
53	9.53 161	35	9.55 831	39	0.44 169	9.97 331	5	7
54	9.53 196	35	9.55 870	40	0.44 130	9.97 326	5	6
55	9.53 231	35	9.55 910	39	0.44 090	9.97 321	4	5
56	9.53 266	35	9.55 949	40	0.44 051	9.97 317	5	4
57	9.53 301	35	9.55 989	39	0.44 011	9.97 312	4	3
58	9.53 336	34	9.56 028	39	0.43 972	9.97 308	5	2
59	9.53 370	35	9.56 067	40	0.43 933	9.97 303	4	1
60	9.53 405		9.56 107		0.43 893	9.97 299		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	'

P. P.

	41	39	40	37	36.5	36	35.5	35	34.5	5	4
6	4.1	3.9	4.0	3.7	3.7	3.6	3.6	3.5	3.5	0.5	0.4
7	4.8	4.6	4.7	4.3	4.3	4.2	4.1	4.1	4.0	0.6	0.5
8	5.5	5.2	5.3	4.9	4.9	4.8	4.7	4.7	4.6	0.7	0.5
9	6.2	5.9	6.0	5.6	5.5	5.4	5.3	5.3	5.2	0.8	0.6
10	6.8	6.5	6.7	6.2	6.1	6.0	5.9	5.8	5.8	0.8	0.7
20	13.7	13.0	13.3	12.3	12.2	12.0	11.8	11.7	11.5	1.7	1.3
30	20.5	19.5	20.0	18.5	18.3	18.0	17.8	17.5	17.3	2.5	2.0
40	27.3	26.0	26.7	24.7	24.3	24.0	23.7	23.3	23.0	3.3	2.7
50	34.2	32.5	33.3	30.8	30.4	30.0	29.6	29.2	28.8	4.2	3.3

′	Log. Sin.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.	d.	
0	9.53 405	35	9.56 106	39	0.43 893	9.97 298	4	60
1	9.53 440	34	9.56 146	39	0.43 854	9.97 294	4	59
2	9.53 474	34	9.56 185	39	0.43 815	9.97 289	4	58
3	9.53 509	35	9.56 224	39	0.43 775	9.97 285	5	57
4	9.53 544	34	9.56 263	39	0.43 736	9.97 280	4	56
5	9.53 578	34	9.56 303	39	0.43 697	9.97 275	4	55
6	9.53 613	34	9.56 342	39	0.43 658	9.97 271	4	54
7	9.53 647	34	9.56 381	39	0.43 619	9.97 266	5	53
8	9.53 682	34	9.56 420	39	0.43 580	9.97 261	4	52
9	9.53 716	34	9.56 459	39	0.43 540	9.97 257	4	51
10	9.53 750	34	9.56 498	39	0.43 501	9.97 252	4	50
11	9.53 785	34	9.56 537	39	0.43 462	9.97 248	5	49
12	9.53 819	34	9.56 576	39	0.43 423	9.97 243	4	48
13	9.53 854	34	9.56 615	38	0.43 384	9.97 238	4	47
14	9.53 888	34	9.56 654	39	0.43 346	9.97 234	5	46
15	9.53 922	34	9.56 693	39	0.43 307	9.97 229	4	45
16	9.53 956	34	9.56 732	39	0.43 268	9.97 224	4	44
17	9.53 990	34	9.56 771	39	0.43 229	9.97 220	5	43
18	9.54 025	34	9.56 810	38	0.43 190	9.97 215	4	42
19	9.54 059	34	9.56 848	39	0.43 151	9.97 210	4	41
20	9.54 093	34	9.56 887	38	0.43 112	9.97 206	5	40
21	9.54 127	34	9.56 926	39	0.43 074	9.97 201	4	39
22	9.54 161	34	9.56 965	38	0.43 035	9.97 196	5	38
23	9.54 195	34	9.57 003	38	0.42 996	9.97 191	4	37
24	9.54 229	33	9.57 042	39	0.42 958	9.97 187	4	36
25	9.54 263	34	9.57 081	38	0.42 919	9.97 182	5	35
26	9.54 297	34	9.57 119	38	0.42 880	9.97 177	4	34
27	9.54 331	34	9.57 158	38	0.42 842	9.97 173	5	33
28	9.54 365	33	9.57 196	38	0.42 803	9.97 168	4	32
29	9.54 398	34	9.57 235	39	0.42 765	9.97 163	4	31
30	9.54 432	34	9.57 274	38	0.42 726	9.97 159	5	30
31	9.54 466	33	9.57 312	38	0.42 687	9.97 154	4	29
32	9.54 500	34	9.57 350	38	0.42 649	9.97 149	5	28
33	9.54 534	33	9.57 389	38	0.42 611	9.97 144	4	27
34	9.54 567	33	9.57 427	38	0.42 572	9.97 140	5	26
35	9.54 601	33	9.57 466	38	0.42 534	9.97 135	4	25
36	9.54 634	34	9.57 504	38	0.42 495	9.97 130	5	24
37	9.54 668	33	9.57 542	38	0.42 457	9.97 125	4	23
38	9.54 702	33	9.57 581	38	0.42 419	9.97 121	5	22
39	9.54 735	33	9.57 619	38	0.42 380	9.97 116	4	21
40	9.54 769	33	9.57 657	38	0.42 342	9.97 111	5	20
41	9.54 802	33	9.57 696	38	0.42 304	9.97 106	4	19
42	9.54 836	33	9.57 734	38	0.42 266	9.97 102	5	18
43	9.54 869	33	9.57 772	38	0.42 227	9.97 097	5	17
44	9.54 902	33	9.57 810	38	0.42 189	9.97 092	4	16
45	9.54 936	33	9.57 848	38	0.42 151	9.97 087	5	15
46	9.54 969	33	9.57 886	38	0.42 113	9.97 082	4	14
47	9.55 002	33	9.57 925	38	0.42 075	9.97 078	5	13
48	9.55 036	33	9.57 963	38	0.42 037	9.97 073	4	12
49	9.55 069	33	9.58 001	38	0.41 999	9.97 068	5	11
50	9.55 102	33	9.58 039	38	0.41 961	9.97 063	5	10
51	9.55 135	33	9.58 077	38	0.41 923	9.97 058	4	9
52	9.55 168	33	9.58 115	38	0.41 885	9.97 054	5	8
53	9.55 202	33	9.58 153	37	0.41 847	9.97 049	5	7
54	9.55 235	33	9.58 190	38	0.41 809	9.97 044	4	6
55	9.55 268	33	9.58 228	38	0.41 771	9.97 039	5	5
56	9.55 301	33	9.58 266	38	0.41 733	9.97 034	5	4
57	9.55 334	33	9.58 304	37	0.41 695	9.97 029	4	3
58	9.55 367	33	9.58 342	38	0.41 658	9.97 025	5	2
59	9.55 400	33	9.58 380	37	0.41 620	9.97 020	5	1
60	9.55 433		9.58 417		0.41 582	9.97 015		0
	Log. Cos.	d.	Log. Cot.	c.d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	39̄	39
6	3.9	3.9
7	4.6	4.5
8	5.2	5.2
9	5.9	5.8
10	6.6	6.5
20	13.1	13.0
30	19.7	19.5
40	26.3	26.0
50	32.9	32.5

	38̄	38	37̄
6	3.8	3.8	3.7
7	4.5	4.4	4.4
8	5.1	5.0	5.0
9	5.8	5.7	5.6
10	6.4	6.3	6.2
20	12.8	12.6	12.5
30	19.2	19.0	18.7
40	25.6	25.3	25.0
50	32.1	31.6	31.2

	35	34̄	34
6	3.5	3.4	3.4
7	4.1	4.0	3.9
8	4.6	4.6	4.5
9	5.2	5.2	5.1
10	5.8	5.7	5.6
20	11.6	11.5	11.3
30	17.5	17.2	17.0
40	23.3	23.0	22.6
50	29.1	28.7	28.3

	33̄	33
6	3.3	3.3
7	3.9	3.8
8	4.4	4.4
9	5.0	4.9
10	5.6	5.5
20	11.1	11.0
30	16.7	16.5
40	22.3	22.0
50	27.9	27.5

	5	4̄
6	0.5	0.4
7	0.6	0.5
8	0.6	0.6
9	0.7	0.7
10	0.8	0.7
20	1.6	1.5
30	2.5	2.2
40	3.3	3.0
50	4.1	3.7

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.57 357	31	9.60 641	36	0.39 359	9.96 716	5	60
1	9.57 389	31	9.60 677	36	0.39 322	9.96 711	5	59
2	9.57 420	31	9.60 713	36	0.39 286	9.96 706	5	58
3	9.57 451	31	9.60 750	36	0.39 250	9.96 701	5	57
4	9.57 482	31	9.60 786	36	0.39 213	9.96 696	5	56
5	9.57 513	31	9.60 822	36	0.39 177	9.96 691	5	55
6	9.57 544	31	9.60 859	36	0.39 141	9.96 686	5	54
7	9.57 576	31	9.60 895	36	0.39 105	9.96 681	5	53
8	9.57 607	31	9.60 931	36	0.39 069	9.96 675	5	52
9	9.57 638	31	9.60 967	36	0.39 032	9.96 670	5	51
10	9.57 669	31	9.61 003	36	0.38 996	9.96 665	5	50
11	9.57 700	31	9.61 039	36	0.38 960	9.96 660	5	49
12	9.57 731	31	9.61 076	36	0.38 924	9.96 655	5	48
13	9.57 762	30	9.61 112	36	0.38 888	9.96 650	5	47
14	9.57 792	31	9.61 148	36	0.38 852	9.96 644	5	46
15	9.57 823	31	9.61 184	36	0.38 816	9.96 639	5	45
16	9.57 854	31	9.61 220	36	0.38 780	9.96 634	5	44
17	9.57 885	30	9.61 256	36	0.38 744	9.96 629	5	43
18	9.57 916	31	9.61 292	36	0.38 708	9.96 624	5	42
19	9.57 947	30	9.61 328	36	0.38 672	9.96 619	5	41
20	9.57 977	31	9.61 364	36	0.38 636	9.96 613	5	40
21	9.58 008	30	9.61 400	36	0.38 600	9.96 608	5	39
22	9.58 039	31	9.61 436	36	0.38 564	9.96 603	5	38
23	9.58 070	30	9.61 472	35	0.38 528	9.96 598	5	37
24	9.58 100	30	9.61 507	36	0.38 492	9.96 593	5	36
25	9.58 131	31	9.61 543	36	0.38 456	9.96 587	5	35
26	9.58 162	30	7.61 579	35	0.38 420	9.96 582	5	34
27	9.58 192	30	9.61 615	36	0.38 385	9.96 577	5	33
28	9.58 223	30	9.61 651	35	0.38 349	9.96 572	5	32
29	9.58 253	30	9.61 686	36	0.38 313	9.96 567	5	31
30	9.58 284	30	9.61 722	35	0.38 277	9.96 561	5	30
31	9.58 314	30	9.61 758	36	0.38 242	9.96 556	5	29
32	9.58 345	30	9.61 794	35	0.38 206	9.96 551	5	28
33	9.58 375	30	9.61 829	35	0.38 170	9.96 546	5	27
34	9.58 406	30	9.61 865	36	0.38 135	9.96 540	5	26
35	9.58 436	30	9.61 901	35	0.38 099	9.96 535	5	25
36	9.58 466	30	9.61 936	35	0.38 063	9.96 530	5	24
37	9.58 497	30	9.61 972	35	0.38 028	9.96 525	5	23
38	9.58 527	30	9.62 007	35	0.37 992	9.96 519	5	22
39	9.58 557	30	9.62 043	35	0.37 957	9.96 514	5	21
40	9.58 587	30	9.62 078	35	0.37 921	9.96 509	5	20
41	9.58 618	30	9.62 114	35	0.37 886	9.96 503	5	19
42	9.58 648	30	9.62 149	35	0.37 850	9.96 498	5	18
43	9.58 678	30	9.62 185	35	0.37 815	9.96 493	5	17
44	9.58 708	30	9.62 220	35	0.37 779	9.96 488	5	16
45	9.58 738	30	9.62 256	35	0.37 744	9.96 482	5	15
46	9.58 769	30	9.62 291	35	0.37 708	9.96 477	5	14
47	9.58 799	30	9.62 327	35	0.37 673	9.96 472	5	13
48	9.58 829	30	9.62 362	35	0.37 637	9.96 466	5	12
49	9.58 859	30	9.62 397	35	0.37 602	9.96 461	5	11
50	9.58 889	30	9.62 433	35	0.37 567	9.96 456	5	10
51	9.58 919	30	9.62 468	35	0.37 531	9.96 450	5	9
52	9.58 949	30	9.62 503	35	0.37 496	9.96 445	5	8
53	9.58 979	30	9.62 539	35	0.37 461	9.96 440	5	7
54	9.59 009	29	9.62 574	35	0.37 426	9.96 434	5	6
55	9.59 038	30	9.62 609	35	0.37 390	9.96 429	5	5
56	9.59 068	30	9.62 644	35	0.37 355	9.96 424	5	4
57	9.59 098	29	9.62 679	35	0.37 320	9.96 418	5	3
58	9.59 128	30	9.62 715	35	0.37 285	9.96 413	5	2
59	9.59 158	30	9.62 750	35	0.37 250	9.96 408	5	1
60	9.59 188		9.62 785		0.37 215	9.96 402		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	36	36
6	3.6	3.
7	4.2	4.
8	4.8	4.
9	5.5	5.
10	6.1	6.
20	12.1	12.
30	18.2	18.
40	24.3	24.
50	30.4	30.

	35	35
6	3.5	3.
7	4.1	4.
8	4.7	4.
9	5.3	5.
10	5.9	5.
20	11.8	11.
30	17.7	17.
40	23.6	23.
50	29.6	29.

	31	31
6	3.1	3.
7	3.7	3.
8	4.2	4.
9	4.7	4.
10	5.2	5.
20	10.5	10.
30	15.7	15.
40	21.0	20.
50	26.2	25.

	30	30
6	3.0	3.0
7	3.5	3.5
8	4.0	4.0
9	4.6	4.5
10	5.1	5.0
20	10.1	10.0
30	15.2	15.0
40	20.3	20.0
50	25.4	25.0

	5	5
6	0.5	0.5
7	0.6	0.
8	0.7	0.
9	0.8	0.
10	0.9	0.
20	1.8	1.
30	2.7	2.
40	3.6	3.
50	4.6	4.

P. P.

'	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	'
0	9.59 188	29	9.62 785	35	0.37 215	9.96 402	5	60
1	9.59 217	30	9.62 820	35	0.37 179	9.96 397	5	59
2	9.59 247	30	9.62 855	35	0.37 144	9.96 392	6	58
3	9.59 277	29	9.62 890	35	0.37 109	9.96 386	5	57
4	9.59 306	30	9.62 925	35	0.37 074	9.96 381	6	56
5	9.59 336	30	9.62 960	35	0.37 039	9.96 375	5	55
6	9.59 366	29	9.62 995	35	0.37 004	9.96 370	5	54
7	9.59 395	30	9.63 030	35	0.36 969	9.96 365	6	53
8	9.59 425	29	9.63 065	35	0.36 934	9.96 359	5	52
9	9.59 454	30	9.63 100	35	0.36 899	9.96 354	6	51
10	9.59 484	29	9.63 135	35	0.36 864	9.96 348	5	50
11	9.59 513	30	9.63 170	35	0.36 829	9.96 343	5	49
12	9.59 543	29	9.63 205	35	0.36 794	9.96 338	6	48
13	9.59 572	30	9.63 240	35	0.36 760	9.96 332	5	47
14	9.59 602	29	9.63 275	35	0.36 725	9.96 327	6	46
15	9.59 631	30	9.63 310	34	0.36 690	9.96 321	5	45
16	9.59 661	29	9.63 344	35	0.36 655	9.96 316	5	44
17	9.59 690	29	9.63 379	35	0.36 620	9.96 311	6	43
18	9.59 719	30	9.63 414	35	0.36 585	9.96 305	5	42
19	9.59 749	29	9.63 449	35	0.36 551	9.96 300	6	41
20	9.59 778	29	9.63 484	34	0.36 516	9.96 294	5	40
21	9.59 807	30	9.63 518	35	0.36 481	9.96 289	6	39
22	9.59 837	29	9.63 553	35	0.36 447	9.96 283	5	38
23	9.59 866	29	9.63 588	34	0.36 412	9.96 278	6	37
24	9.59 895	29	9.63 622	35	0.36 377	9.96 272	5	36
25	9.59 924	29	9.63 657	35	0.36 343	9.96 267	6	35
26	9.59 953	29	9.63 692	34	0.36 308	9.96 261	5	34
27	9.59 982	30	9.63 726	35	0.36 273	9.96 256	5	33
28	9.60 012	29	9.63 761	34	0.36 239	9.96 251	6	32
29	9.60 041	29	9.63 795	35	0.36 204	9.96 245	5	31
30	9.60 070	29	9.63 830	34	0.36 170	9.96 240	6	30
31	9.60 099	29	9.63 864	35	0.36 135	9.96 234	5	29
32	9.60 128	29	9.63 899	34	0.36 101	9.96 229	6	28
33	9.60 157	29	9.63 933	35	0.36 066	9.96 223	5	27
34	9.60 186	29	9.63 968	34	0.36 032	9.96 218	6	26
35	9.60 215	29	9.64 002	35	0.35 997	9.96 212	6	25
36	9.60 244	29	9.64 037	34	0.35 963	9.96 206	5	24
37	9.60 273	28	9.64 071	35	0.35 928	9.96 201	5	23
38	9.60 301	29	9.64 106	34	0.35 894	9.96 196	6	22
39	9.60 330	29	9.64 140	34	0.35 859	9.96 190	6	21
40	9.60 359	29	9.64 174	35	0.35 825	9.96 184	5	20
41	9.60 388	29	9.64 209	34	0.35 791	9.96 179	6	19
42	9.60 417	28	9.64 243	34	0.35 756	9.96 173	5	18
43	9.60 445	29	9.64 277	35	0.35 722	9.96 168	6	17
44	9.60 474	29	9.64 312	34	0.35 688	9.96 162	5	16
45	9.60 503	29	9.64 346	34	0.35 653	9.96 157	6	15
46	9.60 532	28	9.64 380	35	0.35 619	9.96 151	5	14
47	9.60 560	29	9.64 415	34	0.35 585	9.96 146	6	13
48	9.60 589	29	9.64 449	34	0.35 551	9.96 140	6	12
49	9.60 618	28	9.64 483	34	0.35 517	9.96 134	5	11
50	9.60 646	29	9.64 517	34	0.35 482	9.96 129	6	10
51	9.60 675	28	9.64 551	34	0.35 448	9.96 123	5	9
52	9.60 703	29	9.64 585	35	0.35 414	9.96 118	6	8
53	9.60 732	28	9.64 620	34	0.35 380	9.96 112	6	7
54	9.60 760	29	9.64 654	34	0.35 346	9.96 106	5	6
55	9.60 789	28	9.64 688	34	0.35 312	9.96 101	6	5
56	9.60 817	29	9.64 722	34	0.35 278	9.96 095	5	4
57	9.60 846	28	9.64 756	34	0.35 244	9.96 090	6	3
58	9.60 874	29	9.64 790	34	0.35 209	9.96 084	6	2
59	9.60 903	28	9.64 824	34	0.35 175	9.96 078	5	1
60	9.60 931		9.64 858		0.35 141	9.96 073		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	'

P. P.

	35	34	30	29	28	6	5
6	3.5	3.4	3.0	2.9	2.8	0.6	0.5
7	4.1	4.0	3.5	3.4	3.3	0.7	0.6
8	4.7	4.5	4.0	3.9	3.7	0.8	0.7
9	5.3	5.1	4.5	4.4	4.2	0.9	0.8
10	5.8	5.7	5.0	4.8	4.7	1.0	0.8
20	11.7	11.3	10.0	9.7	9.3	2.0	1.7
30	17.5	17.0	15.0	14.5	14.0	3.0	2.5
40	23.3	22.7	20.0	19.3	18.7	4.0	3.3
50	29.2	28.3	25.0	24.2	23.3	5.0	4.2

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.69 897	22	9.76 144	29	0.23 856	9.93 753	7	60
1	9.69 919	22	9.76 173	29	0.23 827	9.93 746	8	59
2	9.69 941	22	9.76 202	29	0.23 798	9.93 738	7	58
3	9.69 963	21	9.76 231	30	0.23 769	9.93 731	7	57
4	9.69 984	22	9.76 261	29	0.23 739	9.93 724	7	56
5	9.70 006	22	9.76 290	29	0.23 710	9.93 717	8	55
6	9.70 028	22	9.76 319	29	0.23 681	9.93 709	7	54
7	9.70 050	22	9.76 348	29	0.23 652	9.93 702	7	53
8	9.70 072	21	9.76 377	29	0.23 623	9.93 695	8	52
9	9.70 093	22	9.76 406	29	0.23 594	9.93 687	7	51
10	9.70 115	22	9.76 435	29	0.23 565	9.93 680	7	50
11	9.70 137	22	9.76 464	29	0.23 536	9.93 673	8	49
12	9.70 159	21	9.76 493	29	0.23 507	9.93 665	7	48
13	9.70 180	22	9.76 522	29	0.23 478	9.93 658	8	47
14	9.70 202	22	9.76 551	29	0.23 449	9.93 650	7	46
15	9.70 224	21	9.76 580	30	0.23 420	9.93 643	7	45
16	9.70 245	22	9.76 610	29	0.23 390	9.93 636	8	44
17	9.70 267	21	9.76 639	29	0.23 361	9.93 628	7	43
18	9.70 288	22	9.76 668	29	0.23 332	9.93 621	7	42
19	9.70 310	22	9.76 697	29	0.23 303	9.93 614	8	41
20	9.70 332	21	9.76 726	29	0.23 274	9.93 606	7	40
21	9.70 353	22	9.76 755	29	0.23 245	9.93 599	8	39
22	9.70 375	21	9.76 784	29	0.23 216	9.93 591	7	38
23	9.70 396	22	9.76 813	28	0.23 187	9.93 584	7	37
24	9.70 418	21	9.76 841	29	0.23 159	9.93 577	8	36
25	9.70 439	22	9.76 870	29	0.23 130	9.93 569	7	35
26	9.70 461	21	9.76 899	29	0.23 101	9.93 562	8	34
27	9.70 482	22	9.76 928	29	0.23 072	9.93 554	7	33
28	9.70 504	21	9.76 957	29	0.23 043	9.93 547	8	32
29	9.70 525	22	9.76 986	29	0.23 014	9.93 539	7	31
30	9.70 547	21	9.77 015	29	0.22 985	9.93 532	7	30
31	9.70 568	22	9.77 044	29	0.22 956	9.93 525	8	29
32	9.70 590	21	9.77 073	28	0.22 927	9.93 517	7	28
33	9.70 611	22	9.77 101	29	0.22 899	9.93 510	8	27
34	9.70 633	21	9.77 130	29	0.22 870	9.93 502	7	26
35	9.70 654	21	9.77 159	29	0.22 841	9.93 495	8	25
36	9.70 675	22	9.77 188	29	0.22 812	9.93 487	7	24
37	9.70 697	21	9.77 217	29	0.22 783	9.93 480	8	23
38	9.70 718	21	9.77 246	29	0.22 754	9.93 472	7	22
39	9.70 739	22	9.77 275	28	0.22 725	9.93 465	8	21
40	9.70 761	21	9.77 303	29	0.22 697	9.93 457	7	20
41	9.70 782	21	9.77 332	29	0.22 668	9.93 450	8	19
42	9.70 803	21	9.77 361	29	0.22 639	9.93 442	7	18
43	9.70 824	22	9.77 390	29	0.22 610	9.93 435	8	17
44	9.70 846	21	9.77 419	28	0.22 581	9.93 427	7	16
45	9.70 867	21	9.77 447	29	0.22 553	9.93 420	8	15
46	9.70 888	21	9.77 476	29	0.22 524	9.93 412	7	14
47	9.70 909	22	9.77 505	29	0.22 495	9.93 405	8	13
48	9.70 931	21	9.77 534	28	0.22 466	9.93 397	7	12
49	9.70 952	21	9.77 562	29	0.22 438	9.93 390	8	11
50	9.70 973	21	9.77 591	29	0.22 409	9.93 382	7	10
51	9.70 994	21	9.77 620	28	0.22 380	9.93 375	8	9
52	9.71 015	21	9.77 648	29	0.22 352	9.93 367	7	8
53	9.71 036	22	9.77 677	29	0.22 323	9.93 360	8	7
54	9.71 058	21	9.77 706	28	0.22 294	9.93 352	8	6
55	9.71 079	21	9.77 734	29	0.22 266	9.93 344	7	5
56	9.71 100	21	9.77 763	29	0.22 237	9.93 337	8	4
57	9.71 121	21	9.77 792	28	0.22 208	9.93 329	7	3
58	9.71 142	21	9.77 820	29	0.22 180	9.93 322	8	2
59	9.71 163	21	9.77 849	28	0.22 151	9.93 314	7	1
60	9.71 184		9.77 877		0.22 123	9.93 307		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	30	29
6	3.0	2.9
7	3.5	3.4
8	4.0	3.9
9	4.5	4.4
10	5.0	4.8
20	10.0	9.7
30	15.0	14.5
40	20.0	19.3
50	25.0	24.2

	21
6	2.1
7	2.5
8	2.8
9	3.2
10	3.5
20	7.0
30	10.5
40	14.0
50	17.5

'	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.73 611	19	9.81 251	28	0.18 748	9.92 359	8	60
1	9.73 630	19	9.81 279	27	0.18 720	9.92 351	8	59
2	9.73 650	19	9.81 307	27	0.18 693	9.92 342	8	58
3	9.73 669	19	9.81 334	28	0.18 665	9.92 334	8	57
4	9.73 688	19	9.81 362	27	0.18 637	9.92 326	8	56
5	9.73 708	19	9.81 390	27	0.18 610	9.92 318	8	55
6	9.73 727	19	9.81 417	27	0.18 582	9.92 310	8	54
7	9.73 746	19	9.81 445	28	0.18 555	9.92 301	8	53
8	9.73 766	19	9.81 473	27	0.18 527	9.92 293	8	52
9	9.73 785	19	9.81 500	27	0.18 499	9.92 285	8	51
10	9.73 805	19	9.81 528	27	0.18 472	9.92 277	8	50
11	9.73 824	19	9.81 555	27	0.18 444	9.92 268	8	49
12	9.73 843	19	9.81 583	27	0.18 417	9.92 260	8	48
13	9.73 862	19	9.81 610	27	0.18 389	9.92 252	8	47
14	9.73 882	19	9.81 638	28	0.18 362	9.92 244	8	46
15	9.73 901	19	9.81 666	27	0.18 334	9.92 235	8	45
16	9.73 920	19	9.81 693	27	0.18 306	9.92 227	8	44
17	9.73 940	19	9.81 721	27	0.18 279	9.92 219	8	43
18	9.73 959	19	9.81 748	27	0.18 251	9.92 210	8	42
19	9.73 978	19	9.81 776	27	0.18 224	9.92 202	8	41
20	9.73 997	19	9.81 803	27	0.18 196	9.92 194	8	40
21	9.74 016	19	9.81 831	27	0.18 169	9.92 185	8	39
22	9.74 036	19	9.81 858	27	0.18 141	9.92 177	8	38
23	9.74 055	19	9.81 886	27	0.18 114	9.92 169	8	37
24	9.74 074	19	9.81 913	27	0.18 086	9.92 160	8	36
25	9.74 093	19	9.81 941	27	0.18 059	9.92 152	8	35
26	9.74 112	19	9.81 968	27	0.18 031	9.92 144	8	34
27	9.74 131	19	9.81 996	27	0.18 004	9.92 135	8	33
28	9.74 151	19	9.82 023	27	0.17 976	9.92 127	8	32
29	9.74 170	19	9.82 051	27	0.17 949	9.92 119	8	31
30	9.74 189	19	9.82 078	27	0.17 921	9.92 110	8	30
31	9.74 208	19	9.82 105	27	0.17 894	9.92 102	8	29
32	9.74 227	19	9.82 133	27	0.17 867	9.92 094	8	28
33	9.74 246	19	9.82 160	27	0.17 839	9.92 085	8	27
34	9.74 265	19	9.82 188	27	0.17 812	9.92 077	8	26
35	9.74 284	19	9.82 215	27	0.17 784	9.92 069	8	25
36	9.74 303	19	9.82 243	27	0.17 757	9.92 060	8	24
37	9.74 322	19	9.82 270	27	0.17 729	9.92 052	8	23
38	9.74 341	18	9.82 297	27	0.17 702	9.92 043	8	22
39	9.74 360	19	9.82 325	27	0.17 675	9.92 035	8	21
40	9.74 379	19	9.82 352	27	0.17 647	9.92 027	8	20
41	9.74 398	19	9.82 380	27	0.17 620	9.92 018	8	19
42	9.74 417	19	9.82 407	27	0.17 593	9.92 010	8	18
43	9.74 436	19	9.82 434	27	0.17 565	9.92 001	8	17
44	9.74 455	19	9.82 462	27	0.17 538	9.91 993	8	16
45	9.74 474	19	9.82 489	27	0.17 510	9.91 984	8	15
46	9.74 493	18	9.82 516	27	0.17 483	9.91 976	8	14
47	9.74 511	19	9.82 544	27	0.17 456	9.91 967	8	13
48	9.74 530	19	9.82 571	27	0.17 428	9.91 959	8	12
49	9.74 549	19	9.82 598	27	0.17 401	9.91 951	8	11
50	9.74 568	18	9.82 626	27	0.17 374	9.91 942	8	10
51	9.74 587	19	9.82 653	27	0.17 347	9.91 934	8	9
52	9.74 606	19	9.82 680	27	0.17 319	9.91 925	8	8
53	9.74 625	18	9.82 708	27	0.17 292	9.91 917	8	7
54	9.74 643	19	9.82 735	27	0.17 265	9.91 908	8	6
55	9.74 662	18	9.82 762	27	0.17 237	9.91 900	8	5
56	9.74 681	19	9.82 789	27	0.17 210	9.91 891	8	4
57	9.74 700	18	9.82 817	27	0.17 183	9.91 883	8	3
58	9.74 718	19	9.82 844	27	0.17 156	9.91 874	8	2
59	9.74 737	18	9.82 871	27	0.17 128	9.91 866	8	1
60	9.74 756		9.82 898		0.17 101	9.91 857		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	'

P. P.

	28	27	27
6	2.8	2.7	2.7
7	3.2	3.2	3.1
8	3.7	3.6	3.6
9	4.2	4.1	4.0
10	4.6	4.6	4.5
20	9.3	9.1	9.0
30	14.0	13.7	13.5
40	18.6	18.3	18.0
50	23.3	22.9	22.5

	19	19	18
6	1.9	1.9	1.8
7	2.3	2.2	2.1
8	2.6	2.5	2.4
9	2.9	2.8	2.8
10	3.2	3.1	3.1
20	6.5	6.3	6.1
30	9.7	9.5	9.2
40	13.0	12.6	12.3
50	16.2	15.8	15.4

	8	8
6	0.8	0.8
7	1.0	0.9
8	1.1	1.0
9	1.3	1.2
10	1.4	1.3
20	2.8	2.6
30	4.2	4.0
40	5.6	5.3
50	7.1	6.6

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.77 946	16	9.87 711	26	0.12 288	9.90 235	9	60
1	9.77 963	17	9.87 737	26	0.12 262	9.90 225	9	59
2	9.77 980	16	9.87 764	26	0.12 236	9.90 216	9	58
3	9.77 996	17	9.87 790	26	0.12 209	9.90 206	10	57
4	9.78 013	16	9.87 816	26	0.12 183	9.90 196	9	56
5	9.78 030	16	9.87 843	26	0.12 157	9.90 187	9	55
6	9.78 046	17	9.87 869	26	0.12 131	9.90 177	9	54
7	9.78 063	16	9.87 895	26	0.12 104	9.90 168	9	53
8	9.78 080	17	9.87 921	26	0.12 078	9.90 158	9	52
9	9.78 097	16	9.87 948	26	0.12 052	9.90 149	9	51
10	9.78 113	16	9.87 974	26	0.12 026	9.90 139	9	50
11	9.78 130	17	9.88 000	26	0.11 999	9.90 130	10	49
12	9.78 147	16	9.88 026	26	0.11 973	9.90 120	9	48
13	9.78 163	16	9.88 053	26	0.11 947	9.90 110	9	47
14	9.78 180	16	9.88 079	26	0.11 921	9.90 101	9	46
15	9.78 196	16	9.88 105	26	0.11 895	9.90 091	9	45
16	9.78 213	17	9.88 131	26	0.11 868	9.90 082	10	44
17	9.78 230	16	9.88 157	26	0.11 842	9.90 072	9	43
18	9.78 246	16	9.88 184	26	0.11 816	9.90 062	9	42
19	9.78 263	16	9.88 210	26	0.11 790	9.90 053	9	41
20	9.78 279	16	9.88 236	26	0.11 763	9.90 043	10	40
21	9.78 296	16	9.88 262	26	0.11 737	9.90 033	9	39
22	9.78 312	16	9.88 288	26	0.11 711	9.90 024	9	38
23	9.78 329	17	9.88 315	26	0.11 685	9.90 014	10	37
24	9.78 346	16	9.88 341	26	0.11 659	9.90 004	9	36
25	9.78 362	16	9.88 367	26	0.11 633	9.89 995	9	35
26	9.78 379	16	9.88 393	26	0.11 606	9.89 985	10	34
27	9.78 395	16	9.88 419	26	0.11 580	9.89 975	9	33
28	9.78 412	16	9.88 445	26	0.11 554	9.89 966	9	32
29	9.78 428	16	9.88 472	26	0.11 528	9.89 956	10	31
30	9.78 444	16	9.88 498	26	0.11 502	9.89 946	9	30
31	9.78 461	16	9.88 524	26	0.11 476	9.89 937	9	29
32	9.78 477	16	9.88 550	26	0.11 449	9.89 927	10	28
33	9.78 494	16	9.88 576	26	0.11 423	9.89 917	9	27
34	9.78 510	16	9.88 602	26	0.11 397	9.89 908	10	26
35	9.78 527	16	9.88 629	26	0.11 371	9.89 898	9	25
36	9.78 543	16	9.88 655	26	0.11 345	9.89 888	10	24
37	9.78 559	16	9.88 681	26	0.11 319	9.89 878	9	23
38	9.78 576	16	9.88 707	26	0.11 293	9.89 869	10	22
39	9.78 592	16	9.88 733	26	0.11 266	9.89 859	9	21
40	9.78 609	16	9.88 759	26	0.11 240	9.89 849	10	20
41	9.78 625	16	9.88 785	26	0.11 214	9.89 839	9	19
42	9.78 641	16	9.88 811	26	0.11 188	9.89 830	10	18
43	9.78 658	16	9.88 838	26	0.11 162	9.89 820	9	17
44	9.78 674	16	9.88 864	26	0.11 136	9.89 810	10	16
45	9.78 690	16	9.88 890	26	0.11 110	9.89 800	9	15
46	9.78 707	16	9.88 916	26	0.11 084	9.89 791	10	14
47	9.78 723	16	9.88 942	26	0.11 058	9.89 781	10	13
48	9.78 739	16	9.88 968	26	0.11 032	9.89 771	9	12
49	9.78 755	16	9.88 994	26	0.11 005	9.89 761	10	11
50	9.78 772	16	9.89 020	26	0.10 979	9.89 751	9	10
51	9.78 788	16	9.89 046	26	0.10 953	9.89 742	10	9
52	9.78 804	16	9.89 072	26	0.10 927	9.89 732	10	8
53	9.78 821	16	9.89 098	26	0.10 901	9.89 722	9	7
54	9.78 837	16	9.89 124	26	0.10 875	9.89 712	10	6
55	9.78 853	16	9.89 150	26	0.10 849	9.89 702	10	5
56	9.78 869	16	9.89 177	26	0.10 823	9.89 692	9	4
57	9.78 885	16	9.89 203	26	0.10 797	9.89 683	10	3
58	9.78 902	16	9.89 229	26	0.10 771	9.89 673	10	2
59	9.78 918	16	9.89 255	26	0.10 745	9.89 663	10	1
60	9.78 934		9.89 281		0.10 719	9.89 653		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	26	26
6	2.6	2.6
7	3.1	3.0
8	3.5	3.4
9	4.0	3.9
10	4.4	4.3
20	8.8	8.6
30	13.2	13.0
40	17.6	17.3
50	22.1	21.6

	17	16	16
6	1.7	1.6	1.6
7	2.0	1.9	1.8
8	2.2	2.2	2.1
9	2.5	2.5	2.4
10	2.8	2.7	2.6
20	5.6	5.5	5.3
30	8.5	8.2	8.0
40	11.3	11.0	10.6
50	14.1	13.7	13.3

	10	9
6	1.0	0.9
7	1.1	1.1
8	1.3	1.2
9	1.5	1.4
10	1.6	1.6
20	3.3	3.1
30	5.0	4.7
40	6.6	6.3
50	8.3	7.9

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.79 887		9.90 837		0.09 163	9.89 050		60
1	9.79 903	16	9.90 863	26	0.09 137	9.89 040	10	59
2	9.79 918	15	9.90 889	26	0.09 111	9.89 030	10	58
3	9.79 934	16	9.90 915	26	0.09 085	9.89 019	11	57
4	9.79 949	15	9.90 940	25	0.09 060	9.89 009	10	56
5	9.79 965	16	9.90 966	26	0.09 034	9.88 999	10	55
6	9.79 980	15	9.90 992	26	0.09 008	9.88 989	10	54
7	9.79 996	16	9.91 018	26	0.08 982	9.88 978	11	53
8	9.80 011	15	9.91 043	25	0.08 957	9.88 968	10	52
9	9.80 027	16	9.91 069	26	0.08 931	9.88 958	10	51
10	9.80 042	15	9.91 095	26	0.08 905	9.88 947	11	50
11	9.80 058	16	9.91 121	26	0.08 879	9.88 937	10	49
12	9.80 073	15	9.91 147	26	0.08 853	9.88 927	10	48
13	9.80 089	16	9.91 172	25	0.08 828	9.88 917	10	47
14	9.80 104	15	9.91 198	26	0.08 802	9.88 906	11	46
15	9.80 120	16	9.91 224	26	0.08 776	9.88 896	10	45
16	9.80 135	15	9.91 250	26	0.08 750	9.88 886	10	44
17	9.80 151	16	9.91 276	26	0.08 724	9.88 875	11	43
18	9.80 166	15	9.91 301	25	0.08 699	9.88 865	10	42
19	9.80 182	16	9.91 327	26	0.08 673	9.88 855	10	41
20	9.80 197	15	9.91 353	26	0.08 647	9.88 844	11	40
21	9.80 213	16	9.91 379	26	0.08 621	9.88 834	10	39
22	9.80 228	15	9.91 404	25	0.08 596	9.88 823	11	38
23	9.80 243	15	9.91 430	26	0.08 570	9.88 813	10	37
24	9.80 259	16	9.91 456	26	0.08 544	9.88 803	10	36
25	9.80 274	15	9.91 482	26	0.08 518	9.88 792	11	35
26	9.80 289	15	9.91 507	25	0.08 493	9.88 782	10	34
27	9.80 305	16	9.91 533	26	0.08 467	9.88 772	10	33
28	9.80 320	15	9.91 559	26	0.08 441	9.88 761	11	32
29	9.80 335	15	9.91 585	26	0.08 415	9.88 751	10	31
30	9.80 351	16	9.91 610	25	0.08 390	9.88 740	11	30
31	9.80 366	15	9.91 636	26	0.08 364	9.88 730	10	29
32	9.80 381	15	9.91 662	26	0.08 338	9.88 720	10	28
33	9.80 397	16	9.91 688	26	0.08 312	9.88 709	11	27
34	9.80 412	15	9.91 713	25	0.08 287	9.88 699	10	26
35	9.80 427	15	9.91 739	26	0.08 261	9.88 688	11	25
36	9.80 443	16	9.91 765	26	0.08 235	9.88 678	10	24
37	9.80 458	15	9.91 790	25	0.08 210	9.88 667	11	23
38	9.80 473	15	9.91 816	26	0.08 184	9.88 657	10	22
39	9.80 488	15	9.91 842	26	0.08 158	9.88 646	11	21
40	9.80 504	16	9.91 867	25	0.08 133	9.88 636	10	20
41	9.80 519	15	9.91 893	26	0.08 107	9.88 625	11	19
42	9.80 534	15	9.91 919	26	0.08 081	9.88 615	10	18
43	9.80 549	15	9.91 945	26	0.08 055	9.88 604	11	17
44	9.80 564	15	9.91 970	25	0.08 030	9.88 594	10	16
45	9.80 580	16	9.91 996	26	0.08 004	9.88 583	11	15
46	9.80 595	15	9.92 022	26	0.07 978	9.88 573	10	14
47	9.80 610	15	9.92 047	25	0.07 953	9.88 562	11	13
48	9.80 625	15	9.92 073	26	0.07 927	9.88 552	10	12
49	9.80 640	15	9.92 099	26	0.07 901	9.88 541	11	11
50	9.80 655	15	9.92 124	25	0.07 876	9.88 531	10	10
51	9.80 671	16	9.92 150	26	0.07 850	9.88 520	11	9
52	9.80 686	15	9.92 176	26	0.07 824	9.88 510	10	8
53	9.80 701	15	9.92 201	25	0.07 799	9.88 499	11	7
54	9.80 716	15	9.92 227	26	0.07 773	9.88 488	11	6
55	9.80 731	15	9.92 253	26	0.07 747	9.88 478	10	5
56	9.80 746	15	9.92 279	26	0.07 721	9.88 467	11	4
57	9.80 761	15	9.92 304	25	0.07 696	9.88 457	10	3
58	9.80 776	15	9.92 330	26	0.07 670	9.88 446	11	2
59	9.80 791	15	9.92 355	25	0.07 645	9.88 436	10	1
60	9.80 807	16	9.92 381	26	0.07 619	9.88 425	11	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	26	25
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

	16	15	15
6	1.6	1.5	1.5
7	1.9	1.8	1.8
8	2.1	2.0	2.0
9	2.4	2.3	2.3
10	2.7	2.5	2.5
20	5.3	5.0	5.0
30	8.0	7.5	7.5
40	10.7	10.0	10.0
50	13.3	12.5	12.5

11	10
1.1	1.0
1.3	1.2
1.5	1.3
1.7	1.5
1.8	1.7
3.7	3.3
5.5	5.0
7.3	6.7
9.2	8.3

P. P.

'	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.80 806	15	9.92 381	25	0.07 618	9.88 425	10	60
1	9.80 822	15	9.92 407	25	0.07 593	9.88 415	11	59
2	9.80 837	15	9.92 432	26	0.07 567	9.88 404	10	58
3	9.80 852	15	9.92 458	25	0.07 541	9.88 393	10	57
4	9.80 867	15	9.92 484	25	0.07 516	9.88 383	10	56
5	9.80 882	15	9.92 509	25	0.07 490	9.88 372	11	55
6	9.80 897	15	9.92 535	26	0.07 465	9.88 361	10	54
7	9.80 912	15	9.92 561	25	0.07 439	9.88 351	10	53
8	9.80 927	15	9.92 586	25	0.07 413	9.88 340	11	52
9	9.80 942	15	9.92 612	26	0.07 388	9.88 329	10	51
10	9.80 957	15	9.92 638	25	0.07 362	9.88 319	10	50
11	9.80 972	15	9.92 663	25	0.07 336	9.88 308	11	49
12	9.80 987	14	9.92 689	25	0.07 311	9.88 297	10	48
13	9.81 001	15	9.92 714	26	0.07 285	9.88 287	10	47
14	9.81 016	15	9.92 740	25	0.07 259	9.88 276	11	46
15	9.81 031	15	9.92 766	25	0.07 234	9.88 265	10	45
16	9.81 046	15	9.92 791	25	0.07 208	9.88 255	10	44
17	9.81 061	15	9.92 817	25	0.07 183	9.88 244	11	43
18	9.81 076	14	9.92 842	26	0.07 157	9.88 233	10	42
19	9.81 091	15	9.92 868	25	0.07 131	9.88 223	11	41
20	9.81 106	15	9.92 894	25	0.07 106	9.88 212	10	40
21	9.81 121	15	9.92 919	25	0.07 080	9.88 201	11	39
22	9.81 136	14	9.92 945	26	0.07 055	9.88 190	10	38
23	9.81 150	15	9.92 971	25	0.07 029	9.88 180	11	37
24	9.81 165	15	9.92 996	25	0.07 003	9.88 169	10	36
25	9.81 180	14	9.93 022	25	0.06 978	9.88 158	11	35
26	9.81 195	15	9.93 047	25	0.06 952	9.88 147	10	34
27	9.81 210	15	9.93 073	25	0.06 927	9.88 137	11	33
28	9.81 225	14	9.93 098	26	0.06 901	9.88 126	10	32
29	9.81 239	15	9.93 124	25	0.06 875	9.88 115	11	31
30	9.81 254	14	9.93 150	25	0.06 850	9.88 104	10	30
31	9.81 269	15	9.93 175	25	0.06 824	9.88 094	11	29
32	9.81 284	15	9.93 201	25	0.06 799	9.88 083	11	28
33	9.81 299	14	9.93 226	25	0.06 773	9.88 072	10	27
34	9.81 313	15	9.93 252	26	0.06 748	9.88 061	11	26
35	9.81 328	14	9.93 278	25	0.06 722	9.88 050	11	25
36	9.81 343	15	9.93 303	25	0.06 696	9.88 039	10	24
37	9.81 358	14	9.93 329	25	0.06 671	9.88 029	11	23
38	9.81 372	14	9.93 354	25	0.06 645	9.88 018	11	22
39	9.81 387	15	9.93 380	25	0.06 620	9.88 007	10	21
40	9.81 402	14	9.93 405	25	0.06 594	9.87 996	11	20
41	9.81 416	15	9.93 431	25	0.06 569	9.87 985	11	19
42	9.81 431	14	9.93 456	25	0.06 543	9.87 974	11	18
43	9.81 446	14	9.93 482	26	0.06 518	9.87 963	10	17
44	9.81 460	15	9.93 508	25	0.06 492	9.87 953	11	16
45	9.81 475	14	9.93 533	25	0.06 466	9.87 942	11	15
46	9.81 490	14	9.93 559	25	0.06 441	9.87 931	11	14
47	9.81 504	15	9.93 584	25	0.06 415	9.87 920	10	13
48	9.81 519	14	9.93 610	25	0.06 390	9.87 909	11	12
49	9.81 534	14	9.93 635	25	0.06 364	9.87 898	11	11
50	9.81 548	14	9.93 661	25	0.06 339	9.87 887	11	10
51	9.81 563	15	9.93 686	25	0.06 313	9.87 876	11	9
52	9.81 578	14	9.93 712	25	0.06 288	9.87 865	11	8
53	9.81 592	14	9.93 737	25	0.06 262	9.87 854	10	7
54	9.81 607	14	9.93 763	25	0.06 237	9.87 844	11	6
55	9.81 621	14	9.93 788	25	0.06 211	9.87 833	11	5
56	9.81 636	14	9.93 814	26	0.06 186	9.87 822	11	4
57	9.81 650	14	9.93 840	25	0.06 160	9.87 811	11	3
58	9.81 665	15	9.93 865	25	0.06 134	9.87 800	11	2
59	9.81 680	14	9.93 891	25	0.06 109	9.87 789	11	1
60	9.81 694		9.93 916		0.06 083	9.87 778		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	'

P. P.

	26	25
6	2.6	2.5
7	3.0	3.0
8	3.4	3.4
9	3.9	3.8
10	4.3	4.2
20	8.6	8.5
30	13.0	12.7
40	17.3	17.0
50	21.6	21.2

	15	15	14
6	1.5	1.5	1.4
7	1.8	1.7	1.7
8	2.0	2.0	1.9
9	2.3	2.2	2.2
10	2.6	2.5	2.4
20	5.1	5.0	4.8
30	7.7	7.5	7.2
40	10.3	10.0	9.6
50	12.9	12.5	12.1

	11	10
6	1.1	1.0
7	1.3	1.2
8	1.4	1.4
9	1.6	1.6
10	1.8	1.7
20	3.6	3.5
30	5.5	5.2
40	7.3	7.0
50	9.1	8.7

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.82 551	14	9.95 443	25	0.04 556	9.87 107	11	60
1	9.82 565	14	9.95 469	25	0.04 531	9.87 096	11	59
2	9.82 579	14	9.95 494	25	0.04 505	9.87 084	11	58
3	9.82 593	14	9.95 520	25	0.04 480	9.87 073	11	57
4	9.82 607	14	9.95 545	25	0.04 454	9.87 062	11	56
5	9.82 621	14	9.95 571	25	0.04 429	9.87 050	11	55
6	9.82 635	14	9.95 596	25	0.04 404	9.87 039	11	54
7	9.82 649	14	9.95 621	25	0.04 378	9.87 027	11	53
8	9.82 663	14	9.95 647	25	0.04 353	9.87 016	11	52
9	9.82 677	14	9.95 672	25	0.04 327	9.87 004	11	51
10	9.82 691	14	9.95 697	25	0.04 302	9.86 993	11	50
11	9.82 705	14	9.95 723	25	0.04 277	9.86 982	11	49
12	9.82 719	14	9.95 748	25	0.04 251	9.86 970	11	48
13	9.82 733	13	9.95 774	25	0.04 226	9.86 959	11	47
14	9.82 746	14	9.95 799	25	0.04 200	9.86 947	11	46
15	9.82 760	14	9.95 824	25	0.04 175	9.86 936	11	45
16	9.82 774	14	9.95 850	25	0.04 150	9.86 924	11	44
17	9.82 788	14	9.95 875	25	0.04 124	9.86 913	11	43
18	9.82 802	13	9.95 901	25	0.04 099	9.86 901	11	42
19	9.82 816	14	9.95 926	25	0.04 074	9.86 890	11	41
20	9.82 830	14	9.95 951	25	0.04 048	9.86 878	11	40
21	9.82 844	14	9.95 977	25	0.04 023	9.86 867	11	39
22	9.82 858	13	9.96 002	25	0.03 997	9.86 855	11	38
23	9.82 871	14	9.96 027	25	0.03 972	9.86 844	11	37
24	9.82 885	14	9.96 053	25	0.03 947	9.86 832	11	36
25	9.82 899	13	9.96 078	25	0.03 921	9.86 821	11	35
26	9.82 913	14	9.96 104	25	0.03 896	9.86 809	11	34
27	9.82 927	13	9.96 129	25	0.03 871	9.86 798	12	33
28	9.82 940	14	9.96 154	25	0.03 845	9.86 786	11	32
29	9.82 954	14	9.96 180	25	0.03 820	9.86 774	11	31
30	9.82 968	13	9.96 205	25	0.03 795	9.86 763	11	30
31	9.82 982	14	9.96 230	25	0.03 769	9.86 751	11	29
32	9.82 996	13	9.96 256	25	0.03 744	9.86 740	11	28
33	9.83 009	14	9.96 281	25	0.03 718	9.86 728	12	27
34	9.83 023	13	9.96 306	25	0.03 693	9.86 716	11	26
35	9.83 037	14	9.96 332	25	0.03 668	9.86 705	11	25
36	9.83 051	13	9.96 357	25	0.03 642	9.86 693	11	24
37	9.83 064	14	9.96 383	25	0.03 617	9.86 682	11	23
38	9.83 078	13	9.96 408	25	0.03 592	9.86 670	12	22
39	9.83 092	14	9.96 433	25	0.03 566	9.86 658	11	21
40	9.83 106	13	9.96 459	25	0.03 541	9.86 647	11	20
41	9.83 119	13	9.96 484	25	0.03 516	9.86 635	12	19
42	9.83 133	14	9.96 509	25	0.03 490	9.86 623	11	18
43	9.83 147	13	9.96 535	25	0.03 465	9.86 612	11	17
44	9.83 160	13	9.96 560	25	0.03 440	9.86 600	12	16
45	9.83 174	14	9.96 585	25	0.03 414	9.86 588	11	15
46	9.83 188	13	9.96 611	25	0.03 389	9.86 577	11	14
47	9.83 201	13	9.96 636	25	0.03 364	9.86 565	12	13
48	9.83 215	14	9.96 661	25	0.03 338	9.86 553	11	12
49	9.83 229	13	9.96 687	25	0.03 313	9.86 542	12	11
50	9.83 242	13	9.96 712	25	0.03 287	9.86 530	11	10
51	9.83 256	13	9.96 737	25	0.03 262	9.86 518	11	9
52	9.83 269	14	9.96 763	25	0.03 237	9.86 507	12	8
53	9.83 283	13	9.96 788	25	0.03 211	9.86 495	11	7
54	9.83 297	13	9.96 813	25	0.03 186	9.86 483	12	6
55	9.83 310	13	9.96 839	25	0.03 161	9.86 471	11	5
56	9.83 324	13	9.96 864	25	0.03 135	9.86 460	12	4
57	9.83 337	13	9.96 889	25	0.03 110	9.86 448	11	3
58	9.83 351	14	9.96 915	25	0.03 085	9.86 436	12	2
59	9.83 365	13	9.96 940	25	0.03 059	9.86 424	12	1
60	9.83 378		9.96 965		0.03 034	9.86 412		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	$\overline{25}$	25
6	2.5	2.5
7	3.0	2.9
8	3.4	3.3
9	3.8	3.7
10	4.2	4.1
20	8.5	8.3
30	12.7	12.5
40	17.0	16.6
50	21.2	20.8

	14	$\overline{13}$
6	1.4	1.3
7	1.6	1.6
8	1.8	1.8
9	2.1	2.0
10	2.3	2.2
20	4.6	4.5
30	7.0	6.7
40	9.3	9.0
50	11.6	11.2

	12	$\overline{11}$	11
6	1.2	1.1	1.1
7	1.4	1.3	1.3
8	1.6	1.5	1.4
9	1.8	1.7	1.6
10	2.0	1.9	1.8
20	4.0	3.8	3.6
30	6.0	5.7	5.5
40	8.0	7.6	7.3
50	10.0	9.6	9.1

P. P.

Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
9.83 378	13	9.96 965	25	0.03 034	9.86 412	11	60
9.83 392	13	9.96 991	25	0.03 009	9.86 401	12	59
9.83 405	13	9.97 016	25	0.02 984	9.86 389	11	58
9.83 419	13	9.97 041	25	0.02 958	9.86 377	12	57
9.83 432	13	9.97 067	25	0.02 933	9.86 365	11	56
9.83 446	13	9.97 092	25	0.02 908	9.86 354	12	55
9.83 459	13	9.97 117	25	0.02 882	9.86 342	12	54
9.83 473	13	9.97 143	25	0.02 857	9.86 330	11	53
9.83 486	13	9.97 168	25	0.02 832	9.86 318	12	52
9.83 500	13	9.97 193	25	0.02 806	9.86 306	12	51
9.83 513	13	9.97 219	25	0.02 781	9.86 294	12	50
9.83 527	13	9.97 244	25	0.02 756	9.86 282	11	49
9.83 540	13	9.97 269	25	0.02 730	9.86 271	12	48
9.83 554	13	9.97 295	25	0.02 705	9.86 259	12	47
9.83 567	13	9.97 320	25	0.02 680	9.86 247	11	46
9.83 580	13	9.97 345	25	0.02 654	9.86 235	12	45
9.83 594	13	9.97 370	25	0.02 629	9.86 223	12	44
9.83 607	13	9.97 396	25	0.02 604	9.86 211	12	43
9.83 621	13	9.97 421	25	0.02 578	9.86 199	12	42
9.83 634	13	9.97 446	25	0.02 553	9.86 187	11	41
9.83 647	13	9.97 472	25	0.02 528	9.86 176	12	40
9.83 661	13	9.97 497	25	0.02 502	9.86 164	12	39
9.83 674	13	9.97 522	25	0.02 477	9.86 152	12	38
9.83 688	13	9.97 548	25	0.02 452	9.86 140	12	37
9.83 701	13	9.97 573	25	0.02 427	9.86 128	12	36
9.83 714	13	9.97 598	25	0.02 401	9.86 116	12	35
9.83 728	13	9.97 624	25	0.02 376	9.86 104	12	34
9.83 741	13	9.97 649	25	0.02 351	9.86 092	12	33
9.83 754	13	9.97 674	25	0.02 325	9.86 080	12	32
9.83 768	13	9.97 699	25	0.02 300	9.86 068	12	31
9.83 781	13	9.97 725	25	0.02 275	9.86 056	12	30
9.83 794	13	9.97 750	25	0.02 249	9.86 044	12	29
9.83 808	13	9.97 775	25	0.02 224	9.86 032	12	28
9.83 821	13	9.97 801	25	0.02 199	9.86 020	12	27
9.83 834	13	9.97 826	25	0.02 174	9.86 008	12	26
9.83 847	13	9.97 851	25	0.02 148	9.85 996	12	25
9.83 861	13	9.97 877	25	0.02 123	9.85 984	12	24
9.83 874	13	9.97 902	25	0.02 098	9.85 972	12	23
9.83 887	13	9.97 927	25	0.02 072	9.85 960	12	22
9.83 900	13	9.97 952	25	0.02 047	9.85 948	12	21
9.83 914	13	9.97 978	25	0.02 022	9.85 936	12	20
9.83 927	13	9.98 003	25	0.01 996	9.85 924	12	19
9.83 940	13	9.98 028	25	0.01 971	9.85 912	12	18
9.83 953	13	9.98 054	25	0.01 946	9.85 900	12	17
9.83 967	13	9.98 079	25	0.01 921	9.85 887	12	16
9.83 980	13	9.98 104	25	0.01 895	9.85 875	12	15
9.83 993	13	9.98 129	25	0.01 870	9.85 863	12	14
9.84 006	13	9.98 155	25	0.01 845	9.85 851	12	13
9.84 019	13	9.98 180	25	0.01 819	9.85 839	12	12
9.84 033	13	9.98 205	25	0.01 794	9.85 827	12	11
9.84 046	13	9.98 231	25	0.01 769	9.85 815	12	10
9.84 059	13	9.98 256	25	0.01 744	9.85 803	12	9
9.84 072	13	9.98 281	25	0.01 718	9.85 791	12	8
9.84 085	13	9.98 306	25	0.01 693	9.85 778	12	7
9.84 098	13	9.98 332	25	0.01 668	9.85 766	12	6
9.84 111	13	9.98 357	25	0.01 642	9.85 754	12	5
9.84 124	13	9.98 382	25	0.01 617	9.85 742	12	4
9.84 138	13	9.98 408	25	0.01 592	9.85 730	12	3
9.84 151	13	9.98 433	25	0.01 567	9.85 718	12	2
9.84 164	13	9.98 458	25	0.01 541	9.85 705	12	1
9.84 177		9.98 483		0.01 516	9.85 693		0
Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	25	25
6	2.5	2.5
7	3.0	2.9
8	3.4	3.3
9	3.8	3.7
10	4.2	4.1
20	8.5	8.3
30	12.7	12.5
40	17.0	16.6
50	21.2	20.8

	13	13
6	1.3	1.3
7	1.6	1.5
8	1.8	1.7
9	2.0	1.9
10	2.2	2.1
20	4.5	4.3
30	6.7	6.5
40	9.0	8.6
50	11.2	10.8

	12	12	11
6	1.2	1.2	1.1
7	1.4	1.4	1.3
8	1.6	1.6	1.5
9	1.9	1.8	1.7
10	2.1	2.0	1.9
20	4.1	4.0	3.8
30	6.2	6.0	5.7
40	8.3	8.0	7.6
50	10.4	10.0	9.6

P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

′	0° Log. Vers.	D	Log. Exsec.	D	1° Log. Vers.	D	Log. Exsec.	D	′
0	— ∞		— ∞		6.18271	1436	6.18278	1436	0
1	2.62642	60206	2.62642	60206	.19707	1412	.19714	1412	1
2	3.22848	35218	3.22848	35218	.21119	1390	.21126	1390	2
3	3.58066	24987	3.58066	24987	.22509	1368	.22516	1368	3
4	3.83054	19382	3.83054	19382	.23877	1346	.23884	1347	4
5	4.02436	15836	4.02436	15836	6.25223	1326	6.25231	1326	5
6	.18272	13389	.18272	13389	.26549	1306	.26557	1307	6
7	.31662	11598	.31662	11598	.27856	1286	.27864	1287	7
8	.43260	10230	.43260	10230	.29142	1268	.29151	1268	8
9	.53490	9151	.53491	9151	.30410	1250	.30419	1250	9
10	4.62642	8278	4.62642	8279	6.31660	1232	6.31669	1232	10
11	.70920	7558	.70921	7557	.32892	1214	.32901	1216	11
12	.78478	6953	.78478	6952	.34107	1198	.34118	1198	12
13	.85431	6437	.85431	6437	.35305	1182	.35315	1182	13
14	.91868	5992	.91868	5993	.36487	1166	.36497	1166	14
15	4.97860	5606	4.97861	5605	6.37653	1150	6.37663	1151	15
16	5.03466	5266	5.03466	5266	.38803	1135	.38814	1135	16
17	.08732	4964	.08732	4964	.39938	1121	.39949	1121	17
18	.13696	4696	.13697	4696	.41059	1106	.41070	1106	18
19	.18393	4455	.18393	4456	.42165	1093	.42177	1093	19
20	5.22848	4238	5.22849	4238	6.43258	1078	6.43270	1079	20
21	.27086	4040	.27087	4040	.44337	1066	.44349	1066	21
22	.31126	3861	.31127	3861	.45403	1053	.45415	1053	22
23	.34987	3697	.34988	3697	.46455	1040	.46468	1040	23
24	.38684	3546	.38685	3546	.47496	1028	.47509	1028	24
25	5.42230	3406	5.42231	3407	6.48524	1016	6.48537	1015	25
26	.45636	3278	.45638	3278	.49539	1004	.49553	1004	26
27	.48916	3158	.48916	3159	.50544	992	.50557	993	27
28	.52073	3048	.52075	3048	.51536	981	.51550	982	28
29	.55121	2944	.55123	2945	.52518	970	.52532	970	29
30	5.58066	2848	5.58068	2848	6.53488	960	6.53503	960	30
31	.60914	2757	.60916	2758	.54448	949	.54463	950	31
32	.63672	2672	.63674	2672	.55397	939	.55413	939	32
33	.66344	2593	.66346	2594	.56336	929	.56352	929	33
34	.68937	2518	.68940	2517	.57265	919	.57281	919	34
35	5.71455	2447	5.71457	2447	6.58184	909	6.58201	909	35
36	.73902	2379	.73904	2380	.59093	900	.59110	900	36
37	.76282	2316	.76284	2316	.59993	891	.60011	891	37
38	.78598	2256	.78601	2256	.60884	882	.60902	882	38
39	.80854	2199	.80857	2199	.61760	873	.61784	873	39
40	5.83053	2145	5.83056	2146	6.62633	864	6.62657	864	40
41	.85198	2093	.85201	2093	.63503	856	.63522	856	41
42	.87291	2044	.87295	2043	.64359	847	.64378	848	42
43	.89335	1996	.89338	1997	.65206	839	.65226	839	43
44	.91332	1952	.91335	1952	.66045	831	.66065	831	44
45	5.93284	1909	5.93288	1909	6.66876	823	6.66897	823	45
46	.95193	1868	.95197	1868	.67700	815	.67720	816	46
47	.97061	1829	.97065	1829	.68515	808	.68536	808	47
48	5.98890	1790	5.98894	1791	.69323	800	.69343	800	48
49	6.00680	1755	6.00685	1755	.70124	793	.70143	794	49
50	6.02435	1720	6.02440	1720	6.70917	786	6.70937	786	50
51	.04155	1686	.04160	1687	.71703	779	.71723	779	51
52	.05842	1654	.05847	1654	.72482	772	.72505	772	52
53	.07496	1623	.07501	1623	.73254	765	.73277	765	53
54	.09120	1594	.09125	1594	.74019	758	.74043	759	54
55	6.10714	1565	6.10719	1565	6.74777	752	6.74802	752	55
56	.12279	1537	.12284	1537	.75529	746	.75554	746	56
57	.13816	1511	.13822	1511	.76275	739	.76300	739	57
58	.15327	1484	.15333	1485	.77014	733	.77040	733	58
59	.16811	1460	.16818	1460	.77747	728	.77773	727	59
60	6.18271		6.18278		6.78474		6.78500		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

	2°				3°			
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D
0	6.78474	721	6.78500	721	7.13687	481	7.13746	481
1	.79195	714	.79221	716	.14168	478	.14228	479
2	.79909	709	.79937	709	.14646	476	.14707	476
3	.80618	703	.80646	703	.15122	473	.15183	474
4	.81322	697	.81350	698	.15595	470	.15657	471
5	6.82019	692	6.82048	692	7.16066	468	7.16129	469
6	.82711	686	82740	687	.16534	466	.16598	466
7	.83398	681	.83427	682	.17000	463	.17064	464
8	.84079	676	.84109	676	.17463	460	.17528	461
9	.84755	670	.84785	671	.17923	458	.17989	459
10	6.85425	665	6.85457	666	7.18382	455	7.18448	456
11	.86091	660	.86123	660	.18837	453	.18905	454
12	.86751	655	.86783	656	.19291	451	.19359	452
13	.87407	650	.87439	651	.19742	448	.19811	449
14	.88057	646	.88090	648	.20191	446	.20260	447
15	6.88703	641	6.88737	641	7.20637	444	7.20707	445
16	.89344	636	.89378	636	.21081	442	.21152	442
17	.89980	631	.90015	632	.21523	440	.21595	440
18	.90612	627	.90647	628	.21963	437	.22035	438
	.91239	622	.91275	623	22400	435	22473	436
	6.91862	618	6.91899	618	7.22836	433	7 22908	434
	.92480	613	.92516	614	.23269	431	.23343	431
	.93093	609	.93131	610	.23700	429	.23775	429
	.93703	605	.93741	605	.24129	426	.24204	427
	.94308	601	.94348	601	.24555	424	.24632	425
	6.94909	597	6 94948	597	7.24980	422	7.25057	423
	.95506	592	95545	593	.25402	420	.25480	421
	.96099	589	.96139	589	.25823	418	.25902	419
	.96688	584	.96728	585	.26241	416	.26321	417
	.97272	581	.97313	581	.26658	414	.26738	415
	6.97853	577	6.97895	577	7.27072	412	7 27153	413
	.98430	573	.98472	574	.27485	410	.27567	411
	.99004	569	.99046	570	.27895	409	.27978	409
	6 99573	565	6 99616	566	.28304	406	.28387	407
	7 00139	562	7.00182	563	.28711	405	.28795	405
	7.00701	558	7.00745	559	7.29116	402	7.29200	404
	.01259	555	.01304	556	.29518	401	.29604	402
	.01814	551	.01860	552	.29919	399	.30006	400
	.02366	548	.02412	548	.30319	397	.30406	398
	.02914	544	02960	545	30716	395	30804	396
	7 03458	541	7 03505	541	7 31112	393	7 31201	394
	.03999	537	.04047	538	.31506	392	.31595	393
	.04537	534	.04585	535	31897	390	.31988	391
	.05071	531	.05120	531	.32288	388	.32379	389
	05603	527	05652	528	32676	386	.32768	388
	7 06130	524	7.06180	525	7 33063	385	7.33156	386
	.06655	521	.06706	522	33448	383	.33542	384
	07177	518	.07228	519	33831	382	.33926	383
	.07695	516	.07747	516	.34213	380	.34309	380
	.08211	512	.08263	513	34593	378	34689	379
	7.08723	509	7.08775	509	7 34971	377	7 35069	377
	.09232	506	.09286	507	.35348	375	.35446	376
	.09739	503	.09793	503	35723	373	.35822	374
	.10242	500	.10297	501	.36097	371	.36196	373
	.10743	497	.10798	498	36468	370	.36569	371
	7.11240	495	7.11297	495	7 36839	368	7.36940	369
	.11735	492	.11792	493	37207	367	.37310	368
	.12227	489	.12285	490	37574	366	.37678	366
	.12716	486	.12775	487	.37940	364	.38044	365
	13203	484	.13262	484	38304	362	38409	363
	7 13687		7 13746		7 38667		7 38773	
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D

8° 9°

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14° 15°

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36° 37°

38° 39°

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41°

′	42° Lg. Vers.	D	42° Log. Exs.	D	43° Lg. Vers.	D	43° Log. Exs.	D	′
0	9.40969	32	9.53861	44	9.42918	32	9.56505	43	0
1	.41001	33	.53906	44	.42950	32	.56549	44	1
2	.41034	33	.53950	44	.42982	32	.56593	44	2
3	.41067	33	.53994	44	.43014	32	.56637	43	3
4	.41100	32	.54038	44	.43046	32	.56680	44	4
5	9.41133	33	9.54083	44	9.43078	32	9.56724	44	5
6	.41166	33	.54127	44	.43110	31	.56768	43	6
7	.41199	32	.54171	44	.43142	32	.56812	44	7
8	.41231	33	.54215	44	.43174	32	.56856	43	8
9	.41264	32	.54259	44	.43206	32	.56899	44	9
10	9.41297	33	9.54304	44	9.43238	32	9.56943	44	10
11	.41330	32	.54348	44	.43270	32	.56987	43	11
12	.41362	33	.54392	44	.43302	32	.57031	44	12
13	.41395	32	.54436	44	.43334	31	.57075	43	13
14	.41428	33	.54480	44	.43365	32	.57118	44	14
15	9.41461	32	9.54525	44	9.43397	32	9.57162	43	15
16	.41493	33	.54569	44	.43429	32	.57206	44	16
17	.41526	32	.54613	44	.43461	31	.57250	43	17
18	.41559	32	.54657	44	.43493	32	.57293	44	18
19	.41591	32	.54701	44	.43525	32	.57337	43	19
20	9.41624	33	9.54745	44	9.43557	31	9.57381	43	20
21	.41657	32	.54790	44	.43588	32	.57424	44	21
22	.41689	32	.54834	44	.43620	31	.57468	43	22
23	.41722	32	.54878	44	.43652	32	.57512	44	23
24	.41754	32	.54922	44	.43684	31	.57556	43	24
25	9.41787	32	9.54966	44	9.43715	32	9.57599	43	25
26	.41819	32	.55010	44	.43747	31	.57643	44	26
27	.41852	33	.55054	44	.43779	31	.57687	43	27
28	.41885	32	.55098	44	.43810	32	.57730	43	28
29	.41917	32	.55142	44	.43842	31	.57774	44	29
30	9.41950	32	9.55186	44	9.43874	32	9.57818	43	30
31	.41982	32	.55230	44	.43906	31	.57861	43	31
32	.42014	32	.55275	44	.43937	31	.57905	44	32
33	.42047	32	.55319	44	.43969	31	.57949	43	33
34	.42079	32	.55363	44	.44000	32	.57992	43	34
35	9.42112	32	9.55407	44	9.44032	31	9.58036	43	35
36	.42144	32	.55451	44	.44064	31	.58079	44	36
37	.42177	32	.55495	44	.44095	31	.58123	43	37
38	.42209	32	.55539	44	.44127	31	.58167	43	38
39	.42241	32	.55583	44	.44158	31	.58210	43	39
40	9.42274	32	9.55627	44	9.44190	31	9.58254	43	40
41	.42306	32	.55671	44	.44221	31	.58297	44	41
42	.42338	32	.55715	44	.44253	31	.58341	43	42
43	.42371	32	.55759	44	.44284	31	.58385	43	43
44	.42403	32	.55803	44	.44316	31	.58428	43	44
45	9.42435	32	9.55847	43	9.44347	31	9.58472	43	45
46	.42467	32	.55890	44	.44379	31	.58515	43	46
47	.42500	32	.55934	44	.44410	31	.58559	43	47
48	.42532	32	.55978	44	.44442	31	.58602	43	48
49	.42564	32	.56022	44	.44473	31	.58646	43	49
50	9.42596	32	9.56066	44	9.44504	31	9.58689	43	50
51	.42629	32	.56110	44	.44536	31	.58733	43	51
52	.42661	32	.56154	43	.44567	31	.58776	44	52
53	.42693	32	.56198	44	.44599	31	.58820	43	53
54	.42725	32	.56242	44	.44630	31	.58864	43	54
55	9.42757	32	9.56286	44	9.44661	31	9.58907	43	55
56	.42789	32	.56330	44	.44693	31	.58951	43	56
57	.42822	32	.56374	43	.44724	31	.58994	43	57
58	.42854	32	.56417	44	.44755	31	.59037	43	58
59	.42886	32	.56461	43	.44787	31	.59081	43	59
60	9.42918		9.56505		9.44818		9.59124		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	44	44
6	4.4	4.4
7	5.2	5.1
8	5.9	5.8
9	6.7	6.6
10	7.4	7.3
20	14.8	14.6
30	22.2	22.0
40	29.6	29.3
50	37.1	36.6

	43	43
6	4.3	4.3
7	5.1	5.0
8	5.8	5.7
9	6.5	6.4
10	7.2	7.1
20	14.5	14.3
30	21.7	21.5
40	29.0	28.6
50	36.2	35.8

	33	32
6	3.3	3.2
7	3.8	3.8
8	4.4	4.3
9	4.9	4.9
10	5.5	5.4
20	11.0	10.8
30	16.5	16.2
40	22.0	21.6
50	27.5	27.1

	32	31
6	3.2	3.1
7	3.7	3.7
8	4.2	4.2
9	4.8	4.7
10	5.3	5.2
20	10.6	10.5
30	16.0	15.7
40	21.3	21.0
50	26.6	26.2

	31
6	3.1
7	3.6
8	4.1
9	4.6
10	5.1
20	10.3
30	15.5
40	20.6
50	25.8

P. P.

44° 45°

46°

47°

48°　　　　49°

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51°

52° 53°

54° 55°

56° 57°

58° 59°

60° 61°

62°   63°

64° 65°

66° 67°

68°	69°

70° 71°

72°　　73°

74° 75°

76°

77°

80° 81°

84° 85°

86° 87°

2° 3°

′	6° Sin.	6° Cos.	6° Tan.	6° Cot.	7° Sin.	7° Cos.	7° Tan.	7° Cot.
0	.10453	.99452	.10510	9.51436	.12187	.99255	.12278	8.14435
1	.10482	.99449	.10540	9.48781	.12216	.99251	.12308	8.12481
2	.10511	.99446	.10569	9.46141	.12245	.99248	.12338	8.10536
3	.10540	.99443	.10599	9.43515	.12274	.99244	.12367	8.08600
4	.10569	.99440	.10628	9.40904	.12302	.99240	.12397	8.06674
5	.10597	.99437	.10657	9.38307	.12331	.99237	.12426	8.04756
6	.10626	.99434	.10687	9.35724	.12360	.99233	.12456	8.02848
7	.10655	.99431	.10716	9.33155	.12389	.99230	.12485	8.00948
8	.10684	.99428	.10746	9.30599	.12418	.99226	.12515	7.99058
9	.10713	.99424	.10775	9.28058	.12447	.99222	.12544	7.97176
10	.10742	.99421	.10805	9.25530	.12476	.99219	.12574	7.95302
11	.10771	.99418	.10834	9 23016	.12504	.99215	.12603	7.93438
12	.10800	.99415	.10863	9.20516	.12533	.99211	.12633	7.91582
13	.10829	.99412	.10893	9.18028	.12562	.99208	.12662	7.89734
14	.10858	.99409	.10922	9.15554	12591	.99204	.12692	7.87895
15	.10887	.99406	.10952	9.13093	.12620	.99200	.12722	7.86064
16	.10916	.99402	.10981	9.10646	.12649	.99197	.12751	7.84242
17	.10945	.99399	.11011	9.08211	.12678	.99193	.12781	7.82428
18	.10973	.99396	.11040	9.05789	.12706	.99189	.12810	7.80622
19	.11002	.99393	.11070	9.03379	.12735	.99186	.12840	7.78825
20	.11031	.99390	.11099	9.00983	.12764	.99182	.12869	7.77035
21	.11060	.99386	.11128	8.98598	.12793	.99178	.12899	7.75254
22	.11089	.99383	.11158	8.96227	.12822	.99175	.12929	7.73480
23	.11118	.99380	.11187	8.93867	.12851	.99171	.12958	7.71715
24	.11147	.99377	.11217	8.91520	.12880	.99167	.12988	7.69957
25	.11176	.99374	.11246	8.89185	.12908	.99163	.13017	7.68208
26	.11205	.99370	.11276	8.86862	.12937	.99160	.13047	7.66466
27	.11234	.99367	.11305	8.84551	.12966	.99156	.13076	7.64732
28	.11263	.99364	.11335	8.82252	.12995	.99152	.13106	7.63005
29	.11291	.99360	.11364	8.79964	.13024	.99148	.13136	7.61287
30	.11320	.99357	.11394	8.77689	.13053	.99144	.13165	7.59575
31	.11349	.99354	.11423	8.75425	.13081	.99141	.13195	7.57872
32	.11378	.99351	.11452	8.73172	.13110	.99137	.13224	7.56176
33	.11407	.99347	.11482	8.70931	.13139	.99133	.13254	7.54487
34	.11436	.99344	.11511	8.68701	.13168	.99129	.13284	7.52806
35	.11465	.99341	.11541	8.66482	.13197	.99125	.13313	7.51132
36	.11494	.99337	.11570	8.64275	.13226	.99122	.13343	7.49465
37	.11523	.99334	.11600	8.62078	.13254	.99118	.13372	7.47806
38	.11552	.99331	.11629	8.59893	.13283	.99114	.13402	7.46154
39	.11580	.99327	.11659	8.57718	.13312	.99110	.13432	7.44509
40	.11609	.99324	.11688	8.55555	.13341	.99106	.13461	7.42871
41	.11638	.99320	.11718	8.53402	.13370	.99102	.13491	7.41240
42	.11667	.99317	.11747	8.51259	.13399	.99098	.13521	7.39616
43	.11696	.99314	.11777	8.49128	.13427	.99094	.13550	7.37999
44	.11725	.99310	.11806	8.47007	.13456	.99091	.13580	7.36389
45	.11754	.99307	.11836	8.44896	.13485	.99087	.13609	7.34786
46	.11783	.99303	.11865	8.42795	.13514	.99083	.13639	7.33190
47	.11812	.99300	.11895	8.40705	.13543	.99079	.13669	7.31600
48	.11840	.99297	.11924	8.38625	.13572	.99075	.13698	7.30018
49	.11869	.99293	.11954	8.36555	.13600	.99071	.13728	7.28442
50	.11898	.99290	.11983	8.34496	.13629	.99067	.13758	7.26873
51	.11927	.99286	.12013	8.32446	.13658	.99063	.13787	7.25310
52	.11956	.99283	.12042	8.30406	.13687	.99059	.13817	7.23754
53	.11985	.99279	.12072	8.28376	.13716	.99055	.13846	7.22204
54	.12014	.99276	.12101	8.26355	.13744	.99051	.13876	7.20661
55	.12043	.99272	.12131	8.24345	.13773	.99047	.13906	7.19125
56	.12071	.99269	.12160	8.22344	.13802	.99043	.13935	7.17594
57	.12100	.99265	.12190	8.20352	.13831	.99039	.13965	7.16071
58	.12129	.99262	.12219	8.18370	.13860	.99035	.13995	7.14553
59	.12158	.99258	.12249	8.16398	.13889	.99031	.14024	7.13042
60	.12187	.99255	.12278	8.14435	.13917	.99027	.14054	7.11537
′	83° Cos.	83° Sin.	83° Cot.	83° Tan.	82° Cos.	82° Sin.	82° Cot.	82° Tan.

81°   741   80°

10° 11°

79° 78°

	12°				13°				
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
0	.20791	.97815	.21256	4.70463	.22495	.97437	.23087	4.33148	60
1	.20820	.97809	.21286	4.69791	.22523	.97430	.23117	4.32573	59
2	.20848	.97803	.21316	4.69121	.22552	.97424	.23148	4.32001	58
3	.20877	.97797	.21347	4.68452	.22580	.97417	.23179	4.31430	57
4	.20905	.97791	.21377	4.67786	.22608	.97411	.23209	4.30860	56
5	.20933	.97784	.21408	4.67121	.22637	.97404	.23240	4.30291	55
6	.20962	.97778	.21438	4.66458	.22665	.97398	.23271	4.29724	54
7	.20990	.97772	.21469	4.65797	.22693	.97391	.23301	4.29159	53
8	.21019	.97766	.21499	4.65138	.22722	.97384	.23332	4.28595	52
9	.21047	.97760	.21529	4.64480	.22750	.97378	.23363	4.28032	51
10	.21076	.97754	.21560	4.63825	.22778	.97371	.23393	4.27471	50
11	.21104	.97748	.21590	4.63171	.22807	.97365	.23424	4.26911	49
12	.21132	.97742	.21621	4.62518	.22835	.97358	.23455	4.26352	48
13	.21161	.97735	.21651	4.61868	.22863	.97351	.23485	4.25795	47
14	.21189	.97729	.21682	4.61219	.22892	.97345	.23516	4.25239	46
15	.21218	.97723	.21712	4.60572	.22920	.97338	.23547	4.24685	45
16	.21246	.97717	.21743	4.59927	.22948	.97331	.23578	4.24132	44
17	.21275	.97711	.21773	4.59283	.22977	.97325	.23608	4.23580	43
18	.21303	.97705	.21804	4.58641	.23005	.97318	.23639	4.23030	42
19	.21331	.97698	.21834	4.58001	.23033	.97311	.23670	4.22481	41
20	.21360	.97692	.21864	4.57363	.23062	.97304	.23700	4.21933	40
21	.21388	.97686	.21895	4.56726	.23090	.97298	.23731	4.21387	39
22	.21417	.97680	.21925	4.56091	.23118	.97291	.23762	4.20842	38
23	.21445	.97673	.21956	4.55458	.23146	.97284	.23793	4.20298	37
24	.21474	.97667	.21986	4.54826	.23175	.97278	.23823	4.19756	36
25	.21502	.97661	.22017	4.54196	.23203	.97271	.23854	4.19215	35
26	.21530	.97655	.22047	4.53568	.23231	.97264	.23885	4.18675	34
27	.21559	.97648	.22078	4.52941	.23260	.97257	.23916	4.18137	33
28	.21587	.97642	.22108	4.52316	.23288	.97251	.23946	4.17600	32
29	.21616	.97636	.22139	4.51693	.23316	.97244	.23977	4.17064	31
30	.21644	.97630	.22169	4.51071	.23345	.97237	.24008	4.16530	30
31	.21672	.97623	.22200	4.50451	.23373	.97230	.24039	4.15997	29
32	.21701	.97617	.22231	4.49832	.23401	.97223	.24069	4.15465	28
33	.21729	.97611	.22261	4.49215	.23429	.97217	.24100	4.14934	27
34	.21758	.97604	.22292	4.48600	.23458	.97210	.24131	4.14405	26
35	.21786	.97598	.22322	4.47986	23486	.97203	.24162	4.13877	25
36	.21814	.97592	.22353	4.47374	.23514	.97196	.24193	4.13350	24
37	.21843	.97585	.22383	4.46764	.23542	.97189	.24223	4.12825	23
38	.21871	.97579	.22414	4.46155	.23571	.97182	.24254	4.12301	22
39	.21899	.97573	.22444	4.45548	.23599	.97176	.24285	4.11778	21
40	.21928	.97566	.22475	4.44942	.23627	.97169	.24316	4.11256	20
41	.21956	.97560	.22505	4.44338	.23656	.97162	.24347	4.10736	19
42	.21985	.97553	.22536	4.43735	.23684	.97155	.24377	4.10216	18
43	.22013	.97547	.22567	4.43134	.23712	.97148	.24408	4.09699	17
44	.22041	.97541	.22597	4.42534	.23740	.97141	.24439	4.09182	16
45	.22070	.97534	.22628	4.41936	.23769	.97134	.24470	4.08666	15
46	.22098	.97528	.22658	4.41340	.23797	.97127	.24501	4.08152	14
47	.22126	.97521	.22689	4.40745	.23825	.97120	.24532	4.07639	13
48	.22155	.97515	.22719	4.40152	.23853	.97113	.24562	4.07127	12
49	.22183	.97508	.22750	4.39560	.23882	.97106	.24593	4.06616	11
50	.22212	.97502	.22781	4.38969	.23910	.97100	.24624	4.06107	10
51	.22240	.97496	.22811	4.38381	.23938	.97093	.24655	4.05599	9
52	.22268	.97489	.22842	4.37793	.23966	.97086	.24686	4.05092	8
53	.22297	.97483	.22872	4.37207	.23995	.97079	.24717	4.04586	7
54	.22325	.97476	.22903	4.36623	.24023	.97072	.24747	4.04081	6
55	.22353	.97470	.22934	4.36040	.24051	.97065	.24778	4.03578	5
56	.22382	.97463	.22964	4.35459	.24079	.97058	.24809	4.03076	4
57	.22410	.97457	.22995	4.34879	.24108	.97051	.24840	4.02574	3
58	.22438	.97450	.23026	4.34300	.24136	.97044	.24871	4.02074	2
59	.22467	.97444	.23056	4.33723	.24164	.97037	.24902	4.01576	1
60	.22495	.97437	.23087	4.33148	.24192	.97030	.24933	4.01078	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′
	77°				76°				

14°

15°

75°

74°

16°

17°

73°

72°

	18°				19°				
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
0	.30902	.95106	.32492	3.07768	.32557	.94552	.34433	2.90421	60
1	.30929	.95097	.32524	3.07464	.32584	.94542	.34465	2.90147	59
2	.30957	.95088	.32556	3.07160	.32612	.94533	.34498	2.89873	58
3	.30985	.95079	.32588	3.06857	.32639	.94523	.34530	2.89600	57
4	.31012	.95070	.32621	3.06554	.32667	.94514	.34563	2.89327	56
5	.31040	.95061	.32653	3.06252	.32694	.94504	.34596	2.89055	55
6	.31068	.95052	.32685	3.05950	.32722	.94495	.34628	2.88783	54
7	.31095	.95043	.32717	3.05649	.32749	.94485	.34661	2.88511	53
8	.31123	.95033	.32749	3.05349	.32777	.94476	.34693	2.88240	52
9	.31151	.95024	.32782	3.05049	.32804	.94466	.34726	2.87970	51
10	.31178	.95015	.32814	3.04749	.32832	.94457	.34758	2.87700	50
11	.31206	.95006	.32846	3.04450	.32859	.94447	.34791	2.87430	49
12	.31233	.94997	.32878	3.04152	.32887	.94438	.34824	2.87161	48
13	.31261	.94988	.32911	3.03854	.32914	.94428	.34856	2.86892	47
14	.31289	.94979	.32943	3.03556	.32942	.94418	.34889	2.86624	46
15	.31316	.94970	.32975	3.03260	.32969	.94409	.34922	2.86356	45
16	.31344	.94961	.33007	3.02963	.32997	.94399	.34954	2.86089	44
17	.31372	.94952	.33040	3.02667	.33024	.94390	.34987	2.85822	43
18	.31399	.94943	.33072	3.02372	.33051	.94380	.35020	2.85555	42
19	.31427	.94933	.33104	3.02077	.33079	.94370	.35052	2.85289	41
20	.31454	.94924	.33136	3.01783	.33106	.94361	.35085	2.85023	40
21	.31482	.94915	.33169	3.01489	.33134	.94351	.35118	2.84758	39
22	.31510	.94906	.33201	3.01196	.33161	.94342	.35150	2.84494	38
23	.31537	.94897	.33233	3.00903	.33189	.94332	.35183	2.84229	37
24	.31565	.94888	.33266	3.00611	.33216	.94322	.35216	2.83965	36
25	.31593	.94878	.33298	3.00319	.33244	.94313	.35248	2.83702	35
26	.31620	.94869	.33330	3.00028	.33271	.94303	.35281	2.83439	34
27	.31648	.94860	.33363	2.99738	.33298	.94293	.35314	2.83176	33
28	.31675	.94851	.33395	2.99447	.33326	.94284	.35346	2.82914	32
29	.31703	.94842	.33427	2.99158	.33353	.94274	.35379	2.82653	31
30	.31730	.94832	.33460	2.98868	.33381	.94264	.35412	2.82391	30
31	.31758	.94823	.33492	2.98580	.33408	.94254	.35445	2.82130	29
32	.31763	.94814	.33524	2.98292	.33436	.94245	.35477	2.81870	28
33	.31813	.94805	.33557	2.98004	.33463	.94235	.35510	2.81610	27
34	.31841	.94795	.33589	2.97717	.33490	.94225	.35543	2.81350	36
35	.31868	.94786	.33621	2.97430	.33518	.94215	.35576	2.81091	25
36	.31896	.94777	.33654	2.97144	.33545	.94206	.35608	2.80833	24
37	.31923	.94768	.33686	2.96858	.33573	.94196	.35641	2.80574	23
38	.31951	.94758	.33718	2.96573	.33600	.94186	.35674	2.80316	22
39	.31979	.94749	.33751	2.96288	.33627	.94176	.35707	2.80059	21
40	.32006	.94740	.33783	2.96004	.33655	.94167	.35740	2.79802	20
41	.32034	.94730	.33816	2.95721	.33682	.94157	.35772	2.79545	19
42	.32061	.94721	.33848	2.95437	.33710	.94147	.35805	2.79289	18
43	.32089	.94712	.33881	2.95155	.33737	.94137	.35838	2.79033	17
44	.32116	.94702	.33913	2.94872	.33764	.94127	.35871	2.78778	16
45	.32144	.94693	.33945	2.94591	.33792	.94118	.35904	2.78523	15
46	.32171	.94684	.33978	2.94309	.33819	.94108	.35937	2.78269	14
47	.32199	.94674	.34010	2.94028	.33846	.94098	.35969	2.78014	13
48	.32227	.94665	.34043	2.93748	.33874	.94088	.36002	2.77761	12
49	.32254	.94656	.34075	2.93468	.33901	.94078	.36035	2.77507	11
50	.32282	.94646	.34108	2.93189	.33929	.94068	.36068	2.77254	10
51	.32309	.94637	.34140	2.92910	.33956	.94058	.36101	2.77002	9
52	.32337	.94627	.34173	2.92632	.33983	.94049	.36134	2.76750	8
53	.32364	.94618	.34205	2.92354	.34011	.94039	.36167	2.76498	7
54	.32392	.94609	.34238	2.92076	.34038	.94029	.36199	2.76247	6
55	.32419	.94599	.34270	2.91799	.34065	.94019	.36232	2.75996	5
56	.32447	.94590	.34303	2.91523	.34093	.94009	.36265	2.75746	4
57	.32474	.94580	.34335	2.91246	.34120	.93999	.36298	2.75496	3
58	.32502	.94571	.34368	2.90971	.34147	.93989	.36331	2.75246	2
59	.32529	.94561	.34400	2.90696	.34175	.93979	.36364	2.74997	1
60	.32557	.94552	.34433	2.90421	.34202	.93969	.36397	2.74748	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′
	71°				70°				

22° 23°

67° 66°

24° 25°

65° 64°

26°				27°				
Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
43837	89879	48773	2 05030	45399	89101	50953	1 96261	60
43863	89867	48809	2 04879	45425	89087	50989	1 96120	59
43889	89854	48845	2 04728	45451	89074	51026	1 95979	58
43916	89841	48881	2 04577	45477	89061	51063	1 95838	57
43942	89828	48917	2 04426	45503	89048	51099	1 95698	56
43968	89816	48953	2 04276	45529	89035	51136	1 95557	55
43994	89803	48989	2 04126	45554	89021	51173	1 95417	54
44020	89790	49026	2 03976	45580	89008	51209	1 95277	53
44046	89777	49062	2 03826	45606	88995	51246	1 95137	52
44072	89764	49098	2 03677	45632	88981	51283	1 94997	51
44098	89752	49134	2 03526	45658	88968	51319	1 94858	50
44124	89739	49170	2 03376	45684	88955	51356	1 94718	49
44151	89726	49206	2 03227	45710	88942	51393	1 94579	48
44177	89713	49242	2 03078	45736	88928	51430	1 94440	47
44203	89700	49278	2 02929	45762	88915	51467	1 94301	46
44229	89687	49315	2 02780	45787	88902	51503	1 94162	45
44255	89674	49351	2 02631	45813	88888	51540	1 94023	44
44281	89662	49387	2 02483	45839	88875	51577	1 93885	43
44307	89649	49423	2 02335	45865	88862	51614	1 93746	42
44333	89636	49459	2 02187	45891	88848	51651	1 93608	41
44359	89623	49495	2 02039	45917	88835	51688	1 93470	40
44385	89610	49532	2 01891	45942	88822	51724	1 93332	39
44411	89597	49568	2 01743	45968	88808	51761	1 93195	38
44437	89584	49604	2 01596	45994	88795	51798	1 93057	37
44464	89571	49640	2 01449	46020	88782	51835	1 92920	36
44490	89558	49677	2 01302	46046	88768	51872	1 92782	35
44516	89545	49713	2 01155	46072	88755	51909	1 92645	34
44542	89532	49749	2 01008	46097	88741	51946	1 92508	33
44568	89519	49786	2 00862	46123	88728	51983	1 92371	32
44594	89506	49822	2 00715	46149	88715	52020	1 92235	31
44620	89493	49858	2 00569	46175	88701	52057	1 92098	30
44646	89480	49894	2 00423	46201	88688	52094	1 91962	29
44672	89467	49931	2 00277	46226	88674	52131	1 91826	28
44698	89454	49967	2 00131	46252	88661	52168	1 91690	27
44724	89441	50004	1 99986	46278	88647	52205	1 91554	26
44750	89428	50040	1 99841	46304	88634	52242	1 91418	25
44776	89415	50076	1 99695	46330	88620	52279	1 91282	24
44802	89402	50113	1 99550	46355	88607	52316	1 91147	23
44828	89389	50149	1 99406	46381	88593	52353	1 91012	22
44854	89376	50185	1 99261	46407	88580	52390	1 90876	21
44880	89363	50222	1 99116	46433	88566	52427	1 90741	20
44906	89350	50258	1 98972	46458	88553	52464	1 90607	19
44932	89337	50295	1 98828	46484	88539	52501	1 90472	18
44958	89324	50331	1 98684	46510	88526	52538	1 90337	17
44984	89311	50368	1 98540	46536	88512	52575	1 90203	16
45010	89298	50404	1 98396	46561	88499	52613	1 90069	15
45036	89285	50441	1 98253	46587	88485	52650	1 89935	14
45062	89272	50477	1 98110	46613	88472	52687	1 89801	13
45088	89259	50514	1 97966	46639	88458	52724	1 89667	12
45114	89245	50550	1 97823	46664	88445	52761	1 89533	11
45140	89232	50587	1 97681	46690	88431	52798	1 89400	10
45166	89219	50623	1 97538	46716	88417	52836	1 89266	9
45192	89206	50660	1 97395	46742	88404	52873	1 89133	8
45218	89193	50696	1 97253	46767	88390	52910	1 89000	7
45243	89180	50733	1 97111	46793	88377	52947	1 88867	6
45269	89167	50769	1 96969	46819	88363	52985	1 88734	5
45295	89153	50806	1 96827	46844	88349	53022	1 88602	4
45321	89140	50843	1 96685	46870	88336	53059	1 88469	3
45347	89127	50879	1 96544	46896	88322	53096	1 88337	2
45373	89114	50916	1 96402	46921	88308	53134	1 88205	1
45399	89101	50953	1 96261	46947	88295	53171	1 88073	0
Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′
63°				62°				

	28°				29°			
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.
0	.46947	.88295	.53171	1.88073	.48481	.87462	.55431	1.80405
1	.46973	.88281	.53208	1.87941	.48506	.87448	.55469	1.80281
2	.46999	.88267	.53246	1.87809	.48532	.87434	.55507	1.80158
3	.47024	.88254	.53283	1.87677	.48557	.87420	.55545	1.80034
4	.47050	.88240	.53320	1.87546	.48583	.87406	.55583	1.79911
5	.47076	.88226	.53358	1.87415	.48608	.87391	.55621	1.79788
6	.47101	.88213	.53395	1.87283	.48634	.87377	.55659	1.79665
7	.47127	.88199	.53432	1.87152	.48659	.87363	.55697	1.79542
8	.47153	.88185	.53470	1.87021	.48684	.87349	.55736	1.79419
9	.47178	.88172	.53507	1.86891	.48710	.87335	.55774	1.79296
10	.47204	.88158	.53545	1.86760	.48735	.87321	.55812	1.79174
11	.47229	.88144	.53582	1.86630	.48761	.87306	.55850	1.79051
12	.47255	.88130	.53620	1.86499	.48786	.87292	.55888	1.78929
13	.47281	.88117	.53657	1.86369	.48811	.87278	.55926	1.78807
14	.47306	.88103	.53694	1.86239	.48837	.87264	.55964	1.78685
15	.47332	.88089	.53732	1.86109	.48862	.87250	.56003	1.78563
16	.47358	.88075	.53769	1.85979	.48888	.87235	.56041	1.78441
17	.47383	.88062	.53807	1.85850	.48913	.87221	.56079	1.78319
18	.47409	.88048	.53844	1.85720	.48938	.87207	.56117	1.78198
19	.47434	.88034	.53882	1.85591	.48964	.87193	.56156	1.78077
20	.47460	.88020	.53920	1.85462	.48989	.87178	.56194	1.77955
21	.47486	.88006	.53957	1.85333	.49014	.87164	.56232	1.77834
22	.47511	.87993	.53995	1.85204	.49040	.87150	.56270	1.77713
23	.47537	.87979	.54032	1.85075	.49065	.87136	.56309	1.77592
24	.47562	.87965	.54070	1.84946	.49090	.87121	.56347	1.77471
25	.47588	.87951	.54107	1.84818	.49116	.87107	.56385	1.77351
26	.47614	.87937	.54145	1.84689	.49141	.87093	.56424	1.77230
27	.47639	.87923	.54183	1.84561	.49166	.87079	.56462	1.77110
28	.47665	.87909	.54220	1.84433	.49192	.87064	.56501	1.76990
29	.47690	.87896	.54258	1.84305	.49217	.87050	.56539	1.76869
30	.47716	.87882	.54296	1.84177	.49242	.87036	.56577	1.76749
31	.47741	.87868	.54333	1.84049	.49268	.87021	.56616	1.76629
32	.47767	.87854	.54371	1.83922	.49293	.87007	.56654	1.76510
33	.47793	.87840	.54409	1.83794	.49318	.86993	.56693	1.76390
34	.47818	.87826	.54446	1.83667	.49344	.86978	.56731	1.76271
35	.47844	.87812	.54484	1.83540	.49369	.86964	.56769	1.76151
36	.47869	.87798	.54522	1.83413	.49394	.86949	.56808	1.76032
37	.47895	.87784	.54560	1.83286	.49419	.86935	.56846	1.75913
38	.47920	.87770	.54597	1.83159	.49445	.86921	.56885	1.75794
39	.47946	.87756	.54635	1.83033	.49470	.86906	.56923	1.75675
40	.47971	.87743	.54673	1.82906	.49495	.86892	.56962	1.75556
41	.47997	.87729	.54711	1.82780	.49521	.86878	.57000	1.75437
42	.48022	.87715	.54748	1.82654	.49546	.86863	.57039	1.75319
43	.48048	.87701	.54786	1.82528	.49571	.86849	.57078	1.75200
44	.48073	.87687	.54824	1.82402	.49596	.86834	.57116	1.75082
45	.48099	.87673	.54862	1.82276	.49622	.86820	.57155	1.74964
46	.48124	.87659	.54900	1.82150	.49647	.86805	.57193	1.74846
47	.48150	.87645	.54938	1.82025	.49672	.86791	.57232	1.74728
48	.48175	.87631	.54975	1.81899	.49697	.86777	.57271	1.74610
49	.48201	.87617	.55013	1.81774	.49723	.86762	.57309	1.74492
50	.48226	.87603	.55051	1.81649	.49748	.86748	.57348	1.74375
51	.48252	.87589	.55089	1.81524	.49773	.86733	.57386	1.[illegible]4257
52	.48277	.87575	.55127	1.81399	.49798	.86719	.57425	1.74140
53	.48303	.87561	.55165	1.81274	.49824	.86704	.57464	1.74022
54	.48328	.87546	.55203	1.81150	.49849	.86690	.57503	1.73905
55	.48354	.87532	.55241	1.81025	.49874	.86675	.57541	1.73788
56	.48379	.87518	.55279	1.80901	.49899	.86661	.57580	1.73671
57	.48405	.87504	.55317	1.80777	.49924	.86646	.57619	1.73555
58	.48430	.87490	.55355	1.80653	.49950	.86632	.57657	1.73438
59	.48456	.87476	.55393	1.80529	.49975	.86617	.57696	1.73321
60	.48481	.87462	.55431	1.80405	.50000	.86603	.57735	1.73205
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.
	61°				60°			

	30°				31°				
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
0	.50000	.86603	.57735	1.73205	.51504	.85717	.60086	1.66428	60
1	.50025	.86588	.57774	1.73089	.51529	.85702	.60126	1.66318	59
2	.50050	.86573	.57813	1.72973	.51554	.85687	.60165	1.66209	58
3	.50076	.86559	.57851	1.72857	.51579	.85672	.60205	1.66099	57
4	.50101	.86544	.57890	1.72741	.51604	.85657	.60245	1.65990	56
5	.50126	.86530	.57929	1.72625	.51628	.85642	.60284	1.65881	55
6	.50151	.86515	.57968	1.72509	.51653	.85627	.60324	1.65772	54
7	.50176	.86501	.58007	1.72393	.51678	.85612	.60364	1.65663	53
8	.50201	.86486	.58046	1.72278	.51703	.85597	.60403	1.65554	52
9	.50227	.86471	.58085	1.72163	.51728	.85582	.60443	1.65445	51
10	.50252	.86457	.58124	1.72047	.51753	.85567	.60483	1.65337	50
11	.50277	.86442	.58162	1.71932	.51778	.85551	.60522	1.65228	49
12	.50302	.86427	.58201	1.71817	.51803	.85536	.60562	1.65120	48
13	.50327	.86413	.58240	1.71702	.51828	.85521	.60602	1.65011	47
14	.50352	.86398	.58279	1.71588	.51852	.85506	.60642	1.64903	46
15	.50377	.86384	.58318	1.71473	.51877	.85491	.60681	1.64795	45
16	.50403	.86369	.58357	1.71358	.51902	.85476	.60721	1.64687	44
17	.50428	.86354	.58396	1.71244	.51927	.85461	.60761	1.64579	43
18	.50453	.86340	.58435	1.71129	.51952	.85446	.60801	1.64471	42
19	.50478	.86325	.58474	1.71015	.51977	.85431	.60841	1.64363	41
20	.50503	.86310	.58513	1.70901	.52002	.85416	.60881	1.64256	40
21	.50528	.86295	.58552	1.70787	.52026	.85401	.60921	1.64148	39
22	.50553	.86281	.58591	1.70673	.52051	.85385	.60960	1.64041	38
23	.50578	.86266	.58631	1.70560	.52076	.85370	.61000	1.63934	37
24	.50603	.86251	.58670	1.70446	.52101	.85355	.61040	1.63826	36
25	.50628	.86237	58709	1.70332	.52126	.85340	.61080	1.63719	35
26	.50654	.86222	.58748	1.70219	.52151	.85325	.61120	1.63612	34
27	.50679	.86207	.58787	1.70106	.52175	.85310	.61160	1.63505	33
28	.50704	.86192	.58826	1.69992	.52200	.85294	.61200	1.63398	32
29	.50729	.86178	.58865	1.69879	.52225	.85279	.61240	1.63292	31
30	.50754	.86163	.58905	1.69766	.52250	.85264	.61280	1.63185	30
31	.50779	.86148	.58944	1.69653	.52275	.85249	.61320	1.63079	29
32	.50804	.86133	.58983	1.69541	.52299	.85234	.61360	1.62972	28
33	.50829	.86119	.59022	1.69428	.52324	.85218	.61400	1.62866	27
34	.50854	.86104	.59061	1.69316	.52349	.85203	.61440	1.62760	26
35	.50879	.86089	.59101	1.69203	.52374	.85188	.61480	1.62654	25
36	.50904	.86074	.59140	1.69091	.52399	.85173	.61520	1.62548	24
37	.50929	.86059	.59179	1.68979	.52423	.85157	.61561	1.62442	23
38	.50954	.86045	59218	1.68866	.52448	.85142	.61601	1.62336	22
39	.50979	.86030	.59258	1.68754	.52473	.85127	.61641	1.62230	21
40	.51004	.86015	.59297	1.68643	.52498	.85112	.61681	1.62125	20
41	.51029	.86000	.59336	1.68531	.52522	.85096	.61721	1.62019	19
42	.51054	.85985	.59376	1.68419	.52547	.85081	.61761	1.61914	18
43	.51079	.85970	.59415	1.68308	.52572	.85066	.61801	1.61808	17
44	.51104	.85956	.59454	1.68196	.52597	.85051	.61842	1.61703	16
45	.51129	.85941	.59494	1.68085	.52621	.85035	.61882	1.61598	15
46	.51154	.85926	.59533	1.67974	.52646	.85020	.61922	1.61493	14
47	.51179	.85911	.59573	1.67863	.52671	.85005	.61962	1.61388	13
48	.51204	.85896	.59612	1.67752	.52696	.84989	.62003	1.61283	12
49	.51229	.85881	.59651	1.67641	.52720	.84974	.62043	1.61179	11
50	.51254	.85866	.59691	1.67530	.52745	.84959	.62083	1.61074	10
51	.51279	.85851	.59730	1.67419	.52770	.84943	.62124	1.60970	9
52	.51304	.85836	.59770	1.67309	.52794	.84928	.62164	1.60865	8
53	.51329	.85821	.59809	1.67198	.52819	.84913	.62204	1.60761	7
54	.51354	.85806	.59849	1.67088	.52844	.84897	.62245	1.60657	6
55	.51379	.85792	.59888	1.66978	.52869	.84882	.62285	1.60553	5
56	.51404	.85777	.59928	1.66867	.52893	.84866	.62325	1.60449	4
57	.51429	.85762	.59967	1.66757	.52918	.84851	.62366	1.60345	3
58	.51454	.85747	.60007	1.66647	.52943	.84836	.62406	1.60241	2
59	.51479	.85732	.60046	1.66538	.52967	.84820	.62446	1.60137	1
60	.51504	.85717	.60086	1.66428	.52992	.84805	.62487	1.60033	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′
	59°				58°				

	32°				33°			
'	Sin.	Cos	Tan.	Cot	Sin.	Cos.	Tan.	Cot.
0	.52992	.84805	.62487	1.60033	.54464	.83867	.64941	1.53986
1	.53017	.84789	.62527	1.59930	.54488	.83851	.64982	1.53888
2	.53041	.84774	.62568	1.59826	.54513	.83835	.65024	1.53791
3	.53066	.84759	.62608	1.59723	.54537	.83819	.65065	1.53693
4	.53091	.84743	.62649	1.59620	.54561	.83804	.65106	1.53595
5	.53115	.84728	.62689	1.59517	.54586	.83788	.65148	1.53497
6	.53140	.84712	.62730	1.59414	.54610	.83772	.65189	1.53400
7	.53164	.84697	.62770	1.59311	.54635	.83756	.65231	1.53302
8	.53189	.84681	.62811	1.59208	.54659	.83740	.65272	1.53205
9	.53214	.84666	.62852	1.59105	.54683	.83724	.65314	1.53107
10	.53238	.84650	.62892	1.59002	.54708	.83708	.65355	1.53010
11	.53263	.84635	.62933	1.58900	.54732	.83692	.65397	1.52913
12	.53288	.84619	.62973	1.58797	.54756	.83676	.65438	1.52816
13	.53312	.84604	.63014	1.58695	.54781	.83660	.65480	1.52719
14	.53337	.84588	.63055	1.58593	.54805	.83645	.65521	1.52622
15	.53361	.84573	.63095	1.58490	.54829	.83629	.65563	1.52525
16	.53386	.84557	.63136	1.58388	.54854	.83613	.65604	1.52429
17	.53411	.84542	.63177	1.58286	.54878	.83597	.65646	1.52332
18	.53435	.84526	.63217	1.58184	.54902	.83581	.65688	1.52235
19	.53460	.84511	.63258	1.58083	.54927	.83565	.65729	1.52139
20	.53484	.84495	.63299	1.57981	.54951	.83549	.65771	1.52043
21	.53509	.84480	.63340	1.57879	.54975	.83533	.65813	1.51946
22	.53534	.84464	.63380	1.57778	.54999	.83517	.65854	1.51850
23	.53558	.84448	.63421	1.57676	.55024	.83501	.65896	1.51754
24	.53583	.84433	.63462	1.57575	.55048	.83485	.65938	1.51658
25	.53607	.84417	.63503	1.57474	.55072	.83469	.65980	1.51562
26	.53632	.84402	.63544	1.57372	.55097	.83453	.66021	1.51466
27	.53656	.84386	.63584	1.57271	.55121	.83437	.66063	1.51370
28	.53681	.84370	.63625	1.57170	.55145	.83421	.66105	1.51275
29	.53705	.84355	.63666	1.57069	.55169	.83405	.66147	1.51179
30	.53730	.84339	.63707	1.56969	.55194	.83389	.66189	1.51084
31	.53754	.84324	.63748	1.56868	.55218	.83373	.66230	1.50988
32	.53779	.84308	.63789	1.56767	.55242	.83356	.66272	1.50893
33	.53804	.84292	.63830	1.56667	.55266	.83340	.66314	1.50797
34	.53828	.84277	.63871	1.56566	.55291	.83324	.66356	1.50702
35	.53853	.84261	.63912	1.56466	.55315	.83308	.66398	1.50607
36	.53877	.84245	.63953	1.56366	.55339	.83292	.66440	1.50512
37	.53902	.84230	.63994	1.56265	.55363	.83276	.66482	1.50417
38	.53926	.84214	.64035	1.56165	.55388	.83260	.66524	1.50322
39	.53951	.84198	.64076	1.56065	.55412	.83244	.66566	1.50228
40	.53975	.84182	.64117	1.55966	.55436	.83228	.66608	1.50133
41	.54000	.84167	.64158	1.55866	.55460	.83212	.66650	1.50038
42	.54024	.84151	.64199	1.55766	.55484	.83195	.66692	1.49944
43	.54049	.84135	.64240	1.55666	.55509	.83179	.66734	1.49849
44	.54073	.84120	.64281	1.55567	.55533	.83163	.66776	1.49755
45	.54097	.84104	.64322	1.55467	.55557	.83147	.66818	1.49661
46	.54122	.84088	.64363	1.55368	.55581	.83131	.66860	1.49566
47	.54146	.84072	.64404	1.55269	.55605	.83115	.66902	1.49472
48	.54171	.84057	.64446	1.55170	.55630	.83098	.66944	1.49378
49	.54195	.84041	.64487	1.55071	.55654	.83082	.66986	1.49284
50	.54220	.84025	.64528	1.54972	.55678	.83066	.67028	1.49190
51	.54244	.84009	.64569	1.54873	.55702	.83050	.67071	1.49097
52	.54269	.83994	.64610	1.54774	.55726	.83034	.67113	1.49003
53	.54293	.83978	.64652	1.54675	.55750	.83017	.67155	1.48909
54	.54317	.83962	.64693	1.54576	.55775	.83001	.67197	1.48816
55	.54342	.83946	.64734	1.54478	.55799	.82985	.67239	1.48722
56	.54366	.83930	.64775	1.54379	.55823	.82969	.67282	1.48629
57	.54391	.83915	.64817	1.54281	.55847	.82953	.67324	1.48536
58	.54415	.83899	.64858	1.54183	.55871	.82936	.67366	1.48442
59	.54440	.83883	.64899	1.54085	.55895	.82920	.67409	1.48349
60	.54464	.83867	.64941	1.53986	.55919	.82904	.67451	1.48256
'	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.
	57°				56°			

	34°				35°				
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
0	.55919	.82904	.67451	1.48256	.57358	.81915	.70021	1.42815	60
1	.55943	.82887	.67493	1.48163	.57381	.81899	.70064	1.42726	59
2	.55968	.82871	.67536	1.48070	.57405	.81882	.70107	1.42638	58
3	.55992	.82855	.67578	1.47977	.57429	.81865	.70151	1.42550	57
4	.56016	.82839	.67620	1.47885	.57453	.81848	.70194	1.42462	56
5	.56040	.82822	.67663	1.47792	.57477	.81832	.70238	1.42374	55
6	.56064	.82806	.67705	1.47699	.57501	.81815	.70281	1.42286	54
7	.56088	.82790	.67748	1.47607	.57524	.81798	.70325	1.42198	53
8	.56112	.82773	.67790	1.47514	.57548	.81782	.70368	1.42110	52
9	.56136	.82757	.67832	1.47422	.57572	.81765	.70412	1.42022	51
10	.56160	.82741	.67875	1.47330	.57596	.81748	.70455	1.41934	50
11	.56184	.82724	.67917	1.47238	.57619	.81731	.70499	1.41847	49
12	.56208	.82708	.67960	1.47146	.57643	.81714	.70542	1.41759	48
13	.56232	.82692	.68002	1.47053	.57667	.81698	.70586	1.41672	47
14	.56256	.82675	.68045	1.46962	.57691	.81681	.70629	1.41584	46
15	.56280	.82659	.68088	1.46870	.57715	.81664	.70673	1.41497	45
16	.56305	.82643	.68130	1.46778	.57738	.81647	.70717	1.41409	44
17	.56329	.82626	.68173	1.46686	.57762	.81631	.70760	1.41322	43
18	.56353	.82610	.68215	1.46595	.57786	.81614	.70804	1.41235	42
19	.56377	.82593	.68258	1.46503	.57810	.81597	.70848	1.41148	41
20	.56401	.82577	.68301	1.46411	.57833	.81580	.70891	1.41061	40
21	.56425	.82561	.68343	1.46320	.57857	.81563	.70935	1.40974	39
22	.56449	.82544	.68386	1.46229	.57881	.81546	.70979	1.40887	38
23	.56473	.82528	.68429	1.46137	.57904	.81530	.71023	1.40800	37
24	.56497	.82511	.68471	1.46046	.57928	.81513	.71066	1.40714	36
25	.56521	.82495	.68514	1.45955	.57952	.81496	.71110	1.40627	35
26	.56545	.82478	.68557	1.45864	.57976	.81479	.71154	1.40540	34
27	.56569	.82462	.68600	1.45773	.57999	.81462	.71198	1.40454	33
28	.56593	.82446	.68642	1.45682	.58023	.81445	.71242	1.40367	32
29	.56617	.82429	.68685	1.45592	.58047	.81428	.71285	1.40281	31
30	.56641	.82413	.68728	1.45501	.58070	.81412	.71329	1.40195	30
31	.56665	.82396	.68771	1.45410	.58094	.81395	.71373	1.40109	29
32	.56689	.82380	.68814	1.45320	.58118	.81378	.71417	1.40022	28
33	.56713	.82363	.68857	1.45229	.58141	.81361	.71461	1.39936	27
34	.56736	.82347	.68900	1.45139	.58165	.81344	.71505	1.39850	26
35	.56760	.82330	.68942	1.45049	.58189	.81327	.71549	1.39764	25
36	.56784	.82314	.68985	1.44958	.58212	.81310	.71593	1.39679	24
37	.56808	.82297	.69028	1.44868	.58236	.81293	.71637	1.39593	23
38	.56832	.82281	.69071	1.44778	.58260	.81276	.71681	1.39507	22
39	.56856	.82264	.69114	1.44688	.58283	.81259	.71725	1.39421	21
40	.56880	.82248	.69157	1.44598	.58307	.81242	.71769	1.39336	20
41	.56904	.82231	.69200	1.44508	.58330	.81225	.71813	1.39250	19
42	.56928	.82214	.69243	1.44418	.58354	.81208	.71857	1.39165	18
43	.56952	.82198	.69286	1.44329	.58378	.81191	.71901	1.39079	17
44	.56976	.82181	.69329	1.44239	.58401	.81174	.71946	1.38994	16
45	.57000	.82165	.69372	1.44149	.58425	.81157	.71990	1.38909	15
46	.57024	.82148	.69416	1.44060	.58449	.81140	.72034	1.38824	14
47	.57047	.82132	.69459	1.43970	.58472	.81123	.72078	1.38738	13
48	.57071	.82115	.69502	1.43881	.58496	.81106	.72122	1.38653	12
49	.57095	.82098	.69545	1.43792	.58519	.81089	.72167	1.38568	11
50	.57119	.82082	.69588	1.43703	.58543	.81072	.72211	1.38484	10
51	.57143	.82065	.69631	1.43614	.58567	.81055	.72255	1.38399	9
52	.57167	.82048	.69675	1.43525	.58590	.81038	.72299	1.38314	8
53	.57191	.82032	.69718	1.43436	.58614	.81021	.72344	1.38229	7
54	.57215	.82015	.69761	1.43347	.58637	.81004	.72388	1.38145	6
55	.57238	.81999	.69804	1.43258	.58661	.80987	.72432	1.38060	5
56	.57262	.81982	.69847	1.43169	.58684	.80970	.72477	1.37976	4
57	.57286	.81965	.69891	1.43080	.58708	.80953	.72521	1.37891	3
58	.57310	.81949	.69934	1.42992	.58731	.80936	.72565	1.37807	2
59	.57334	.81932	.69977	1.42903	.58755	.80919	.72610	1.37722	1
60	.57358	.81915	.70021	1.42815	.58779	.80902	.72654	1.37638	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′
	55°				54°				

	36°				37°				
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
0	.58779	.80902	.72654	1.37638	.60182	.79864	.75355	1.32704	60
1	.58802	.80885	.72699	1.37554	.60205	.79846	.75401	1.32624	59
2	.58826	.80867	.72743	1.37470	.60228	.79829	.75447	1.32544	58
3	.58849	.80850	.72788	1.37386	.60251	.79811	.75492	1.32464	57
4	.58873	.80833	.72832	1.37302	.60274	.79793	.75538	1.32384	56
5	.58896	.80816	.72877	1.37218	.60298	.79776	.75584	1.32304	55
6	.58920	.80799	.72921	1.37134	.60321	.79758	.75629	1.32224	54
7	.58943	.80782	.72966	1.37050	.60344	.79741	.75675	1.32144	53
8	.58967	.80765	.73010	1.36967	.60367	.79723	.75721	1.32064	52
9	.58990	.80748	.73055	1.36883	.60390	.79706	.75767	1.31984	51
10	.59014	.80730	.73100	1.36800	.60414	.79688	.75812	1.31904	50
11	.59037	.80713	.73144	1.36716	.60437	.79671	.75858	1.31825	49
12	.59061	.80696	.73189	1.36633	.60460	.79653	.75904	1.31745	48
13	.59084	.80679	.73234	1.36549	.60483	.79635	.75950	1.31666	47
14	.59108	.80662	.73278	1.36466	.60506	.79618	.75996	1.31586	46
15	.59131	.80644	.73323	1.36383	.60529	.79600	.76042	1.31507	45
16	.59154	.80627	.73368	1.36300	.60553	.79583	.76088	1.31427	44
17	.59178	.80610	.73413	1.36217	.60576	.79565	.76134	1.31348	43
18	.59201	.80593	.73457	1.36134	.60599	.79547	.76180	1.31269	42
19	.59225	.80576	.73502	1.36051	.60622	.79530	.76226	1.31190	41
20	.59248	.80558	.73547	1.35968	.60645	.79512	.76272	1.31110	40
21	.59272	.80541	.73592	1.35885	.60668	.79494	.76318	1.31031	39
22	.59295	.80524	.73637	1.35802	.60691	.79477	.76364	1.30952	38
23	.59318	.80507	.73681	1.35719	.60714	.79459	.76410	1.30873	37
24	.59342	.80489	.73726	1.35637	.60738	.79441	.76456	1.30795	36
25	.59365	.80472	.73771	1.35554	.60761	.79424	.76502	1.30716	35
26	.59389	.80455	.73816	1.35472	.60784	.79406	.76548	1.30637	34
27	.59412	.80438	.73861	1.35389	.60807	.79388	.76594	1.30558	33
28	.59436	.80420	.73906	1.35307	.60830	.79371	.76640	1.30480	32
29	.59459	.80403	.73951	1.35224	.60853	.79353	.76686	1.30401	31
30	.59482	.80386	.73996	1.35142	.60876	.79335	.76733	1.30323	30
31	.59506	.80368	.74041	1.35060	.60899	.79318	.76779	1.30244	29
32	.59529	.80351	.74086	1.34978	.60922	.79300	.76825	1.30166	28
33	.59552	.80334	.74131	1.34896	.60945	.79282	.76871	1.30087	27
34	.59576	.80316	.74176	1.34814	.60968	.79264	.76918	1.30009	26
35	.59599	.80299	.74221	1.34732	.60991	.79247	.76964	1.29931	25
36	.59622	.80282	.74267	1.34650	.61015	.79229	.77010	1.29853	24
37	.59646	.80264	.74312	1.34568	.61038	.79211	.77057	1.29775	23
38	.59669	.80247	.74357	1.34487	.61061	.79193	.77103	1.29696	22
39	.59693	.80230	.74402	1.34405	.61084	.79176	.77149	1.29618	21
40	.59716	.80212	.74447	1.34323	.61107	.79158	.77196	1.29541	20
41	.59739	.80195	.74492	1.34242	.61130	.79140	.77242	1.29463	19
42	.59763	.80178	.74538	1.34160	.61153	.79122	.77289	1.29385	18
43	.59786	.80160	.74583	1.34079	.61176	.79105	.77335	1.29307	17
44	.59809	.80143	.74628	1.33998	.61199	.79087	.77382	1.29229	16
45	.59832	.80125	.74674	1.33916	.61222	.79069	.77428	1.29152	15
46	.59856	.80108	.74719	1.33835	.61245	.79051	.77475	1.29074	14
47	.59879	.80091	.74764	1.33754	.61268	.79033	.77521	1.28997	13
48	.59902	.80073	.74810	1.33673	.61291	.79016	.77568	1.28919	12
49	.59926	.80056	.74855	1.33592	.61314	.78998	.77615	1.28842	11
50	.59949	.80038	.74900	1.33511	.61337	.78980	.77661	1.28764	10
51	.59972	.80021	.74946	1.33430	.61360	.78962	.77708	1.28687	9
52	.59995	.80003	.74991	1.33349	.61383	.78944	.77754	1.28610	8
53	.60019	.79986	.75037	1.33268	.61406	.78926	.77801	1.28533	7
54	.60042	.79968	.75082	1.33187	.61429	.78908	.77848	1.28456	6
55	.60065	.79951	.75128	1.33107	.61451	.78891	.77895	1.28379	5
56	.60089	.79934	.75173	1.33026	.61474	.78873	.77941	1.28302	4
57	.60112	.79916	.75219	1.32946	.61497	.78855	.77988	1.28225	3
58	.60135	.79899	.75264	1.32865	.61520	.78837	.78035	1.28148	2
59	.60158	.79881	.75310	1.32785	.61543	.78819	.78082	1.28071	1
60	.60182	.79864	.75355	1.32704	.61566	.78801	.78129	1.27994	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′
	53°				52°				

TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.

′	38°				39°				′
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
0	61566	78801	78129	1 27994	62932	77715	80978	1 23490	60
1	61589	78783	78175	1 27917	62955	77696	81027	1 23416	59
2	61612	78765	78222	1 27841	62977	77678	81075	1 23343	58
3	61635	78747	78269	1 27764	63000	77660	81123	1 23270	57
4	61658	78729	78316	1 27688	63022	77641	81171	1 23196	56
5	61681	78711	78363	1 27611	63045	77623	81220	1 23123	55
6	61704	78694	78410	1 27535	63068	77605	81268	1 23050	54
7	61726	78676	78457	1 27458	63090	77586	81316	1 22977	53
8	61749	78658	78504	1 27382	63113	77568	81364	1 22904	52
9	61772	78640	78551	1 27306	63135	77550	81413	1 22831	51
10	61795	78622	78598	1 27230	63158	77531	81461	1 22758	50
11	61818	78604	78645	1 27153	63180	77513	81510	1 22685	49
12	61841	78586	78692	1 27077	63203	77494	81558	1 22612	48
13	61864	78568	78739	1 27001	63225	77476	81606	1 22539	47
14	61887	78550	78786	1 26925	63248	77458	81655	1 22467	46
15	61909	78532	78834	1 26849	63271	77439	81703	1 22394	45
16	61932	78514	78881	1 26774	63293	77421	81752	1 22321	44
17	61955	78496	78928	1 26698	63316	77402	81800	1 22249	43
18	61978	78478	78975	1 26622	63338	77384	81849	1 22176	42
19	62001	78460	79022	1 26546	63361	77366	81898	1 22104	41
20	62024	78442	79070	1 26471	63383	77347	81946	1 22031	40
21	62046	78424	79117	1 26395	63406	77329	81995	1 21959	39
22	62069	78405	79164	1 26319	63428	77310	82044	1 21886	38
23	62092	78387	79212	1 26244	63451	77292	82092	1 21814	37
24	62115	78369	79259	1 26169	63473	77273	82141	1 21742	36
25	62138	78351	79306	1 26093	63496	77255	82190	1 21670	35
26	62160	78333	79354	1 26018	63518	77236	82238	1 21598	34
27	62183	78315	79401	1 25943	63540	77218	82287	1 21526	33
28	62206	78297	79449	1 25867	63563	77199	82336	1 21454	32
29	62229	78279	79496	1 25792	63585	77181	82385	1 21382	31
30	62251	78261	79544	1 25717	63608	77162	82434	1 21310	30
31	62274	78243	79591	1 25642	63630	77144	82483	1 21238	29
32	62297	78225	79639	1 25567	63653	77125	82531	1 21166	28
33	62320	78206	79686	1 25492	63675	77107	82580	1 21094	27
34	62342	78188	79734	1 25417	63698	77088	82629	1 21023	26
35	62365	78170	79781	1 25343	63720	77070	82678	1 20951	25
36	62388	78152	79829	1 25268	63742	77051	82727	1 20879	24
37	62411	78134	79877	1 25193	63765	77033	82776	1 20808	23
38	62433	78116	79924	1 25118	63787	77014	82825	1 20736	22
39	62456	78098	79972	1 25044	63810	76996	82874	1 20665	21
40	62479	78079	80020	1 24969	63832	76977	82923	1 20593	20
41	62502	78061	80067	1 24895	63854	76959	82972	1 20522	19
42	62524	78043	80115	1 24820	63877	76940	83022	1 20451	18
43	62547	78025	80163	1 24746	63899	76921	83071	1 20379	17
44	62570	78007	80211	1 24672	63922	76903	83120	1 20308	16
45	62592	77988	80258	1 24597	63944	76884	83169	1 20237	15
46	62615	77970	80306	1 24523	63966	76866	83218	1 20166	14
47	62638	77952	80354	1 24449	63989	76847	83268	1 20095	13
48	62660	77934	80402	1 24375	64011	76828	83317	1 20024	12
49	62683	77916	80450	1 24301	64033	76810	83366	1 19953	11
50	62706	77897	80498	1 24227	64056	76791	83415	1 19882	10
51	62728	77879	80546	1 24153	64078	76772	83465	1 19811	9
52	62751	77861	80594	1 24079	64100	76754	83514	1 19740	8
53	62774	77843	80642	1 24005	64123	76735	83564	1 19669	7
54	62796	77824	80690	1 23931	64145	76717	83613	1 19599	6
55	62819	77806	80738	1 23858	64167	76698	83662	1 19528	5
56	62842	77788	80786	1 23784	64190	76679	83712	1 19457	4
57	62864	77769	80834	1 23710	64212	76661	83761	1 19387	3
58	62887	77751	80882	1 23637	64234	76642	83811	1 19316	2
59	62909	77733	80930	1 23563	64256	76623	83860	1 19246	1
60	62932	77715	80978	1 23490	64279	76604	83910	1 19175	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′
	51°				50°				

	40°				41°				
′	Sin.	Cos.	Tan.	Cot.	Sin.	Cos.	Tan.	Cot.	′
0	.64279	.76604	.83910	1.19175	.65606	.75471	.86929	1.15037	60
1	.64301	.76586	.83960	1.19105	.65628	.75452	.86980	1.14969	59
2	.64323	.76567	.84009	1.19035	.65650	.75433	.87031	1.14902	58
3	.64346	.76548	.84059	1.18964	.65672	.75414	.87082	1.14834	57
4	.64368	.76530	.84108	1.18894	.65694	.75395	.87133	1.14767	56
5	.64390	.76511	.84158	1.18824	.65716	.75375	.87184	1.14699	55
6	.64412	.76492	.84208	1.18754	.65738	.75356	.87236	1.14632	54
7	.64435	.76473	.84258	1.18684	.65759	.75337	.87287	1.14565	53
8	.64457	.76455	.84307	1.18614	.65781	.75318	.87338	1.14498	52
9	.64479	.76436	.84357	1.18544	.65803	.75299	.87389	1.14430	51
10	.64501	.76417	.84407	1.18474	.65825	.75280	.87441	1.14363	50
11	.64524	.76398	.84457	1.18404	.65847	.75261	.87492	1.14296	49
12	.64546	.76380	.84507	1.18334	.65869	.75241	.87543	1.14229	48
13	.64568	.76361	.84556	1.18264	.65891	.75222	.87595	1.14162	47
14	.64590	.76342	.84606	1.18194	.65913	.75203	.87646	1.14095	46
15	.64612	.76323	.84656	1.18125	.65935	.75184	.87698	1.14028	45
16	.64635	.76304	.84706	1.18055	.65956	.75165	.87749	1.13961	44
17	.64657	.76286	.84756	1.17986	.65978	.75146	.87801	1.13894	43
18	.64679	.76267	.84806	1.17916	.66000	.75126	.87852	1.13828	42
19	.64701	.76248	.84856	1.17846	.66022	.75107	.87904	1.13761	41
20	.64723	.76229	.84906	1.17777	.66044	.75088	.87955	1.13694	40
21	.64746	.76210	.84956	1.17708	.66066	.75069	.88007	1.13627	39
22	.64768	.76192	.85006	1.17638	.66088	.75050	.88059	1.13561	38
23	.64790	.76173	.85057	1.17569	.66109	.75030	.88110	1.13494	37
24	.64812	.76154	.85107	1.17500	.66131	.75011	.88162	1.13428	36
25	.64834	.76135	.85157	1.17430	.66153	.74992	.88204	1.13361	35
26	.64856	.76116	.85207	1.17361	.66175	.74973	.88265	1.13295	34
27	.64878	.76097	.85257	1.17292	.66197	.74953	.88317	1.13228	33
28	.64901	.76078	.85308	1.17223	.66218	.74934	.88369	1.13162	32
29	.64923	.76059	.85358	1.17154	.66240	.74915	.88421	1.13096	31
30	.64945	.76041	.85408	1.17085	.66262	.74896	.88473	1.13029	30
31	.64967	.76022	.85458	1.17016	.66284	.74876	.88524	1.12963	29
32	.64989	.76003	.85509	1.16947	.66306	.74857	.88576	1.12897	28
33	.65011	.75984	.85559	1.16878	.66327	.74838	.88628	1.12831	27
34	.65033	.75965	.85609	1.16809	.66349	.74818	.88680	1.12765	26
35	.65055	.75946	.85660	1.16741	.66371	.74799	.88732	1.12699	25
36	.65077	.75927	.85710	1.16672	.66393	.74780	.88784	1.12633	24
37	.65100	.75908	.85761	1.16603	.66414	.74760	.88836	1.12567	23
38	.65122	.75889	.85811	1.16535	.66436	.74741	.88888	1.12501	22
39	.65144	.75870	.85862	1.16466	.66458	.74722	.88940	1.12435	21
40	.65166	.75851	.85912	1.16398	.66480	.74703	.88992	1.12369	20
41	.65188	.75832	.85963	1.16329	.66501	.74683	.89045	1.12303	19
42	.65210	.75813	.86014	1.16261	.66523	.74664	.89097	1.12238	18
43	.65232	.75794	.86064	1.16192	.66545	.74644	.89149	1.12172	17
44	.65254	.75775	.86115	1.16124	.66566	.74625	.89201	1.12106	16
45	.65276	.75756	.86166	1.16056	.66588	.74606	.89253	1.12041	15
46	.65298	.75738	86216	1.15987	.66610	.74586	.89306	1.11975	14
47	.65320	.75719	.86267	1.15919	.66632	.74567	.89358	1.11909	13
48	.65342	.75700	.86318	1.15851	.66653	.74548	.89410	1.11844	12
49	.65364	.75680	.86368	1.15783	.66675	.74528	.89463	1.11778	11
50	.65386	.75661	.86419	1.15715	.66697	.74509	.89515	1.11713	10
51	.65408	.75642	.86470	1.15647	.66718	.74489	.89567	1.11648	9
52	.65430	.75623	.86521	1.15579	.66740	.74470	.89620	1.11582	8
53	.65452	.75604	.86572	1.15511	.66762	.74451	.89672	1.11517	7
54	.65474	.75585	.86623	1.15443	.66783	.74431	.89725	1.11452	6
55	.65496	.75566	.86674	1.15375	.66805	.74412	.89777	1.11387	5
56	.65518	.75547	.86725	1.15308	.66827	.74392	.89830	1.11321	4
57	.65540	.75528	.86776	1.15240	.66848	.74373	.89883	1.11256	3
58	.65562	.75509	.86827	1.15172	.66870	.74353	.89935	1.11191	2
59	.65584	.75490	.86878	1.15104	.66891	.74334	.89988	1.11126	1
60	.65606	.75471	.86929	1.15037	.66913	.74314	.90040	1.11061	0
′	Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	′
	49°				48°				

42° Sin.	Cos.	Tan.	Cot.	43° Sin.	Cos.	Tan.	Cot.	'
.66913	.74314	.90040	1.11061	.68200	.73135	.93252	1.07237	60
.66935	.74295	.90093	1.10996	.68221	.73116	.93306	1.07174	59
.66956	.74276	.90146	1.10931	.68242	.73096	.93360	1.07112	58
.66978	.74256	.90199	1.10867	.68264	.73076	.93415	1.07049	57
.66999	.74237	.90251	1.10802	.68285	.73056	.93469	1.06987	56
.67021	.74217	.90304	1.10737	.68306	.73036	.93524	1.06925	55
.67043	.74198	.90357	1.10672	.68327	.73016	.93578	1.06862	54
.67064	.74178	.90410	1.10607	.68349	.72996	.93633	1.06800	53
.67086	.74159	.90463	1.10543	.68370	.72976	.93688	1.06738	52
.67107	.74139	.90516	1.10478	.68391	.72957	.93742	1.06676	51
.67129	.74120	.90569	1.10414	.68412	.72937	.93797	1.06613	50
.67151	.74100	.90621	1.10349	.68434	.72917	.93852	1.06551	49
.67172	.74080	.90674	1.10285	.68455	.72897	.93906	1.06489	48
.67194	.74061	.90727	1.10220	.68476	.72877	.93961	1.06427	47
.67215	.74041	.90781	1.10156	.68497	.72857	.94016	1.06365	46
.67237	.74022	.90834	1.10091	.68518	.72837	.94071	1.06303	45
.67258	.74002	.90887	1.10027	.68539	.72817	.94125	1.06241	44
.67280	.73983	.90940	1.09963	.68561	.72797	.94180	1.06179	43
.67301	.73963	.90993	1.09899	.68582	.72777	.94235	1.06117	42
.67323	.73944	.91046	1.09834	.68603	.72757	.94290	1.06056	41
.67344	.73924	.91099	1.09770	.68624	.72737	.94345	1.05994	40
.67366	.73904	.91153	1.09706	.68645	.72717	.94400	1.05932	39
.67387	.73885	.91206	1.09642	.68666	.72697	.94455	1.05870	38
.67409	.73865	.91259	1.09578	.68688	.72677	.94510	1.05809	37
.67430	.73846	.91313	1.09514	.68709	.72657	.94565	1.05747	36
.67452	.73826	.91366	1.09450	.68730	.72637	.94620	1.05685	35
.67473	.73806	.91419	1.09386	.68751	.72617	.94676	1.05624	34
.67495	.73787	.91473	1.09322	.68772	.72597	.94731	1.05562	33
.67516	.73767	.91526	1.09258	.68793	.72577	.94786	1.05501	32
.67538	.73747	.91580	1.09195	.68814	.72557	.94841	1.05439	31
.67559	.73728	.91633	1.09131	.68835	.72537	.94896	1.05378	30
.67580	.73708	.91687	1.09067	.68857	.72517	.94952	1.05317	29
.67602	.73688	.91740	1.09003	.68878	.72497	.95007	1.05255	28
.67623	.73669	.91794	1.08940	.68899	.72477	.95062	1.05194	27
.67645	.73649	.91847	1.08876	.68920	.72457	.95118	1.05133	26
.67666	.73629	.91901	1.08813	.68941	.72437	.95173	1.05072	25
.67688	.73610	.91955	1.08749	.68962	.72417	.95229	1.05010	24
.67709	.73590	.92008	1.08686	.68983	.72397	.95284	1.04949	23
.67730	.73570	.92062	1.08622	.69004	.72377	.95340	1.04888	22
.67752	.73551	.92116	1.08559	.69025	.72357	.95395	1.04827	21
.67773	.73531	.92170	1.08496	.69046	.72337	.95451	1.04766	20
.67795	.73511	.92224	1.08432	.69067	.72317	.95506	1.04705	19
.67816	.73491	.92277	1.08369	.69088	.72297	.95562	1.04644	18
.67837	.73472	.92331	1.08306	.69109	.72277	.95618	1.04583	17
.67859	.73452	.92385	1.08243	.69130	.72257	.95673	1.04522	16
.67880	.73432	.92439	1.08179	.69151	.72236	.95729	1.04461	15
.67901	.73413	.92493	1.08116	.69172	.72216	.95785	1.04401	14
.67923	.73393	.92547	1.08053	.69193	.72196	.95841	1.04340	13
.67944	.73373	.92601	1.07990	.69214	.72176	.95897	1.04279	12
.67965	.73353	.92655	1.07927	.69235	.72156	.95952	1.04218	11
.67987	.73333	.92709	1.07864	.69256	.72136	.96008	1.04158	10
.68008	.73314	.92763	1.07801	.69277	.72116	.96064	1.04097	9
.68029	.73294	.92817	1.07738	.69298	.72095	.96120	1.04036	8
.68051	.73274	.92872	1.07676	.69319	.72075	.96176	1.03976	7
.68072	.73254	.92926	1.07613	.69340	.72055	.96232	1.03915	6
.68093	.73234	.92980	1.07550	.69361	.72035	.96288	1.03855	5
.68115	.73215	.93034	1.07487	.69382	.72015	.96344	1.03794	4
.68136	.73195	.93088	1.07425	.69403	.71995	.96400	1.03734	3
.68157	.73175	.93143	1.07362	.69424	.71974	.96457	1.03674	2
.68179	.73155	.93197	1.07299	.69445	.71954	.96513	1.03613	1
.68200	.73135	.93252	1.07237	.69466	.71934	.96569	1.03553	0
Cos.	Sin.	Cot.	Tan.	Cos.	Sin.	Cot.	Tan.	
47°				46°				

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	0° Vers.	0° Ex. sec.	1° Vers.	1° Ex. sec.	2° Vers.	2° Ex. sec.	3° Vers.	3° Ex. sec.	′
0	.00000	.00000	.00015	.00015	.00061	.00061	.00137	.00137	0
1	.00000	.00000	.00016	.00016	.00062	.00062	.00139	.00139	1
2	.00000	.00000	.00016	.00016	.00063	.00063	.00140	.00140	2
3	.00000	.00000	.00017	.00017	.00064	.00064	.00142	.00142	3
4	.00000	.00000	.00017	.00017	.00065	.00065	.00143	.00143	4
5	.00000	.00000	.00018	.00018	.00066	.00066	.00145	.00145	5
6	.00000	.00000	.00018	.00018	.00067	.00067	.00146	.00147	6
7	.00000	.00000	.00019	.00019	.00068	.00068	.00148	.00148	7
8	.00000	.00000	.00020	.00020	.00069	.00069	.00150	.00150	8
9	.00000	.00000	.00020	.00020	.00070	.00070	.00151	.00151	9
10	.00000	.00000	.00021	.00021	.00071	.00072	.00153	.00153	10
11	.00001	.00001	.00021	.00021	.00073	.00073	.00154	.00155	11
12	.00001	.00001	.00022	.00022	.00074	.00074	.00156	.00156	12
13	.00001	.00001	.00023	.00023	.00075	.00075	.00158	.00158	13
14	.00001	.00001	.00023	.00023	.00076	.00076	.00159	.00159	14
15	.00001	.00001	.00024	.00024	.00077	.00077	.00161	.00161	15
16	.00001	.00001	.00024	.00024	.00078	.00078	.00162	.00163	16
17	.00001	.00001	.00025	.00025	.00079	.00079	.00164	.00164	17
18	.00001	.00001	.00026	.00026	.00081	.00081	.00166	.00166	18
19	.00002	.00002	.00026	.00026	.00082	.00082	.00168	.00168	19
20	.00002	.00002	.00027	.00027	.00083	.00083	.00169	.00169	20
21	.00002	.00002	.00028	.00028	.00084	.00084	.00171	.00171	21
22	.00002	.00002	.00028	.00028	00085	.00085	.00173	.00173	22
23	.00002	.00002	.00029	.00029	.00087	.00087	.00174	.00175	23
24	.00002	.00002	.00030	.00030	.00088	.00088	.00176	.00176	24
25	.00003	.00003	.00031	.00031	.00089	.00089	.0017[illegible]	.00178	25
26	.00003	.00003	.00031	.00031	.00090	.00090	.00179	.00180	26
27	.00003	.00003	.00032	.00032	.00091	.00091	.00181	.00182	27
28	.00003	.00003	.00033	.00033	.00093	.00093	.00183	.00183	28
29	.00004	.00004	00034	.00034	.00094	.00094	.00185	.00185	29
30	.00004	.00004	.00034	.00034	00095	.00095	00187	.00187	30
31	.00004	.00004	.00035	.00035	.00096	.00097	00188	.00189	31
32	.00004	.00004	.00036	.00036	.00098	.00098	.00190	.00190	32
33	.00005	.00005	.00037	.00037	.00099	.00099	.00192	.00192	33
34	.00005	.00005	.00037	.00037	.00100	.00100	.00194	.00194	34
35	.00005	.00005	.00038	.00038	.00102	.00102	.00196	.00196	35
36	.00005	.00005	.00039	.00039	.00103	.00103	.00197	.00198	36
37	.00006	.00006	.00040	.00040	.00104	.00104	.00199	.00200	37
38	.00006	00006	.00041	.00041	.00106	.00106	.00201	.00201	38
39	.00006	.00006	.00041	.00041	.00107	.00107	.00203	.00203	39
40	.00007	.00007	.00042	.00042	.00108	.00108	.00205	.00205	40
41	.00007	.00007	.00043	.00043	.00110	.00110	.00207	.00207	41
42	.00007	.00007	.00044	.00044	.00111	.00111	.00208	.00209	42
43	.00008	.00008	.00045	.00045	.00112	.00113	.00210	.00211	43
44	.00008	.00008	.00046	.00046	.00114	.00114	00212	.00213	44
45	.00009	.00009	.00047	.00047	.00115	.00115	.00214	00215	45
46	00009	.00009	.00048	.00048	.00117	.00117	.00216	.0216	46
47	.00009	.00009	.00048	.00048	.00118	.00118	.00218	.00218	47
47	.00010	.00010	.00049	.00049	.00119	.00120	.00220	.00220	48
49	00010	.00010	.00050	.00050	.00121	.00121	.00222	.00222	49
50	.00011	.00011	.00051	.00051	.00122	.00122	.00224	.00224	50
51	.00011	.00011	.00052	.00052	.00124	.00124	.00226	.00226	51
52	.00011	.00011	.00053	.00053	.00125	.00125	.00228	.00228	52
53	.00012	.00012	.00054	.00054	.00127	.00127	.00230	.00230	53
54	.00012	.00012	.00055	.00055	.00128	.00128	.00232	.00232	54
55	.00013	.00013	.00056	.00056	.00130	.00130	.00234	.00234	55
56	.00013	.00013	.00057	.00057	.00131	.00131	.00236	.00236	56
57	.00014	.00014	.00058	.00058	.00133	00133	.00238	.00238	57
58	.00014	.00014	.00059	.00059	.00134	.00134	.00240	.00240	58
59	.00015	.00015	.00060	00060	.00136	.00136	.00242	.00242	59
30	.00015	.00015	.00061	.00061	.00137	.00137	.00244	.00244	60

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	4°		5°		6°		7°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	.00244	.00244	.00381	.00382	.00548	.00551	.00745	.00751	0
1	.00246	.00246	.00383	.00385	.00551	.00554	.00749	.00755	1
2	.00248	.00248	.00386	.00387	.00554	.00557	.00752	.00758	2
3	.00250	.00250	.00388	.00390	.00557	.00560	.00756	.00762	3
4	.00252	.00252	.00391	.00392	.00560	.00563	.00760	.00765	4
5	.00254	.00254	.00393	.00395	.00563	.00566	.00763	.00769	5
6	.00256	.00257	.00396	.00397	.00566	.00569	.00767	.00773	6
7	.00258	.00259	.00398	.00400	.00569	.00573	.00770	.00776	7
8	.00260	.00261	.00401	.00403	.00572	.00576	.00774	.00780	8
9	.00262	00263	.00404	.00405	.00576	.00579	.00778	.00784	9
10	.00264	.00265	.00406	.00408	.00579	.00582	.00781	.00787	10
11	.00266	.00267	.00409	.00411	.00582	.00585	.00785	.00791	11
12	.00269	.00269	.00412	.00413	.00585	.00588	.00789	.00795	12
13	.00271	.00271	.00414	.00416	00588	.00592	00792	.00799	13
14	.00273	.00274	.00417	.00419	.00591	.00595	.00796	.00802	14
15	.00275	.00276	.00420	.00421	.00594	.00598	.00800	.00806	15
16	.00277	.00278	.00422	.00424	.00598	.00601	.00803	.00810	16
17	.00279	.00280	.00425	.00427	.00601	.00604	.00807	.00813	17
18	.00281	.00282	.00428	.00429	.00604	.00608	.00811	.00817	18
19	.00284	.00284	.00430	.00432	.00607	.00611	.00814	.00821	19
20	.00286	.00287	.00433	.00435	.00610	.00614	.00818	.00825	20
21	.00288	.00289	.00436	.00438	.00614	.00617	.00822	.00828	21
22	.00290	.00291	.00438	.00440	.00617	.00621	.00825	.00832	22
23	.00293	.00293	.00441	.00443	.00620	.00624	.00829	.00836	23
24	.00295	.00296	.00444	.00446	.00623	.00627	.00833	.00840	24
25	.00297	.00298	.00447	.00449	.00626	.00630	.00837	.00844	25
26	.00299	.00300	.00449	.00451	.00630	.00634	.00840	.00848	26
27	.00301	.00302	.00452	.00454	.00633	.00637	.00844	.00851	27
28	.00304	.00305	.00455	.00457	.00636	.00640	.00848	.00855	28
29	.00306	.00307	.00458	.00460	.00640	.00644	.00852	.00859	29
30	.00308	.00309	.00460	.00463	.00643	.00647	.00856	.00863	30
31	.00311	.00312	.00463	.00465	.00646	.00650	.00859	.00867	31
32	.00313	.00314	.00466	.00468	.00649	.00654	.00863	.00871	32
33	.00315	.00316	.00469	.00471	.00653	.00657	.00867	.00875	33
34	.00317	.00318	.00472	.00474	.00656	.00660	.00871	.00878	34
35	.00320	.00321	.00474	.00477	.00659	.00664	.00875	.00882	35
36	.00322	.00323	.00477	.00480	.00663	.00667	.00878	.00886	36
37	.00324	.00326	.00480	.00482	.00666	.00671	.00882	.00890	37
38	.00327	.00328	.00483	.00485	.00669	.00674	.00886	.00894	38
39	.00329	.00330	.00486	.00488	.00673	.00677	.00890	.00898	39
40	.00332	.00333	.00489	.00491	.00676	.00681	.00894	.00902	40
41	.00334	.00335	.00492	.00494	.00680	.00684	.00898	.00906	41
42	.00336	.00337	.00494	.00497	.00683	.00688	.00902	.00910	42
43	.00339	.00340	.00497	.00500	.00686	.00691	.00906	.00914	43
44	.00341	.00342	.00500	.00503	.00690	.00695	.00909	.00918	44
45	.00343	.00345	.00503	.00506	.00693	.00698	.00913	.00922	45
46	.00346	.00347	.00506	.00509	.00607	.00701	.00917	.00926	46
47	.00348	.00350	.00509	.00512	.00700	.00705	00921	.00930	47
48	00351	.00352	.00512	.00515	.00703	.00708	00925	.00934	48
49	00353	.00354	.00515	.00518	.00707	.00712	.00929	.00938	49
50	.00356	.00357	.00518	.00521	.00710	.00715	.00933	.00942	50
51	.00358	.00359	.00521	.00524	.00714	.00719	00937	.00946	51
52	.00361	.00362	.00524	.00527	.00717	.00722	.00941	.00950	52
53	.00363	.00364	.00527	.00530	.00721	.00726	.00945	.00954	53
54	.00365	.00367	.00530	.00533	.00724	.00730	.00949	.00958	54
55	.00368	.00369	.00533	.00536	.00728	.00733	.00953	.00962	55
56	.00370	.00372	.00536	.00539	.00731	.00737	.00957	.00966	56
57	.00373	00374	.00539	.00542	.00735	.00740	.00961	.00970	57
58	.00375	00377	.00542	.00545	.00738	.00744	.00965	.00975	58
59	.00378	.00379	00545	00548	.00742	.00747	.00969	.00979	59
60	.00381	.00382	00548	.00551	.00745	.00751	.00973	.00983	60

′	8° Vers.	8° Ex. sec.	9° Vers.	9° Ex. sec.	10° Vers.	10° Ex. sec.	11° Vers.	11° Ex. sec.	′
0	.00973	.00983	.01231	.01247	.01519	.01543	.01837	.01872	0
1	.00977	.00987	.01236	.01251	.01524	.01548	.01843	.01877	1
2	.00981	.00991	.01240	.01256	.01529	.01553	.01848	.01883	2
3	.00985	.00995	.01245	.01261	.01534	.01558	.01854	.01889	3
4	.00989	.00999	.01249	.01265	.01540	.01564	.01860	.01895	4
5	.00994	.01004	.01254	.01270	.01545	.01569	.01865	.01901	5
6	.00998	.01008	.01259	.01275	.01550	.01574	.01871	.01906	6
7	.01002	.01012	.01263	.01279	.01555	.01579	.01876	.01912	7
8	.01006	.01016	.01268	.01284	.01560	.01585	.01882	.01918	8
9	.01010	.01020	.01272	.01289	.01565	.01590	.01888	.01924	9
10	.01014	.01024	.01277	.01294	.01570	.01595	.01893	.01930	10
11	.01018	.01029	.01282	.01298	.01575	.01601	.01899	.01936	11
12	.01022	.01033	.01286	.01303	.01580	.01606	.01904	.01941	12
13	.01027	.01037	.01291	.01308	.01586	.01611	.01910	.01947	13
14	.01031	.01041	.01296	.01313	.01591	.01616	.01916	.01953	14
15	.01035	.01046	.01300	.01318	.01596	.01622	.01921	.01959	15
16	.01039	.01050	.01305	.01322	.01601	.01627	.01927	.01965	16
17	.01043	.01054	.01310	.01327	.01606	.01633	.01933	.01971	17
18	.01047	.01059	.01314	.01332	.01612	.01638	.01939	.01977	18
19	.01052	.01063	.01319	.01337	.01617	.01643	.01944	.01983	19
20	.01056	.01067	.01324	.01342	.01622	.01649	.01950	.01989	20
21	.01060	.01071	.01329	.01346	.01627	.01654	.01956	.01995	21
22	.01064	.01076	.01333	.01351	.01632	.01659	.01961	.02001	22
23	.01069	.01080	.01338	.01356	.01638	.01665	.01967	.02007	23
24	.01073	.01084	.01343	.01361	.01643	.01670	.01973	.02013	24
25	.01077	.01089	.01348	.01366	.01648	.01676	.01979	.02019	25
26	.01081	.01093	.01352	.01371	.01653	.01681	.01984	.02025	26
27	.01086	.01097	.01357	.01376	.01659	.01687	.01990	.02031	27
28	.01090	.01102	.01362	.01381	.01664	.01692	.01996	.02037	28
29	.01094	.01106	.01367	.01386	.01669	.01698	.02002	.02043	29
30	.01098	.01111	.01371	.01391	.01675	.01703	.02008	.02049	30
31	.01103	.01115	.01376	.01395	.01680	.01709	.02013	.02055	31
32	.01107	.01119	.01381	.01400	.01685	.01714	.02019	.02061	32
33	.01111	.01124	.01386	.01405	.01690	.01720	.02025	.02067	33
34	.01116	.01128	.01391	.01410	.01696	.01725	.02031	.02073	34
35	.01120	.01133	.01396	.01415	.01701	.01731	.02037	.02079	35
36	.01124	.01137	.01400	.01420	.01706	.01736	.02042	.02085	36
37	.01129	.01142	.01405	.01425	.01712	.01742	.02048	.02091	37
38	.01133	.01146	.01410	.01430	.01717	.01747	.02054	.02097	38
39	.01137	.01151	.01415	.01435	.01723	.01753	.02060	.02103	39
40	.01142	.01155	.01420	.01440	.01728	.01758	.02066	.02110	40
41	.01146	.01160	.01425	.01445	.01733	.01764	.02072	.02116	41
42	.01151	.01164	.01430	.01450	.01739	.01769	.02078	.02122	42
43	.01155	.01169	.01435	.01455	.01744	.01775	.02084	.02128	43
44	.01159	.01173	.01439	.01461	.01750	.01781	.02090	.02134	44
45	.01164	.01178	.01444	.01466	.01755	.01786	.02095	.02140	45
46	.01168	.01182	.01449	.01471	.01760	.01792	.02101	.02147	46
47	.01173	.01187	.01454	.01476	.01766	.01798	.02107	.02153	47
48	.01177	.01191	.01459	.01481	.01771	.01803	.02113	.02159	48
49	.01182	.01196	.01464	.01486	.01777	.01809	.02119	.02165	49
50	.01186	.01200	.01469	.01491	.01782	.01815	.02125	.02171	50
51	.01191	.01205	.01474	.01496	.01788	.01820	.02131	.02178	51
52	.01195	.01209	.01479	.01501	.01793	.01826	.02137	.02184	52
53	.01200	.01214	.01484	.01506	.01799	.01832	.02143	.02190	53
54	.01204	.01219	.01489	.01512	.01804	.01837	.02149	.02196	54
55	.01209	.01223	.01494	.01517	.01810	.01843	.02155	.02203	55
56	.01213	.01228	.01499	.01522	.01815	.01849	.02161	.02209	56
57	.01218	.01233	.01504	.01527	.01821	.01854	.02167	.02215	57
58	.01222	.01237	.01509	.01532	.01826	.01860	.02173	.02221	58
59	.01227	.01242	.01514	.01537	.01832	.01866	.02179	.02228	59
60	.01231	.01247	.01519	.01543	.01837	.01872	.02185	.02234	60

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECA

′	12° Vers.	12° Ex. sec.	13° Vers.	13° Ex. sec.	14° Vers.	14° Ex. sec.	15° Vers.	15° Ex. sec.
0	.02185	.02234	.02563	.02630	.02970	.03061	.03407	.03528
1	.02191	.02240	.02570	.02637	.02977	.03069	.03415	.03536
2	.02197	.02247	.02576	.02644	.02985	.03076	.03422	.03544
3	.02203	.02253	.02583	.02651	.02992	.03084	.03430	.03552
4	.02210	.02259	.02589	.02658	.02999	.03091	.03438	.03560
5	.02216	.02266	.02596	.02665	.03006	.03099	.03445	.03568
6	.02222	.02272	.02602	.02672	.03013	.03106	.03453	.03576
7	.02228	.02279	.02609	.02679	.03020	.03114	03460	.03584
8	.02234	.02285	.02616	.02686	.03027	.03121	.03468	.03592
9	.02240	.02291	.02622	.02693	.03034	.03129	.03476	.03601
10	.02246	.02298	.02629	.02700	.03041	.03137	.03483	.03609
11	.02252	.02304	.02635	.02707	.03048	.03144	.03491	.03617
12	.02258	.02311	.02642	.02714	.03055	.03152	.03498	.03625
13	.02265	.02317	.02649	.02721	.03063	.03159	.03506	.03633
14	.02271	.02323	.02655	.02728	.03070	.03167	.03514	.03642
15	.02277	.02330	.02662	.02735	.03077	.03175	.03521	.03650
16	.02283	.02336	.02669	.02742	.03084	.03182	.03529	.03658
17	.02289	.02343	.02675	.02749	.03091	.03190	.03537	.03666
18	.02295	.02349	.02682	.02756	.03098	.03198	.03544	.03674
19	.02302	.02356	.02689	.02763	.03106	.03205	.03552	.03683
20	.02308	.02362	.02696	.02770	.03113	.03213	.03560	.03691
21	.02314	.02369	.02702	.02777	.03120	.03221	.03567	.03699
22	.02320	.02375	.02709	.02784	.03127	.03228	.03575	.03708
23	.02327	.02382	.02716	.02791	.03134	.03236	.03583	.03716
24	.02333	.02388	.02722	.02799	.03142	.03244	.03590	.03724
25	.02339	.02395	.02729	.02806	.03149	.03251	.03598	.03732
26	.02345	.02402	.02736	.02813	.03156	.03259	.03606	.03741
27	.02352	.02408	.02743	.02820	.03163	.03267	.03614	.03749
28	.02358	.02415	.02749	.02827	.03171	.03275	.03621	.03758
29	.02364	.02421	.02756	.02834	.03178	.03282	.03629	.03766
30	.02370	.02428	.02763	.02842	.03185	.03290	.03637	.03774
31	.02377	.02435	.02770	.02849	.03193	.03298	.03645	.03783
32	.02383	.02441	.02777	.02856	.03200	.03306	.03653	.03791
33	.02389	.02448	.02783	.02863	.03207	.03313	.03660	.03799
34	.02396	.02454	.02790	.02870	.03214	.03321	.03668	.03808
35	.02402	.02461	.02797	.02878	.03222	.03329	.03676	.03816
36	.02408	.02468	.02804	.02885	.03229	.03337	.03684	.03825
37	.02415	.02474	.02811	.02892	.03236	.03345	.03692	.03833
38	.02421	.02481	.02818	.02899	.03244	.03353	.03699	.03842
39	.02427	.02488	.02824	.02907	.03251	.03360	.03707	.03850
40	.02434	.02494	.02831	.02914	.03258	.03368	03715	.03858
41	.02440	.02501	.02838	.02921	.03266	.03376	.03723	.03867
42	.02447	.02508	.02845	.02928	.03273	.03384	.03731	.03875
43	.02453	.02515	02852	.02936	.03281	.03392	.03739	.03884
44	.02459	.02521	.02859	.02943	.03288	.03400	.03747	.03892
45	.02466	.02528	.02866	.02950	.03295	.03408	.03754	.03901
46	.02472	.02535	.02873	.02958	.03303	.03416	.03762	.03909
47	.02479	.02542	.02880	.02965	.03310	.03424	.03770	.03918
48	.02485	.02548	.02887	.02972	.03318	.03432	.03778	.03927
49	.02492	.02555	.02894	.02980	.03325	.03439	.03786	.03935
50	.02498	.02562	.02900	.02987	.03333	.03447	.03794	.03944
51	.02504	.02569	.02907	.02994	.03340	.03455	.03802	.03952
52	.02511	.02576	.02914	.03002	.03347	.03463	.03810	.03961
53	.02517	.02582	.02921	.03009	.03355	.03471	.03818	.03969
54	.02524	.02589	.02928	.03017	.03362	.03479	.03826	.03978
55	.02530	.02596	.02935	.03024	.03370	.03487	.03834	.03987
56	.02537	.02603	.02942	.03032	.03377	.03495	.03842	.03995
57	.02543	.02610	.02949	.03039	.03385	.03503	.03850	.04004
58	.02550	.02617	.02956	03046	.03392	.03512	.03858	.04013
59	.02556	.02624	.02963	.03054	.03400	.03520	.03866	.04021
60	.02563	.02630	.02970	.03061	03407	.03528	.03874	.04030

16°		17°		18°		19°		
Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	'
03874	04030	04370	04569	04894	05146	05448	05762	0
03882	04039	04378	04578	04903	05156	05458	05773	1
03890	04047	04387	04588	04912	05166	05467	05783	2
03898	04056	04395	04597	04921	05176	05477	05794	3
03906	04065	04404	04606	04930	05186	05486	05805	4
03914	04073	04412	04616	04939	05196	05496	05815	5
03922	04082	04421	04625	04948	05206	05505	05826	6
03930	04091	04429	04635	04957	05216	05515	05836	7
03938	04100	04438	04644	04967	05226	05524	05847	8
03946	04108	04446	04653	04976	05236	05534	05858	9
03954	04117	04455	04663	04985	05246	05543	05869	10
03963	04126	04464	04672	04994	05256	05553	05879	11
03971	04135	04472	04682	05003	05266	05562	05890	12
03979	04144	04481	04691	05012	05276	05572	05901	13
03987	04152	04489	04700	05021	05286	05582	05911	14
03995	04161	04498	04710	05030	05297	05591	05922	15
04003	04170	04507	04719	05039	05307	05601	05933	16
04011	04179	04515	04729	05048	05317	05610	05944	17
04019	04188	04524	04738	05057	05327	05620	05955	18
04028	04197	04533	04748	05067	05337	05630	05965	19
04036	04206	04541	04757	05076	05347	05639	05976	20
04044	04214	04550	04767	05085	05357	05649	05987	21
04052	04223	04559	04776	05094	05367	05658	05998	22
04060	04232	04567	04786	05103	05378	05668	06009	23
04069	04241	04576	04795	05112	05388	05678	06020	24
04077	04250	04585	04805	05122	05398	05687	06030	25
04085	04259	04593	04815	05131	05408	05697	06041	26
04093	04268	04602	04824	05140	05418	05707	06052	27
04102	04277	04611	04834	05149	05429	05716	06063	28
04110	04286	04620	04843	05158	05439	05726	06074	29
04118	04295	04628	04853	05168	05449	05736	06085	30
04126	04304	04637	04863	05177	05460	05746	06096	31
04135	04313	04646	04872	05186	05470	05755	06107	32
04143	04322	04655	04882	05195	05480	05765	06118	33
04151	04331	04663	04891	05205	05490	05775	06129	34
04159	04340	04672	04901	05214	05501	05785	06140	35
04168	04349	04681	04911	05223	05511	05794	06151	36
04176	04358	04690	04920	05232	05521	05804	06162	37
04184	04367	04699	04930	05242	05532	05814	06173	38
04193	04376	04707	04940	05251	05542	05824	06184	39
04201	04385	04716	04950	05260	05552	05833	06195	40
04209	04394	04725	04959	05270	05563	05843	06206	41
04218	04403	04734	04969	05279	05573	05853	06217	42
04226	04413	04743	04979	05288	05584	05863	06228	43
04234	04422	04752	04989	05298	05594	05873	06239	44
04243	04431	04760	04998	05307	05604	05882	06250	45
04251	04440	04769	05008	05316	05615	05892	06261	46
04260	04449	04778	05018	05326	05625	05902	06272	47
04268	04458	04787	05028	05335	05636	05912	06283	48
04276	04468	04796	05038	05344	05646	05922	06295	49
04285	04477	04805	05047	05354	05657	05932	06306	50
04293	04486	04814	05057	05363	05667	05942	06317	51
04302	04495	04823	05067	05373	05678	05951	06328	52
04310	04504	04832	05077	05382	05688	05961	06339	53
04319	04514	04841	05087	05391	05699	05971	06350	54
04327	04523	04850	05097	05401	05709	05981	06362	55
04336	04532	04858	05107	05410	05720	05991	06373	56
04344	04541	04867	05116	05420	05730	06001	06384	57
04353	04551	04876	05126	05429	05741	06011	06395	58
04361	04560	04885	05136	05439	05751	06021	06407	59
04370	04569	04894	05146	05448	05762	06031	06418	60

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

	20°		21°		22°		23°		
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
0	.06031	.06418	.06642	.07115	.07282	.07853	.07950	.08636	0
1	.06041	.06429	.06652	.07126	.07293	.07866	.07961	.08649	1
2	.06051	.06440	.06663	.07138	.07303	.07879	.07972	.08663	2
3	.06061	.06452	.06673	.07150	.07314	.07892	.07984	.08676	3
4	.06071	06463	.06684	.07162	.07325	.07904	.07995	.08690	4
5	.06081	.06474	.06694	.07174	.07336	.07917	.08006	.08703	5
6	.06091	.06486	.06705	.07186	.07347	.07930	.08018	.08717	6
7	.06101	06497	.06715	.07199	.07358	.07943	.08029	.08730	7
8	.06111	.06508	.06726	.07211	.07369	.07955	.08041	.08744	8
9	.06121	.06520	.06736	.07223	.07380	.07968	.08052	.08757	9
10	.06131	.06531	.06747	.07235	.07391	.07981	.08064	.08771	10
11	.06141	.06542	.06757	.07247	.07402	.07994	.08075	.08784	11
12	.06151	.06554	.06768	.07259	.07413	.08006	.08086	.08798	12
13	.06161	.06565	.06778	.07271	.07424	.08019	.08098	.08811	13
14	.06171	.06577	.06789	.07283	.07435	.08032	.08109	.08825	14
15	.06181	.06588	.06799	.07295	.07446	.08045	.08121	.08839	15
16	.06191	.06600	.06810	.07307	.07457	.08058	.08132	.08852	16
17	.06201	.06611	.06820	.07320	.07468	.08071	.08144	.08866	17
18	.06211	.06622	.06831	.07332	.07479	.08084	.08155	.08880	18
19	.06221	.06634	.06841	.07344	.07490	.08097	.08167	.08893	19
20	.06231	.06645	.06852	.07356	.07501	.08109	.08178	.08907	20
21	.06241	.06657	.06863	.07368	.07512	.08122	.08190	.08921	21
22	.06252	.06668	.06873	.07380	.07523	.08135	.08201	08934	22
23	.06262	.06680	.06884	.07393	.07534	.08148	.08213	.08948	23
24	.06272	.06691	.06894	.07405	.07545	.08161	.08225	.08962	24
25	.06282	.06703	.06905	.07417	.07556	.08174	.08236	.08975	25
26	.06292	.06715	.06916	.07429	.07568	.08087	08248	.08989	26
27	.06302	.06726	.06926	.07442	.07579	.08200	.08259	.09003	27
28	.06312	.06738	.06937	.07454	.07590	.08213	.08271	.09017	28
29	.06323	.06749	.06948	.07466	.07601	.08226	.08282	.09030	29
30	.06333	.06761	.06958	.07479	.07612	.08239	.08294	.09044	30
31	.06343	.06773	.06969	.07491	.07623	.08252	.08306	.09058	31
32	.06353	.06784	.06980	.07503	.07634	.08265	.08317	.09072	32
33	.06363	.06796	.06990	.07516	.07645	.08278	.08329	.09086	33
34	.06374	.06807	.07001	.07528	.07657	.08291	.08340	.09099	34
35	.06384	.06819	.07012	.07540	.07668	.08305	.08352	.09113	35
36	.06394	.06831	.07022	.07553	.07679	.08318	.08364	.09127	36
37	.06404	.06843	.07033	.07565	.07690	.08331	.08375	.09141	37
38	.06415	.06854	.07044	.07578	.07701	.08344	.08387	.09155	38
39	.06425	.06866	.07055	.07590	.07713	.08357	.08399	.09169	39
40	.06435	.06878	.07065	.07602	.07724	.08370	.08410	.09183	40
41	.06445	.06889	.07076	.07615	.07735	.08383	.08422	.09197	41
42	.06456	.06901	.07087	.07627	.07746	.08397	08434	.09211	42
43	.06466	.06913	.07098	.07640	.07757	.08410	.08445	.09224	43
44	.06476	.06925	.07108	.07652	.07769	.08423	.08457	.09238	44
45	.06486	.06936	.07119	.07665	.07780	.08436	.08469	.09252	45
46	.06497	.06948	.07130	.07677	.07791	.08449	.08481	.09266	46
47	.06507	.06960	.07141	.07690	.07802	.08463	.08492	.09280	47
48	.06517	.06972	.07151	.07702	.07814	.08476	.08504	.09294	48
49	.06528	.06984	.07162	.07715	.07825	.08489	.08516	09308	49
50	.06538	.06995	.07173	.07727	.07836	.08503	.08528	.09323	50
51	.06548	.07007	.07184	.07740	07848	.08516	.08539	.09337	51
52	.06559	.07019	.07195	.07752	07859	.08529	.08551	.09351	52
53	.06569	.07031	.07206	.07765	.07870	.08542	.08563	.09365	53
54	.06580	.07043	.07216	.07778	.07881	.08556	.08575	09379	54
55	.06590	.07055	.07227	.07790	.07893	.08569	.08586	.09393	55
56	.06600	.07067	.07238	.07803	.07904	.08582	.08598	.09407	56
57	.06611	.07079	.07249	.07816	.07915	.08596	.08610	.09421	57
58	.06621	.07091	.07260	.07828	.07927	.08069	.08622	.09435	58
59	.06632	.07103	.07271	.07841	.07938	.08623	.08634	.09449	59
60	.06642	.07115	.07282	.07853	.07950	.08636	.08645	.09464	60

24°		25°		26°		27°		
Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
08645	.09464	.09869	.10338	.10121	.11260	.10899	.12233	0
08657	.09478	.09882	.10353	.10133	.11276	.10913	.12249	1
08669	.09492	.09894	.10368	.10146	.11292	.10926	.12266	2
08681	.09506	.09406	.10383	.10159	.11308	.10939	.12283	3
08693	.09520	.09418	.10398	.10172	.11323	.10952	.12299	4
08705	.09535	.09431	.10413	.10184	.11339	.10965	.12316	5
08717	.09549	.09443	.10428	.10197	11355	.10979	.12333	6
08728	.09563	.09455	.10443	.10210	.11371	.10992	.12349	7
08740	.09577	09468	.10458	.10223	.11387	.11005	.12366	8
08752	.09592	.09480	.10473	.10236	.11403	.11019	.12383	9
08764	.09606	.09493	.10488	.10248	.11419	.11032	.12400	10
08776	.09620	.09505	.10503	.10261	.11435	.11045	.12416	11
08788	.09635	.09517	.10518	.10274	.11451	.11058	.12433	12
08800	.09649	.09530	.10533	.10287	.11467	11072	.12450	13
08812	.09663	.09542	.10549	.10300	.11483	.11085	.12467	14
08824	.09678	09554	.10564	.10313	.11499	.11098	.12484	15
08836	.09692	.09567	.10579	.10326	11515	.11112	.12501	16
08848	.09707	.09579	.10594	.10338	.11531	.11125	.12518	17
08860	.09721	.09592	.10609	.10351	.11547	.11138	.12534	18
08872	.09735	.09604	.10625	.10364	.11563	.11152	.12551	19
08884	.09750	.09617	.10640	.10377	.11579	.11165	.12568	20
08896	.09764	.09629	.10655	.10390	.11595	.11178	.12585	21
08908	.09779	.09642	.10670	.10403	.11611	.11192	.12602	22
08920	.09793	.09654	.10686	.10416	.11627	.11205	.12619	23
08932	.09808	.09666	.10701	10429	.11643	.11218	12636	24
08944	.09822	.09679	.10716	.10442	.11659	.11232	.12653	25
08956	.09837	.09691	.10731	.10455	.11675	.11245	.12670	26
08968	.09851	.09704	.10747	.10468	.11691	.11259	.12687	27
08980	.09866	.09716	.10762	.10481	.11708	.11272	.12704	28
08992	.09880	.09729	.10777	.10494	.11724	.11285	.12721	29
09004	.09895	.09741	.10793	.10507	.11740	.11299	.12738	30
09016	.09909	.09754	.10808	.10520	.11756	.11312	.12755	31
09028	.09924	.09767	.10824	.10533	.11772	.11326	.12772	32
09040	.09939	.09779	.10839	.10546	.11789	.11339	.12789	33
09052	.09953	.09792	.10854	.10559	.11805	.11353	.12807	34
09064	.09968	.09804	.10870	.10572	.11821	.11366	.12824	35
09076	.09982	09817	.10885	.10585	.11838	.11380	.12841	36
09089	.09997	09829	.10901	.10598	.11854	.11393	.12858	37
09101	.10012	.09842	.10916	.10611	.11870	.11407	.12875	38
09113	.10026	.09854	.10932	.10624	.11886	.11420	.12892	39
09125	.10041	.09867	.10947	.10637	.11903	.11434	.12910	40
09137	.10056	.09880	.10963	.10650	.11919	.11447	.12927	41
09149	.10071	.09892	.10978	.10663	.11936	.11461	.12944	42
09161	.10085	.09905	.10994	.10676	.11952	.11474	.12961	43
09174	.10100	.09918	.11009	.10689	.11968	.11488	.12979	44
09186	.10115	.09930	.11025	.10702	.11985	.11501	.12996	45
09198	.10130	.09943	.11041	.10715	.12001	.11515	.13013	46
09210	.10144	.09955	.11056	.10728	.12018	.11528	.13031	47
09222	.10159	.09968	.11072	.10741	.12034	.11542	.13048	48
09234	.10174	.09981	.11087	.10755	.12051	.11555	.13065	49
09247	.10189	.09993	.11103	.10768	.12067	.11569	.13083	50
09259	.10204	.10006	.11119	.10781	.12084	.11583	.13100	51
09271	.10218	.10019	.11134	10794	.12100	.11596	.13117	52
09283	.10233	.10032	.11150	.10807	.12117	11610	.13135	53
09296	.10248	.10044	.11166	10820	.12133	.11623	.13152	54
09308	.10263	.10057	.11181	.10833	.12150	.11637	.13170	55
09320	.10278	.10070	.11197	.10847	.12166	.11651	.13187	56
09332	.10293	.10082	.11213	.10860	.12183	.11664	.13205	57
09345	.10308	.10095	.11229	.10873	.12199	.11678	.13222	58
09357	.10323	.10108	.11244	.10886	.12216	.11692	.13240	59
09369	.10338	.10121	.11260	.10899	.12233	.11705	.13257	60

28°	29°	30°	31°

32°		33°		34°		35°		
Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
.15195	.17918	.16133	.19236	.17096	.20622	.18085	.22077	0
.15211	.17939	.16149	.19259	.17113	.20645	.18101	.22102	1
.15226	.17961	.16165	.19281	.17129	.20669	.18118	.22127	2
.15241	.17982	.16181	.19304	.17145	.20693	.18135	.22152	3
.15257	.18004	.16196	.19327	.17161	.20717	.18152	.22177	4
.15272	.18025	.16212	.19349	.17178	.20740	.18168	.22202	5
.15288	.18047	.16228	.19372	.17194	.20764	.18185	.22227	6
.15303	.18068	.16244	.19394	.17210	.20788	.18202	.22252	7
.15319	.18090	.16260	.19417	.17227	.20812	.18218	.22277	8
.15334	.18111	.16276	.19440	.17243	.20836	.18235	.22302	9
.15350	.18133	.16292	.19463	.17259	.20859	.18252	.22327	10
.15365	.18155	.16308	.19485	.17276	.20883	.18269	.22352	11
.15381	.18176	.16324	.19508	.17292	.20907	.18286	.22377	12
.15396	.18198	.16340	.19531	.17308	.20931	.18302	.22402	13
.15412	.18220	.16355	.19554	.17325	.20955	.18319	.22428	14
.15427	.18241	.16371	.19576	.17341	.20979	.18336	.22453	15
.15443	.18263	.16387	.19599	.17357	.21003	.18353	.22478	16
.15458	.18285	.16403	.19622	.17374	.21027	.18369	.22503	17
.15474	.18307	.16419	.19645	.17390	.21051	.18386	.22528	18
.15489	.18328	.16435	.19668	.17407	.21075	.18403	.22554	19
.15505	.18350	.16451	.19691	.17423	.21099	.18420	.22579	20
.15520	.18372	.16467	.19713	.17439	.21123	.18437	.22604	21
.15536	.18394	.16483	.19736	.17456	.21147	.18454	.22629	22
.15552	.18416	.16499	.19759	.17472	.21171	.18470	.22655	23
.15567	.18437	.16515	.19782	.17489	.21195	.18487	.22680	24
.15583	.18459	.16531	.19805	.17505	.21220	.18504	.22706	25
.15598	.18481	.16547	.19828	.17522	.21244	.18521	.22731	26
.15614	.18503	.16563	.19851	.17538	.21268	.18538	.22756	27
.15630	.18525	.16579	.19874	.17554	.21292	.18555	.22782	28
.15645	.18547	.16595	.19897	.17571	.21316	.18572	.22807	29
.15661	.18569	.16611	.19920	.17587	.21341	.18588	.22833	30
.15676	.18591	.16627	.19944	.17604	.21365	.18605	.22858	31
.15692	.18613	.16644	.19967	.17620	.21389	.18622	.22884	32
.15708	.18635	.16660	.19990	.17637	.21414	.18639	.22909	33
.15723	.18657	.16676	.20013	.17653	.21438	.18656	.22935	34
.15739	.18679	.16692	.20036	.17670	.21462	.18673	.22960	35
.15755	.18701	.16708	.20059	.17686	.21487	.18690	.22986	36
.15770	.18723	.16724	.20083	.17703	.21511	.18707	.23012	37
.15786	.18745	.16740	.20106	.17719	.21535	.18724	.23037	38
.15802	.18767	.16756	.20129	.17736	.21560	.18741	.23063	39
.15818	.18790	.16772	.20152	.17752	.21584	.18758	.23089	40
.15833	.18812	.16788	.20176	.17769	.21609	.18775	.23114	41
.15849	.18834	.16805	.20199	.17786	.21633	.18792	.23140	42
.15865	.18856	.16821	.20222	.17802	.21658	.18809	.23166	43
.15880	.18878	.16837	.20246	.17819	.21682	.18826	.23192	44
.15896	.18901	.16853	.20269	.17835	.21707	.18843	.23217	45
.15912	.18923	.16869	.20292	.17852	.21731	.18860	.23243	46
.15928	.18945	.16885	.20316	.17868	.21756	.18877	.23269	47
.15943	.18967	.16902	.20339	.17885	.21781	.18894	.23295	48
.15959	.18990	.16918	.20363	.17902	.21805	.18911	.23321	49
.15975	.19012	.16934	.20386	.17918	.21830	.18928	.23347	50
.15991	.19034	.16950	.20410	.17935	.21855	.18945	.23373	51
.16006	.19057	.16966	.20433	.17952	.21879	.18962	.23399	52
.16022	.19079	.16983	.20457	.17968	.21904	.18979	.23424	53
.18038	.19102	.16999	.20480	.17985	.21929	.18996	.23450	54
.16054	.19124	.17015	.20504	.18001	.21953	.19013	.23476	55
.16070	.19146	.17031	.20527	.18018	.21978	.19030	.23502	56
.16085	.19169	.17047	.20551	.18035	.22003	.19047	.23529	57
.16101	.19191	.17064	.20575	.18051	.22028	.19064	.23555	58
.16117	.19214	.17080	.20598	.18068	.22053	.19081	.23581	59
.16133	.19236	.17096	.20622	.18085	.22077	.19098	.23607	60

	36°		37°		38°		39°	
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.
0	.19098	.23607	.20136	.25214	.21199	.26902	.22285	.28676
1	.19115	.23633	.20154	.25241	.21217	.26931	.22304	.28706
2	.19133	.23659	.20171	.25269	.21235	.26960	.22322	.28737
3	.19150	.23685	.20189	.25296	.21253	.26988	.22340	.28767
4	.19167	.23711	.20207	.25324	.21271	.27017	.22359	.28797
5	.19184	.23738	.20224	.25351	.21289	.27046	.22377	.28828
6	.19201	.23764	.20242	.25379	.21307	.27075	.22395	.28858
7	.19218	.23790	.20259	.25406	.21324	.27104	.22414	.28889
8	.19235	.23816	.20277	.25434	.21342	.27133	.22432	.28919
9	.19252	.23843	.20294	.25462	.21360	.27162	.22450	.28950
10	.19270	.23869	.20312	.25489	.21378	.27191	.22469	.28980
11	.19287	.23895	.20329	.25517	.21396	.27221	.22487	.29011
12	.19304	.23922	.20347	.25545	.21414	.27250	.22506	.29042
13	.19321	.23948	.20365	.25572	.21432	.27279	.22524	.29072
14	.19338	.23975	.20382	.25600	.21450	.27308	.22542	.29103
15	.19356	.24001	.20400	.25628	.21468	.27337	.22561	.29133
16	.19373	.24028	.20417	.25656	.21486	.27366	.22579	.29164
17	.19390	.24054	.20435	.25683	.21504	.27396	.22598	.29195
18	.19407	.24081	.20453	.25711	.21522	.27425	.22616	.29226
19	.19424	.24107	.20470	.25739	.21540	.27454	.22634	.29256
20	.19442	.24134	.20488	.25767	.21558	.27483	.22653	.29287
21	.19459	.24160	.20506	.25795	.21576	.27513	.22671	.29318
22	.19476	.24187	.20523	.25823	.21595	.27542	.22690	.29349
23	.19493	.24213	.20541	.25851	.21613	.27572	.22708	.29380
24	.19511	.24240	.20559	.25879	.21631	.27601	.22727	.29411
25	.19528	.24267	.20576	.25907	.21649	.27630	.22745	.29442
26	.19545	.24293	.20594	.25935	.21667	.27660	.22764	.29473
27	.19562	.24320	.20612	.25963	.21685	.27689	.22782	.29504
28	.19580	.24347	.20629	.25991	.21703	.27719	.22801	.29535
29	.19597	.24373	.20647	.26019	.21721	.27748	.22819	.29566
30	.19614	.24400	.20665	.26047	.21739	.27778	.22838	.29597
31	.19632	.24427	.20682	.26075	.21757	.27807	.22856	.29628
32	.19649	.24454	.20700	.26104	.21775	.27837	.22875	.29659
33	.19666	.24481	.20718	.26132	.21794	.27867	.22893	.29690
34	.19684	.24508	.20736	.26160	.21812	.27896	.22912	.29721
35	.19701	.24534	.20753	.26188	.21830	.27926	.22930	.29752
36	.19718	.24561	.20771	.26216	.21848	.27956	.22949	.29784
37	.19736	.24588	.20789	.26245	.21866	.27985	.22967	.29815
38	.19753	.24615	.20807	.26273	.21884	.28015	.22986	.29846
39	.19770	.24642	.20824	.26301	.21902	.28045	.23004	.29877
40	.19788	.24669	.20842	.26330	.21921	.28075	.23023	.29909
41	.19805	.24696	.20860	.26358	.21939	.28105	.23041	.29940
42	.19822	.24723	.20878	.26387	.21957	.28134	.23060	.29971
43	.19840	.24750	.20895	.26415	.21975	.28164	.23079	.30003
44	.19857	.24777	.20913	.26443	.21993	.28194	.23097	.30034
45	.19875	.24804	.20931	.26472	.22012	.28224	.23116	.30066
46	.19892	.24832	.20949	.26500	.22030	.28254	.23134	.30097
47	.19909	24859	.20967	.26529	.22048	.28284	.23153	.30129
48	.19927	.24886	.20985	.26557	.22066	.28314	.23172	.30160
49	.19944	.24913	.21002	.26586	.22084	.28344	.23190	.30192
50	.19962	.24940	.21020	.26615	.22103	.28374	.23209	.30223
51	.19979	.24967	.21038	.26643	.22121	.28404	.23228	.30255
52	.19997	.24995	.21056	.26672	.22139	.28434	.23246	.30287
53	.20014	.25022	.21074	.26701	.22157	.28464	.23265	.30318
54	.20032	.25049	.21092	.26729	.22176	.28495	.23283	.30350
55	.20049	.25077	.21109	.26758	.22194	.28525	.23302	.30382
56	.20066	.25104	.21127	.26787	.22212	.28555	.23321	.30413
57	.20084	.25131	.21146	.26815	.22231	.28585	.23339	.30445
58	.20101	.25159	.21163	.26844	.22249	.28615	.23358	.30477
59	.20119	.25186	.21181	.26873	.22267	.28646	.23377	.30509
60	.20136	.25214	.21199	.26902	.22285	.28676	.23396	.30541

LE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

40°		41°		42°		43°		
Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
23396	30541	24529	32501	25686	34563	26865	36733	0
23414	30573	24548	32535	25705	34599	26884	36770	1
23433	30605	24567	32568	25724	34634	26904	36807	2
23452	30636	24586	32602	25744	34669	26924	36844	3
23470	30668	24605	32636	25763	34704	26944	36881	4
23489	30700	24625	32669	25783	34740	26964	36919	5
23508	30732	24644	32703	25802	34775	26984	36956	6
23527	30764	24663	32737	25822	34811	27004	36993	7
23545	30796	24682	32770	25841	34846	27024	37030	8
23564	30829	24701	32804	25861	34882	27043	37068	9
23583	30861	24720	32838	25880	34917	27063	37105	10
23602	30893	24739	32872	25900	34953	27083	37143	11
23620	30925	24759	32905	25920	34988	27103	37180	12
23639	30957	24778	32939	25939	35024	27123	37218	13
23658	30989	24797	32973	25959	35060	27143	37255	14
23677	31022	24816	33007	25978	35095	27163	37293	15
23696	31054	24835	33041	25998	35131	27183	37330	16
23714	31086	24854	33075	26017	35167	27203	37368	17
23733	31119	24874	33109	26037	35203	27223	37406	18
23752	31151	24893	33143	26056	35238	27243	37443	19
23771	31183	24912	33177	26076	35274	27263	37481	20
23790	31216	24931	33211	26096	35310	27283	37519	21
23808	31248	24950	33245	26115	35346	27303	37556	22
23827	31281	24970	33279	26135	35382	27323	37594	23
23846	31313	24989	33314	26154	35418	27343	37632	24
23865	31346	25008	33348	26174	35454	27363	37670	25
23884	31378	25027	33382	26194	35490	27383	37708	26
23903	31411	25047	33416	26213	35526	27403	37746	27
23922	31443	25066	33451	26233	35562	27423	37784	28
23941	31476	25085	33485	26253	35598	27443	37822	29
23959	31509	25104	33519	26272	35634	27463	37860	30
23978	31541	25124	33554	26292	35670	27483	37898	31
23997	31574	25143	33588	26312	35707	27503	37936	32
24016	31607	25162	33622	26331	35743	27523	37974	33
24035	31640	25182	33657	26351	35779	27543	38012	34
24054	31672	25201	33691	26371	35815	27563	38051	35
24073	31705	25220	33726	26390	35852	27583	38089	36
24092	31738	25240	33760	26410	35888	27603	38127	37
24111	31771	25259	33795	26430	35924	27623	38165	38
24130	31804	25278	33830	26449	35961	27643	38204	39
24149	31837	25297	33864	26469	35997	27663	38242	40
24168	31870	25317	33899	26489	36034	27683	38280	41
24187	31903	25336	33934	26509	36070	27703	38319	42
24206	31936	25356	33968	26528	36107	27723	38357	43
24225	31969	25375	34003	26548	36143	27743	38396	44
24244	32002	25394	34038	26568	36180	27764	38434	45
24262	32035	25414	34073	26588	36217	27784	38473	46
24281	32068	25433	34108	26607	36253	27804	38512	47
24300	32101	25452	34142	26627	36290	27824	38550	48
24320	32134	25472	34177	26647	36327	27844	38589	49
24339	32168	25491	34212	26667	36363	27864	38628	50
24358	32201	25511	34247	26686	36400	27884	38666	51
24377	32234	25530	34282	26706	36437	27905	38705	52
24396	32267	25549	34317	26726	36474	27925	38744	53
24415	32301	25569	34352	26746	36511	27945	38783	54
24434	32334	25588	34387	26766	36548	27965	38822	55
24453	32368	25608	34423	26785	36585	27985	38860	56
24472	32401	25627	34458	26805	36622	28005	38899	57
24491	32434	25647	34493	26825	36659	28026	38938	58
24510	32468	25666	34528	26845	36696	28046	38977	59
24529	32501	25686	34563	26865	36733	28066	39016	60

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS

′	44° Vers.	44° Ex. sec.	45° Vers.	45° Ex. sec.	46° Vers.	46° Ex. sec.	47° Vers.	47° Ex. sec.	′
0	.28066	.39016	.29289	.41421	.30534	.43956	.31800	.46628	0
1	.28086	.39055	.29310	.41463	.30555	.44000	.31821	.46674	1
2	.28106	.39095	.29330	.41504	.30576	.44042	.31842	.46719	2
3	.28127	.39134	.29351	.41545	.30597	.44086	.31864	.46765	3
4	.28147	.39173	.29372	.41586	.30618	.44129	.31885	.46811	4
5	.28167	.39213	.29392	.41627	.30639	.44173	.31907	.46857	5
6	.28187	.39251	.29413	.41669	.30660	.44217	.31928	.46903	6
7	.28208	.39291	.29433	.41710	.30681	.44260	.31949	.46949	7
8	.28228	.39330	.29454	.41752	.30702	.44304	.31971	.46995	8
9	.28248	[illegible]	.29475	.41793	.30723	.44347	.31992	.47041	9
10	.28268	.39409	.29496	.41835	.30744	.44391	.32013	.47087	10
11	.28289	.39448	.29516	.41876	.30765	.44435	.32035	.47134	11
12	.28309	.39487	.29537	.41918	.30786	.44479	.32056	.47180	12
13	.28329	.39527	.29557	.41960	.30807	.44523	.32077	.47226	13
14	.28350	.39566	.29578	.42001	.30828	.44567	.32099	.47272	14
15	.28370	.39606	.29599	.42043	.30849	.44610	.32120	.47319	15
16	.28390	.39646	.29619	.42084	.30870	.44654	.32141	.47366	16
17	.28410	.39685	.29640	.42126	.30891	.44698	.32163	.47411	17
18	.28431	.39725	.29661	.42168	.30912	.44742	.32184	.47458	18
19	.28451	.39764	.29681	.42210	.30933	.44787	.32206	.47504	19
20	.28471	.39804	.29702	.42251	[illegible]	.44831	.32227	.47551	20
21	.28492	.39844	.29723	.42293	.30976	.44875	.32248	.47598	21
22	.28512	[illegible]	.29743	.42335	.30996	[illegible]	.32270	.47644	22
23	.28532	.39924	.29764	.42377	.31017	.44963	.32291	.47691	23
24	.28553	.39963	.29785	.42419	.31038	.45007	.32312	.47738	24
25	.28573	.40003	.29805	.42461	.31059	.45052	[illegible]	.47784	25
26	.28593	[illegible]	.29826	.42503	.31080	.45096	.32355	.47831	26
27	.28614	.40083	.29847	.42545	.31101	.45141	.32377	.47878	27
28	.28634	.40123	.29868	.42587	.31122	[illegible]	.32398	.47925	28
29	.28655	.40163	.29888	.42630	.31143	.45229	.32420	.47972	29
30	.28675	.40203	.29909	.42672	.31165	.45274	.32441	.48019	30
31	.28695	[illegible]	.29930	.42714	.31186	.45319	[illegible]	.48066	31
32	.28716	.40283	.29951	.42756	.31207	.45363	[illegible]	.48113	32
33	[illegible]	.40324	.29971	.42799	.31228	.45408	.32506	.48160	33
34	.28757	.40364	.29992	.42841	.31249	[illegible]	.32527	.48207	34
35	.28777	.40404	.30013	.42883	.31270	.45497	.32548	.48254	35
36	.28797	.40444	.30034	.42926	.31291	.45542	.32570	.48301	36
37	.28818	.40485	.30054	.42968	.31312	.45587	.32591	.48349	37
38	.28838	.40525	.30075	.43011	.31334	.45631	.32613	.48396	38
39	.28859	.40565	.30096	.43053	.31355	[illegible]	.32634	.48443	39
40	.28879	.40606	.30117	.43096	.31376	.45721	.32656	.48491	40
41	.28900	.40646	.30138	.43139	.31397	.45766	.32677	.48538	41
42	.28920	.40687	.30158	.43181	.31418	.45811	.32699	.48586	42
43	.28941	.40727	.30179	.43224	.31439	.45856	.32720	[illegible]	43
44	.28961	.40768	[illegible]	.43267	.31461	.45901	.32742	.48681	44
45	.28981	.40808	.30221	.43310	.31482	.45946	.32763	.48729	45
46	.29002	.40849	.30242	.43352	.31503	.45992	.32785	.48776	46
47	.29022	.40890	.30263	.43395	.31524	.46037	.32806	.48824	47
48	.29043	.40930	.30283	.43438	.31545	.46082	.32828	.48871	48
49	.29063	.40971	.30304	.43481	.31567	.46127	.32849	.48919	49
50	.29084	.41012	.30325	.43524	.31588	.46172	.32871	.48967	50
51	.29104	.41053	.30346	.43567	.31609	.46218	[illegible]	[illegible]	51
52	.29125	[illegible]	.30367	.43610	.31630	.46263	.32914	.49063	52
53	.29145	.41134	[illegible]	.43653	.31651	.46309	.32935	.49111	53
54	.29166	.41175	.30409	.43696	.31673	.46354	.32957	.49159	54
55	.29187	.41216	.30430	.43739	.31694	.46400	.32978	.49207	55
56	.29207	.41257	.30451	.43783	.31715	.46445	.33001	.49255	56
57	.29228	.41298	.30471	.43826	.31736	.46491	.33022	.49303	57
58	.29248	.41339	.30492	.43869	.31758	.46537	.33044	.49351	58
59	.29269	.41380	.30513	.43912	.31779	.46582	.33065	.49399	59
60	.29289	.41421	.30534	[illegible]	.31800	.46628	.33087	[illegible]	60

′	48° Vers.	48° Ex. sec.	49° Vers.	49° Ex. sec.	50° Vers.	50° Ex. sec.	51° Vers.	51° Ex. sec.	′
0	.33087	.49448	.34394	.52425	.35721	.55572	.37068	.58902	0
1	.33109	.49496	.34416	.52476	.35744	.55626	.37091	.58960	1
2	.33130	.49544	.34438	.52527	.35766	.55680	.37113	.59016	2
3	.33152	.49593	.34460	.52579	.35788	.55734	.37136	.59073	3
4	.33173	.49641	.34482	.52630	.35810	.55789	.37158	.59130	4
5	.33195	.49690	.34504	.52681	.35833	.55843	.37181	.59188	5
6	.33217	.49738	.34526	.52732	.35855	.55897	.37204	.59245	6
7	.33238	.49787	.34548	.52784	.35877	.55951	.37226	.59302	7
8	.33260	.49835	.34570	.52835	.35900	.56006	.37249	.59360	8
9	.33282	.49884	.34592	.52886	.35922	.56060	.37271	.59418	9
10	.33303	.49933	.34614	.52938	.35944	.56114	.37294	.59475	10
11	.33325	.49981	.34636	.52989	.35967	.56169	.37317	.59533	11
12	.33347	.50030	.34658	.53041	.35989	.56223	.37339	.59590	12
13	.33368	.50079	.34680	.53093	.36011	.56278	.37362	.59648	13
14	.33390	.50128	.34702	.53144	.36034	.56332	.37384	.59706	14
15	.33412	.50177	.34724	.53196	.36056	.56387	.37407	.59764	15
16	.33434	.50226	.34746	.53247	.36078	.56442	.37430	.59822	16
17	.33455	.50275	.34768	.53299	.36101	.56497	.37452	.59880	17
18	.33477	.50324	.34790	.53351	.36123	.56551	.37475	.59938	18
19	.33499	.50373	.34812	.53403	.36146	.56606	.37498	.59996	19
20	.33520	.50422	.34834	.53455	.36168	.56661	.37521	.60054	20
21	.33542	.50471	.34856	.53507	.36190	.56716	.37543	.60112	21
22	.33564	.50521	.34878	.53559	.36213	.56771	.37566	.60171	22
23	.33586	.50570	.34900	.53611	.36235	.56826	.37589	.60229	23
24	.33607	.50619	.34922	.53663	.36258	.56881	.37612	.60287	24
25	.33629	.50669	.34944	.53715	.36280	.56937	.37634	.60346	25
26	.33651	.50718	.34966	.53768	.36302	.56992	.37657	.60404	26
27	.33673	.50767	.34988	.53820	.36325	.57047	.37680	.60463	27
28	.33694	.50817	.35011	.53872	.36347	.57103	.37703	.60521	28
29	.33716	.50866	.35033	.53924	.36370	.57158	.37725	.60580	29
30	.33738	.50916	.35055	.53977	.36392	.57213	.37748	.60639	30
31	.33760	.50966	.35077	.54029	.36415	.57269	.37771	.60698	31
32	.33782	.51015	.35099	.54082	.36437	.57324	.37794	.60756	32
33	.33803	.51065	.35121	.54134	.36460	.57380	.37817	.60815	33
34	.33825	.51115	.35144	.54187	.36482	.57436	.37840	.60874	34
35	.33847	.51165	.35166	.54240	.36504	.57491	.37862	.60933	35
36	.33869	.51215	.35188	.54292	.36527	.57547	.37885	.60992	36
37	.33891	.51265	.35210	.54345	.36549	.57603	.37908	.61051	37
38	.33912	.51314	.35232	.54398	.36572	.57659	.37931	.61111	38
39	.33934	.51364	.35254	.54451	.36594	.57715	.37954	.61170	39
40	.33956	.51415	.35277	.54504	.36617	.57771	.37976	.61229	40
41	.33978	.51465	.35299	.54557	.36639	.57827	.37999	.61288	41
42	.34000	.51515	.35321	.54610	.36662	.57883	.38022	.61348	42
43	.34022	.51565	.35343	.54663	.36684	.57939	.38045	.61407	43
44	.34044	.51615	.35365	.54716	.36707	.57995	.38068	.61467	44
45	.34065	.51665	.35388	.54769	.36729	.58051	.38091	.61526	45
46	.34087	.51716	.35410	.54822	.36752	.58108	.38113	.61586	46
47	.34109	.51766	.35432	.54876	.36775	.58164	.38136	.61646	47
48	.34131	.51817	.35454	.54929	.36797	.58221	.38159	.61706	48
49	.34153	.51867	.35476	.54982	.36820	.58277	.38182	.61766	49
50	.34175	.51918	.35499	.55036	.36842	.58333	.38205	.61826	50
51	.34197	.51968	.35521	.55089	.36865	.58390	.38228	.61886	51
52	.34219	.52019	.35543	.55143	.36887	.58447	.38251	.61946	52
53	.34241	.52069	.35565	.55196	.36910	.58503	.38274	.62006	53
54	.34262	.52120	.35588	.55250	.36932	.58560	.38296	.62066	54
55	.34284	.52171	.35610	.55303	.36955	.58617	.38319	.62126	55
56	.34306	.52222	.35632	.55357	.36978	.58674	.38342	.62186	56
57	.34328	.52273	.35654	.55411	.37000	.58731	.38365	.62247	57
58	.34350	.52323	.35677	.55465	.37023	.58788	.38388	.62307	58
59	.34372	.52374	.35699	.55518	.37045	.58845	.38411	.62368	59
60	.34394	.52425	.35721	.55572	.37068	.58902	.38434	.62427	60

52°	53°	54°	55°

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	56° Vers.	56° Ex. sec.	57° Vers.	57° Ex. sec.	58° Vers.	58° Ex. sec.	59° Vers.	59° Ex. sec.	′
0	.44081	.78829	.45536	.83608	.47008	.88708	.48496	.94160	0
1	.44105	.78906	.45560	.83690	.47033	.88796	.48521	.94254	1
2	.44129	.78984	.45585	.83773	.47057	.88884	.48546	.94349	2
3	.44153	.79061	.45609	.83855	.47082	.88972	.48571	.94443	3
4	.44177	.79138	.45634	.83938	.47107	.89060	.48596	.94537	4
5	.44201	.79216	.45658	.84020	.47131	.89148	.48621	.94632	5
6	.44225	.79293	.45683	.84103	.47156	.89237	.48646	.94726	6
7	.44250	.79371	.45707	.84186	.47181	.89325	.48671	.94821	7
8	.44274	.79449	.45731	.84269	.47206	.89414	.48696	.94916	8
9	.44298	.79527	.45756	.84352	.47230	.89503	.48721	.95011	9
10	.44322	.79604	.45780	.84435	.47255	.89591	.48746	.95106	10
11	.44346	.79682	.45805	.84518	.47280	.89680	.48771	.95201	11
12	.44370	.79761	.45829	.84601	.47304	.89769	.48796	.95296	12
13	.44395	.79839	.45854	.84685	.47329	.89858	.48821	.95392	13
14	.44419	.79917	.45878	.84768	.47354	.89948	.48846	.95487	14
15	.44443	.79995	.45903	.84852	.47379	.90037	.48871	.95583	15
16	.44467	.80074	.45927	.84935	.47403	.90126	.48896	.95678	16
17	.44491	.80152	.45951	.85019	.47428	.90216	.48921	.95774	17
18	.44516	.80231	.45976	.85103	.47453	.90305	.48946	.95870	18
19	.44540	.80309	.46000	.85187	.47478	.90395	.48971	.95966	19
20	.44564	.80388	.46025	.85271	.47502	.90485	.48996	.96062	20
21	.44588	.80467	.46049	.85355	.47527	.90575	.49021	.96158	21
22	.44612	.80546	.46074	.85439	.47552	.90665	.49046	.96255	22
23	.44637	.80625	.46098	.85523	.47577	.90755	.49071	.96351	23
24	.44661	.80704	.46123	.85608	.47601	.90845	.49096	.96448	24
25	.44685	.80783	.46147	.85692	.47626	.90935	.49121	.96544	25
26	.44709	.80862	.46172	.85777	.47651	.91026	.49146	.96641	26
27	.44734	.80942	.46196	.85861	.47676	.91116	.49171	.96738	27
28	.44758	.81021	.46221	.85946	.47701	.91207	.49196	.96835	28
29	.44782	.81101	.46246	.86031	.47725	.91297	.49221	.96932	29
30	.44806	.81180	.46270	.86116	.47750	.91388	.49246	.97029	30
31	.44831	.81260	.46295	.86201	.47775	.91479	.49271	.97127	31
32	.44855	.81340	.46319	.86286	.47800	.91570	.49296	.97224	32
33	.44879	.81419	.46344	.86371	.47825	.91661	.49321	.97322	33
34	.44903	.81499	.46368	.86457	.47849	.91752	.49346	.97420	34
35	.44928	.81579	.46393	.86542	.47874	.91844	.49372	.97517	35
36	.44952	.81659	.46417	.86627	.47899	.91935	.49397	.97615	36
37	.44976	.81740	.46442	.86713	.47924	.92027	.49422	.97713	37
38	.45001	.81820	.46466	.86799	.47949	.92118	.49447	.97811	38
39	.45025	.81900	.46491	.86885	.47974	.92210	.49472	.97910	39
40	.45049	.81981	.46516	.86970	.47998	.92302	.49497	.98008	40
41	.45073	.82061	.46540	.87056	.48023	.92394	.49522	.98107	41
42	.45098	.82142	.46565	.87142	.48048	.92486	.49547	.98205	42
43	.45122	.82222	.46589	.87229	.48073	.92578	.49572	.98304	43
44	.45146	.82303	.46614	.87315	.48098	.92670	.49597	.98403	44
45	.45171	.82384	.46639	.87401	.48123	.92762	.49623	.98502	45
46	.45195	.82465	.46663	.87488	.48148	.92855	.49648	.98601	46
47	.45219	.82546	.46688	.87574	.48172	.92947	.49673	.98700	47
48	.45244	.82627	.46712	.87661	.48197	.93040	.49698	.98799	48
49	.45268	.82709	.46737	.87748	.48222	.93133	.49723	.98899	49
50	.45292	.82790	.46762	.87834	.48247	.93226	.49748	.98998	50
51	.45317	.82871	.46786	.87921	.48272	.93319	.49773	.99098	51
52	.45341	.82953	.46811	.88008	.48297	.93412	.49799	.99198	52
53	.45365	.83034	.46836	.88095	.48322	.93505	.49824	.99298	53
54	.45390	.83116	.46860	.88183	.48347	.93598	.49849	.99398	54
55	.45414	.83198	.46885	.88270	.48372	.93692	.49874	.99498	55
56	.45439	.83280	.46909	.88357	.48396	.93785	.49899	.99598	56
57	.45463	.83362	.46934	.88445	.48421	.93879	.49924	.99698	57
58	.45487	.83444	.46959	.88532	.48446	.93973	.49950	.99799	58
59	.45512	.83526	.46983	.88620	.48471	.94066	.49975	.99899	59
60	.45536	.83608	.47008	.88708	.48496	.94160	.50000	1.00000	60

′	60° Vers.	60° Ex. sec.	61° Vers.	61° Ex. sec.	62° Vers.	62° Ex. sec.	63° Vers.	63° Ex. sec.
0	.50000	1.00000	.51519	1.06267	.53053	1.13005	.54601	1.20269
1	.50025	1.00101	.51544	1.06375	.53079	1.13122	.54627	1.20395
2	.50050	1.00202	.51570	1.06483	.53104	1.13239	.54653	1.20521
3	.50076	1.00303	.51595	1.06592	.53130	1.13356	.54679	1.20647
4	.50101	1.00404	.51621	1.06701	.53156	1.13473	.54705	1.20773
5	.50126	1.00505	.51646	1.06809	.53181	1.13590	.54731	1.20900
6	.50151	1.00607	.51672	1.06918	.53207	1.13707	.54757	1.21026
7	.50176	1.00708	.51697	1.07027	.53233	1.13825	.54782	1.21153
8	.50202	1.00810	.51723	1.07137	.53258	1.13942	.54808	1.21280
9	.50227	1.00912	.51748	1.07246	.53284	1.14060	.54834	1.21407
10	.50252	1.01014	.51774	1.07356	.53310	1.14178	.54860	1.21535
11	.50277	1.01116	.51799	1.07465	.53336	1.14296	.54886	1.21662
12	.50303	1.01218	.51825	1.07575	.53361	1.14414	.54912	1.21790
13	.50328	1.01320	.51850	1.07685	.53387	1.14533	.54938	1.21918
14	.50353	1.01422	.51876	1.07795	.53413	1.14651	.54964	1.22045
15	.50378	1.01525	.51901	1.07905	.53439	1.14770	.54990	1.22174
16	.50404	1.01628	.51927	1.08015	.53464	1.14889	.55016	1.22302
17	.50429	1.01730	.51952	1.08126	.53490	1.15008	.55042	1.22430
18	.50454	1.01833	.51978	1.08236	.53516	1.15127	.55068	1.22559
19	.50479	1.01936	.52003	1.08347	.53542	1.15246	.55094	1.22688
20	.50505	1.02039	.52029	1.08458	.53567	1.15366	.55120	1.22817
21	.50530	1.02143	.52054	1.08569	.53593	1.15485	.55146	1.22946
22	.50555	1.02246	.52080	1.08680	.53619	1.15605	.55172	1.23075
23	.50581	1.02349	.52105	1.08791	.53645	1.15725	.55198	1.23205
24	.50606	1.02453	.52131	1.08903	.53670	1.15845	.55224	1.23334
25	.50631	1.02557	.52156	1.09014	.53696	1.15965	.55250	1.23464
26	.50656	1.02661	.52182	1.09126	.53722	1.16085	.55276	1.23594
27	.50682	1.02765	.52207	1.09238	.53748	1.16206	.55302	1.23724
28	.50707	1.02869	.52233	1.09350	.53774	1.16326	.55328	1.23855
29	.50732	1.02973	.52259	1.09462	.53799	1.16447	.55354	1.23985
30	.50758	1.03077	.52284	1.09574	.53825	1.16568	.55380	1.24116
31	.50783	1.03182	.52310	1.09686	.53851	1.16689	.55406	1.24247
32	.50808	1.03286	.52335	1.09799	.53877	1.16810	.55432	1.24378
33	.50834	1.03391	.52361	1.09911	.53903	1.16932	.55458	1.24509
34	.50859	1.03496	.52386	1.10024	.53928	1.17053	.55484	1.24640
35	.50884	1.03601	.52412	1.10137	.53954	1.17175	.55510	1.24772
36	.50910	1.03706	.52438	1.10250	.53980	1.17297	.55536	1.24903
37	.50935	1.03811	.52463	1.10363	.54006	1.17419	.55563	1.25035
38	.50960	1.03916	.52489	1.10477	.54032	1.17541	.55589	1.25167
39	.50986	1.04022	.52514	1.10590	.54058	1.17663	.55615	1.25300
40	.51011	1.04128	.52540	1.10704	.54083	1.17786	.55641	1.25432
41	.51036	1.04233	.52566	1.10817	.54109	1.17909	.55667	1.25565
42	.51062	1.04339	.52591	1.10981	.54135	1.18031	.55693	1.25697
43	.51087	1.04445	.52617	1.11045	.54161	1.18154	.55719	1.25830
44	.51113	1.04551	.52642	1.11159	.54187	1.18277	.55745	1.25963
45	.51138	1.04658	.52668	1.11274	.54213	1.18401	.55771	1.26097
46	.51163	1.04764	.52694	1.11388	.54238	1.18524	.55797	1.26230
47	.51189	1.04870	.52719	1.11503	.54264	1.18648	.55823	1.26364
48	.51214	1.04977	.52745	1.11617	.54290	1.18772	.55849	1.26498
49	.51239	1.05084	.52771	1.11732	.54316	1.18895	.55876	1.26632
50	.51265	1.05191	.52796	1.11847	.54342	1.19019	.55902	1.26766
51	.51290	1.05298	.52822	1.11963	.54368	1.19144	.55928	1.26900
52	.51316	1.05405	.52848	1.12078	.54394	1.19268	.55954	1.27035
53	.51341	1.05512	.52873	1.12193	.54420	1.19393	.55980	1.27169
54	.51366	1.05619	.52899	1.12309	.54446	1.19517	.56006	1.27304
55	.51392	1.05727	.52924	1.12425	.54471	1.19642	.56032	1.27439
56	.51417	1.05835	.52950	1.12540	.54497	1.19767	.56058	1.27574
57	.51443	1.05942	.52976	1.12657	.54523	1.19892	.56084	1.27710
58	.51468	1.06050	.53001	1.12773	.54549	1.20018	.56111	1.27845
59	.51494	1.06158	.53027	1.12889	.54575	1.20143	.56137	1.27981
60	.51519	1.06267	.53053	1.13005	.54601	1.20269	.56163	1.28117

64°		65°		66°		67°		
Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
56163	1.28117	.57738	1.36620	.59326	1.45859	.60927	1.55930	0
56189	1.28253	.57765	1.36768	.59353	1.46020	.60954	1.56106	1
56215	1.28390	.57791	1.36916	.59379	1.46181	.60980	1.56282	2
56241	1.28526	.57817	1.37064	.59406	1.46342	.61007	1.56458	3
56267	1.28663	.57844	1.37212	.59433	1.46504	.61034	1.56634	4
56294	1.28800	.57870	1.37361	.59459	1.46665	.61061	1.56811	5
56320	1.28937	.57896	1.37509	.59486	1.46827	.61088	1.56988	6
56346	1.29074	.57923	1.37658	.59512	1.46989	.61114	1.57165	7
56372	1.29211	.57949	1.37808	.59539	1.47152	.61141	1.57342	8
56398	1.29349	.57976	1.37957	.59566	1.47314	.61168	1.57520	9
56425	1.29487	.58002	1.38107	.59592	1.47477	.61195	1.57698	10
56451	1.29625	.58028	1.38256	.59619	1.47640	.61222	1.57876	11
56477	1.29763	.58055	1.38406	.59645	1.47804	.61248	1.58054	12
56503	1.29901	.58081	1.38556	.59672	1.47967	.61275	1.58233	13
56529	1.30040	.58108	1.38707	.59699	1.48131	.61302	1.58412	14
56555	1.30179	.58134	1.38857	.59725	1.48295	.61329	1.58591	15
56582	1.30318	.58160	1.39008	.59752	1.48459	.61356	1.58771	16
56608	1.30457	.58187	1.39159	.59779	1.48624	.61383	1.58950	17
56634	1.30596	.58213	1.39311	.59805	1.48789	.61409	1.59130	18
56660	1.30735	.58240	1.39462	.59832	1.48954	.61436	1.59311	19
56687	1.30875	.58266	1.39614	.59859	1.49119	.61463	1.59491	20
56713	1.31015	.58293	1.39766	.59885	1.49284	.61490	1.59672	21
56739	1.31155	.58319	1.39918	.59912	1.49450	.61517	1.59853	22
56765	1.31295	.58345	1.40070	.59938	1.49616	.61544	1.60035	23
56791	1.31436	.58372	1.40222	.59965	1.49782	.61570	1.60217	24
56818	1.31576	.58398	1.40375	.59992	1.49948	.61597	1.60399	25
56844	1.31717	.58425	1.40528	.60018	1.50115	.61624	1.60581	26
56870	1.31858	.58451	1.40681	.60045	1.50282	.61651	1.60763	27
56896	1.31999	.58478	1.40835	.60072	1.50449	.61678	1.60946	28
56923	1.32140	.58504	1.40988	.60098	1.50617	.61705	1.61129	29
56949	1.32282	.58531	1.41142	.60125	1.50784	.61732	1.61313	30
56975	1.32424	.58557	1.41296	.60152	1.50952	.61759	1.61496	31
57001	1.32566	.58584	1.41450	.60178	1.51120	.61785	1.61680	32
57028	1.32708	.58610	1.41605	.60205	1.51289	.61812	1.61864	33
57054	1.32850	.58637	1.41760	.60232	1.51457	.61839	1.62049	34
57080	1.32993	.58663	1.41914	.60259	1.51626	.61866	1.62234	35
57106	1.33135	.58690	1.42070	.60285	1.51795	.61893	1.62419	36
57133	1.33278	.58716	1.42225	.60312	1.51965	.61920	1.62604	37
57159	1.33422	.58743	1.42380	.60339	1.52134	.61947	1.62790	38
57185	1.33565	.58769	1.42536	.60365	1.52304	.61974	1.62976	39
57212	1.33708	.58796	1.42692	.60392	1.52474	.62001	1.63162	40
57238	1.33852	.58822	1.42848	.60419	1.52645	.62027	1.63348	41
57264	1.33996	.58849	1.43005	.60445	1.52815	.62054	1.63535	42
57291	1.34140	.58875	1.43162	.60472	1.52986	.62081	1.63722	43
57317	1.34284	.58902	1.43318	.60499	1.53157	.62108	1.63909	44
57343	1.34429	.58928	1.43476	.60526	1.53329	.62135	1.64097	45
57369	1.34573	.58955	1.43633	.60552	1.53500	.62162	1.64285	46
57396	1.34718	.58981	1.43790	.60579	1.53672	.62189	1.64473	47
57422	1.34863	.59008	1.43948	.60606	1.53845	.62216	1.64662	48
.57448	1.35009	.59034	1.44106	.60633	1.54017	.62243	1.64851	49
.57475	1.35154	.59061	1.44264	.60659	1.54190	.62270	1.65040	50
.57501	1.35300	.59087	1.44423	.60686	1.54363	.62297	1.65229	51
.57527	1.35446	.59114	1.44582	.60713	1.54536	.62324	1.65419	52
.57554	1.35592	.59140	1.44741	.60740	1.54709	.62351	1.65609	53
.57580	1.35738	.59167	1.44900	.60766	1.54883	.62378	1.65799	54
.57606	1.35885	.59194	1.45059	.60793	1.55057	.62405	1.65989	55
.57633	1.36031	.59220	1.45219	.60820	1.55231	.62431	1.66180	56
.57659	1.36178	.59247	1.45378	.60847	1.55405	.62458	1.66371	57
.57685	1.36325	.59273	1.45539	.60873	1.55580	.62485	1.66563	58
.57712	1.36473	.59300	1.45699	.60900	1.55755	.62512	1.66755	59
.57738	1.36620	.59326	1.45859	.60927	1.55930	.62539	1.66947	60

	68°		69°		70°		71°	
′	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. se
0	.62539	1.66947	.64163	1.79043	.65798	1.92380	.67443	2.0715
1	.62566	1.67139	.64190	1.79254	.65825	1.92614	.67471	2.0741
2	.62593	1.67332	.64218	1.79466	.65853	1.92849	.67498	2.0767
3	.62620	1.67525	.64245	1.79679	.65880	1.93083	.67526	2.0793
4	.62647	1.67718	.64272	1.79891	.65907	1.93318	.67553	2.0819
5	.62674	1.67911	.64299	1.80104	.65935	1.93554	.67581	2.0845
6	.62701	1.68105	.64326	1.80318	.65962	1.93790	.67608	2.0872
7	.62728	1.68299	.64353	1.80531	.65989	1.94026	.67636	2.0898
8	.62755	1.68494	.64381	1.80746	.66017	1.94263	.67663	2.0924
9	.62782	1.68689	.64408	1.80960	.66044	1.94500	.67691	2.0951
10	.62809	1.68884	.64435	1.81175	.66071	1.94737	.67718	2.0977
11	.62836	1.69079	.64462	1.81390	.66099	1.94975	.67746	2.1003
12	.62863	1.69275	.64489	1.81605	.66126	1.95213	.67773	2.1030
13	.62890	1.69471	.64517	1.81821	.66154	1.95452	.67801	2.1056
14	.62917	1.69667	.64544	1.82037	.66181	1.95691	.67829	2.1083
15	.62944	1.69864	.64571	1.82254	.66208	1.95931	.67856	2.1110
16	.62971	1.70061	.64598	1.82471	.66236	1.96171	.67884	2.1136
17	.62998	1.70258	.64625	1.82688	.66263	1.96411	.67911	2.1163
18	.63025	1.70455	.64653	1.82906	.66290	1.96652	.67939	2.1190
19	.63052	1.70653	.64680	1.83124	.66318	1.96893	.67966	2.1217
20	.63079	1.70851	.64707	1.83342	.66345	1.97135	.67994	2.1244
21	.63106	1.71050	.64734	1.83561	.66373	1.97377	.68021	2.1270
22	.63133	1.71249	.64761	1.83780	.66400	1.97619	.68049	2.1297
23	.63161	1.71448	.64789	1.83999	.66427	1.97862	.68077	2.1324
24	.63188	1.71647	.64816	1.84219	.66455	1.98106	.68104	2.1352
25	.63215	1.71847	.64843	1.84439	.66482	1.98349	.68132	2.1379
26	.63242	1.72047	.64870	1.84659	.66510	1.98594	.68159	2.1406
27	.63269	1.72247	.64898	1.84880	.66537	1.98838	.68187	2.1433
28	.63296	1.72448	.64925	1.85102	.66564	1.99083	.68214	2.1460
29	.63323	1.72649	.64952	1.85323	.66592	1.99329	.68242	2.1488
30	.63350	1.72850	.64979	1.85545	.66619	1.99574	.68270	2.1515
31	.63377	1.73052	.65007	1.85767	.66647	1.99821	.68297	2.1542
32	.63404	1.73254	.65034	1.85990	.66674	2.00067	.68325	2.1570
33	.63431	1.73456	.65061	1.86213	.66702	2.00315	.68352	2.1597
34	.63458	1.73659	.65088	1.86437	.66729	2.00562	.68380	2.1625
35	.63485	1.73862	.65116	1.86661	.66756	2.00810	.68408	2.1653
36	.63512	1.74065	.65143	1.86885	.66784	2.01059	.68435	2.1680
37	.63539	1.74269	.65170	1.87109	.66811	2.01308	.68463	2.1708
38	.63566	1.74473	.65197	1.87334	.66839	2.01557	.68490	2.1736
39	.63594	1.74677	.65225	1.87560	.66866	2.01807	.68518	2.1764
40	.63621	1.74881	.65252	1.87785	.66894	2.02057	.68546	2.1792
41	.63648	1.75086	.65279	1.88011	.66921	2.02308	.68573	2.1819
42	.63675	1.75292	.65306	1.88238	.66949	2.02559	.68601	2.1847
43	.63702	1.75497	.65334	1.88465	.66976	2.02810	.68628	2.1875
44	.63729	1.75703	.65361	1.88692	.67003	2.03062	.68656	2.1904
45	.63756	1.75909	.65388	1.88920	.67031	2.03315	.68684	2.1932
46	.63783	1.76116	.65416	1.89148	.67059	2.03568	.68711	2.1960
47	.63810	1.76323	.65443	1.89376	.67086	2.03821	.68739	2.1988
48	.63838	1.76530	.65470	1.89605	.67113	2.04075	.68767	2.2016
49	.63865	1.76737	.65497	1.89834	.67141	2.04329	.68794	2.2045
50	.63892	1.76945	.65525	1.90063	.67168	2.04584	.68822	2.2073
51	.63919	1.77154	.65552	1.90293	.67196	2.04839	.68849	2.2102
52	.63946	1.77362	.65579	1.90524	.67223	2.05094	.68877	2.2130
53	.63973	1.77571	.65607	1.90754	.67251	2.05350	.68905	2.2159
54	.64000	1.77780	.65634	1.90986	.67278	2.05607	.68932	2.2187
55	.64027	1.77990	.65661	1.91217	.67306	2.05864	.68960	2.2216
56	.64055	1.78200	.65689	1.91449	.67333	2.06121	.68988	2.2245
57	.64082	1.78410	.65716	1.91681	.67361	2.06379	.69015	2.2274
58	.64109	1.78621	.65743	1.91914	.67388	2.06637	.69043	2.2302
59	.64136	1.78832	.65771	1.92147	.67416	2.06896	.69071	2.2331
60	.64163	1.79043	.65798	1.92380	.67443	2.07155	.69098	2.2360

.E X.—NATURAL VERSED SINES AND EXTERNAL SECANTS

72° 73° 74° 75°

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	76° Vers.	76° Ex. sec.	77° Vers.	77° Ex. sec.	78° Vers.	78° Ex. sec.	79° Vers.	79° Ex. sec.	′
0	.75808	3.13357	.77505	3.44541	.79209	3.80973	.80919	4.24084	0
1	.75836	3.13839	.77533	3.45102	.79237	3.81633	.80948	4.24870	1
2	.75864	3.14323	.77562	3.45664	.79266	3.82294	.80976	4.25658	2
3	.75892	3.14809	.77590	3.46228	.79294	3.82956	.81005	4.26448	3
4	.75921	3.15295	.77618	3.46793	.79323	3.83621	.81033	4.27241	4
5	.75949	3.15782	.77647	3.47360	.79351	3.84288	.81062	4.28036	5
6	.75977	3.16271	.77675	3.47928	.79380	3.84956	.81090	4.28833	6
7	.76005	3.16761	.77703	3.48498	.79408	3.85627	.81119	4.29634	7
8	.76034	3.17252	.77732	3.49069	.79437	3.86299	.81148	4.30436	8
9	.76062	3.17744	.77760	3.49642	.79465	3.86973	.81176	4.31241	9
10	.76090	3.18238	.77788	3.50216	.79493	3.87649	.81205	4.32049	10
11	.76118	3.18733	.77817	3.50791	.79522	3.88327	.81233	4.32859	11
12	.76147	3.19228	.77845	3.51368	.79550	3.89007	.81262	4.33671	12
13	.76175	3.19725	.77874	3.51947	.79579	3.89689	.81290	4.34486	13
14	.76203	3.20224	.77902	3.52527	.79607	3.90373	.81319	4.35304	14
15	.76231	3.20723	.77930	3.53109	.79636	3.91058	.81348	4.36124	15
16	.76260	3.21224	.77959	3.53692	.79664	3.91746	.81376	4.36947	16
17	.76288	3.21726	.77987	3.54277	.79693	3.92436	.81405	4.37772	17
18	.76316	3.22229	.78015	3.54863	.79721	3.93128	.81433	4.38600	18
19	.76344	3.22734	.78044	3.55451	.79750	3.93821	.81462	4.39430	19
20	.76373	3.23239	.78072	3.56041	.79778	3.94517	.81491	4.40263	20
21	.76401	3.23746	.78101	3.56632	.79807	3.95215	.81519	4.41099	2.
22	.76429	3.24255	.78129	3.57224	.79835	3.95914	.81548	4.41937	2
23	.76458	3.24764	.78157	3.57819	.79864	3.96616	.81576	4.42778	23
24	.76486	3.25275	.78186	3.58414	.79892	3.97320	.81605	4.43622	24
25	.76514	3.25787	.78214	3.59012	.79921	3.98025	.81633	4.44468	25
26	.76542	3.26300	.78242	3.59611	.79949	3.98733	.81662	4.45317	26
27	.76571	3.26814	.78271	3.60211	.79978	3.99443	.81691	4.46169	27
28	.76599	3.27330	.78299	3.60813	.80006	4.00155	.81719	4.47023	28
29	.76627	3.27847	.78328	3.61417	.80035	4.00869	.81748	4.47881	29
30	.76655	3.28366	.78356	3.62023	.80063	4.01585	.81776	4.48740	30
31	.76684	3.28885	.78384	3.62630	.80092	4.02303	.81805	4.49603	31
32	.76712	3.29406	.78413	3.63238	.80120	4.03024	.81834	4.50468	32
33	.76740	3.29929	.78441	3.63849	.80149	4.03746	.81862	4.51337	33
34	.76769	3.30452	.78470	3.64461	.80177	4.04471	.81891	4.52208	34
35	.76797	3.30977	.78498	3.65074	.80206	4.05197	.81919	4.53081	35
36	.76825	3.31503	.78526	3.65690	.80234	4.05926	.81948	4.53958	36
37	.76854	3.32031	.78555	3.66307	.80263	4.06657	.81977	4.54837	37
38	.76882	3.32560	.78583	3.66925	.80291	4.07390	.82005	4.55720	38
39	.76910	3.33090	.78612	3.67545	.80320	4.08125	.82034	4.56605	39
40	.76938	3.33622	.78640	3.68167	.80348	4.08863	.82063	4.57493	40
41	.76967	3.34154	.78669	3.68791	.80377	4.09602	.82091	4.58383	41
42	.76995	3.34689	.78697	3.69417	.80405	4.10344	.82120	4.59277	42
43	.77023	3.35224	.78725	3.70044	.80434	4.11088	.82148	4.60174	43
44	.77052	3.35761	.78754	3.70673	.80462	4.11835	.82177	4.61073	44
45	.77080	3.36299	.78782	3.71303	.80491	4.12583	.82206	4.61976	45
46	.77108	3.36839	.78811	3.71935	.80520	4.13334	.82234	4.62881	46
47	.77137	3.37380	.78839	3.72569	.80548	4.14087	.82263	4.63790	47
48	.77165	3.37923	.78868	3.73205	.80577	4.14842	.82292	4.64701	48
49	.77193	3.38466	.78896	3.73843	.80605	4.15599	.82320	4.65616	49
50	.77222	3.39012	.78924	3.74482	.80634	4.16359	.82349	4.66533	50
51	.77250	3.39558	.78953	3.75123	.80662	4.17121	.82377	4.67454	51
52	.77278	3.40106	.78981	3.75766	.80691	4.17886	.82406	4.68377	52
53	.77307	3.40656	.79010	3.76411	.80719	4.18652	.82435	4.69304	53
54	.77335	3.41206	.79038	3.77057	.80748	4.19421	.82463	4.70234	54
55	.77363	3.41759	.79067	3.77705	.80776	4.20193	.82492	4.71166	55
56	.77392	3.42312	.79095	3.78355	.80805	4.20966	.82521	4.72102	56
57	.77420	3.42867	.79123	3.79007	.80833	4.21742	.82549	4.73041	57
58	.77448	3.43424	.79152	3.79661	.80862	4.22521	.82578	4.73983	58
59	.77477	3.43982	.79180	3.80316	.80891	4.23301	.82607	4.74929	59
60	.77505	3.44541	.79209	3.80973	.80919	4.24084	.82635	4.75877	60

80°		81°		82°		83°		
Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
.82635	4.75877	.84357	5.39245	.86083	6.18530	.87813	7.20551	0
.82664	4.76829	.84385	5.40422	.86112	6.20020	.87842	7.22500	1
.82692	4.77784	.84414	5.41602	.86140	6.21517	.87871	7.24457	2
.82721	4.78742	.84443	5.42787	.86169	6.23019	.87900	7.26425	3
.82750	4.79703	.84471	5.43977	.86198	6.24529	.87929	7.28402	4
.82778	4.80667	.84500	5.45171	.86227	6.26044	.87957	7.30388	5
.82807	4.81635	.84529	5.46369	.86256	6.27566	.87986	7.32384	6
.82836	4.82606	.84558	5.47572	.86284	6.29095	.88015	7.34390	7
.82864	4.83581	.84586	5.48779	.86313	6.30630	.88044	7.36405	8
.82893	4.84558	.84615	5.49991	.86342	6.32171	.88073	7.38431	9
.82922	4.85539	.84644	5.51208	.86371	6.33719	.88102	7.40466	10
.82950	4.86524	.84673	5.52429	.86400	6.35274	.88131	7.42511	11
.82979	4.87511	.84701	5.53655	.86428	6.36835	.88160	7.44566	12
.83008	4.88502	.84730	5.54886	.86457	6.38403	.88188	7.46632	13
.83036	4.89497	.84759	5.56121	.86486	6.39978	.88217	7.48707	14
.83065	4.90495	.84788	5.57361	.86515	6.41560	.88246	7.50793	15
.83094	4.91496	.84816	5.58606	.86544	6.43148	.88275	7.52889	16
.83122	4.92501	.84845	5.59855	.86573	6.44743	.88304	7.54996	17
.83151	4.93509	.84874	5.61110	.86601	6.46346	.88333	7.57113	18
.83180	4.94521	.84903	5.62369	.86630	6.47955	.88362	7.59241	19
.83208	4.95536	.84931	5.63633	.86659	6.49571	.88391	7.61379	20
.83237	4.96555	.84960	5.64902	.86688	6.51194	.88420	7.63528	21
.83266	4.97577	.84989	5.66176	.86717	6.52825	.88448	7.65688	22
.83294	4.98603	.85018	5.67454	.86746	6.54462	.88477	7.67859	23
.83323	4.99633	.85046	5.68738	.86774	6.56107	.88506	7.70041	24
.83352	5.00666	.85075	5.70027	.86803	6.57759	.88535	7.72234	25
.83380	5.01703	.85104	5.71321	.86832	6.59418	.88564	7.74438	26
.83409	5.02743	.85133	5.72620	.86861	6.61085	.88593	7.76653	27
.83438	5.03787	.85162	5.73924	.86890	6.62759	.88622	7.78880	28
.83467	5.04834	.85190	5.75233	.86919	6.64441	.88651	7.81118	29
.83495	5.05886	.85219	5.76547	.86947	6.66130	.88680	7.83367	30
.83524	5.06941	.85248	5.77866	.86976	6.67826	.88709	7.85628	31
.83553	5.08000	.85277	5.79191	.87005	6.69530	.88737	7.87901	32
.83581	5.09062	.85305	5.80521	.87034	6.71242	.88766	7.90186	33
.83610	5.10129	.85334	5.81856	.87063	6.72962	.88795	7.92482	34
.83639	5.11199	.85363	5.83196	.87092	6.74689	.88824	7.94791	35
.83667	5.12273	.85392	5.84542	.87120	6.76424	.88853	7.97111	36
.83696	5.13350	.85420	5.85893	.87149	6.78167	.88882	7.99444	37
.83725	5.14432	.85449	5.87250	.87178	6.79918	.88911	8.01788	38
.83754	5.15517	.85478	5.88612	.87207	6.81677	.88940	8.04146	39
.83782	5.16607	.85507	5.89979	.87236	6.83443	.88969	8.06515	40
.83811	5.17700	.85536	5.91352	.87265	6.85218	.88998	8.08897	41
.83840	5.18797	.85564	5.92731	.87294	6.87001	.89027	8.11292	42
.83868	5.19898	.85593	5.94115	.87322	6.88792	.89055	8.13699	43
.83897	5.21004	.85622	5.95505	.87351	6.90592	.89084	8.16120	44
.83926	5.22113	.85651	5.96900	.87380	6.92400	.89113	8.18553	45
.83954	5.23226	.85680	5.98301	.87409	6.94216	.89142	8.20999	46
.83983	5.24343	.85708	5.99708	.87438	6.96040	.89171	8.23459	47
.84012	5.25464	.85737	6.01120	.87467	6.97873	.89200	8.25931	48
.84041	5.26590	.85766	6.02538	.87496	6.99714	.89229	8.28417	49
.84069	5.27719	.85795	6.03962	.87524	7.01565	.89258	8.30917	50
.84098	5.28853	.85823	6.05392	.87553	7.03423	.89287	8.33430	51
.84127	5.29991	.85852	6.06828	.87582	7.05291	.89316	8.35957	52
.84155	5.31133	.85881	6.08269	.87611	7.07167	.89345	8.38497	53
.84184	5.32279	.85910	6.09717	.87640	7.09052	.89374	8.41052	54
.84213	5.33429	.85939	6.11171	.87669	7.10946	.89403	8.43620	55
.84242	5.34584	.85967	6.12630	.87698	7.12849	.89431	8.46203	56
.84270	5.35743	.85996	6.14096	.87726	7.14760	.89460	8.48800	57
.84299	5.36906	.86025	6.15568	.87755	7.16681	.89489	8.51411	58
.84328	5.38073	.86054	6.17046	.87784	7.18612	.89518	8.54037	59
.84357	5.39245	.86083	6.18530	.87813	7.20551	.89547	8.56677	60

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

′	84° Vers.	84° Ex. sec.	85° Vers.	85° Ex. sec.	86° Vers.	86° Ex. sec.	′
0	.89547	8.56677	.91284	10.47371	.93024	13.33559	0
1	.89576	8.59332	.91313	10.51199	.93053	13.39547	1
2	.89605	8.62002	.91342	10.55052	.93082	13.45586	2
3	.89634	8.64687	.91371	10.58932	.93111	13.51676	3
4	.89663	8.67387	.91400	10.62837	.93140	13.57817	4
5	.89692	8.70103	.91429	10.66769	.93169	13.64011	5
6	.89721	8.72833	.91458	10.70728	.93198	13.70258	6
7	.89750	8.75579	.91487	10.74714	.93227	13.76558	7
8	.89779	8.78341	.91516	10.78727	.93257	13.82913	8
9	.89808	8.81119	.91545	10.82768	.93286	13.89323	9
10	.89836	8.83912	.91574	10.86837	.93315	13.95788	10
11	.89865	8.86722	.91603	10.90934	.93344	14.02310	11
12	.89894	8.89547	.91632	10.95060	.93373	14.08890	12
13	.89923	8.92389	.91661	10.99214	.93402	14.15527	13
14	.89952	8.95248	.91690	11.03397	.93431	14.22223	14
15	.89981	8.98123	.91719	11.07610	.93460	14.28979	15
16	.90010	9.01015	.91748	11.11852	.93489	14.35795	16
17	.90039	9.03923	.91777	11.16125	.93518	14.42672	17
18	.90068	9.06849	.91806	11.20427	.93547	14.49611	18
19	.90097	9.09792	.91835	11.24761	.93576	14.56614	19
20	.90126	9.12752	.91864	11.29125	.93605	14.63679	20
21	.90155	9.15730	.91893	11.33521	.93634	14.70810	21
22	.90184	9.18725	.91922	11.37948	.93663	14.78005	22
23	.90213	9.21739	.91951	11.42408	.93692	14.85268	23
24	.90242	9.24770	.91980	11.46900	.93721	14.92597	24
25	.90271	9.27819	.92009	11.51424	.93750	14.99995	25
26	.90300	9.30887	.92038	11.55982	.93779	15.07462	26
27	.90329	9.33973	.92067	11.60572	.93808	15.14999	27
28	.90358	9.37077	.92096	11.65197	.93837	15.22607	28
29	.90386	9.40201	.92125	11.69856	.93866	15.30287	29
30	.90415	9.43343	.92154	11.74550	.93895	15.38041	30
31	.90444	9.46505	.92183	11.79278	.93924	15.45869	31
32	.90473	9.49685	.92212	11.84042	.93953	15.53772	32
33	.90502	9.52886	.92241	11.88841	.93982	15.61751	33
34	.90531	9.56106	.92270	11.93677	.94011	15.69808	34
35	.90560	9.59346	.92299	11.98549	.94040	15.77944	35
36	.90589	9.62605	.92328	12.03458	.94069	15.86159	36
37	.90618	9.65885	.92357	12.08404	.94098	15.94456	37
38	.90647	9.69186	.92386	12.13388	.94127	16.02835	38
39	.90676	9.72507	.92415	12.18411	.94156	16.11297	39
40	.90705	9.75849	.92444	12.23472	.94186	16.19843	40
41	.90734	9.79212	.92473	12.28572	.94215	16.28476	41
42	.90763	9.82596	.92502	12.33712	.94244	16.37196	42
43	.90792	9.86001	.92531	12.38891	.94273	16.46005	43
44	.90821	9.89428	.92560	12.44112	.94302	16.54903	44
45	.90850	9.92877	.92589	12.49373	.94331	16.63893	45
46	.90879	9.96348	.92618	12.54676	.94360	16.72975	46
47	.90908	9.99841	.92647	12.60021	.94389	16.82152	47
48	.90937	10.03356	.92676	12.65408	.94418	16.91424	48
49	.90966	10.06894	.92705	12.70838	.94447	17.00794	49
50	.90995	10.10455	.92734	12.76312	.94476	17.10262	50
51	.91024	10.14039	.92763	12.81829	.94505	17.19830	51
52	.91053	10.17646	.92792	12.87391	.94534	17.29501	52
53	.91082	10.21277	.92821	12.92999	.94563	17.39274	53
54	.91111	10.24932	.92850	12.98651	.94592	17.49153	54
55	.91140	10.28610	.92879	13.04350	.94621	17.59139	55
56	.91169	10.32313	.92908	13.10096	.94650	17.69233	56
57	.91197	10.36040	.92937	13.15889	.94679	17.79438	57
58	.91226	10.39792	.92966	13.21730	.94708	17.89755	58
59	.91255	10.43569	.92995	13.27620	.94737	18.00185	59
60	.91284	10.47371	.93024	13.33559	.94766	18.10732	60

87°		88°		89°		
Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	′
.94766	18.10732	.96510	27.65371	.98255	56.29869	0
.94795	18.21397	.96539	27.89440	.98284	57.26976	1
.94825	18.32182	.96568	28.13917	.98313	58.27431	2
.94854	18.43088	.96597	28.38812	.98342	59.31411	3
.94883	18.54119	.96626	28.64137	.98371	60.39105	4
.94912	18.65275	.96655	28.89903	.98400	61.50715	5
.94941	18.76560	.96684	29.16120	.98429	62.66460	6
.94970	18.87976	.96714	29.42802	.98458	63.86572	7
.94999	18.99524	.96743	29.69960	.98487	65.11304	8
.95028	19.11208	.96772	29.97607	.98517	66.40927	9
.95057	19.23028	.96801	30.25758	.98546	67.75736	10
.95086	19.34989	.96830	30.54425	.98575	69.16047	11
.95115	19.47093	.96859	30.83623	.98604	70.62205	12
.95144	19.59341	.96888	31.13366	.98633	72.14583	13
.95173	19.71737	.96917	31.43671	.98662	73.73586	14
.95202	19.84283	.96946	31.74554	.98691	75.39655	15
.95231	19.96982	.96975	32.06030	.98720	77.13274	16
.95260	20.09838	.97004	32.38118	.98749	78.94968	17
.95289	20.22852	.97033	32.70835	.98778	80.85315	18
.95318	20.36027	.97062	33.04199	.98807	82.84947	19
.95347	20.49368	.97092	33.38232	.98836	84.94561	20
.95377	20.62876	.97121	33.72952	.98866	87.14924	21
.95406	20.76555	.97150	34.08380	.98895	89.46886	22
.95435	20.90409	.97179	34.44539	.98924	91.91387	23
.95464	21.04440	.97208	34.81452	.98953	94.49471	24
.95493	21.18653	.97237	35.19141	.98982	97.22303	25
.95522	21.33050	.97266	35.57633	.99011	100.1119	26
.95551	21.47635	.97295	35.96953	.99040	103.1757	27
.95580	21.62413	.97324	36.37127	.99069	106.4311	28
.95609	21.77386	.97353	36.78185	.99098	109.8966	29
.95638	21.92559	.97382	37.20155	.99127	113.5930	30
.95667	22.07935	.97411	37.63068	.99156	117.5444	31
.95696	22.23520	.97440	38.06957	.99186	121.7780	32
.95725	22.39316	.97470	38.51855	.99215	126.3253	33
.95754	22.55328	.97499	38.97797	.99244	131.2223	34
.95783	22.71563	.97528	39.44820	.99273	136.5111	35
.95812	22.88022	.97557	39.92963	.99302	142.2406	36
.95842	23.04712	.97586	40.42266	.99331	148.4684	37
.95871	23.21637	.97615	40.92772	.99360	155.2623	38
.95900	23.38802	.97644	41.44525	.99389	162.7033	39
.95929	23.56212	.97673	41.97571	.99418	170.8883	40
.95958	23.73873	.97702	42.51961	.99447	179.9350	41
.95987	23.91790	.97731	43.07746	.99476	189.9868	42
.96016	24.09969	.97760	43.64980	.99505	201.2212	43
.96045	24.28414	.97789	44.23720	.99535	213.8600	44
.96074	24.47134	.97819	44.84026	.99564	228.1839	45
.96103	24.66132	.97848	45.45963	.99593	244.5540	46
.96132	24.85417	.97877	46.09596	.99622	263.4427	47
.96161	25.04994	.97906	46.74997	.99651	285.4795	48
.96190	25.24869	.97935	47.42241	.99680	311.5230	49
.96219	25.45051	.97964	48.11406	.99709	342.7752	50
.96248	25.65546	.97993	48.82576	.99738	380.9723	51
.96277	25.86360	.98022	49.55840	.99767	428.7187	52
.96307	26.07503	.98051	50.31290	.99796	490.1070	53
.96336	26.28961	.98080	51.09027	.99825	571.9581	54
.96365	26.50804	.98109	51.89156	.99855	686.5496	55
.96394	26.72978	.98138	52.71790	.99884	858.4369	56
.96423	26.95513	.98168	53.57046	.99913	1144.916	57
.96452	27.18417	.98197	54.45053	.99942	1717.874	58
.96481	27.41700	.98226	55.35946	.99971	3436.747	59
.96510	27.65371	.98255	56.29869	1.00000	Infinite	60

TABLE XI.—REDUCTION OF BAROMETER READING TO 32° F.

Temp. ° Fahr.	Inches.										
	26.0	26.5	27.0	27.5	28.0	28.5	29.0	29.5	30.0	30.5	31.0
45	−.039	−.039	−.040	−.041	−.042	.042	−.043	−.044	−.045	−.045	−.046
46	.041	.042	.043	.043	.044	.045	.046	.046	.047	.048	.049
47	.043	.044	.045	.046	.047	.048	.048	.049	.050	.051	.052
48	.046	.047	.047	.048	.049	.050	.051	.052	.053	.053	.054
49	.048	.049	.050	.051	.052	.052	.054	.054	.055	.056	.057
50	.050	.051	.052	.053	.054	.055	.056	.057	.058	.059	.060
51	.053	.054	.055	.056	.057	.058	.059	.060	.061	.062	.063
52	.055	.056	.057	.058	.059	.060	.061	.062	.064	.065	.066
53	.057	.058	.060	.061	.062	.063	.064	.065	.066	.067	.068
54	.060	.061	.062	.063	.064	.065	.067	.068	.069	.070	.071
55	.062	.063	.064	.065	.066	.068	.069	.070	.071	.073	.074
56	.064	.065	.067	.068	.069	.070	.072	.073	.074	.075	.077
57	.067	.068	.069	.070	.072	.073	.075	.076	.077	.078	.080
58	.069	.070	.071	.073	.074	.076	.077	.078	.080	.081	.082
59	.072	.073	.074	.075	.077	.078	.080	.081	.083	.084	.085
60	.074	.076	.077	.078	.079	.081	.082	.084	.085	.086	.088
61	.076	.077	.079	.080	.082	.083	.085	.086	.088	.089	.091
62	.079	.080	.082	.083	.085	.086	.088	.089	.091	.092	.094
63	.081	.082	.084	.085	.087	.088	.090	.091	.093	.095	.096
64	.083	.085	.086	.088	.090	.091	.093	.094	.096	.097	.099
65	.086	.087	.089	.090	.092	.093	.095	.097	.099	.100	.102
66	.088	.089	.091	.093	.095	.096	.098	.099	.101	.103	.105
67	.090	.092	.094	.095	.097	.099	.101	.102	.104	.106	.108
68	.093	.094	.096	.098	.100	.101	.103	.105	.107	.108	.110
69	.095	.097	.099	.100	.102	.104	.106	.107	.110	.111	.113
70	.097	.099	.101	.103	.105	.106	.109	.110	.112	.114	.116
71	.100	.101	.103	.105	.107	.109	.111	.113	.115	.117	.119
72	.102	.104	.106	.108	.110	.112	.114	.116	.118	.120	.122
73	.104	.106	.108	.110	.112	.114	.116	.118	.120	.122	.124
74	.107	.109	.111	.113	.115	.117	.119	.121	.123	.125	.127
75	.109	.111	.113	.115	.117	.119	.122	.124	.126	.128	.130
76	.111	.113	.116	.118	.120	.122	.124	.126	.128	.130	.133
77	.114	.116	.118	.120	.122	.124	.127	.129	.131	.133	.136
78	.116	.118	.120	.122	.125	.127	.129	.131	.134	.136	.138
79	.118	.120	.123	.125	.127	.129	.132	.134	.137	.139	.141
80	.121	.123	.125	.127	.130	.132	.135	.137	.139	.141	.144
81	.123	.125	.128	.130	.132	.134	.137	.139	.142	.144	.147
82	.125	.128	.130	.132	.135	.137	.140	.142	.145	.147	.149
83	.128	.130	.133	.135	.138	.140	.142	.145	.147	.149	.152
84	.130	.132	.135	.138	.140	.142	.145	.147	.150	.152	.155
85	.132	.134	.137	.140	.143	.145	.148	.150	.153	.155	.158
86	.135	.137	.140	.142	.145	.148	.150	.153	.155	.158	.161
87	.137	.139	.142	.144	.148	.150	.153	.155	.158	.161	.163
88	.139	.142	.145	.147	.150	.152	.155	.158	.161	.163	.166
89	.142	.144	.147	.150	.153	.155	.158	.161	.164	.166	.169
90	.144	.147	.150	.153	.155	.158	.161	.164	.166	.169	.172
91	−.146	−.149	−.152	−.155	−.158	−.160	−.163	−.166	−.169	−.172	−.175

TABLE XII.—BAROMETRIC ELEVATIONS.*

B	*A*	Diff. for .01.	*B*	*A*	Diff. for .01.	*B*	*A*	Diff. for .01.
Inches.	Feet.	Feet.	Inches.	Feet.	Feet.	Inches.	Feet.	Feet.
20·0	11,047	−13·6	23·7	6,423	−11·5	27·4	2,470	−9·9
20·1	10,911	13·5	23·8	6,308	11·4	27·5	2,371	9·9
20·2	10,776	13·4	23·9	6,194	11·4	27·6	2,272	9·9
20·3	10,642	13·4	24·0	6,080	11·3	27·7	2,173	9·8
20·4	10,508	13·3	24·1	5,967	11·3	27·8	2,075	9·8
20·5	10,375	13·3	24·2	5,854	11·3	27·9	1,977	9·7
20·6	10,242	13·2	24·3	5,741	11·2	28·0	1,880	9·7
20·7	10,110	13·1	24·4	5,629	11·1	28·1	1,783	9·7
20·8	9,979	13·1	24·5	5,518	11·1	28·2	1 686	9·7
20·9	9,848	13·0	24·6	5.407	11·1	28·3	1,589	9·6
21·0	9,718	12·9	24·7	5,296	11·0	28·4	1,493	9·6
21·1	9,589	12·9	24·8	5,186	10·9	28·5	1,397	9·5
21·2	9,460	12·8	24·9	5,077	10·9	28·6	1,302	9·5
21·3	9,332	12·8	25·0	4.968	10·9	28·7	1,207	9·5
21·4	9.204	12·7	25·1	4,859	10·8	28·8	1,112	9·4
21·5	9,077	12·6	25·2	4,751	10·8	28·9	1,018	9·4
21·6	8,951	12·6	25·3	4,643	10·8	29·0	924	9·4
21·7	8,825	12·5	25·4	4.535	10·7	29·1	830	9·4
21·8	8,700	12·5	25·5	4,428	10·7	29·2	736	9·3
21·9	8,575	12·4	25·6	4,321	10·6	29·3	643	9·3
22·0	8,451	12·4	25·7	4,215	10·6	29·4	550	9·2
22·1	8,327	12·3	25·8	4.109	10·5	29·5	458	9·2
22·2	8,204	12·2	25·9	4,004	10·5	29·6	366	9·2
22·3	8,082	12·2	26·0	3 899	10·4	29·7	274	9·2
22·4	7,960	12·2	26·1	3,794	10·4	29·8	182	9·1
22·5	7,838	12·1	26·2	3 690	10·4	29·9	91	9·1
22·6	7,717	12·0	26·3	3,586	10·3	30·0	0	9·1
22·7	7,597	12·0	26·4	3,483	10·3	30·1	−91	9·0
22·8	7,477	11·9	26·5	3,380	10·3	30·2	181	9·0
22·9	7,358	11·9	26·6	3,277	10·2	30·3	271	9·0
23·0	7,239	11·8	26·7	3,175	10·2	30·4	361	9·0
23·1	7,121	11·7	26·8	3,073	10·1	30·5	451	8·9
23·2	7,004	11·7	26·9	2·972	10·1	30·6	540	8·9
23·3	6,887	11·7	27·0	2·871	10·1	30·7	629	8·8
23·4	6,770	11·6	27·1	2 770	10·0	30·8	717	8·8
23·5	6,554	11·6	27·2	2,670	10·0	30·9	805	−8·8
23·6	6,538	−11·5	27·3	2,570	−10·0	31·0	−893	
23·7	6,423		27·4	2.470				

* Compiled from Report of U. S. C. & G. Survey for 1881, App. 10 ·Table XI.

TABLE XIII.—COEFFICIENTS FOR CORRECTIONS FOR TEMPERATURE AND HUMIDITY.*

$t+t'$	*C*	Diff. for 1°.	$t+t'$	*C*	Diff. for 1°.	$t+t'$	*C*	Diff. for 1°.
0°	−.1024	10·9	60°	−.0380	10·7	120°	+.0262	10·6
10	.0915	10·9	70	.0273	10·7	130	.0368	10·4
20	.0806	10·8	80	.0166	10·8	140	.0472	10·3
30	.0698	10·6	90	−.0058	10·7	150	.0575	10·2
40	.0592	10·6	100	+.0049	10·7	160	.0677	10·2
50	.0486	10·6	110	.0156	10·6	170	.0779	10·0
60	−.0380		120	+.0262		180	+.0879	

* Compiled from Report of U. S. C. & G. Survey for 1881, App. 10, Tables I, IV.

1	$\sin a$	$=\dfrac{1}{\operatorname{cosec} a}=\dfrac{\tan a}{\sqrt{1+\tan^2 a}}=\sqrt{\dfrac{1-\cos 2a}{2}}=\dfrac{1}{\sqrt{1+\cot^2 a}}$
		$=\cos a \tan a=\sqrt{1-\cos^2 a}=2\sin\frac{1}{2}a\cos\frac{1}{2}a$
		$=\dfrac{1+\cos a}{\cot\frac{1}{2}a}=\dfrac{2\tan\frac{1}{2}a}{1+\tan^2\frac{1}{2}a}=\operatorname{vers} a\cot\frac{1}{2}a.$
2	$\cos a$	$=\dfrac{1}{\sec a}=\dfrac{\cot a}{\sqrt{1+\cot^2 a}}=\dfrac{1}{\sqrt{1+\tan^2 a}}$
		$=1-\operatorname{vers} a=\sin a\cot a=\sqrt{1-\sin^2 a}=2\cos^2\frac{1}{2}a-1$
		$=\sin a\cot\frac{1}{2}a-1=\cos^2\frac{1}{2}a-\sin^2\frac{1}{2}a=1-2\sin^2\frac{1}{2}a.$
3	$\tan a$	$=\dfrac{1}{\cot a}=\dfrac{\sin a}{\cos a}=\dfrac{\sec a}{\operatorname{cosec} a}=\dfrac{1}{\sqrt{\operatorname{cosec}^2 a-1}}$
		$=\operatorname{vers} 2a\operatorname{cosec} 2a=\cot a-2\cot 2a=\sin a\sec a$
		$=\dfrac{\sin 2a}{1+\cos 2a}=\operatorname{exsec} a\cot\frac{1}{2}a=\operatorname{exsec} 2a\cot 2a.$
4	$\cot a$	$=\dfrac{1}{\tan a}=\dfrac{\cos a}{\sin a}=\dfrac{\sin 2a}{1-\cos 2a}=\dfrac{1+\cos 2a}{\sin 2a}$
		$=\sqrt{\operatorname{cosec}^2 a-1}=\cot\frac{1}{2}a-\operatorname{cosec} a.$
5	$\operatorname{vers} a$	$=1-\cos a=\sin a\tan\frac{1}{2}a=2\sin^2\frac{1}{2}a=\cos a\operatorname{exsec} a.$
6	$\operatorname{exsec} a$	$=\sec a-1=\tan a\tan\frac{1}{2}a=\operatorname{vers} a\sec a.$
7	$\sin\frac{1}{2}a$	$=\sqrt{\dfrac{\operatorname{vers} a}{2}}=\dfrac{\sin a}{2\cos\frac{1}{2}a}=\dfrac{\operatorname{vers} a\cos\frac{1}{2}a}{\sin a}.$
8	$\cos\frac{1}{2}a$	$=\sqrt{\dfrac{1+\cos a}{2}}=\dfrac{\sin a}{2\sin\frac{1}{2}a}=\dfrac{\sin a\sin\frac{1}{2}a}{\operatorname{vers} a}.$
9	$\tan\frac{1}{2}a$	$=\operatorname{vers} a\operatorname{cosec} a=\operatorname{cosec} a-\cot a=\dfrac{\tan a}{1+\sec a}.$
10	$\cot\frac{1}{2}a$	$=\dfrac{1+\cos a}{\sin a}=\operatorname{cosec} a+\cot a=\dfrac{\tan a}{\operatorname{exsec} a}=\dfrac{1}{\operatorname{cosec} a-\cot a}.$
11	$\operatorname{vers}\frac{1}{2}a$	$=1-\sqrt{\frac{1}{2}(1+\cos a)}.$
12	$\operatorname{exsec}\frac{1}{2}a$	$=\dfrac{1}{\sqrt{\frac{1}{2}(1+\cos a)}}-1.$

13 $\sin 2a = 2\sin a\cos a = \dfrac{2\tan a}{1+\tan^2 a}$.

14 $\cos 2a = \cos^2 a - \sin^2 a = 1 - 2\sin^2 a = 2\cos^2 a - 1$

$= \dfrac{1-\tan^2 a}{1+\tan^2 a}$.

15 $\tan 2a = \dfrac{2\tan a}{1-\tan^2 a}$.

16 $\cot 2a = \frac{1}{2}\cot a - \frac{1}{2}\tan a = \dfrac{\cot^2 a - 1}{2\cot a} = \dfrac{1-\tan^2 a}{2\tan a}$.

17 $\operatorname{vers} 2a = 2\sin^2 a = 1 - \cos 2a = 2\sin a\cos a\tan a$.

18 $\operatorname{exsec} 2a = \dfrac{\tan 2a}{\cot a} = \dfrac{2\tan^2 a}{1-\tan^2 a} = \dfrac{2\sin^2 a}{1-2\sin^2 a}$.

19 $\sin(a \pm b) = \sin a\cos b \pm \cos a\sin b$.

20 $\cos(a \pm b) = \cos a\cos b \mp \sin a\sin b$.

21 $\sin a + \sin b = 2\sin\frac{1}{2}(a+b)\cos\frac{1}{2}(a-b)$.

22 $\sin a - \sin b = 2\sin\frac{1}{2}(a-b)\cos\frac{1}{2}(a+b)$.

23 $\cos a + \cos b = 2\cos\frac{1}{2}(a+b)\cos\frac{1}{2}(a-b)$.

24 $\cos a - \cos b = -2\sin\frac{1}{2}(a+b)\sin\frac{1}{2}(a-b)$.

Call the sides of any triangle A, B, C, and the opposite angles a, b, and c. Call $s = \frac{1}{2}(A+B+C)$.

25 $\tan\frac{1}{2}(a-b) = \dfrac{A-B}{A+B}\tan\frac{1}{2}(a+b) = \dfrac{A-B}{A+B}\cot\frac{1}{2}c$.

26 $C = (A+B)\dfrac{\cos\frac{1}{2}(a+b)}{\cos\frac{1}{2}(a-b)} = (A-B)\dfrac{\sin\frac{1}{2}(a+b)}{\sin\frac{1}{2}(a-b)}$.

27 $\sin\frac{1}{2}a = \sqrt{\dfrac{(s-B)(s-C)}{BC}}$.

28 $\cos\frac{1}{2}a = \sqrt{\dfrac{s(s-A)}{BC}}$.

29 $\operatorname{vers} a = \dfrac{2(s-B)(s-C)}{BC}$.

30 $\text{Area} = \sqrt{s(s-A)(s-B)(s-C)} = A^2\dfrac{\sin b\sin c}{2\sin a}$.

		Logarithm.
Circumference of a circle (radius $= r$) $= 2\pi r$.		
Area of a circle $= \pi r^2$.		
Area of sector (length of arc $= l$) $= \frac{1}{2} lr$.		
" " " (angle of arc $= a°$) $= \frac{a}{360} \pi r^2$.		
Area of segment (chord $= c$, mid. ord. $= m$) $= \frac{2}{3} cm$ (approx.).		
Area of a circle to radius 1 / Circumference of a circle to diameter 1 / Surface of a sphere to diameter 1 $\} = \pi =$	3.1415927	0.497 1499
Volume of a sphere to radius 1 $= 4\pi \div 3 =$	4.1887902	0.622 0886
Arc equal to radius expressed in degrees =	57.2957795	1.758 1226
Arc equal to radius expressed in minutes =	3437.7467708	3.536 2739
Arc equal to radius expressed in seconds =	206264.8062471	5.314 4251
Length of arc of 1°, radius unity	0.01745329	8.241 8774
Sine of one second = 0.0000048481		4.685 5749
Cubic inches in United States standard gallon = 231		2.363 6120
Weight of one cubic foot of water at maximum density (therm. 39°.8 F., barom. 30″)	62.379	1.795 0384
Weight of one cubic foot of water at ordinary temperature (therm. 62° F.)	62.321	1.794 6349
Acceleration due to gravity at latitude of New York in feet per square second	32.15945	1.507 3086
Feet in one metre	3.280869	0.515 9889
Metres in one foot	0.304797	9.484 0111

TABLE XVI.—SQUARES, CUBES, SQUARE ROOTS,

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
1	1	1	1.0000000	1.0000000	1.000000000
2	4	8	1.4142136	1.2599210	.500000000
3	9	27	1.7320508	1.4422496	.333333333
4	16	64	2.0000000	1.5874011	.250000000
5	25	125	2.2360680	1.7099759	.200000000
6	36	216	2.4494897	1.8171206	.166666667
[illegible]	49	343	2.6457513	1.9129312	.142857143
8	64	512	2.8284271	2.0000000	.125000000
9	81	729	3.0000000	2.0800837	.111111111
10	100	1000	3.1622777	2.1544347	.100000000
11	121	1331	3.3166248	2.2239801	.090909091
12	144	1728	3.4641016	2.2894286	.083333333
13	169	2197	3.6055513	2.3513347	.076923077
14	196	2744	3.7416574	2.4101422	.071428571
15	225	3375	3.8729833	2.4[illegible]21	.066666667
16	256		4.0000000	2.5198421	.062500000
17	289		4.1231056	2.5712816	.058823529
18	324		4.2426407	2.6207414	.055555556
19	361		4 3588989	2.6684016	.052631579
20	400		4.4721360	2.7144177	.050000000
21	441		4.5825757	2.7589243	.047619048
22	48[illegible]		4.6904158	2.8020393	.045454545
23	529		4.7958315	2.8438670	.043478261
24	576		4.8989795	2.8844991	.041666667
25	625		5.0000000	2.9240177	.040000000
26	676		5.0990195	2.9624960	.038461538
27	729		5.1961524	3.0000000	.037037037
28	784		5.2915026	3.0365889	.035714286
29	841		5.3851648	3.0723168	.034482759
30	900		5.4772256	3.1072325	.033333333
31	961	29791	5.5677644	3.1413806	.032258065
32	1024	32768	5.6568542	3.1748021	.031250000
33	1089	35937	5.7445626	3.2075343	.030303030
34	1156	[illegible]	5.[illegible]	3.2396118	.029411765
35	1225	42875	5.9160798	3.2710663	.028571429
36	1296	46656	6.0000000	3.3019272	.027777778
37	1369		6.0827625	3.3322218	.027027027
38	[illegible]		6.1644140	3.36[illegible]4	.026315789
39	1521		6.2449980	3.39[illegible]4	.025641026
40	1600		6.3245553	3 4199519	.025000000
41	1681	68921	6.4031242	3.4482172	.024390244
42	1764	74088	6.4807407	3.4760266	.023809524
43	1849	79507	6.5574385	3.5033981	.023255814
44	1936	85184	6.6332496	3.5303483	.022727273
45	2025	[illegible]	6.7082039	3.5568933	.022222222
46	2116	97336	6.7823300	3.5830479	.021739130
47	2209	103823	6.8556546	3.6088261	.021276600
48	2304	110592	6.9282032	3.6342411	.020833333
49	2401	117649	7.0000000	3.6593057	.020408163
50	2500	125000	7.0710678	3.6840314	.020000000
51	2601	132651	7.1414284	3.7084298	.019607843
52	2704	[illegible]	7.2111026	3.7325111	.019230769
53	2809	148877	7.2801099	3.7562858	.018867925
54	2916	[illegible]	7.[illegible]	3.7797631	.018518519
55	3025	166375	7.[illegible]	3.8029525	.018181818
56	3136	175616	7.[illegible]	3.8258624	.017857143
57	3249	185193	7.5498344	3.8485011	.017543860
58	3364	195112	7.6157731	3.8708766	.017241379
59	3481	205379	7.[illegible]	3.8929965	.016949153
60	3600	216000	7.[illegible]	3.9148676	.016666667

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
61	3721	226981	7.8102497	3.9364972	.016393443
62	3844	238328	7.8740079	3.9578915	.016129032
63	3969	250047	7.9372539	3.9790571	.015873016
64	4096	262144	8.0000000	4.0000000	.015625000
65	4225	274625	8.0622577	4.0207256	.015384615
66	4356	287496	8.1240384	4.0412401	.015151515
67	4489	300763	8.1853528	4.0615480	.014925373
68	4624	314432	8.2462113	4.0816551	.014705882
69	4761	328509	8.3066239	4.1015661	.014492754
70	4900	343000	8.3666003	4.1212853	.014285714
71	5041	357911	8.4261498	4.1408178	.014084507
72	5184	373248	8.4852814	4.1601676	.013888889
73	5329	389017	8.5440037	4.1793390	.013698630
74	5476	405224	8.6023253	4.1983364	.013513514
75	5625	421875	8.6602540	4.2171633	.013333333
76	5776	438976	8.7177979	4.2358236	.013157895
77	5929	456533	8.7749644	4.2543210	.012987013
78	6084	474552	8.8317609	4.2726586	.012820513
79	6241	493039	8.8881944	4.2908404	.012658228
80	6400	512000	8.9442719	4.3088695	.012500000
81	6561	531441	9.0000000	4.3267487	.012345679
82	6724	551368	9.0553851	4.3444815	.012195122
83	6889	571787	9.1104336	4.3620707	.012048193
84	7056	592704	9.1651514	4.3795191	.011904762
85	7225	614125	9.2195445	4.3968296	.011764706
86	7396	636056	9.2736185	4.4140049	.011627907
87	7569	658503	9.3273791	4.4310476	.011494253
88	7744	681472	9.3808315	4.4479602	.011363636
89	7921	704969	9.4339811	4.4647451	.011235955
90	8100	729000	9.4868330	4.4814047	.011111111
91	8281	753571	9.5393920	4.4979414	.010989011
92	8464	778688	9.5916630	4.5143574	.010869565
93	8649	804357	9.6436508	4.5306549	.010752688
94	8836	830584	9.6953597	4.5468359	.010638298
95	9025	857375	9.7467943	4.5629026	.010526316
96	9216	884736	9.7979590	4.5788570	.010416667
97	9409	912673	9.8488578	4.5947009	.010309278
98	9604	941192	9.8994949	4.6104363	.010204082
99	9801	970299	9.9498744	4.6260650	.010101010
100	10000	1000000	10.0000000	4.6415888	.010000000
101	10201	1030301	10.0498756	4.6570095	.009900990
102	10404	1061208	10.0995049	4.6723287	.009803922
103	10609	1092727	10.1488916	4.6875482	.009708738
104	10816	1124864	10.1980390	4.7026694	.009615385
105	11025	1157625	10.2469508	4.7176940	.009523810
106	11236	1191016	10.2956301	4.7326235	.009433962
107	11449	1225043	10.3440804	4.7474594	.009345794
108	11664	1259712	10.3923048	4.7622032	.009259259
109	11881	1295029	10.4403065	4.7768562	.009174312
110	12100	1331000	10.4880885	4.7914199	.009090909
111	12321	1367631	10.5356538	4.8058955	.009009009
112	12544	1404928	10.5830052	4.8202845	.008928571
113	12769	1442897	10.6301458	4.8345881	.008849558
114	12996	1481544	10.6770783	4.8488076	.008771930
115	13225	1520875	10.7238053	4.8629442	.008695652
116	13456	1560896	10.7703296	4.8769990	.008620690
117	13689	1601613	10.8166538	4.8909732	.008547009
118	13924	1643032	10.8627805	4.9048681	.008474576
119	14161	1685159	10.9087121	4.9186847	.008403361
120	14400	1728000	10.9544512	4.9324242	.008333333

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
181	32761	5929741	13.4536240	5.6566528	.005524862
182	33124	6028568	13.4907376	5.6670511	.005494505
183	33489	6128487	13.5277493	5.6774114	.005464481
184	33856	6229504	13.5646600	5.6877340	.005434783
185	34225	6331625	13.6014705	5.6980192	.005405405
186	34596	6434856	13.6381817	5.7082675	.005376344
187	34969	6539203	13.6747943	5.7184791	.005347594
188	35344	6644672	13.7113092	5.7286543	.005319149
189	35721	6751269	13.7477271	5.7387936	.005291005
190	36100	6859000	13.7840488	5.7488971	.005263158
191	36481	6967871	13.8202750	5.7589652	.005235602
192	36864	7077888	13.8564065	5.7689982	.005208333
193	37249	7189057	13.8924440	5.7789966	.005181347
194	37636	7301384	13.9283883	5.7889604	.005154639
195	38025	7414875	13.9642400	5.7988900	.005128205
196	38416	7529536	14.0000000	5.8087857	.005102041
197	38809	7645373	14.0356688	5.8186479	.005076142
198	39204	7762392	14.0712473	5.8284767	.005050505
199	39601	7880599	14.1067360	5.8382725	.005025126
200	40000	8000000	14.1421356	5.8480355	.005000000
201	40401	8120601	14.1774469	5.8577660	.004975124
202	40804	8242408	14.2126704	5.8674643	.004950495
203	41209	8365427	14.2478068	5.8771307	.004926108
204	41616	8489664	14.2828569	5.8867653	.004901961
205	42025	8615125	14.3178211	5.8963685	.004878049
206	42436	8741816	14.3527001	5.9059406	.004854369
207	42849	8869743	14.3874946	5.9154817	.004830918
208	43264	8998912	14.4222051	5.9249921	.004807692
209	43681	9129329	14.4568323	5.9344721	.004784689
210	44100	9261000	14.4913767	5.9439220	.004761905
211	44521	9393931	14.5258390	5.9533418	.004739336
212	44944	9528128	14.5602198	5.9627320	.004716981
213	45369	9663597	14.5945195	5.9720926	.004694836
214	45796	9800344	14.6287388	5.9814240	.004672897
215	46225	9938375	14.6628783	5.9907264	.004651163
216	46656	10077696	14.6969385	6.0000000	.004629630
217	47089	10218313	14.7309199	6.0092450	.004608295
218	47524	10360232	14.7648231	6.0184617	.004587156
219	47961	10503459	14.7986486	6.0276502	.004566210
220	48400	10648000	14.8323970	6.0368107	.004545455
221	48841	10793861	14.8660687	6.0459435	.004524887
222	49284	10941048	14.8996644	6.0550489	.004504505
223	49729	11089567	14.9331845	6.0641270	.004484305
224	50176	11239424	14.9666295	6.0731779	.004464286
225	50625	11390625	15.0000000	6.0822020	.004444444
226	51076	11543176	15.0332964	6.0911994	.004424779
227	51529	11697083	15.0665192	6.1001702	.004405286
228	51984	11852352	15.0996689	6.1091147	.004385965
229	52441	12008989	15.1327460	6.1180332	.004366812
230	52900	12167000	15.1657509	6.1269257	.004347826
231	53361	12326391	15.1986842	6.1357924	.004329004
232	53824	12487168	15.2315462	6.1446337	.004310345
233	54289	12649337	15.2643375	6.1534495	.004291845
234	54756	12812904	15.2970585	6.1622401	.004273504
235	55225	12977875	15.3297097	6.1710058	.004255319
236	55696	13144256	15.3622915	6.1797466	.004237288
237	56169	13312053	15.3948043	6.1884628	.004219409
238	56644	13481272	15.4272486	6.1971544	.004201681
239	57121	13651919	15.4596248	6.2058218	.004184100
240	57600	13824000	15.4919334	6.2144650	.004166667

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
301	90601	27270901	17.3493516	6.7017593	.003322259
302	91204	27543608	17.3781472	6.7091729	.003311258
303	91809	27818127	17.4068952	6.7165700	.003300330
304	92416	28094464	17.4355958	6.7239508	.003289474
305	93025	28372625	17.4642492	6.7313155	.003278689
306	93636	28652616	17.4928557	6.7386641	.003267974
307	94249	28934443	17.5214155	6.7459967	.003257329
308	94864	29218112	17.5499288	6.7533134	.003246753
309	95481	29503629	17.5783958	6.7606143	.003236246
310	96100	29791000	17.6068169	6.7678995	.003225806
311	96721	30080231	17.6351921	6.7751690	.003215434
312	97344	30371328	17.6635217	6.7824229	.003205128
313	97969	30664297	17.6918060	6.7896613	.003194888
314	98596	30959144	17.7200451	6.7968844	.003184713
315	99225	31255875	17.7482393	6.8040921	.003174603
316	99856	31554496	17.7763888	6.8112847	.003164557
317	100489	31855013	17.8044938	6.8184620	.003154574
318	101124	32157432	17.8325545	6.8256242	.003144654
319	101761	32461759	17.8605711	6.8327714	.003134796
320	102400	32768000	17.8885438	6.8399037	.003125000
321	103041	33076161	17.9164729	6.8470213	.003115265
322	103684	33386248	17.9443584	6.8541240	.003105590
323	104329	33698267	17.9722008	6.8612120	.003095975
324	104976	34012224	18.0000000	6.8682855	.003086420
325	105625	34328125	18.0277564	6.8753443	.003076923
326	106276	34645976	18.0554701	6.8823888	.003067485
327	106929	34965783	18.0831413	6.8894188	.003058104
328	107584	35287552	18.1107703	6.8964345	.003048780
329	108241	35611289	18.1383571	6.9034359	.003039514
330	108900	35937000	18.1659021	6.9104232	.003030303
331	109561	36264691	18.1934054	6.9173964	.003021148
332	110224	36594368	18.2208672	6.9243556	.003012048
333	110889	36926037	18.2482876	6.9313008	.003003003
334	111556	37259704	18.2756669	6.9382321	.002994012
335	112225	37595375	18.3030052	6.9451496	.002985075
336	112896	37933056	18.3303028	6.9520533	.002976190
337	113569	38272753	18.3575598	6.9589434	.002967359
338	114244	38614472	18.3847763	6.9658198	.002958580
339	114921	38958219	18.4119526	6.9726826	.002949853
340	115600	39304000	18.4390889	6.9795321	.002941176
341	116281	39651821	18.4661853	6.9863681	.002932551
342	116964	40001688	18.4932420	6.9931906	.002923977
343	117649	40353607	18.5202592	7.0000000	.002915452
344	118336	40707584	18.5472370	7.0067962	.002906977
345	119025	41063625	18.5741756	7.0135791	.002898551
346	119716	41421736	18.6010752	7.0203490	.002890173
347	120409	41781923	18.6279360	7.0271058	.002881844
348	121104	42144192	18.6547581	7.0338497	.002873563
349	121801	42508549	18.6815417	7.0405806	.002865330
350	122500	42875000	18.7082869	7.0472987	.002857143
351	123201	43243551	18.7349940	7.0540041	.002849003
352	123904	43614208	18.7616630	7.0606967	.002840909
353	124609	43986977	18.7882942	7.0673767	.002832861
354	125316	44361864	18.8148877	7.0740440	.002824859
355	126025	44738875	18.8414437	7.0806988	.002816901
356	126736	45118016	18.8679623	7.0873411	.002808989
357	127449	45499293	18.8944436	7.0939709	.002801120
358	128164	45882712	18.9208879	7.1005885	.002793296
359	128881	46268279	18.9472953	7.1071937	.002785515
360	129600	46656000	18.9736660	7.1137[illegible]66	.002777778

Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
130321	47045881	19.0000000	7.1203674	.002770083
131044	47437928	19.0262976	7.1269360	.002762431
131769	47832147	19.0525589	7.1334925	.002754821
132496	48228544	19.0787840	7.1400370	.002747253
133225	48627125	19.1049732	7.1465695	.002739726
133956	49027896	19.1311265	7.1530901	.002732240
134689	49430863	19.1572441	7.1595988	.002724796
135424	49836032	19.1833261	7.1660957	.002717391
136161	50243409	19.2093727	7.1725809	.002710027
136900	50653000	19.2353841	7.1790544	.002702703
137641	51064811	19.2613603	7.1855162	.002695418
138384	51478848	19.2873015	7.1919663	.002688172
139129	51895117	19.3132079	7.1984050	.002680965
139876	52313624	19.3390796	7.2048322	.002673797
140625	52734375	19.3649167	7.2112479	.002666667
141376	53157376	19.3907194	7.2176522	.002659574
142129	53582633	19.4164878	7.2240450	.002652520
142884	54010152	19.4422221	7.2304268	.002645503
143641	54439939	19.4679223	7.2367972	.002638522
144400	54872000	19.4935887	7.2431565	.002631579
145161	55306341	19.5192213	7.2495045	.002624672
145924	55742968	19.5448203	7.2558415	.002617801
146689	56181887	19.5703858	7.2621675	.002610966
147456	56623104	19.5959179	7.2684824	.002604167
148225	57066625	19.6214169	7.2747864	.002597403
148996	57512456	19.6468827	7.2810794	.002590674
149769	57960603	19.6723156	7.2873617	.002583979
150544	58411072	19.6977156	7.2936330	.002577320
151321	58863869	19.7230829	7.2998936	.002570694
152100	59319000	19.7484177	7.3061436	.002564103
152881	59776471	19.7737199	7.3123828	.002557545
153664	60236288	19.7989899	7.3186114	.002551020
154449	60698457	19.8242276	7.3248295	.002544529
155236	61162984	19.8494332	7.3310369	.002538071
156025	61629875	19.8746069	7.3372339	.002531646
156816	62099136	19.8997487	7.3434205	.002525253
157609	62570773	19.9248588	7.3495966	.002518892
158404	63044792	19.9499373	7.3557624	.002512563
159201	63521199	19.9749844	7.3619178	.002506266
160000	64000000	20.0000000	7.3680630	.002500000
160801	64481201	20.0249844	7.3741979	.002493766
161604	64964808	20.0499377	7.3803227	.002487562
162409	65450827	20.0748599	7.3864373	.002481390
163216	65939264	20.0997512	7.3925418	.002475248
164025	66430125	20.1246118	7.3986363	.002469136
164836	66923416	20.1494417	7.4047206	.002463054
165649	67419143	20.1742410	7.4107950	.002457002
166464	67917312	20.1990099	7.4168595	.002450980
167281	68417929	20.2237484	7.4229142	.002444988
168100	68921000	20.2484567	7.4289589	.002439024
168921	69426531	20.2731349	7.4349938	.002433090
169744	69934528	20.2977831	7.4410189	.002427184
170569	70444997	20.3224014	7.4470342	.002421308
171396	70957944	20.3469899	7.4530399	.002415459
172225	71473375	20.3715488	7.4590359	.002409639
173056	71991296	20.3960781	7.4650223	.002403846
173889	72511713	20.4205779	7.4709991	.002398082
174724	73034632	20.4450483	7.4769664	.002392344
175561	73560059	20.4694895	7.4829242	.002386635
176400	74088000	20.4939015	7.4888724	.002380952

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
421	177241	74618461	20.5182845	7.4948113	.002375297
422	178084	75151448	20.5426386	7.5007406	.002369668
423	178929	75686967	20.5669638	7.5066607	.002364066
424	179776	76225024	20.5912603	7.5125715	.002358491
425	180625	76765625	20.6155281	7.5184730	.002352941
426	181476	77308776	20.6397674	7.5243652	.002347418
427	182329	77854483	20.6639783	7.5302482	.002341920
428	183184	78402752	20.6881609	7.5361221	.002336449
429	184041	78953589	20.7123152	7.5419867	.002331002
430	184900	79507000	20.7364414	7.5478423	.002325581
431	185761	80062991	20.7605395	7.5536888	.002320186
432	186624	80621568	20.7846097	7.5595263	.002314815
433	187489	81182737	20.8086520	7.5653548	.002309469
434	188356	81746504	20.8326667	7.5711743	.002304147
435	189225	82312875	20.8566536	7.5769849	.002298851
436	190096	82881856	20.8806130	7.5827865	.002293578
437	190969	83453453	20.9045450	7.5885793	.002288330
438	191844	84027672	20.9284495	7.5943633	.002283105
439	192721	84604519	20.9523268	7.6001385	.002277904
440	193600	85184000	20.9761770	7.6059049	.002272727
441	194481	85766121	21.0000000	7.6116626	.002267574
442	195364	86350888	21.0237960	7.6174116	.002262443
443	196249	86938307	21.0475652	7.6231519	.002257336
444	197136	87528384	21.0713075	7.6288837	.002252252
445	198025	88121125	21.0950231	7.6346067	.002247191
446	198916	88716536	21.1187121	7.6403213	.002242152
447	199809	89314623	21.1423745	7.6460272	.002237136
448	200704	89915392	21.1660105	7.6517247	.002232143
449	201601	90518849	21.1896201	7.6574138	.002227171
450	202500	91125000	21.2132034	7.6630943	.002222222
451	203401	91733851	21.2367606	7.6687665	.002217295
452	204304	92345408	21.2602916	7.6744303	.002212389
453	205209	92959677	21.2837967	7.6800857	.002207506
454	206116	93576664	21.3072758	7.6857328	.002202643
455	207025	94196375	21.3307290	7.6913717	.002197802
456	207936	94818816	21.3541565	7.6970023	.002192982
457	208849	95443993	21.3775583	7.7026246	.002188184
458	209764	96071912	21.4009346	7.7082388	.002183406
459	210681	96702579	21.4242853	7.7138448	.002178649
460	211600	97336000	21.4476106	7.7194426	.002173913
461	212521	97972181	21.4709106	7.7250325	.002169197
462	213444	98611128	21.4941853	7.7306141	.002164502
463	214369	99252847	21.5174348	7.7361877	.002159827
464	215296	99897344	21.5406592	7.7417532	.002155172
465	216225	100544625	21.5638587	7.7473109	.002150538
466	217156	101194696	21.5870331	7.7528606	.002145923
467	218089	101847563	21.6101828	7.7584023	.002141328
468	219024	102503232	21.6333077	7.7639361	.002136752
469	219961	103161709	21.6564078	7.7694620	.002132196
470	220900	103823000	21.6794834	7.7749801	.002127660
471	221841	104487111	21.7025344	7.7804904	.002123142
472	222784	105154048	21.7255610	7.7859928	.002118644
473	223729	105823817	21.7485632	7.7914875	.002114165
474	224676	106496424	21.7715411	7.7969745	.002109705
475	225625	107171875	21.7944947	7.8024538	.002105263
476	226576	107850176	21.8174242	7.8079254	.002100840
477	227529	108531333	21.8403297	7.8133892	.002096436
478	228484	109215352	21.8632111	7.8188456	.002092050
479	229441	109902239	21.8860686	7.8242942	.002087683
480	230400	110592000	21.9089023	7.8297353	.002083333

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
541	292681	158340421	23.2594067	8.1482765	.001848429
542	293764	159220088	23.2808935	8.1532939	.001845018
543	294849	160103007	23.3023604	8.1583051	.001841621
544	295936	160989184	23.3238076	8.1633102	.001838235
545	297025	161878625	23.3452351	8.1683092	.001834862
546	298116	162771336	23.3666429	8.1733020	.001831502
547	299209	163667323	23.3880311	8.1782888	.001828154
548	300304	164566592	23.4093998	8.1832695	.001824818
549	301401	165469149	23.4307490	8.1882441	.001821494
550	302500	166375000	23.4520788	8.1932127	.001818182
551	303601	167284151	23.4733892	8.1981753	.001814882
552	304704	168196608	23.4946802	8.2031319	.001811594
553	305809	169112377	23.5159520	8.2080825	.001808318
554	306916	170031464	23.5372046	8.2130271	.001805054
555	308025	170953875	23.5584380	8.2179657	.001801802
556	309136	171879616	23.5796522	8.2228985	.001798561
557	310249	172808693	23.6008474	8.2278254	.001795332
558	311364	173741112	23.6220236	8.2327463	.001792115
559	312481	174676879	23.6431808	8.2376614	.001788909
560	313600	175616000	23.6643191	8.2425706	.001785714
561	314721	176558481	23.6854386	8.2474740	.001782531
562	315844	177504328	23.7065392	8.2523715	.001779359
563	316969	178453547	23.7276210	8.2572633	.001776199
564	318096	179406144	23.7486842	8.2621492	.001773050
565	319225	180362125	23.7697286	8.2670294	.001769912
566	320356	181321496	23.7907545	8.2719039	.001766784
567	321489	182284263	23.8117618	8.2767726	.001763668
568	322624	183250432	23.8327506	8.2816355	.001760563
569	323761	184220009	23.8537209	8.2864928	.001757469
570	324900	185193000	23.8746728	8.2913444	.001754386
571	326041	186169411	23.8956063	8.2961903	.001751313
572	327184	187149248	23.9165215	8.3010304	.001748252
573	328329	188132517	23.9374184	8.3058651	.001745201
574	329476	189119224	23.9582971	8.3106941	.001742160
575	330625	190109375	23.9791576	8.3155175	.001739130
576	331776	191102976	24.0000000	8.3203353	.001736111
577	332929	192100033	24.0208243	8.3251475	.001733102
578	334084	193100552	24.0416306	8.3299542	.001730104
579	335241	194104539	24.0624188	8.3347553	.001727116
580	336400	195112000	24.0831891	8.3395509	.001724138
581	337561	196122941	24.1039416	8.3443410	.001721170
582	338724	197137368	24.1246762	8.3491256	.001718213
583	339889	198155287	24.1453929	8.3539047	.001715266
584	341056	199176704	24.1660919	8.3586784	.001712329
585	342225	200201625	24.1867732	8.3634466	.001709402
586	343396	201230056	24.2074369	8.3682095	.001706485
587	344569	202262003	24.2280829	8.3729668	.001703578
588	345744	203297472	24.2487113	8.3777188	.001700680
589	346921	204336469	24.2693222	8.3824653	.001697793
590	348100	205379000	24.2899156	8.3872065	.001694915
591	349281	206425071	24.3104916	8.3919423	.001692047
592	350464	207474688	24.3310501	8.3966729	.001689189
593	351649	208527857	24 3515913	8.4013981	.001686341
594	352836	209584584	24.3721152	8.4061180	.001683502
595	354025	210644875	24.3926218	8.4108326	.001680672
596	355216	211708736	24.4131112	8.4155419	.001677852
597	356409	212776173	24.4335834	8.4202460	.001675042
598	357604	213847192	24.4540385	8.4249448	.001672241
599	358801	214921799	24.4744765	8.4296383	.001669449
600	360000	216000000	24.4948974	8.4343267	.001666667

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
661	436921	288804781	25·7099208	8·7109827	.001512859
662	438244	290117528	25·7293607	8·7153734	.001510574
663	439569	291434247	25·7487864	8·7197596	.001508296
664	440896	292754944	25·7681975	8·7241414	.001506024
665	442225	294079625	25·7875939	8·7285187	.001503759
666	443556	295408296	25·8069758	8·7328918	.001501502
667	444889	296740963	25·8263431	8·7372604	.001499250
668	446224	298077632	25·8456960	8·7416246	.001497006
669	447561	299418309	25·8650343	8·7459846	.001494768
670	448900	300763000	25·8843582	8·7503401	.001492537
671	450241	302111711	25·9036677	8·7546913	.001490313
672	451584	303464448	25·9229628	8·7590383	.001488095
673	452929	304821217	25·9422435	8·7633809	.001485884
674	454276	306182024	25·9615100	8·7677192	.001483680
675	455625	307546875	25·9807621	8·7720532	.001481481
676	456976	308915776	26·0000000	8·7763830	.001479290
677	458329	310288733	26·0192237	8·7807084	.001477105
678	459684	311665752	26·0384331	8·7850296	.001474926
679	461041	313046839	26·0576284	8·7893466	.001472754
680	462400	314432000	26·0768096	8·7936593	.001470588
681	463761	315821241	26·0959767	8·7979679	.001468429
682	465124	317214568	26·1151297	8·8022721	.001466276
683	466489	318611987	26·1342687	8·8065722	.001464129
684	467856	320013504	26·1533937	8·8108681	.001461988
685	469225	321419125	26·1725047	8·8151598	.001459854
686	470596	322828856	26·1916047	8·8194474	.001457726
687	471969	324242703	26·2106848	8·8237307	.001455604
688	473344	325660672	26·2297541	8·8280099	.001453488
689	474721	327082769	26·2488095	8·8322850	.001451379
690	476100	328509000	26·2678511	8·8365559	.001449275
691	477481	329939371	26·2868889	8·8408227	.001447178
692	478864	331373888	26·3058929	8·8450854	.001445087
693	480249	332812557	26·3248932	8·8493440	.001443001
694	481636	334255384	26·3438797	8·8535985	.001440922
695	483025	335702375	26·3628527	8·8578489	.001438849
696	484416	337153536	26·3818119	8·8620952	.001436782
697	485809	338608873	26·4007576	8·8663375	.001434720
698	487204	340068392	26·4196896	8·8705757	.001432665
699	488601	341532099	26·4386081	8·8748099	.001430615
700	490000	343000000	26·4575131	8·8790400	.001428571
701	491401	344472101	26·4764046	8·8832661	.001426534
702	492804	345948408	26·4952826	8·8874882	.001424501
703	494209	347428927	26·5141472	8·8917063	.001422475
704	495616	348913664	26·5329983	8·8959204	.001420455
705	497025	350402625	26·5518361	8·9001304	.001418440
706	498436	351895816	26·5706605	8·9043366	.001416431
707	499849	353393243	26·5894716	8·9085387	.001414427
708	501264	354894912	26·6082694	8·9127369	.001412429
709	502681	356400829	26·6270539	8·9169311	.001410437
710	504100	357911000	26·6458252	8·9211214	.001408451
711	505521	359425431	26·6645833	8·9253078	.001406470
712	506944	360944128	26·6833281	8·9294902	.001404494
713	508369	362467097	26·7020598	8·9336687	.001402525
714	509796	363994344	26·7207784	8·9378433	.001400560
715	511225	365525875	26·7394839	8·9420140	.001398601
716	512656	367061696	26·7581763	8·9461809	.001396648
717	514089	368601813	26·7768557	8·9503438	.001394700
718	515524	370146232	26·7955220	8·9545029	.001392758
719	516961	371694959	26·8141754	8·9586581	.001390821
720	518400	373248000	26·8328157	8·9628095	.001388889

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
721	519841	374805361	26.8514432	8.9669570	.001386963
722	521284	376367048	26.8700577	8.9711007	.001385042
723	522729	377933067	26.8886593	8.9752406	.001383126
724	524176	379503424	26.9072481	8.9793766	.001381215
725	525625	381078125	26.9258240	8.9835089	.001379310
726	527076	382657176	26.9443872	8.9876373	.001377410
727	528529	384240583	26.9629375	8.9917620	.001375516
728	529984	385828352	26.9814751	8.9958829	.001373626
729	531441	387420489	27.0000000	9.0000000	.001371742
730	532900	389017000	27.0185122	9.0041134	.001369863
731	534361	390617891	27.0370117	9.0082229	.001367989
732	535824	392223168	27.0554985	9.0123288	.001366120
733	537289	393832837	27.0739727	9.0164309	.001364256
734	538756	395446904	27.0924344	9.0205293	.001362398
735	540225	397065375	27.1108834	9.0246239	.001360544
736	541696	398688256	27.1293199	9.0287149	.001358696
737	543169	400315553	27.1477439	9.0328021	.001356852
738	544644	401947272	27.1661554	9.0368857	.001355014
739	546121	403583419	27.1845544	9.0409655	.001353180
740	547600	405224000	27.2029410	9.0450419	.001351351
741	549081	406869021	27.2213152	9.0491142	.001349528
742	550564	408518488	27.2396769	9.0531831	.001347709
743	552049	410172407	27.2580263	9.0572482	.001345895
744	553536	411830784	27.2763634	9.0613098	.001344086
745	555025	413493625	27.2946881	9.0653677	.001342282
746	556516	415160936	27.3130006	9.0694220	.001340483
747	558009	416832723	27.3313007	9.0734726	.001338688
748	559504	418508992	27.3495887	9.0775197	.001336898
749	561001	420189749	27.3678644	9.0815631	.001335113
750	562500	421875000	27.3861279	9.0856030	.001333333
751	564001	423564751	27.4043792	9.0896392	.001331558
752	565504	425259008	27.4226184	9.0936719	.001329787
753	567009	426957777	27.4408455	9.0977010	.001328021
754	568516	428661064	27.4590604	9.1017265	.001326260
755	570025	430368875	27.4772633	9.1057485	.001324503
756	571536	432081216	27.4954542	9.1097669	.001322751
757	573049	433798093	27.5136330	9.1137818	.001321004
758	574564	435519512	27.5317998	9.1177931	.001319261
759	576081	437245479	27.5499546	9.1218010	.001317523
760	577600	438976000	27.5680975	9.1258053	.001315789
761	579121	440711081	27.5862284	9.1298061	.001314060
762	580644	442450728	27.6043475	9.1338034	.001312336
763	582169	444194947	27.6224546	9.1377971	.001310616
764	583696	445943744	27.6405499	9.1417874	.001308901
765	585225	447697125	27.6586334	9.1457742	.001307190
766	586756	449455096	27.6767050	9.1497576	.001305483
767	588289	451217663	27.6947648	9.1537375	.001303781
768	589824	452984832	27.7128129	9.1577139	.001302083
769	591361	454756609	27.7308492	9.1616869	.001300390
770	592900	456533000	27.7488739	9.1656565	.001298701
771	594441	458314011	27.7668868	9.1696225	.001297017
772	595984	460099648	27.7848880	9.1735852	.001295337
773	597529	461889917	27.8028775	9.1775445	.001293661
774	599076	463684824	27.8208555	9.1815003	.001291990
775	600625	465484375	27.8388218	9.1854527	.001290323
776	602176	467288576	27.8567766	9.1894018	.001288660
777	603729	469097433	27.8747197	9.1933474	.001287001
778	605284	470910952	27.8926514	9.1972897	.001285347
779	606841	472729139	27.9105715	9.2012286	.001283697
780	608400	474552000	27.9284801	9.2051641	.001282051

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
781	609961	476379541	27.9463772	9.2090962	.001280410
782	611524	478211768	27.9642629	9.2130250	.001278772
783	613089	480048687	27.9821372	9.2169505	.001277139
784	614656	481890304	28.0000000	9.2208726	.001275510
785	616225	483736625	28.0178515	9.2247914	.001273885
786	617796	485587656	28.0356915	9.2287068	.001272265
787	619369	487443403	28.0535203	9.2326189	.001270648
788	620944	489303872	28.0713377	9.2365277	.001269036
789	622521	491169069	28.0891438	9.2404333	.001267427
790	624100	493039000	28.1069386	9.2443355	.001265823
791	625681	494913671	28.1247222	9.2482344	.001264223
792	627264	496793088	28.1424946	9.2521300	.001262626
793	628849	498677257	28.1602557	9.2560224	.001261034
794	630436	500566184	28.1780056	9.2599114	.001259446
795	632025	502459875	28.1957444	9.2637973	.001257862
796	633616	504358336	28.2134720	9.2676798	.001256281
797	635209	506261573	28.2311884	9.2715592	.001254705
798	636804	508169592	28.2488938	9.2754352	.001253133
799	638401	510082399	28.2665881	9.2793081	.001251564
800	640000	512000000	28.2842712	9.2831777	.001250000
801	641601	513922401	28.3019434	9.2870440	.001248439
802	643204	515849608	28.3196045	9.2909072	.001246883
803	644809	517781627	28.3372546	9.2947671	.001245330
804	646416	519718464	28.3548938	9.2986239	.001243781
805	648025	521660125	28.3725219	9.3024775	.001242236
806	649636	523606616	28.3901391	9.3063278	.001240695
807	651249	525557943	28.4077454	9.3101750	.001239157
808	652864	527514112	28.4253408	9.3140190	.001237624
809	654481	529475129	28.4429253	9.3178598	.001236094
810	656100	531441000	28.4604989	9.3216976	.001234568
811	657721	533411731	28.4780617	9.3255320	.001233046
812	659344	535387328	28.4956137	9.3293634	.001231527
813	660969	537367797	28.5131549	9.3331916	.001230012
814	662596	539353144	28.5306852	9.3370167	.001228501
815	664225	541343375	28.5482048	9.3408386	.001226994
816	665856	543338496	28.5657137	9.3446575	.001225490
817	667489	545338513	28.5832119	9.3484731	.001223990
818	669124	547343432	28.6006993	9.3522857	.001222494
819	670761	549353259	28.6181760	9.3560952	.001221001
820	672400	551368000	28.6356421	9.3599016	.001219512
821	674041	553387661	28.6530976	9.3637049	.001218027
822	675684	555412248	28.6705424	9.3675051	.001216545
823	677329	557441767	28.6879766	9.3713022	.001215067
824	678976	559476224	28.7054002	9.3750963	.001213592
825	680625	561515625	28.7228132	9.3788873	.001212121
826	682276	563559976	28.7402157	9.3826752	.001210654
827	683929	565609283	28.7576077	9.3864600	.001209190
828	685584	567663552	28.7749891	9.3902419	.001207729
829	687241	569722789	28.7923601	9.3940206	.001206273
830	688900	571787000	28.8097206	9.3977964	.001204819
831	690561	573856191	28.8270706	9.4015691	.001203369
832	692224	575930368	28.8444102	9.4053387	.001201923
833	693889	578009537	28.8617394	9.4091054	.001200480
834	695556	580093704	28.8790582	9.4128690	.001199041
835	697225	582182875	28.8963666	9.4166297	.001197605
836	698896	584277056	28.9136646	9.4203873	.001196172
837	700569	586376253	28.9309523	9.4241420	.001194743
838	702244	588480472	28.9482297	9.4278936	.001193317
839	703921	590589719	28.9654967	9.4316423	.001191895
840	705600	592704000	28.9827535	9.4353880	.001190476

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
841	707281	594823321	29.0000000	9.4391307	.001189061
842	708964	596947688	29.0172363	9.4428704	.001187648
843	710649	599077107	29.0344623	9.4466072	.001186240
844	712336	601211584	29.0516781	9.4503410	.001184834
845	714025	603351125	29.0688837	9.4540719	.001183432
846	715716	605495736	29.0860791	9.4577999	.001182033
847	717409	607645423	29.1032644	9.4615249	.001180638
848	719104	609800192	29.1204396	9.4652470	.001179245
849	720801	611960049	29.1376046	9.4689661	.001177856
850	722500	614125000	29.1547595	9.4726824	.001176471
851	724201	616295051	29.1719043	9.4763957	.001175088
852	725904	618470208	29.1890390	9.4801061	.001173709
853	727609	620650477	29.2061637	9.4838136	.001172333
854	729316	622835864	29.2232784	9.4875182	.001170960
855	731025	625026375	29.2403830	9.4912200	.001169591
856	732736	627222016	29.2574777	9.4949188	.001168224
857	734449	629422793	29.2745623	9.4986147	.001166861
858	736164	631628712	29.2916370	9.5023078	.001165501
859	737881	633839779	29.3087018	9.5059980	.001164144
860	739600	636056000	29.3257566	9.5096854	.001162791
861	741321	638277381	29.3428015	9.5133699	.001161440
862	743044	640503928	29.3598365	9.5170515	.001160093
863	744769	642735647	29.3768616	9.5207303	.001158749
864	746496	644972544	29.3938769	9.5244063	.001157407
865	748225	647214625	29.4108823	9.5280794	.001156069
866	749956	649461896	29.4278779	9.5317497	.001154734
867	751689	651714363	9.4448637	9.5354172	.001153403
868	753424	653972032	29.4618397	9.5390818	.001152074
869	755161	656234909	29.4788059	9.5427437	.001150748
870	756900	658503000	29.4957624	9.5464027	.001149425
871	758641	660776311	29.5127091	9.5500589	.001148106
872	760384	663054848	29.5296461	9.5537123	.001146789
873	762129	665338617	29.5465734	9.5573630	.001145475
874	763876	667627624	29.5634910	9.5610108	.001144165
875	765625	669921875	29.5803989	9.5646559	.001142857
876	767376	672221376	29.5972972	9.5682982	.001141553
877	769129	674526133	29.6141858	9.5719377	.001140251
878	770884	676836152	29.6310648	9.5755745	.001138952
879	772641	679151439	29.6479342	9.5792085	.001137656
880	774400	681472000	29.6647939	9.5828397	.001136364
881	776161	683797841	29.6816442	9.5864682	.001135074
882	777924	686128968	29.6984848	9.5900939	.001133787
883	779689	688465387	29.7153159	9.5937169	.001132503
884	781456	690807104	29.7321375	9.5973373	.001131222
885	783225	693154125	29.7489496	9.6009548	.001129944
886	784996	695506456	29.7657521	9.6045696	.001128668
887	786769	697864103	29.7825452	9.6081817	.001127396
888	788544	700227072	29.7993289	9.6117911	.001126126
889	790321	702595369	29.8161030	9.6153977	.001124859
890	792100	704969000	29.8328678	9.6190017	.001123596
891	793881	707347971	29.8496231	9.6226030	.001122334
892	795664	709732288	29.8663690	9.6262016	.001121076
893	797449	712121957	29.8831056	9.6297975	.001119821
894	799236	714516984	29.8998328	9.6333907	.001118568
895	801025	716917375	29.9165506	9.6369812	.001117318
896	802816	719323136	29.9332591	9.6405690	.001116071
897	804609	721734273	29.9499583	9.6441542	.001114827
898	806404	724150792	29.9666481	9.6477367	.001113586
899	808201	726572699	29.9833287	9.6513166	.001112347
900	810000	729000000	30.0000000	9.6548938	.001111111

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
901	811801	731432701	30.0166620	9.6584684	.001109878
902	813604	733870808	30.0333148	9.6620403	.001108647
903	815409	736314327	30.0499584	9.6656096	.001107420
904	817216	738763264	30.0665928	9.6691762	.001106195
905	819025	741217625	30.0832179	9.6727403	.001104972
906	820836	743677416	30.0998339	9.6763017	.001103753
907	822649	746142643	30.1164407	9.6798604	.001102536
908	824464	748613312	30.1330383	9.6834166	.001101322
909	826281	751089429	30.1496269	9.6869701	.001100110
910	828100	753571000	30.1662063	9.6905211	.001098901
911	829921	756058031	30.1827765	9.6940694	.001097695
912	831744	758550528	30.1993377	9.6976151	.001096491
913	833569	761048497	30.2158899	9.7011583	.001095290
914	835396	763551944	30.2324329	9.7046989	.001094092
915	837225	766060875	30.2489669	9.7082369	.001092896
916	839056	768575296	30.2654919	9.7117723	.001091703
917	840889	771095213	30.2820079	9.7153051	.001090513
918	842724	773620632	30.2985148	9.7188354	.001089325
919	844561	776151559	30.3150128	9.7223631	.001088139
920	846400	778688000	30.3315018	9.7258883	.001086957
921	848241	781229961	30.3479818	9.7294109	.001085776
922	850084	783777448	30.3644529	9.7329309	.001084599
923	851929	786330467	30.3809151	9.7364484	.001083424
924	853776	788889024	30.3973683	9.7399634	.001082251
925	855625	791453125	30.4138127	9.7434758	.001081081
926	857476	794022776	30.4302481	9.7469857	.001079914
927	859329	796597983	30.4466747	9.7504930	.001078749
928	861184	799178752	30.4630924	9.7539979	.001077586
929	863041	801765089	30.4795013	9.7575002	.001076426
930	864900	804357000	30.4959014	9.7610001	.001075269
931	866761	806954491	30.5122926	9.7644974	.001074114
932	868624	809557568	30.5286750	9.7679922	.001072961
933	870489	812166237	30.5450487	9.7714845	.001071811
934	872356	814780504	30.5614136	9.7749743	.001070664
935	874225	817400375	30.5777697	9.7784617	.001069519
936	876096	820025856	30.5941171	9.7819466	.001068376
937	877969	822656953	30.6104557	9.7854288	.001067236
938	879844	825293672	30.6267857	9.7889087	.001066098
939	881721	827936019	30.6431069	9.7923861	.001064963
940	883600	830584000	30.6594194	9.7958611	.001063830
941	885481	833237621	30.6757233	9.7993336	.001062699
942	887364	835896888	30.6920185	9.8028036	.001061571
943	889249	838561807	30.7083051	9.8062711	.001060445
944	891136	841232384	30.7245830	9.8097362	.001059322
945	893025	843908625	30.7408523	9.8131989	.001058201
946	894916	846590536	30.7571130	9.8166591	.001057082
947	896809	849278123	30.7733651	9.8201169	.001055966
948	898704	851971392	30.7896086	9.8235723	.001054852
949	900601	854670349	30.8058436	9.8270252	.001053741
950	902500	857375000	30.8220700	9.8304757	.001052632
951	904401	860085351	30.8382879	9.8339238	.001051525
952	906304	862801408	30.8544972	9.8373695	.001050420
953	908209	865523177	30.8706981	9.8408127	.001049318
954	910116	868250664	30.8868904	9.8442536	.001048218
955	912025	870983875	30.9030743	9.8476920	.001047120
956	913936	873722816	30.9192497	9.8511280	.001046025
957	915849	876467493	30.9354166	9.8545617	.001044932
958	917764	879217912	30.9515751	9.8579929	.001043841
959	919681	881974079	30.9677251	9.8614218	.001042753
960	921600	884736000	30.9838668	9.8648483	.001041667

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
961	923521	887503681	31.0000000	9.8682724	.001040583
962	925444	890277128	31.0161248	9.8716941	.001039501
963	927369	893056347	31.0322413	9.8751135	.001038422
964	929296	895841344	31.0483494	9.8785305	.001037344
965	931225	898632125	31.0644491	9.8819451	.001036269
966	933156	901428696	31.0805405	9.8853574	.001035197
967	935089	904231063	31.0966236	9.8887673	.001034126
968	937024	907039232	31.1126984	9.8921749	.001033058
969	938961	909853209	31.1287648	9.8955801	.001031992
970	940900	912673000	31.1448230	9.8989830	.001030928
971	942841	915498611	31.1608729	9.9023835	.001029866
972	944784	918330048	31.1769145	9.9057817	.001028807
973	946729	921167317	31.1929479	9.9091776	.001027749
974	948676	924010424	31.2089731	9.9125712	.001026694
975	950625	926859375	31.2249900	9.9159624	.001025641
976	952576	929714176	31.2409987	9.9193513	.001024590
977	954529	932574833	31.2569992	9.9227379	.001023541
978	956484	935441352	31.2729915	9.9261222	.001022495
979	958441	938313739	31.2889757	9.9295042	.001021450
980	960400	941192000	31.3049517	9.9328839	.001020408
981	962361	944076141	31.3209195	9.9362613	.001019368
982	964324	946966168	31.3368792	9.9396363	.001018330
983	966289	949862087	31.3528308	9.9430092	.001017294
984	968256	952763904	31.3687743	9.9463797	.001016260
985	970225	955671625	31.3847097	9.9497479	.001015228
986	972196	958585256	31.4006369	9.9531138	.001014199
987	974169	961504803	31.4165561	9.9564775	.001013171
988	976144	964430272	31.4324673	9.9598389	.001012146
989	978121	967361669	31.4483704	9.9631981	.001011122
990	980100	970299000	31.4642654	9.9665549	.001010101
991	982081	973242271	31.4801525	9.9699095	.001009082
992	984064	976191488	31.4960315	9.9732619	.001008065
993	986049	979146657	31.5119025	9.9766120	.001007049
994	988036	982107784	31.5277655	9.9799599	.001006036
995	990025	985074875	31.5436206	9.9833055	.001005025
996	992016	988047936	31.5594677	9.9866488	.001004016
997	994009	991026973	31.5753068	9.9899900	.001003009
998	996004	994011992	31.5911380	9.9933289	.001002004
999	998001	997002999	31.6069613	9.9966656	.001001001
1000	1000000	1000000000	31.6227766	10.0000000	.001000000
1001	1002001	1003003001	31.6385840	10.0033322	.0009990010
1002	1004004	1006012008	31.6543836	10.0066622	.0009980040
1003	1006009	1009027027	31.6701752	10.0099899	.0009970090
1004	1008016	1012048064	31.6859590	10.0133155	.0009960159
1005	1010025	1015075125	31.7017349	10.0166389	.0009950249
1006	1012036	1018108216	31.7175030	10.0199601	.0009940358
1007	1014049	1021147343	31.7332633	10.0232791	.0009930487
1008	1016064	1024192512	31.7490157	10.0265958	.0009920635
1009	1018081	1027243729	31.7647603	10.0299104	.0009910803
1010	1020100	1030301000	31.7804972	10.0332228	.0009900990
1011	1022121	1033364331	31.7962262	10.0365330	.0009891197
1012	1024144	1036433728	31.8119474	10.0398410	.0009881423
1013	1026169	1039509197	31.8276609	10.0431469	.0009871668
1014	1028196	1042590744	31.8433666	10.0464506	.0009861933
1015	1030225	1045678375	31.8590646	10.0497521	.0009852217
1016	1032256	1048772096	31.8747549	10.0530514	.0009842520
1017	1034289	1051871913	31.8904374	10.0563485	.0009832842
1018	1036324	1054977832	31.9061123	10.0596435	.0009823183
1019	1038361	1058089859	31.9217794	10.0629364	.0009813543
1020	1040400	1061208000	31.9374388	10.0662271	.0009803922

TABLE XVII.—CUBIC YARDS PER 100 FEET OF LEVEL SECTIONS. SLOPE 1 : 1.

Depth, *d*	Base 12 feet.	Base 14 feet.	Base 16 feet.	Base 18 feet.	Base 20 feet.	Base 28 feet.	Base 30 feet.	Base 32 feet.
1	48	56	63	70	78	107	115	122
2	104	119	133	148	163	222	237	252
3	167	189	211	233	256	344	367	389
4	237	267	296	326	356	474	504	533
5	315	352	389	426	463	611	648	685
6	400	444	489	533	578	756	800	844
7	493	544	596	648	700	907	959	1011
8	593	652	711	770	830	1067	1126	1185
9	700	767	833	900	967	1233	1300	1367
10	815	889	963	1037	1111	1407	1481	1556
11	937	1019	1100	1181	1263	1589	1670	1752
12	1067	1156	1244	1333	1422	1778	1867	1956
13	1204	1300	1396	1493	1589	1974	2070	2167
14	1348	1452	1556	1659	1763	2178	2281	2385
15	1500	1611	1722	1833	1944	2389	2500	2611
16	1659	1778	1896	2015	2133	2607	2726	2844
17	1826	1952	2078	2204	2330	2833	2959	3085
18	2000	2133	2267	2400	2533	3067	3200	3333
19	2181	2322	2463	2604	2744	3307	3448	3589
20	2370	2519	2667	2815	2963	3556	3704	3852
21	2567	2722	2878	3033	3189	3811	3967	4122
22	2770	2933	3096	3259	3422	4074	4237	4400
23	2981	3152	3322	3493	3663	4344	4515	4685
24	3200	3378	3556	3733	3911	4622	4800	4978
25	3426	3611	3796	3981	4167	4907	5093	5278
26	3659	3852	4044	4237	4430	5200	5393	5585
27	3900	4100	4300	4500	4700	5500	5700	5900
28	4148	4356	4563	4770	4978	5807	6015	6222
29	4404	4619	4833	5048	5263	6122	6337	6552
30	4667	4889	5111	5333	5556	6444	6667	6889
31	4937	5167	5396	5626	5856	6774	7004	7233
32	5215	5452	5689	5926	6163	7111	7348	7585
33	5500	5744	5989	6233	6478	7456	7700	7944
34	5793	6044	6296	6548	6800	7807	8059	8311
35	6093	6352	6611	6870	7130	8167	8426	8685
36	6400	6667	6933	7200	7467	8533	8800	9067
37	6715	6989	7263	7537	7811	8907	9181	9456
38	7037	7319	7600	7881	8163	9289	9570	9852
39	7367	7656	7944	8233	8522	9678	9967	10256
40	7704	8000	8296	8593	8889	10074	10370	10667
41	8048	8352	8656	8959	9263	10478	10781	11085
42	8400	8711	9022	9333	9644	10889	11200	11511
43	8759	9078	9396	9715	10033	11307	11626	11944
44	9126	9452	9778	10104	10430	11733	12059	12385
45	9500	9833	10167	10500	10833	12167	12500	12833
46	9881	10222	10563	10904	11244	12607	12948	13289
47	10270	10619	10967	11315	11663	13056	13404	13752
48	10667	11022	11378	11733	12089	13511	13867	14222
49	11070	11433	11796	12159	12522	13974	14337	14700
50	11481	11852	12222	12593	12963	14444	14815	15185
51	11900	12278	12656	13033	13411	14922	15300	15678
52	12326	12711	13096	13481	13867	15407	15793	16178
53	12759	13152	13544	13937	14330	15900	16293	16685
54	13200	13600	14000	14400	14800	16400	16800	17200
55	13648	14056	14463	14870	15278	16907	17315	17722
56	14104	14519	14933	15348	15763	17422	17837	18252
57	14567	14989	15411	15833	16256	17944	18367	18789
58	15037	15467	15896	16326	16756	18474	18904	19333
59	15515	15952	16389	16826	17263	19011	19448	19885
60	16000	16444	16889	17333	17778	19556	20000	20444

TABLE XVII.—CUBIC YARDS PER 100 FEET OF LEVEL SECTIONS. SLOPE 1.5 : 1.

Depth d	Base 12 feet.	Base 14 feet.	Base 16 feet.	Base 18 feet.	[illegible] 20 feet.	Base 28 feet.	Base 30 feet.	Base 32 feet.
1	50	7	65	72	80	109	17	124
2	11	6	141	156	170	230	44	[illegible]
3	83	6	228	250	272	361	83	
4	67	4	326	356	385	504	33	
5	61	[illegible]	435	472	509	657	94	
6	67	1	556	600	644	822	67	
7	83	5	687	739	791	998	50	
8	11	0	[illegible]	889	948	1185	44	
9	50	7	983	1050	1117	[illegible]	50	
10	00	4	[illegible]	1222	1296	1593	[illegible]	
11	1161	1243	1324	1406	[illegible]	1818	[illegible]	1976
12	1333	1422	[illegible]	1600	1689	2044	2133	2222
13	1517	1613	1709	1806	1902	2287	2383	2480
14	1711	1815	1919	2022	2126	[illegible]	2644	2748
15	1917	2028	2139	2250	2361	2806	2917	3028
16	2133	2252	2370	2489	2607	3081	3200	3319
17	2361	2487	2613	2739	2865	3369	3494	3620
18	2600	2733	2867	3000	3133	3667	3800	3933
19	2850	2991	3131	3272	[illegible]	[illegible]	[illegible]	4257
20	3111	3259	3407	3556	[illegible]	4296	4444	4593
21	83	3539	3694	3850	4006	28	83	
22	67	3830	3993	4156	4319	70	33	
23	61	4131	4302	4472	4642	24	94	
24	67	4444	4622	4800	4978	89	67	
25	83	4769	4954	5139	5324	65	50	
26	11	5104	5296	5489	5681	52	44	
27	50	5450	5650	5850	6050	50	50	
28	00	5807	6015	6222	6430	59	67	
29	61	6176	6391	6606	6820	80	94	
30	[illegible]	6556	6778	7000	7222	11	33	
31	6717	6946	7176	7406	7635	8554	8783	
32	[illegible]	7348	7585	7822	8059	9007	9244	
33	7517	7761	8006	8250	8494	9472	9717	
34	7933	8185	8437	8689	8941	9948	10200	
35	8361	8620	8880	9139	[illegible]	10435	10694	
36	8800	9067	9333	9600	[illegible]	10933	11200	
37	9250	9524	9798	10072	10346	11443	11717	
38	9711	9993	10274	10556	10837	11963	12244	
39	10183	10472	10761	11050	11339	12494	12783	
40	10667	10963	11259	11556	[illegible]	13037	13333	
41	[illegible]	11465	11769	72	12376	13591	[illegible]	1
42	11667	11978	12289	00	12911	14156	14467	1
43	12183	12502	12820	39	13457	14731	15050	1
44	12711	13037	13363	89	[illegible]	15319	15644	1
45	13250	13583	13917	50	14583	[illegible]	16250	1
46	13800	14141	14481	22	15163	16526	16867	1
47	14361	14709	15057	06	15754	17146	17494	1
48	14933	15289	15644	00	16356	17778	[illegible]	1
49	15517	15880	16243	06	16969	18420	18783	1
50	16111	16481	16852	22	17593	19074	19444	1
51	16717	17094	17472	17850	18228	19739	20117	2
52	17333	17719	18104	18489	18874	20415	20800	2
53	17961	18354	18746	19139	19531	21102	[illegible]	2
54	18600	19000	19400	19800	20200	21800	22200	2
55	19250	19657	20065	20472	20880	22509	22917	2
56	19911	20326	20741	21156	21570	23230	23644	2
57	20583	21006	21428	21850	22272	23961	24383	2
58	21267	21696	22126	22556	22985	24704	25133	2
59	21961	22398	22835	23272	23709	25457	25894	2
60	22667	23111	23556	24000	24444	26222	26667	2

TABLE XVII.—CORRECTIVE *PERCENTAGE* FACTORS FOR TABLES OF LEVEL SECTIONS.

To be applied when cross-sections are *not* level. See § 95.

Side slope = 1.5 : 1 or $\beta = 33°41'$.

Transverse surface slope.		b = 12 feet and d =			b = 20 feet and d =			b = 30 feet and d =		
$\alpha°$	Per-cent	10 feet.	20 feet.	50 feet.	10 feet.	20 feet.	50 feet.	10 feet.	20 feet.	50 feet.
		%	%	%	%	%	%	%	%	%
5	9	1.9	1.8	1.8	2.1	1.8	1.8	2.3	2.0	1.8
10	18	8.2	7.7	7.5	9.0	8.0	7.6	10.0	8.4	7.7
15	27	21	20	19	23	21	20	26	22	20
20	36	46	44	43	51	45	44	57	48	44
30	57	327	324	317	358	336	321	400	354	326

Side slope = 1 : 1 or $\beta = 45°$.

Transverse surface slope.		b = 12 feet and d =			b = 20 feet and d =			b = 30 feet and d =		
$\alpha°$	Per-cent	10 feet.	20 feet.	50 feet.	10 feet.	20 feet.	50 feet.	10 feet.	20 feet.	50 feet.
		%	%	%	%	%	%	%	%	%
5	9	0.9	0.8	0.8	1.0	0.9	0.8	1.2	0.9	0.8
10	18	3.7	3.4	3.2	4.3	3.6	3.3	5.0	4.0	3.4
15	27	9.0	8.2	7.8	10.3	8.7	8.0	12.1	9.5	8.2
20	36	18	16	15	20	17	16	24	19	16
30	57	58	53	50	67	56	51	78	61	58

TABLE XVIII.—ANNUAL CHARGE AGAINST A TIE, BASED ON THE ORIGINAL COST AND ASSUMED LIFE OF THE TIE; INTEREST COMPOUNDED AT 5%. (See § 217.)

Original cost of tie in cents.	Life of tie in years.																	
	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
20	7.34	5.64	4.62	3.94	3.46	3.09	2.81	2.59	2.41	2.26	2.13	2.02	1.93	1.85	1.77	1.71	1.65	1.60
25	9.18	7.05	5.77	4.92	4.32	3.87	3.52	3.24	3.01	2.82	2.66	2.53	2.41	2.31	2.22	2.14	2.07	2.01
30	11.02	8.46	6.93	5.91	5.18	4.64	4.22	3.89	3.61	3.38	3.19	3.03	2.89	2.77	2.66	2.57	2.48	2.41
35	12.85	9.87	8.08	6.90	6.05	5.42	4.92	4.53	4.21	3.95	3.73	3.54	3.37	3.23	3.10	2.99	2.90	2.81
40	14.69	11.28	9.24	7.88	6.91	6.19	5.63	5.18	4.81	4.51	4.26	4.04	3.85	3.79	3.55	3.42	3.31	3.21
45	16.52	12.69	10.39	8.87	7.78	6.96	6.33	5.83	5.42	5.08	4.79	4.55	4.34	4.15	3.99	3.85	3.72	3.61
50	18.36	14.10	11.55	9.85	8.64	7.74	7.03	6.48	6.02	5.64	5.32	5.05	4.82	4.61	4.43	4.28	4.14	4.01
55	20.20	15.51	12.70	10.84	9.51	8.51	7.74	7.12	6.62	6.21	5.86	5.56	5.30	5.07	4.88	4.71	4.55	4.41
60	22.03	16.92	13.86	11.82	10.37	9.28	8.44	7.77	7.22	6.77	6.39	6.06	5.78	5.54	5.32	5.13	4.96	4.81
65	23.87	18.33	15.01	12.81	11.23	10.06	9.14	8.42	7.83	7.33	6.92	6.57	6.26	6.00	5.77	5.56	5.38	5.22
70	25.70	19.74	16.17	13.79	12.10	10.83	9.85	9.07	8.43	7.90	7.45	7.07	6.74	6.46	6.21	5.99	5.79	5.62
75	27.54	21.15	17.32	14.78	12.96	11.60	10.55	9.72	9.03	8.46	7.98	7.58	7.22	6.92	6.65	6.42	6.20	6.02
80	29.38	22.56	18.48	15.76	13.83	12.38	11.25	10.36	9.63	9.03	8.52	8.08	7.71	7.38	7.10	6.84	6.62	6.42
85	31.21	23.97	19.63	16.75	14.69	13.15	11.96	11.01	10.23	9.59	9.05	8.59	8.19	7.84	7.54	7.27	7.03	6.82
90	33.05	25.38	20.79	17.73	15.55	13.92	12.66	11.66	10.84	10.15	9.58	9.09	8.67	8.30	7.98	7.70	7.45	7.22
95	34.88	26.79	21.94	18.71	16.42	14.70	13.37	12.30	11.44	10.72	10.12	9.60	9.15	8.76	8.42	8.12	7.86	7.62
100	36.72	28.20	23.10	19.70	17.28	15.47	14.07	12.95	12.04	11.28	10.65	10.10	9.63	9.23	8.87	8.55	8.27	8.02
For each 5 cents, add	1.836	1.410	1.155	.985	.864	.774	.703	.648	.602	.564	.532	.505	.482	.461	.443	.428	.414	.401

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